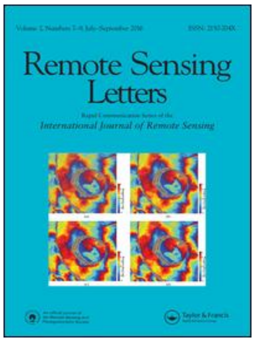

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Estimators of compound Gaussian clutter with log-normal texture

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ABSTRACT
Estimators of clutter models parameters based upon higher order moments estimator (HOME) produce usually poor results in particular for low sample sizes. In an attempt to remedy this situation, closed forms of [zlog(z)] and fractional order moments estimator (FOME) are derived in this work and yield a good estimation accuracy of parameters of the compound Gaussian clutter with log-normal texture (CG-LNT). Using simulated and real data, estimation comparisons show that best values of mean square error (MSE) and bias are achieved using the proposed procedures.

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1. Introduction

Sea clutter, which is back scattering from the sea surface itself is regarded as undesirable interference and limits the ability of airborne or shipborne radar to detect targets. To obtain the best fitting to real data, sea clutter models have been based upon compound Gaussian distributions without thermal noise. These models with different laws of the texture component provide accurate prediction results. For instance, texture distributions which characterize the fluctuation of the clutter power follow gamma, the inverse gamma, the inverse Gaussian and the log-normal distributions (Sahed and Mezache 2017; Mezache et al. 2016; Ollila et al. 2012; Carretero et al. 2010).

Construction of many radar signal processing algorithms necessitates the knowledge of the estimation of clutter model parameters. In the open literature, numerous approaches have been based principally on the maximum likelihood estimator (MLE), [zlog(z)], FOME and artificial intelligence methods. For the estimation of *K*-clutter parameters, accurate results have been obtained by the MLE method compared to other methods, but the computational is too expensive (Joughin, Percival, and Winebrenner 1993). The authors in (Iskander and Zoubir 1999) proposed an alternative estimation procedure based on higher order and fractional moments where the computational complexity is much lower than that of the MLE method given in (Joughin, Percival, and Winebrenner 1993). The [zlog(z)] estimator was originally obtained in (Blacknell and

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Tough 2001) and provides best estimates in all cases compared to fractional moments estimator. Recently, in (Mezache and Sahed 2018), a closed form estimator of compound Gaussian clutter with inverse Gaussian (CGIG) texture parameters is derived after the combination of two statistical ratios with positive and negative order moments.

This letter will investigate the parameter estimation of CG-LNT clutter without thermal noise. Estimators based upon HOME produce commonly poor results particularly when low sample sizes are at hand. In an attempt to remedy this situation, closed forms of [zlog(z)] and FOME methods are derived in this paper and yield a good estimation accuracy. Using simulated and real data, estimation comparisons show that best values of MSE and bias are achieved using the proposed procedures.

2. CG-LNT distribution and HOME method

Compound Gaussian models are characterized by two components; speckle and texture. The probability density function (PDF) of the CG-LNT model is (Carretero et al. 2010)

$$p(z) = \frac{z}{\sqrt{2\,\pi\,\sigma^2}} \int_0^{+\infty} \frac{2}{y^2}\, exp\left(- \frac{[ln(y/\delta)]^2}{2\sigma^2} - \frac{z^2}{y} \right) dy \tag{1}$$

where z is the amplitude and y is termed the 'texture' component which follows a log-normal pdf. Parameters δ and σ are respectively the median value of y and the standard deviation of $ln(y^2)$ which is related to the radar's illuminated patch area (i.e. shape parameter). As σ is related to the spikness of sea clutter, it is worth noting in Figure 1 that high values of σ control heavy-tailed CG-LNT distribution. For small values of σ ($\sigma \approx 1/8$), the CG-LNT is nearly close to the Rayleigh distribution.

The moment expression of order n is derived from (1) and is given by

$$\langle z^n \rangle = \frac{1}{M} \sum_{i=1}^{M} {z_i}^n = \delta^{n/2}\, \Gamma\,(1 + n/2) exp\left(\frac{1}{2} \left(\frac{n\sigma}{2} \right)^2 \right) \tag{2}$$

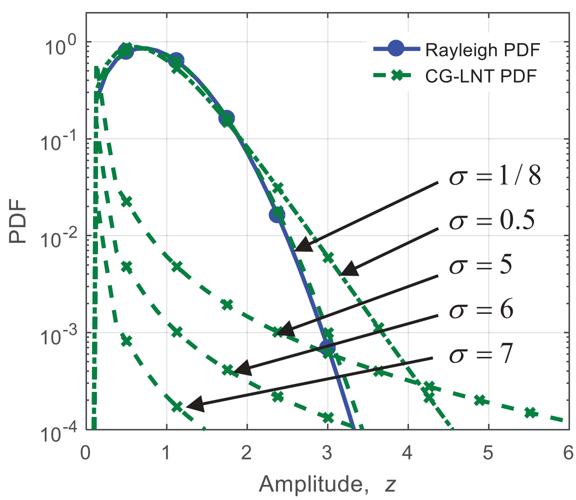

Figure 1. Comparison of heavy-tailed CG-LNT and Rayleigh models.

where, $\langle . \rangle$ denotes the expectation.

From (2), the HOME method is given by means of the two first even order moments

$$\begin{cases} \hat{\sigma} = \sqrt{ln\left(\frac{\langle z^4 \rangle}{2\langle z^2 \rangle^2}\right)} \\ \hat{\delta} = \langle z^2 \rangle exp\left(-\frac{\hat{\sigma}^2}{2}\right) \end{cases} \quad (3)$$

3. Estimators

In the following, the [zlog(z)] and FOME procedures are constructed under the assumption of independent and identically distributed (iid) CG-LNT clutter samples, z_i, $i = 1, \ldots, M$.

3.1. [zlog(z)] estimator

To develop the [zlog(z)] estimator, derivatives of left and right hand sides of (2) with respect to n are of prime importance. Hence

$$\frac{d\langle z^n \rangle}{dn} = \langle z^n lnz \rangle = \frac{d\left(\delta^{n/2}\Gamma(1+n/2)exp\left(\frac{1}{2}\left(\frac{n\sigma}{2}\right)^2\right)\right)}{dn} \quad (4)$$

with $n = 0$, $n = 1$ and $\langle z \rangle = 0.5\sqrt{\delta\,\pi}\exp(\sigma^2/8)$, (4) is rewritten as

$$\begin{cases} \langle lnz \rangle = \frac{1}{2}(ln\delta + \Psi(1)) \\ \frac{\langle z\,lnz \rangle}{\langle z \rangle} = \frac{1}{2}\left(ln\delta + \Psi(1+1/2) + \frac{\sigma^2}{2}\right) \end{cases} \quad (5)$$

where $\Psi(.)$ is the digamma function. Using the recurrence relation, $\Psi(x+1) = \Psi(x) + \frac{1}{x}$ with $\Psi(1) = -\gamma$ and $\Psi(1/2) = -\gamma - 2ln2$, subtraction side-by-side of (5) gives the closed form of [zlog(z)] estimator as

$$\hat{\sigma} = 2\sqrt{\frac{\langle z\,lnz \rangle}{\langle z \rangle} - \langle lnz \rangle - 1 + ln2} \quad (6)$$

where γ is the Euler's constant, $\langle lnz \rangle = \frac{1}{M}\sum_{i=1}^{M} lnz_i$ and $\langle z\,lnz \rangle = \frac{1}{M}\sum_{i=1}^{M} z_i\,lnz_i$.

3.2. FOME method

In order to eliminate δ in (2), the FOME method is based on the manipulation of the statistical ratio, $\langle z^{n+1} \rangle / \langle z \rangle \langle z^n \rangle$. Hence

$$\frac{\langle z^{n+1} \rangle}{\langle z \rangle \langle z^n \rangle} = \frac{\delta^{(n+1)/2}}{\delta^{1/2}\delta^{n/2}} \frac{\Gamma(1+(n+1)/2)}{\Gamma(1+1/2)\Gamma(1+n/2)} \frac{exp\left(\frac{1}{2}\left(\frac{(n+1)\sigma}{2}\right)^2\right)}{exp\left(\frac{1}{2}\left(\frac{\sigma}{2}\right)^2\right)exp\left(\frac{1}{2}\left(\frac{n\sigma}{2}\right)^2\right)} \quad (7)$$

simplification of (7) offers closed form formula of the FOME as

$$\hat{\sigma} = 2\sqrt{\frac{1}{n}ln\left(\frac{\langle z^{n+1}\rangle}{\langle z\rangle\langle z^n\rangle}\frac{0.5n\sqrt{\pi}\Gamma(0.5n)}{(n+1)\Gamma(0.5n+0.5)}\right)} \quad (8)$$

Once $\hat{\sigma}$ is given using either the [zlog(z)] or the FOME method, $\hat{\delta}$ is then computed in terms of fractional order moment,$\langle z^n\rangle$

$$\hat{\delta} = \left(\frac{\langle z^n\rangle}{\Gamma(1+n/2)}\exp\left(-\frac{n^2\sigma^2}{8}\right)\right)^{\frac{2}{n}} \quad (9)$$

Due to the integral form of the CG-LNT model of (1), the MLE method can not be obtained in a closed form. This estimator is relatively slow because numerical integration is required to calculate the likelihood function, along with numerical optimization to find the maximum. Generally, estimation results obtained by the [zlog(z)] are close to the ones given by the MLE method (Blacknell and Tougth 2001). For the purpose of real life applications, the proposed estimators which are given in closed forms are compared in the following section with the standard HOME method.

4. Estimation results

This section focuses on parameters estimation of CG-LNT clutter model and validation of [zlog(z)] and FOME based methods using simulated and real data.

4.1. Estimation using simulated data

Usually, before incorporating estimation procedures in real life applications, simulation studies must be carried out first. To this effect, parameter estimation is assessed for cases of non-spiky and spiky clutter situations (i.e. Rayleigh and non-Rayleigh pdfs). In a previous work (Greco, Gini and Rangaswamy 2006), estimated values of σ are obtained between 0.6 and 6 using IPIX (Intelligent PIxel Processing) database with different cell resolutions and radar parameters. In this work, σ is varied in the interval [0.5–7] so that the CG-LNT pdf takes different forms of heavy-tailed pdf. To evaluate the accuracy of underling estimators, MSE and bias criteria are considered over L= 10 000 independent trials and the clutter power is normalized to unity. A small value of fractional order moment with $n = 0.1$that exhibits the best estimates was used for the FOME method.

First, the effect of the number of samples M= 500, 1000 and 10 000 on the estimation precision is shown in Figure 2(a,b) which depict MSE and bias values in terms of true values of σ respectively. Here, two comparisons are made together; estimation results versus M and σ. As expected, the [zlog(z)] exhibits the best estimation results. Another investigation of these curves reveals that if small and high values of σ are taken into account, there are remarkable estimation errors for both [zlog(z)] and FOME methods. Specifically, when the spikiness of the clutter increases (σ> 1), the shape parameter is not estimated accurately by any of the three methods. In other words, it is difficult to estimate σ for small sample size (M= 500) and large shape parameter values, because the clutter distribution is very spiky and the performances diminish. This conclusion was noticed in (Mezache et al. 2016; Iskander and Zoubir 1999). On the other hand, it is also difficult to estimate σ if true values of the shape parameter is less than 1, because the

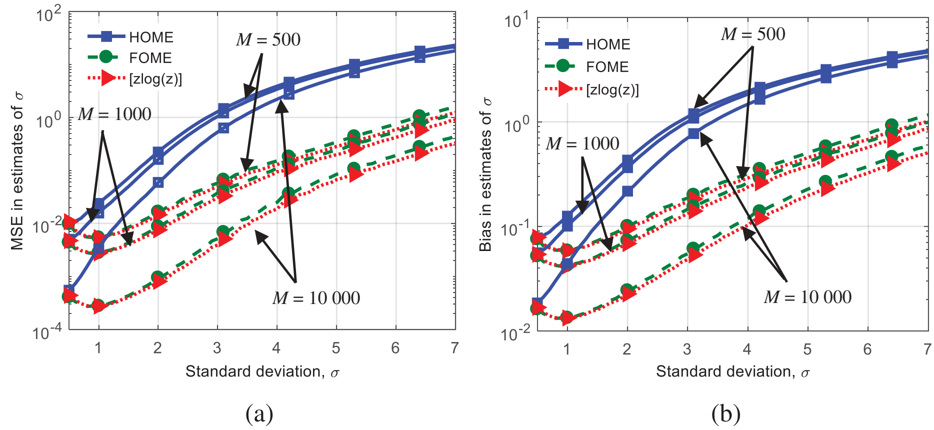

Figure 2. Estimation results of σ; (a) MSE in estimates of σ, (b) Bias in estimates of σ.

Rayleigh distribution is dominated by the tail, which is entirely due to Gaussian clutter. In this case, the estimation error increases with decreasing shape parameter. It is reasonable to believe that the performance of our estimators will be degraded if σ is smaller than 1, as shown in Figure. 2(a,b).

On the other hand, the estimation performances in terms of δ are also investigated according to (9) with σ= 2 (spiky clutter case). Compared to HOME method, the [zlog(z)] and FOME methods provide better estimation precision as shown in Figure 3, but useful values of estimation results are always obtained by the [zlog(z)] based approach. It is also noticed that high values of *M* improves the estimation performance for all methods.

4.2. Estimation using real data

Estimation study using real life data can also evaluate the robustness of statistical estimation procedures under several conditions. To do this, the McMaster IPIX clutter is considered (Greco, Gini, and Rangaswamy 2006). In order to investigate the estimation

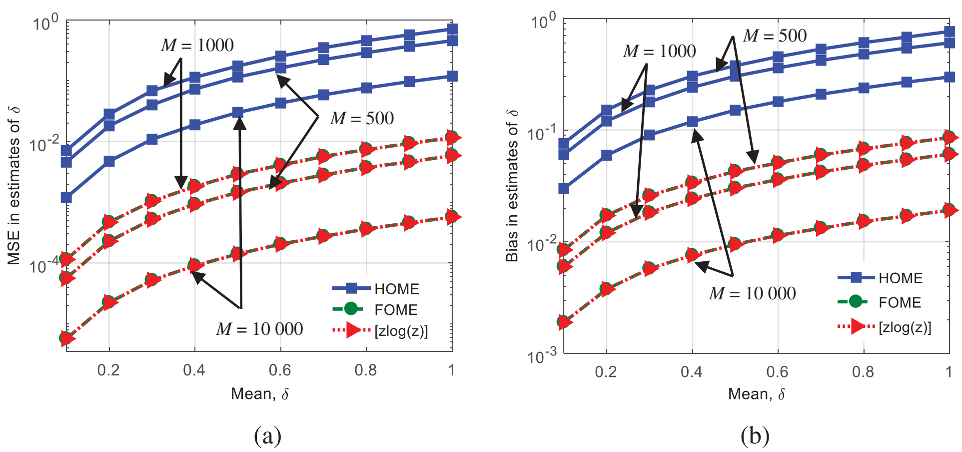

Figure 3. Estimation results of δ for σ= 2; (a) MSE in estimates of δ, (b) Bias in estimates of δ.

performance, we will compare empirical PDF and the complementary cumulative distributed function (CCDF) of the data with their theoretical CG-LNT model. From (1), the CCDF is expressed as a function of normalised threshold T by

$$CCDF(T) = \int_{0}^{+\infty} exp\left(-\frac{T^2}{y}\right) \frac{1}{y\sqrt{2\,\pi\,\sigma^2}} exp\left(-\frac{(\ln(y/\delta))^2}{2\sigma^2}\right) dy \quad (10)$$

First, in the case of HH radar antennas polarization, cell resolution of 15 m and 7th range cell, PDFs and CCDFs curves are plotted in terms of amplitude z and T respectively (see Figure. 4(a,b)). MSE and CG-LNT model parameters values are given in Table 1. We observe that the [zlog(z)], FOME and HOME based approaches give almost the same tail fitting to real PDF and real CCDF. Here, a complete overlap of fitted theoretical slopes to empirical data is not observed, because recorded IPIX data deviate relatively from the CG-LNT distribution. Previous modeling study in (Mezache et al. 2016; Ollila et al. 2012) indicated that it is difficult to make any specific clutter model as a function of different datasets scenarios of sea clutter.

Second, for a low resolution case with 30 m, VV antennas polarization and 4th range cell. Table 1 reports also estimated parameters and MSE values for each method. Note that the [zlog(z)] and FOME based approaches achieve the best fit to real PDF and real CCDF than the HOME approach. Modeling cases given in Figures 4

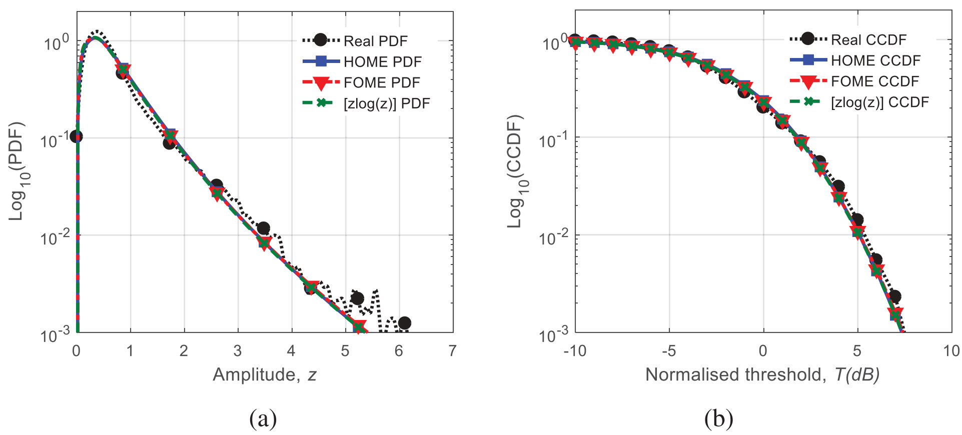

Figure 4. Modeling performances for HH polarization, resolution of 15 m and 7th range cell. (a) Empirical and theoretical PDFs, (b) Empirical and theoretical CCDFs.

Table 1. MSE estimates by HOME, FOME and [zlog(z)] methods using real data.

IPIX data sets		HOME	FOME	[zlog(z)]
HH Polarization, resolution 15 m and 7th range cell	PDF MSE	0.0021	0.0023	0.0022
	CCDF MSE	0.7913×10^{-4}	0.5213×10^{-4}	0.6095×10^{-4}
	Parameters	$\hat{\sigma}$= 1.1585	$\hat{\sigma}$= 1.1899	$\hat{\sigma}$= 1.1798
		$\hat{\delta}$= 0.5112	$\hat{\delta}$= 0.4859	$\hat{\delta}$= 0.4862
VV Polarization, resolution 30 m and 4th range cell	PDF MSE	0.0027	0.0013	0.0013
	CCDF MSE	0.7536×10^{-4}	0.3912×10^{-4}	0.4200×10^{-4}
	Parameters	$\hat{\sigma}$= 1.0472	$\hat{\sigma}$= 1.1999	$\hat{\sigma}$= 1.1965
		$\hat{\delta}$= 0.5779	$\hat{\delta}$= 0.4904	$\hat{\delta}$= 0.4905

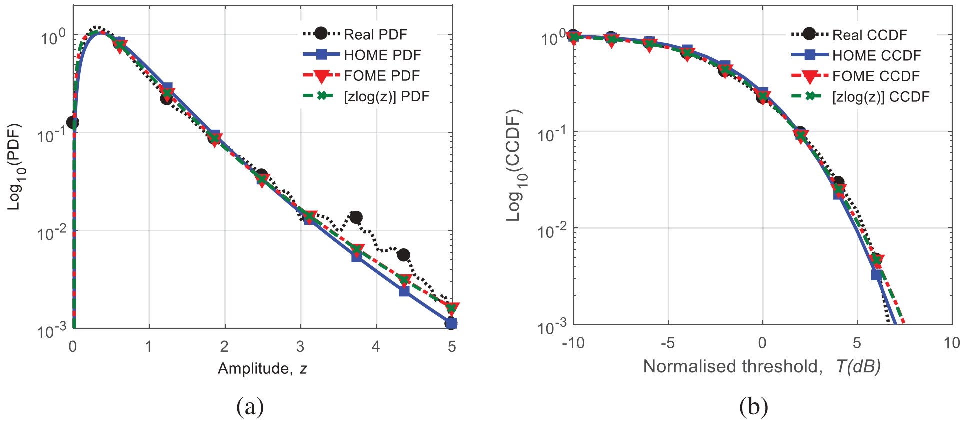

Figure 5. Modeling results for VV polarization, resolution of 30 m and 4th range cell. (a) Real and theoretical PDFs, (b) Real and theoretical CCDFs.

and 5 show the convergence of fitted PDFs and CCDFs slopes to real data. As a conclusion, the CG-LNT law is a candidate model characterizing such sea radar echoes. Examples illustrated in Figures 4 and 5 showed the approximation of CG-LNT model to empirical IPIX data. In this work, it is not possible to test all fitting cases, but *K*, Pareto type 2 and CGIG distributions with and without thermal noise can provide competitive fitting to real data. Moreover, during the examination of model fitting to IPIX data, the above distributions give different fitting errors for HH polarization case. For VV polarization, the modeling complexity is reduced especially when datasets with a low resolution (30 m) are at hand. With different cell resolutions, the fitting error is varied and CG-LNT model can provide approximate performances in some cases compared to the existing statistical models.

5. Conclusions

This letter has been concerned with parameter estimation of CG-LNT clutter model without thermal noise. The proposed [zlog(z)] and FOME methods were given in closed forms, and do not need any numerical computation routines. Simulated and IPIX real data were performed to investigate the performances of proposed estimation approaches. Estimation results given here show a little superiority of [zlog(z)] method with respect to the FOME method. With decreasing the fractional order, almost similar estimates can be obtained by means of the two methods. When either log moments or fractional moments is probably required in real life applications, the [zlog(z)] or the FOME method is preferred.

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