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وزارة التعليم العالي والبحث العلمي
Ministry of Higher Education and Scientific Research
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كلية التكنولوجيا
FACULTY OF TECHNOLOGY

Faculty of Technology
Vice Deanship of Post-Graduation, Scientific Research and External Relations

كـلية التكنـــولــوجيـــا
نيابة العمادة لما بعد التدرج والبحث العلمي والعلاقات الخارجية

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شهادة ادارية

المصادقة على تقارير خبرة للموافقة على مطبوعة بيداغوجية

بعد الإطلاع على تقارير لجنة الخبراء للموافقة على المطبوعة البيداغوجية للأستاذ : **برابح فؤاد – أستاذ محاضر قسم أ** ، بقسم الهندسة المدنية بكلية التكنولوجيا بجامعة محمد بوضياف بالمسيلة والتي كانت كلها ايجابية.

تمّ تقرير التّالي:

1-المصادقة على تقارير لجنة الخبراء للموافقة المطبوعة البيداغوجية والمعنونة بـــ:

Soil Mechanics
Second year of License-Civil engineering

2- حيث تمّ تشكيل هذه اللجنة بناءاً على إجتماع اللجنة العلمية لقسم الهندسة المدنية المنعقد بتاريخ: 2026/04/27.

المكونة من السّادة الآتية أسماؤهم:

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وتمت الموافقة بالاجماع على هذه المطبوعة.

رئيس المجلس العلمي للكلية

المجلس العلمي

د. علي جريوي

Mohamed Boudiaf University - M’sila

Faculty of Technology

Department of Civil Engineering

Lecture notes on:

Soil Mechanics

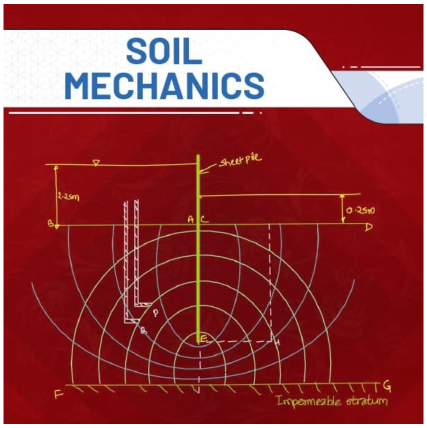
Course and exercises

by :

Dr. Fouad BERRABAH

Intended for:

2nd Year Civil Engineering Students



Academic Year 2025-2026

Preface

Soil is a natural and complex material whose mechanical behavior differs significantly from conventional construction materials such as steel or concrete. Its properties are highly variable and influenced by environmental factors, particularly the presence and movement of water. For this reason, understanding soil behavior is essential in civil engineering practice.

Soil mechanics is a fundamental discipline, as most engineering structures are either founded on or built with soils. Inadequate evaluation of soil properties may lead to serious problems such as excessive settlement, instability, or seepage-related failures. Therefore, it is crucial to introduce students to the basic principles governing soil behavior at an early stage of their training.

This course, "**Soil Mechanics**", is designed for second-year civil engineering students. It aims to provide a solid foundation in the fundamental concepts of geotechnical engineering, combining theoretical knowledge with practical applications.

The course is organized into four main chapters. The first chapter introduces soil formation and basic characteristics. The second focuses on soil identification and classification methods. The third addresses soil compaction and its importance in construction. The fourth examines the role of water in soils, including permeability, seepage, capillarity, and related phenomena. Each chapter is supported by practical examples and exercises to enhance understanding and develop analytical skills.

This document provides the fundamental knowledge required for advanced studies in geotechnical engineering and professional practice.

Fouad BERRABAH
M'sila, 2026

Contents

Chapter 1
Introduction to soil mechanics

1.1 Purpose of soil mechanics

Civil engineering structures rely on the ground as the element that transmits structural loads to the underlying soil layers capable of supporting them safely.

Therefore, the success of a structure largely depends on the design and performance of its foundation system. Depending on the type of structure and its design method, the ground may serve either as:

- a supporting base for the entire structure, such as roads, tunnels, gravity dams, retaining walls, and airfields, or
- a supporting medium for structural elements, such as building foundations, bridges, and arch dams.

Soil mechanics is the science that combines the knowledge and techniques required to:

- identify the properties that govern the mechanical behavior of soils,
- analyze soil–structure interaction,
- ensure the safe design and construction of underground and earth structures.

In practice, soil mechanics addresses problems related to:

- foundations,
- retaining structures,
- embankments and earth structures,
- tunnels and underground excavations,
- mining works.

1.1.1 Disciplines related to soil mechanics

To achieve the objectives described above, several disciplines contribute to geotechnical engineering.

a) Engineering geology

Engineering geology plays a crucial role in identifying:

- the different soil and rock layers,
- their thickness and inclination,
- the possible presence of groundwater.

Geological investigations provide qualitative descriptions of soils, help understand the formation and history of deposits, and guide preliminary site investigations.

b) Physicochemical characteristics

The study of the physical and chemical properties of soils is essential for predicting and interpreting soil behavior. Most of these properties are determined through laboratory tests or in-situ tests.

c) Hydrogeology and hydraulic studies

The presence of water in soil layers has a major influence on soil behavior. Determining the groundwater level and studying groundwater flow conditions allow engineers to:

- select appropriate dewatering systems,
- prevent problems such as piping or quicksand conditions.

The chemical composition of groundwater must also be evaluated in order to design appropriate waterproofing systems for underground structures.

d) Mechanical characteristics

The mechanical behavior of soils is determined based on the results of geological, physical, and hydraulic studies, together with laboratory and in-situ tests. This discipline allows engineers to determine:

- soil strength,
- bearing capacity,
- appropriate foundation types,
- and dimensions of underground structural elements.

It also allows the prediction of settlements and ground deformations under structural loads.

e) Theoretical research and numerical modeling

To better understand complex soil behavior, numerous theoretical models have been developed. These models describe geotechnical problems using mathematical formulations, whose solutions often require advanced numerical and computational techniques.

Such approaches represent an important area of modern geotechnical research.

f) Design and construction

This field concerns the practical techniques used in the design and construction of geotechnical structures. It includes:

- the selection of appropriate construction methods,
- evaluation of alternative technical solutions,
- cost analysis,

- compliance with engineering standards and safety regulations during both construction and operation of the structure.

1.1.2 History of soil mechanics

The development of soil mechanics can be traced through the evolution of its major theories (Table 1.1).

Table 1.1 Major contributors to soil mechanics

Century	Author	Theory
18th	Coulomb	Theory of soil shear strength
19th	Collin	Stability of clay slopes
	Darcy	Law of water flow in porous media
	Rankine	Earth pressure on retaining walls
	Gregory	Horizontal drainage and railway slope stabilization
20th	Atterberg	Consistency limits of clays
	Terzaghi	Founder of modern soil mechanics
	Casagrande	Development of the liquid limit test

1.1.3 Can we build on this soil?

This question cannot be answered immediately without proper investigation. A rational approach requires answering several preliminary questions:

- What is going to be built?

A dam, a dike, a road, a runway, a retaining wall, or a building?

- Where will it be built?

In a dry or humid region?

- How will it be built?

What construction techniques and equipment are available?

Soils can serve several purposes:

1. As construction materials

Examples include:

- adobe,
- stabilized or fired clay bricks,
- earth dams and dikes,
- earth roadways.

In such cases, it is necessary to select appropriate soils based on the available borrow areas, the type of structure, and the construction method. Proper quality control during construction is also essential.

2. As foundation support

Soils may also support:

- building foundations,
- engineering structures,
- embankments.

In this case, the engineer must select the appropriate foundation system, considering:

- the loads to be supported,
- the mechanical properties of the soil,
- the groundwater level.

In particular, it is necessary to predict settlement magnitudes and verify that they remain within acceptable limits for the proper functioning of the structure. This highlights the importance of geotechnical engineering, whose objective is to study the mechanical behavior of soils and to determine whether a given soil is suitable for a specific construction project.

1.2 Definition of soils

In geotechnical engineering, the materials forming the Earth's crust are generally classified into two main categories:

- **Rocks:** Aggregates of mineral grains bonded together by strong and permanent cohesive forces that remain intact even after prolonged immersion in water. Their behavior is studied in rock mechanics.
- **Soils:** A soil is a natural heterogeneous assembly of mineral particles or crystals with widely varying properties such as size, shape, and physicochemical characteristics. The particles can be separated by relatively small mechanical forces. The mechanical behavior of soils is studied in soil mechanics.

Materials that exhibit properties intermediate between soils and rocks are often referred to as HSSR (Hard Soils and Soft Rocks).

Soils generally have the following characteristics:

- they are loose and porous materials,
- they are heterogeneous,
- they may exhibit anisotropic mechanical behavior,
- they consist of mineral or organic particles of varying size and shape.

1.2.1 Constitutive elements of soil

Soil is a three-phase system composed of (Figure 1.1):

- solid particles forming the soil skeleton,
- water occupying part of the void space,
- air or gas filling the remaining voids.

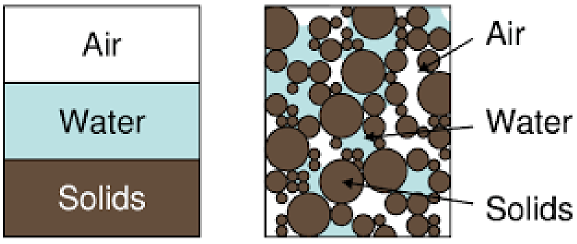


Figure 1.1 Three phases of soil (soil constituents)

Soil = solid phase + liquid phase + gas phase

The spaces between soil particles, called voids or pores, may be filled with water, air, or both simultaneously.

The gas contained in these voids is generally air when the soil is dry, or a mixture of air and water vapor when the soil is moist, which is the most common situation.

Water may fill part or all of the pore spaces and may either remain static or flow through the soil.

- When all the voids are filled with water, the soil is said to be saturated.
- When no water is present, the soil is considered dry.

In temperate regions, most soils located a few meters below the ground surface are fully saturated due to the presence of groundwater.

The behavior of unsaturated soils, which involve interactions between the solid, water, and air phases, is significantly more complex and constitutes a specialized field within soil mechanics.

1.3 Origin and formation of soils

Soils originate primarily from the weathering and decomposition of rocks and organic materials. Two main processes contribute to soil formation.

1.3.1 Weathering of rocks

Rocks are gradually broken down by mechanical and chemical weathering under the action of natural agents such as:

- fracturing caused by stress relief, temperature variations, or freeze–thaw cycles,

- mechanical abrasion and fragmentation resulting from transport by glaciers, rivers, waves, wind, or gravity,
- chemical alteration caused by circulating water containing dissolved acids or bases.

1.3.2 Decomposition of living organisms

Soils may also form from the decomposition and accumulation of biological materials, including:

- plant remains, which may form peat,
- biogenic carbonate deposits produced by marine organisms, such as chalk and limestone.

Residual and transported soils

Depending on the formation process, soils are commonly classified into two main categories:

- **Residual soils:** soils that remain at the location where the parent rock has weathered.
- **Transported soils:** soils formed from weathered materials that have been transported and deposited by natural agents such as water, wind, glaciers, or gravity.

Transported soils often present greater variability in their properties, which may create additional challenges for geotechnical design.

In addition, soils may contain varying amounts of organic matter, depending on the environmental conditions during their formation and deposition.

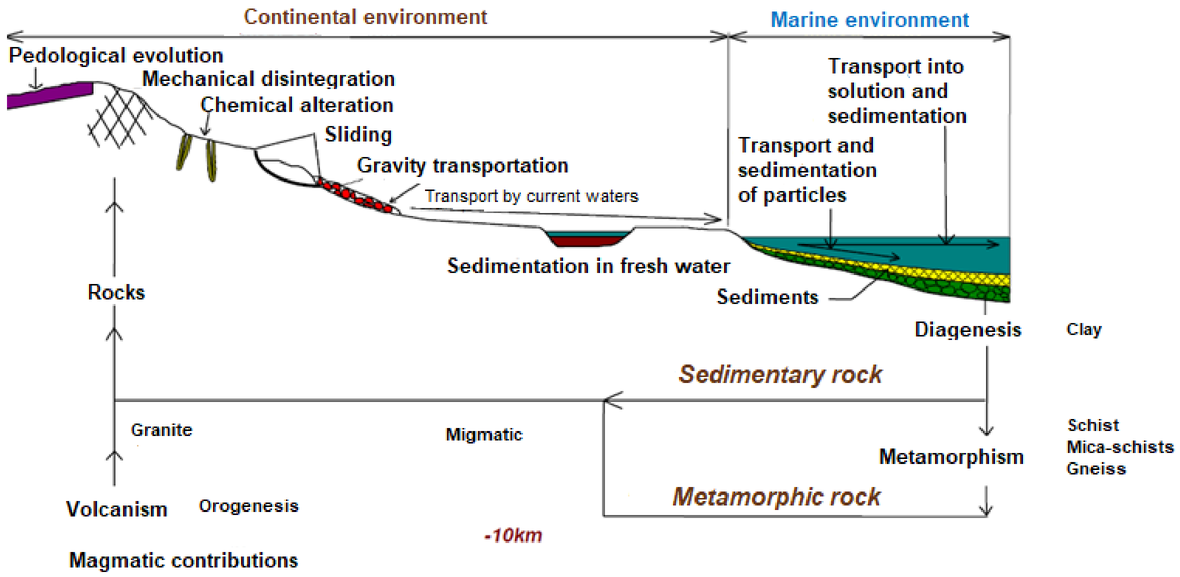


Figure 1.2 Origin of soils

It should be noted that mechanical weathering alone generally cannot reduce particle sizes below about 10-20 μm. As particle size decreases, the effectiveness of mechanical fragmentation becomes limited because the forces generated by impact and friction decrease with particle mass.

Below this size range, further particle reduction occurs mainly through chemical weathering, which breaks chemical bonds within minerals and leads to the formation of clay minerals. This process also greatly increases the specific surface area of soil particles, making them more susceptible to further chemical reactions.

1.4 Soil structure (coarse-grained and fine-grained soils)

Soil is a particulate material composed of discrete solid particles. The size of these particles may vary widely, ranging from large gravel or pebbles several centimeters in diameter to extremely fine particles smaller than one micrometer.

1.4.1 Main physical properties of soil particles

The main physical characteristics of soil particles are:

- particle size
- particle shape
- specific surface area

These properties strongly influence the hydraulic and mechanical behavior of soils.

a) Particle size

Particle size is generally expressed using an equivalent diameter.

For coarse particles, the equivalent diameter corresponds to the size of the square sieve opening through which the particle can pass during sieve analysis.

For fine particles that cannot be separated by sieving, the equivalent diameter is determined using sedimentation methods based on Stokes' law.

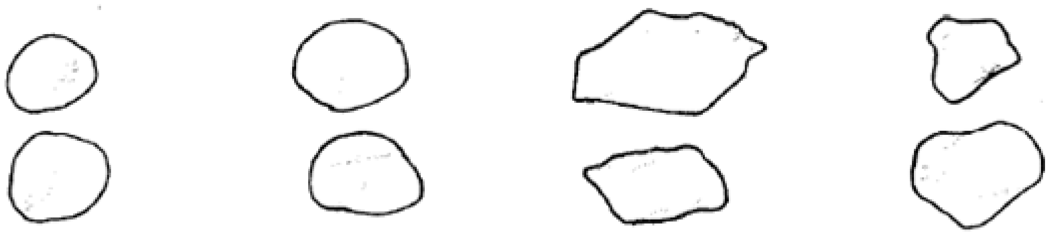
b) Particle shape

Although soil particles can have many shapes, two general forms are commonly distinguished.

- **Bulky or equidimensional particles**

This shape is typical of gravel, sand, and most silt particles. These particles are generally approximately equidimensional, meaning their length, width, and thickness are of similar magnitude.

Their surfaces may be rounded, subrounded, or angular depending on the transport and weathering processes that formed them.



rounded subrounded angular subangular

Figure 1.3 Typical shapes of coarse soil particles

- **Plate-like particles**

Fine clay minerals typically have a plate-like or sheet-like structure. A particle is considered plate-like when its length-to-thickness ratio exceeds about 10. This structure is characteristic of clay minerals such as kaolinite, illite, and montmorillonite.

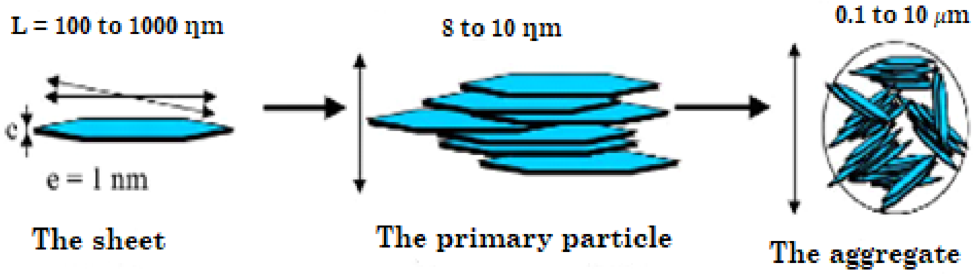


Figure 1.4 Plate-like clay particles and clay aggregates

c) Specific surface area

The specific surface area of soil particles represents the total surface area of particles per unit mass of soil.

It is defined as:

$$S_s = \frac{A_s}{M}$$

where:

A_s = total surface area of particles

M = mass of soil particles

The specific surface area is usually expressed in: m^2/kg

It can also be expressed per unit volume: $m^2/m^3 = m^{-1}$

Fine soils, particularly clays, have extremely large specific surface areas, which strongly influence their water adsorption capacity, plasticity, and compressibility. Table 1.2 shows typical values of the specific surface area for different soil types.

Table 1.2 Specific surface area of particles for different soil types

Type of soil	**Equivalent diameter (mm)**	**Typical thickness(ηm)**	**Average Specific Surface (m^2/kg)**
Sand	1 - 2	-	1.5
Sand	0.25 - 0.5	-	6
Silt	0.002 - 0.05	-	82.5
Clay (kaolinite)	0.0003 - 0.002	50 - 100	15 000
Clay (illite)	0.0001 - 0.002	30	90 000
Clay (montmorillonite)	0.0001 - 0.001	3	800 000

1.4.2 Soil types

Soils are commonly classified according to the size of their constituent particles. In soil mechanics, soils are generally grouped into three major classes:

- Coarse-grained soils
- Fine-grained soils
- Organic soils

a) Coarse-grained soils

Coarse-grained soils consist mainly of gravel and sand particles. Very large particles such as boulders and cobbles (riprap) may also be present and typically have diameters greater than 80 mm. These soils generally exhibit high permeability.

- **Gravel:** particle size between 2 mm and 80 mm
- **Sand:** particle size between 0.08 mm and 2 mm

Coarse-grained soils usually have good drainage properties and relatively low compressibility.

b) Fine-grained soils

Fine-grained soils include silt and clay, whose particles are much smaller.

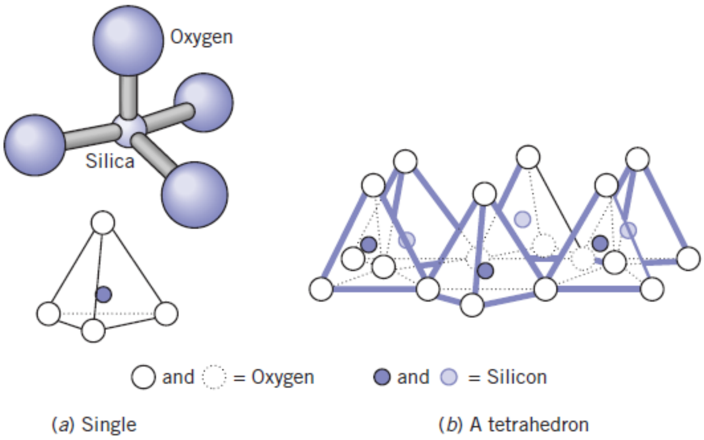
- **Silt:** particle diameter between 0.002 mm and 0.08 mm. Silt particles can be observed using an optical microscope.
- **Clay:** particle diameter smaller than 0.002 mm. Clay particles originate mainly from the chemical weathering of minerals and consist largely of aluminum, magnesium, or iron silicates.

Because of their extremely small size, clay particles can only be observed using advanced techniques such as a Scanning Electron Microscope (SEM).

Structure of clay minerals

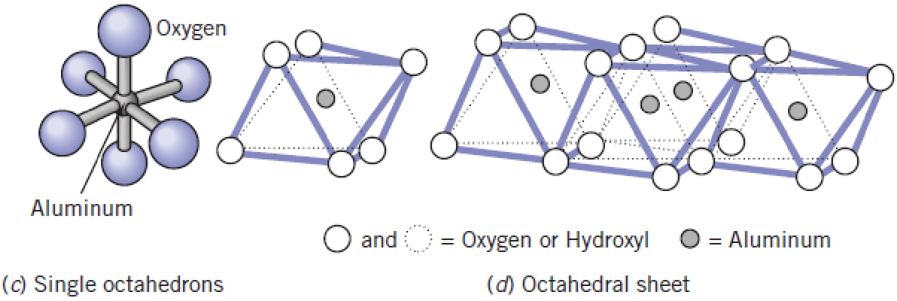
Clay minerals consist of microscopic crystalline sheets formed by the stacking of basic structural units called tetrahedral and octahedral sheets. The thickness of these sheets is extremely small, typically on the order of 0.5 nm. Two fundamental structural units exist:

- Tetrahedral sheet
- Octahedral sheet



(*a*) Single (*b*) A tetrahedron

Figure 1.5 Tetrahedral sheet structure



(*c*) Single octahedrons (*d*) Octahedral sheet

Figure 1.6 Octahedral sheet structure

Main families of clay minerals

Three main groups of clay minerals are commonly distinguished:

- **Kaolinite** (1:1 structure – T–O)

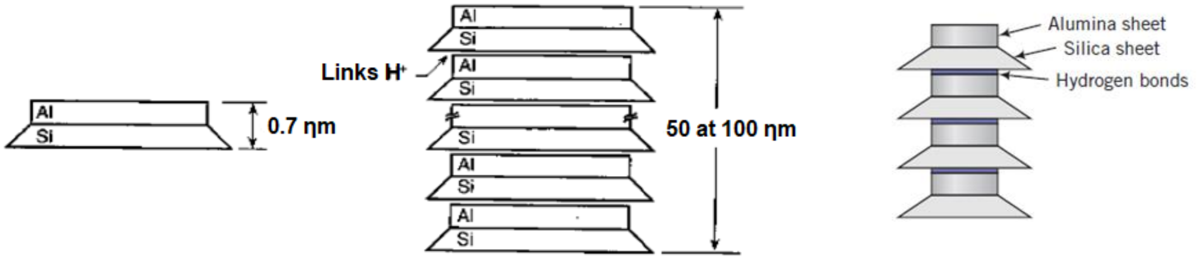
  Low plasticity and low swelling potential.

- **Illite** (2:1 structure – T–O–T)

  Moderate plasticity and moderate swelling.

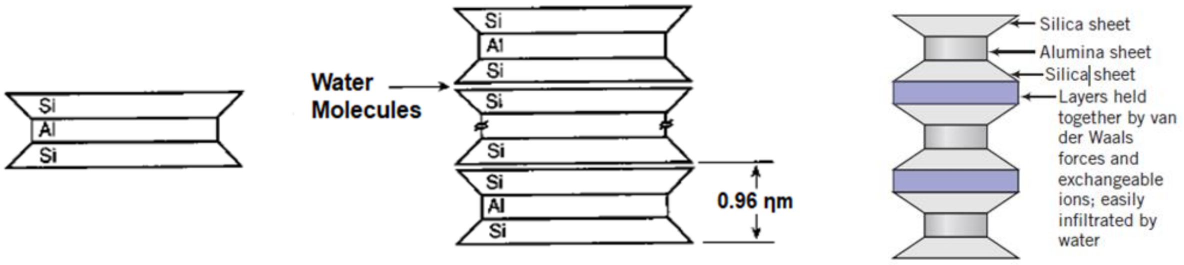
- **Montmorillonite** (2:1 structure – T–O–T)

  Very high swelling and shrinkage potential, which can cause serious engineering problems.



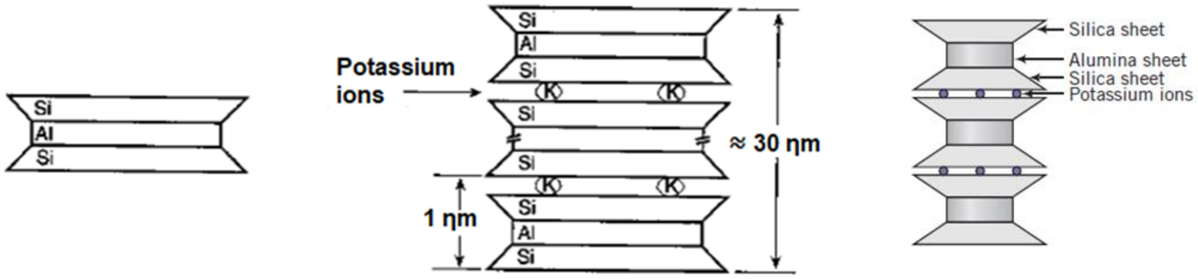
(a) structural layer **(b)** kaolinite particle

Figure 1.7 Structure of kaolinite



(a) structural layer **(b)** montmorillonite particle

Figure 1.8 Structure of montmorillonite



(a) structural layer **(b)** illite particle

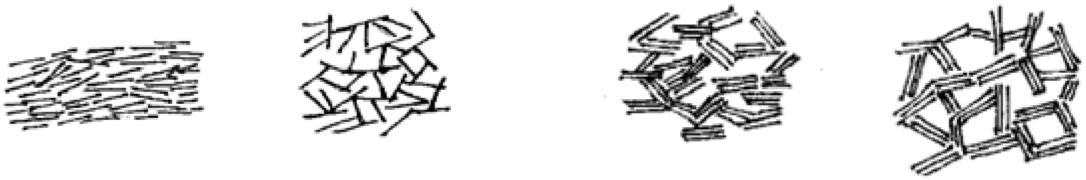
Figure 1.9 Structure of illite

Clay particle arrangements

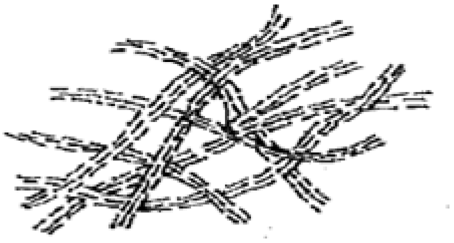
Clay particles have a plate-like shape, and their arrangement may vary depending on physicochemical conditions. Common soil fabrics include:

- flocculated structure
- dispersed structure
- honeycomb structure

These structures significantly influence the strength, compressibility, and permeability of fine-grained soils.

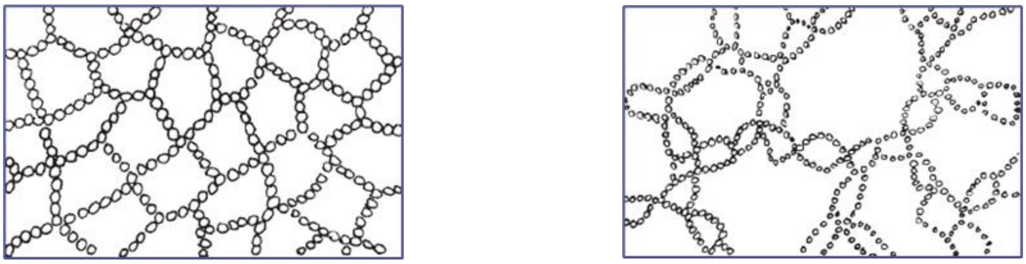


(a) clay plate arrangement **(b)** clay platelet grouping arrangement



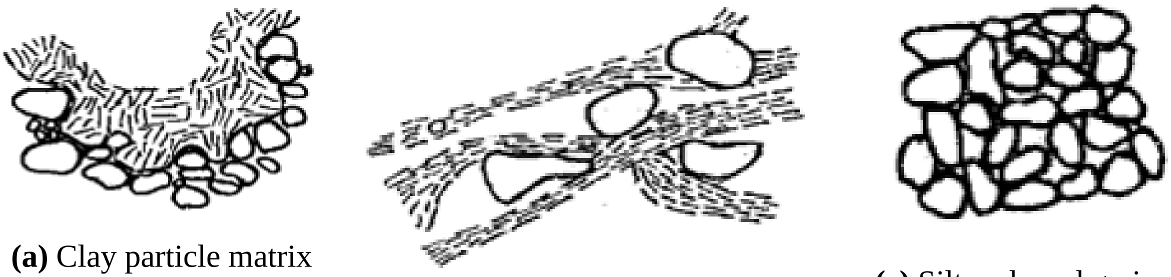
(c) Tangle of clay piles

Figure 1.10 Fabric of fine-grained soils



(a) Honey-comb structure **(b)** Flocculent structure

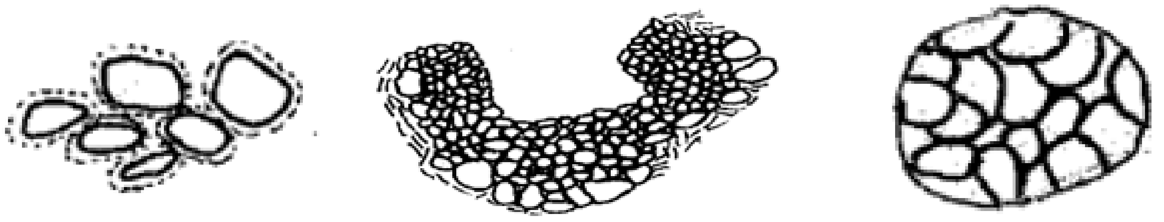
Fig. 1.11 Arrangement of clay platelets



(a) Clay particle matrix

(b) Tangle of clay piles with silt inclusions

(c) Silt and sand grains



(d) Silt plates and grains of sand

(e) Granular particle matrix

(f) Partially discernible matrix between particles

Figure 1.12 Arrangement of particles in soils with mixed grain sizes

When soils contain particles of various sizes, complex structures may develop, including:

- aggregates
- clusters
- particle matrices

c) Organic soils

Organic soils contain significant amounts of organic matter (OM). Typical classification:

- OM < 3%: inorganic soil
- 3% < OM < 10%: slightly organic soil
- 10% < OM < 30%: organic soil

Highly organic soils such as peat may contain much larger percentages of organic matter. Organic soils usually exhibit high compressibility and low strength, which may create significant geotechnical challenges.

1.5 Exercises

Questions

1) Why is geology important in geotechnical engineering?
2) What is engineering soil?
3) What is the composition of soils?
4) What are the main minerals in soils?
5) How is soil described?
6) What are the differences between coarse-grained and fine-grained soils?
7) What is a grading curve?
8) How do you determine the particle size distribution in soils?
9) How do you interpret a grading curve?

Chapter 2
Identification and classification of soils

2.1 Physical characteristics

2.1.1 Basic soil model

A soil is composed of solid particles, water, and air. Each phase occupies a portion of the total volume of the soil (Figure 2.1).

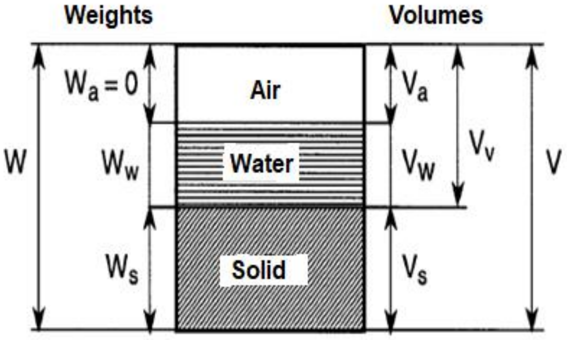


Figure 2.1 Conventional representation of a soil volume (weights and volumes of the phases)

Notations:

W: total weight

W_s: weight of the solids

Ww: weight of water

V: total volume

V_s: volume of solids

V_v: volume of voids

V_w: volume of water

V_a: volume of air

Relationships:

$$W = W_s + W_w$$

$$V_v = V_w + V_a$$

$$V = V_s + V_v = V_s + V_w + V_a$$

a) Dimensional parameters (unit weights):

- **Unit weight of solids**, denoted γ_s, is the weight of the solid particles per unit volume of solids only, excluding any voids or water:

$$\gamma s = \frac{W_s}{V_s}$$

- **Unit weight of water**, denoted γ_w, is the weight of water per unit volume:

$$\gamma_w = \frac{W_w}{V_w} = 9.81 \text{ kN/m}^3 \approx 10\ kN/m^3$$

(Using 10 kN/m^3 introduces a relative error of about 2%.)

- **Unit weight** is the weight of a soil per unit volume. We will use the term *bulk unit weight*, γ, to denote unit weight:

$$\gamma = \frac{W}{V}$$

Special Cases

(a) Dry unit weight (S = 0)

$$\gamma_d = \frac{Ws}{V}$$

(b) Saturated unit weight (S = 1)

$$\gamma_{sat} = \frac{(W_s + \gamma_w V_v)}{\text{V}}$$

- **Effective or buoyant unit weight** is the weight of a saturated soil, surrounded by water, per unit volume of soil:

$$\gamma' = \gamma_{sat} - \gamma_w$$

- **Specific gravity (G_s)** is the ratio of the weight of the soil solids to the weight of water of equal volume:

$$G_s = \frac{\gamma_s}{\gamma_w}$$

b) Dimensionless parameters (state parameters):

- **Porosity (n)** is the ratio of the volume of voids to the total volume. Porosity is usually expressed as a percentage.

$$n = \frac{V_v}{V}$$

- **Void ratio (e)** is the ratio of the volume of void space to the volume of solids. Void ratio is usually expressed as a decimal quantity.

$$e = \frac{V_v}{V_s}$$

- **Water content (w)** is the ratio, often expressed as a percentage, of the weight of water to the weight of solids:

$$w = \frac{W_w}{Ws} \times 100\%$$

- **Degree of saturation (*S*)** is the ratio, often expressed as a percentage, of the volume of water to the volume of voids:

$$S = \frac{V_w}{V_v}$$

If $S = 1$ or 100%, the soil is saturated. If $S = 0$, the soil is bone-dry. It is practically impossible to obtain a soil with $S = 0$.

- **Relative density (D_r)** is an index that quantifies the degree of packing between the loosest and densest possible state of coarse-grained soils as determined by experiments:

$$D_r = \frac{e_{max} - e}{e_{max} - e_{min}}$$

where e_{max} is the maximum void ratio (loosest condition), e_{min} is the minimum void ratio (densest condition), and e is the current void ratio.

The relative density can also be written as

$$D_r = \frac{\gamma_{dmax}}{\gamma_d} \times \frac{(\gamma_d - \gamma_{dmin})}{(\gamma_{dmax} - \gamma_{dmin})}$$

2.1.2 Relationships between parameters

The parameters defined previously are not independent. Several relationships exist between them and are widely used in soil mechanics.

[1] $n = \frac{V_v}{V}$	[2] $e = \frac{V_v}{V_s}$	[3] $\mathrm{w} = \frac{W_w}{W_s}$
[4] $n = \frac{e}{1+e}$	[5] $e = \frac{n}{1-n}$	[6] $w = e.S.\frac{\gamma_w}{\gamma_s}$
[7] $\mathrm{n} = 1 - \frac{\gamma_d}{\gamma_s}$	[8] $\mathrm{e} = \frac{\gamma_s}{\gamma_d} - 1$	[9] $w = \frac{\gamma}{\gamma_d} - 1$
[10] $\mathrm{n} = \frac{\gamma_s - \gamma_{sat}}{\gamma_s - \gamma_w}$	[11] $\mathrm{e} = \frac{\gamma_s - \gamma_{sat}}{\gamma_{sat} - \gamma_w}$	[12] $\mathrm{w} = S \,.\, \gamma_w \left(\frac{1}{\gamma_d} - \frac{1}{\gamma_s}\right)$
[13] $S = \frac{V_w}{V_v}$	[14] $S = \frac{\gamma_s}{\gamma_w} . \frac{\mathrm{w}}{\mathrm{e}}$	[15] $S = \frac{\mathrm{w}}{\mathrm{w}_{sat}} (\gamma_d \text{ constant})$
[16] $\gamma = (1+w)(1-n).\gamma_s$	[17] $\gamma = \frac{1+w}{1+e}.\gamma_s$	[18] $\gamma = (1+w)\gamma_d$
[19] $\gamma = \gamma_d + n.S.\gamma_w$	[20] $\gamma = \frac{\gamma_s + e.S.\gamma_w}{1+e}$	[21] $\gamma = (1 - n).\gamma_s + n.S.\gamma_w$
[22] $\gamma_d = (1 - n)\,\gamma_s$	[23] $\gamma_d = \frac{\gamma_s}{1+e}$	[24] $\gamma' = \gamma_{sat} - \gamma_w$
[25] $\gamma' = (1-n)(\gamma_s - \gamma_w)$	[26] $\gamma' = \frac{\gamma_s - \gamma_w}{1+e}$	[27] $\gamma' = \frac{\gamma_s - \gamma_w}{\gamma_s}.\gamma_d$

2.2 Particle-Size Characteristics

2.2.1 Particle-size distribution

To properly describe a soil, its particle-size distribution must be known, that is, the distribution of its particles according to their equivalent diameters. Two laboratory tests are commonly used to determine the particle-size distribution of soils (Figure 2.2):

- Sieve analysis
- Sedimentation analysis

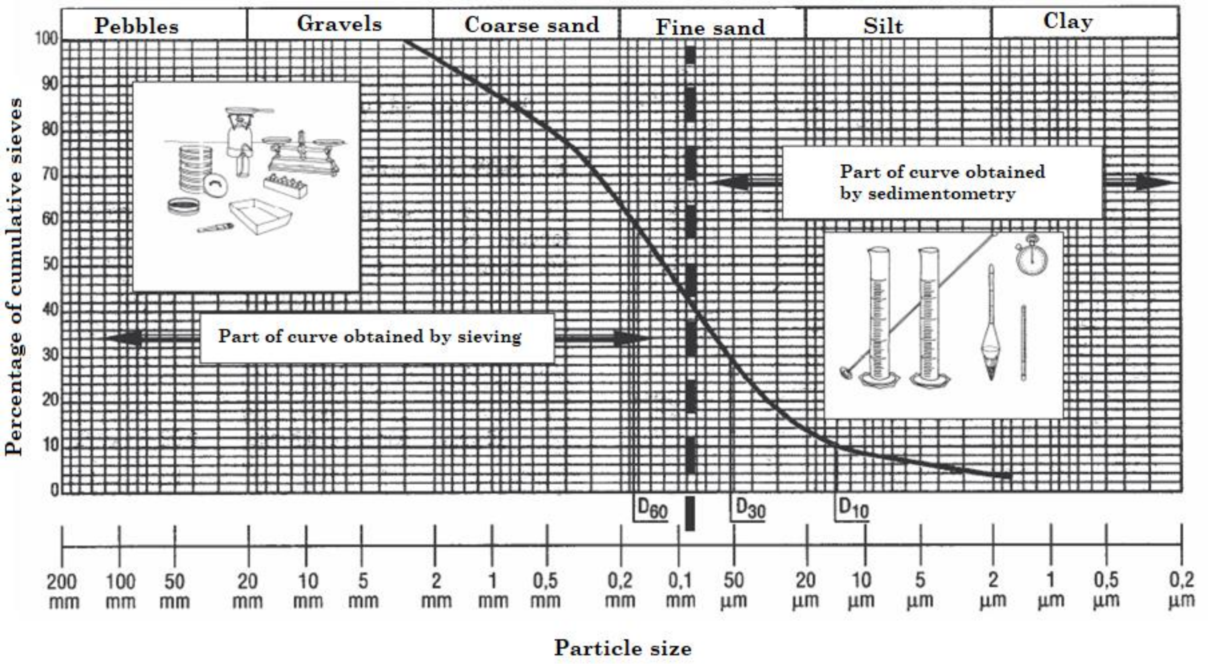


Figure 2.2 Particle-size distribution curve

a) Sieve analysis

This test consists of passing a representative soil sample through a stack of sieves with decreasing mesh openings from top to bottom. The largest particles are retained on the upper sieves, while the finer particles pass through to the lower sieves.

When the masses retained on each sieve become constant, the sieving operation is complete, and the material retained on each sieve is weighed. The mass retained on each sieve is then compared with the total mass of the sample in order to calculate the cumulative retained percentage and the percentage passing. The results are plotted on a semi-logarithmic graph to obtain the particle-size distribution curve.

b) Sedimentation analysis

To determine the particle-size distribution of silt and clay particles, sedimentation analysis is used.

This method is based on measuring the settling velocity of particles in a column of water. Using Stokes' law, the particle diameter can be determined:

$$v = \frac{9.8D^2\,(D_{rs} - D_{rl})}{3\eta} \Rightarrow D = \sqrt{\frac{3v\eta}{9.8\,(D_{rs} - D_{rl})}}$$

Where:

D: particle diameter (mm)

v: settling velocity (cm/min)

D_{rs}: relative density of the particle

D_{rl}: relative density of the liquid

H: dynamic viscosity of the liquid (Pa.s)

2.2.2 Interpretation of particle-size distribution curves

A particle-size distribution curve makes it possible to identify the different soil fractions present in the analyzed sample. By examining the curve of a soil composed of gravel, sand, silt, and clay, the respective proportions of these fractions can be determined as percentages.

Once these proportions are known, the soil can be named according to the convention shown in Table 2.1. For example, if a soil is composed of 27% gravel, 38% sand, 29% silt, and 6% clay, it may be described as gravelly silty sand with traces of clay.

Table 2.1 Soil naming based on the proportion of soil fractions

Proportion of soil types	Terminology	Examples
> 35%	Name	Gravel, sand, silt, etc.
20% to 35%	Adjective	Gravely, sandy, etc.
10% to 20%	Slightly / with some	Slightly silt, with some sand, etc.
< 10%	With traces of	With traces of clay, silt, etc.

The grading of a soil can be characterized by the uniformity coefficient and the coefficient of curvature.

a) Uniformity coefficient

The uniformity coefficient, also called Hazen’s coefficient, expresses the spread of the grading curve:

$$C_u = \frac{D_{60}}{D_{10}}$$

Where:

D_{60}: particle diameter of corresponding to 60% passing

D_{10}: particle diameter corresponding to 10% passing

A larger value of C_u generally indicates a wider range of particle sizes.

Based on the value of the uniformity coefficient, the degree of grading of a soil can be qualitatively assessed, as shown in Table 2.2.

Table 2.2 Degree of grading based on the uniformity coefficient

Uniformity coefficient	**Degree of grading**
$C_u \leq 2$	Uniform (very poorly graded)
$2 < C_u \leq 5$	Poorly graded
$5 < C_u \leq 20$	Moderately graded
$20 < C_u \leq 200$	Well graded
$200 < C_u$	Very Well graded

b) Coefficient of curvature

The coefficient of curvature describes the shape of the grading curve:

$$C_c = \frac{(D_{30})^2}{D_{10} \times D_{60}}$$

Where:

D_{30}: particle diameter corresponding to 30% passing

A soil is considered well graded when its particle-size distribution is continuous over a wide range of sizes, without a predominance of a particular fraction. For coarse-grained soils, the usual criteria are:

- Gravel: $C_u \geq 4$ and $1 < C_c < 3$
- Sand: $C_u \geq 6$ and $1 < C_c < 3$

If these conditions are not satisfied, the soil is considered poorly graded. Well-graded soils generally form denser natural deposits, have higher bearing capacity, can be compacted more easily in embankments, and tend to form more stable slopes.

2.3 Consistency of fine-grained soils (Atterberg limits)

2.3.1 Definition

Consistency refers to the degree of firmness of a soil, which is mainly governed by the cohesive forces between particles. It therefore concerns fine-grained (cohesive) soils and strongly influences their resistance to deformation.

Consistency is closely related to the water content and the void ratio of the soil. As the water content increases, the void ratio generally increases, leading to greater interparticle spacing.

$w \uparrow \Rightarrow e \uparrow \Rightarrow$ particles farther apart $\Rightarrow$ softer consistency $\Rightarrow$ soil deforms more easily.

2.3.2 States of consistency

Four states of consistency are distinguished (Figure 2.3):

a) Solid state

In the solid state, soil particles are in close contact, and the adsorbed water films are very thin. There is essentially no free water between particles. Drying does not cause further volume change. The soil exhibits high shear strength and behaves in a brittle manner, similar to a solid material.

b) Semi-solid state

In the semi-solid state, the soil contains a small amount of water. The adsorbed water films slightly separate the particles. Further drying leads to volume reduction (shrinkage), and deformation is often accompanied by cracking.

c) Plastic state

In the plastic state, the water content is higher, and the particles are more separated. The adsorbed water films are thicker and allow particles to slide relative to each other. The soil can undergo significant deformation without cracking and can be molded by hand.

d) Liquid state

In the liquid state, the water content is very high, and cohesive forces between particles become negligible. The particles are separated by free water. The soil behaves like a viscous fluid, with a consistency ranging from a thick paste to a liquid suspension.

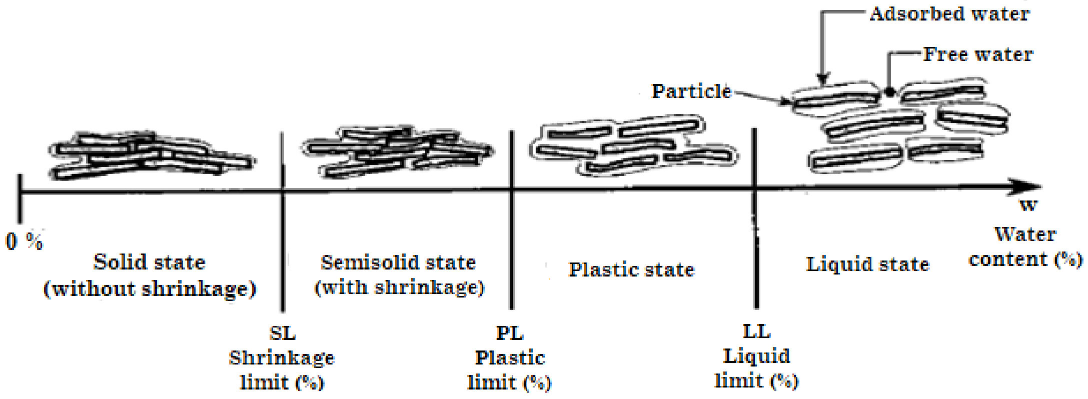


Figure 2.3 States of consistency

2.3.3 Atterberg limits

The water contents that define the different states of consistency are called the Atterberg limits or consistency limits. These limits, expressed as percentages, are:

a) **Shrinkage limit (SL):** the maximum water content at which a soil can lose water without a decrease in volume. It separates the solid state from the semi-solid state.

b) **Plastic limit (PL):** the water content at which a soil begins to lose its plastic behavior and starts to crumble when rolled into threads. It separates the semi-solid state from the plastic state.

c) **Liquid limit (LL):** the water content at which a soil changes from the plastic state to the liquid state.

The liquid limit and plastic limit are widely used for the identification and classification of fine-grained soils. The shrinkage limit is particularly useful for studying soils that undergo significant volume changes due to variations in water content, especially expansive clays rich in montmorillonite.

2.3.4 Plasticity and consistency indices

a) Plasticity index (PI):

Expressed as a percentage, it is defined as:

$$PI = LL - PL$$

It defines the range of water contents over which the soil remains in the plastic state (Figure 2.4).

b) Liquidity index (LI):

It indicates the position of the natural water content *w* relative to the plastic and liquid limits:

$$LI = \frac{w - PL}{LL - PL} = \frac{w - PL}{PI}$$

It provides an indication of whether the soil is in a solid, semi-solid, plastic, or liquid state.

c) Consistency index (CI):

It indicates the relative firmness of the soil at its natural water content and is defined by:

$$CI = \frac{LL - w}{PI} = \frac{LL - w}{LL - PL}$$

Changes in soil states as a function of water content and soil volume are shown in Figure 2.5.

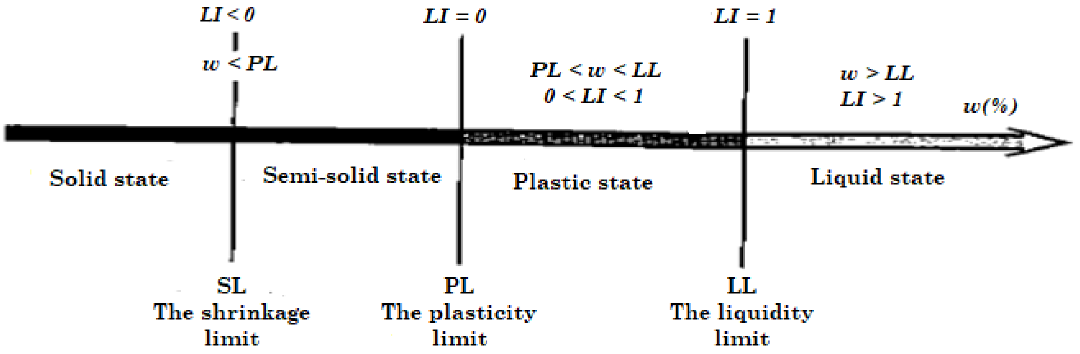


Figure 2.4 Atterberg limits and liquidity index

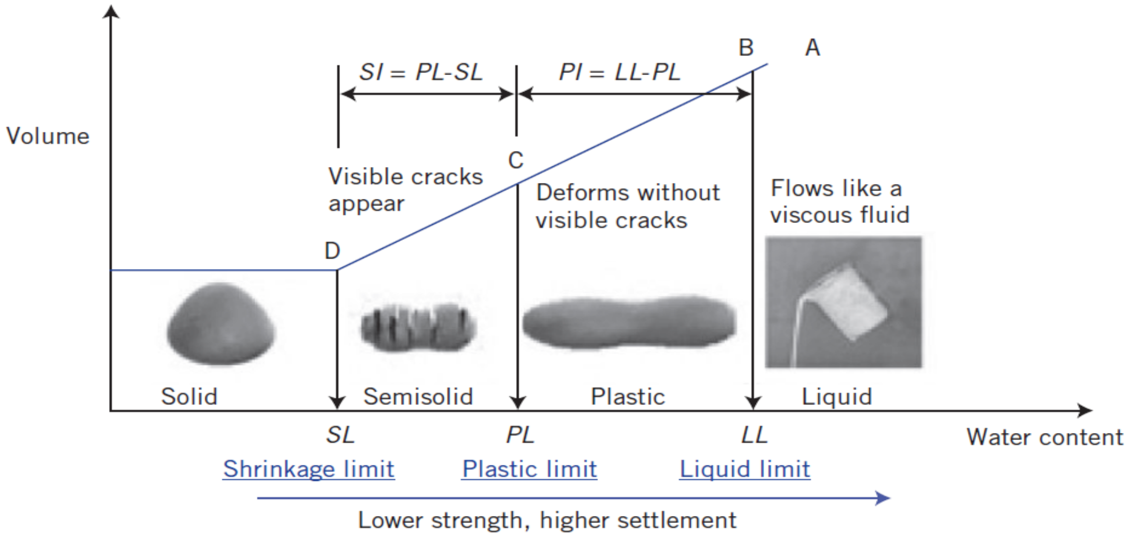


Figure 2.5 Changes in soil states as a function of water content and soil volume

2.3.5 Activity of clays

The activity of a clay, denoted *A*, is defined as the ratio of the plasticity index (PI) to the percentage of clay-sized particles in the soil:

$$A = \frac{PI}{F_{clay}}$$

where:

PI = plasticity index (%)

F_{clay} = percentage by weight of particles with diameter smaller than 0.002 mm

The clay fraction represents the mass percentage of particles finer than 0.002 mm. The activity of a soil reflects the type of clay minerals present and their ability to absorb water and develop plasticity. Typical ranges of clay activity are presented in Table 2.1. These values show that activity is not strictly constant, but depends mainly on the mineralogical composition of the clay fraction.

Table 2.1 Activity of clay-rich soils

Activity	**Description**
Inactive	< 0.75
Normal	0.75–1.25
Active	1.25–2
Very (highly) active (e.g., montmorillonite or bentonite)	> 6
Minerals	
Kaolinite	0.3–0.5
Illite	0.5–1.3
Na-montmorillonite	4–7
Ca-montmorillonite	0.5–2.0

2.3.6 Plasticity chart

In 1932, Casagrande proposed the plasticity chart, which is widely used to classify fine-grained soils based on their Atterberg limits (Figure 2.5).

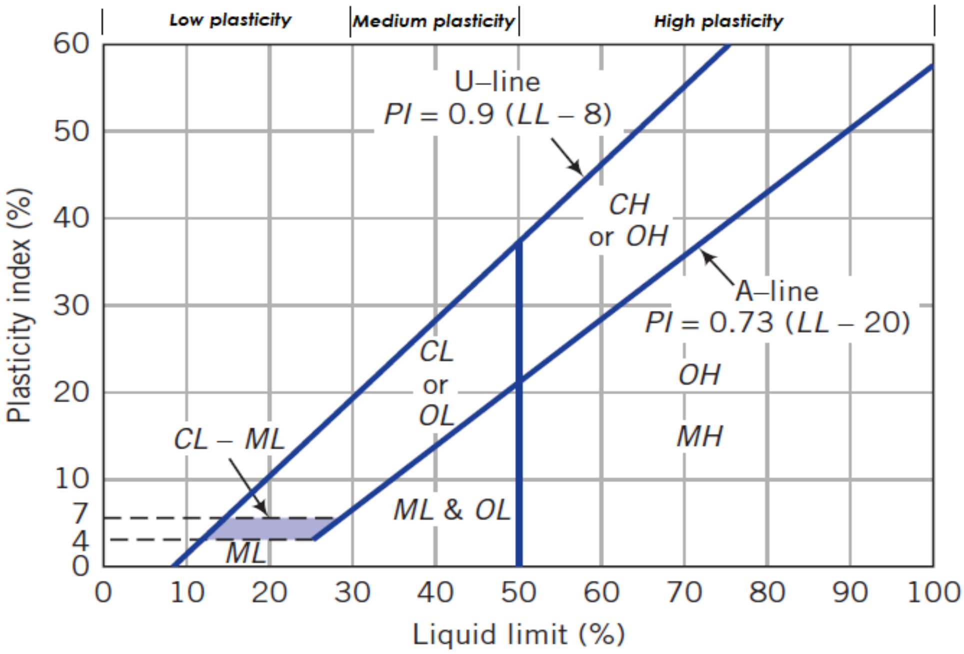


Figure 2.5 Plasticity chart

The chart is constructed with:

- Liquid Limit (LL) on the horizontal axis
- Plasticity Index (PI) on the vertical axis

The diagram is divided into two main regions by the A-line:

- Above the A-line: clays (C)

- Below the A-line: silts (M)

In addition, soils are classified according to their plasticity level:

- Low plasticity (L): LL < 50
- High plasticity (H): LL ≥ 50

The plasticity chart is a fundamental tool in the Unified Soil Classification System (USCS) for identifying and classifying fine-grained soils.

2.4 Geotechnical soil classification

2.4.1 Principles of soil classification

Soil classification systems were developed to provide civil engineers with reliable information on soil behavior, enabling them to make rapid and effective design decisions, particularly in road construction, airfields, and dam engineering. The objective of these systems is to group soils into families with similar geotechnical properties. They allow large numbers of samples collected during site investigations to be organized and used to construct geotechnical profiles of the ground. These profiles are essential for engineering design, as they complement geological data. Indeed, soils with the same geological origin may exhibit very different geotechnical properties, and conversely. However, soil classification systems cannot replace site investigations or laboratory testing, which are necessary to determine the mechanical properties of soils.

Several soil classification systems exist:

- Some are based on the suitability of soils for specific engineering applications. These systems are generally limited to the particular use for which they were developed.
- Others are based on identification tests, such as particle-size distribution and Atterberg limits. Some of these systems rely only on particle-size distribution (e.g., triangular diagrams), while others combine both particle-size distribution and plasticity.

2.4.2 Triangular classification

Triangular classification systems are represented by triangular diagrams in which the sides correspond to the proportions of sand, silt, and clay in a soil sample (Figure 2.6). The triangle is divided into zones, each corresponding to a specific soil type based on the relative proportions of these three components.

Example of application

Consider a soil composed of:

- 41% clay
- 42% sand

  - 17% silt

Although the sand fraction is slightly higher than the clay fraction, the corresponding point on the triangular diagram lies within the clay zone. Therefore, the soil is classified as clay.

This classification highlights the dominant influence of the clay matrix on soil behavior, even when it is not the largest fraction.

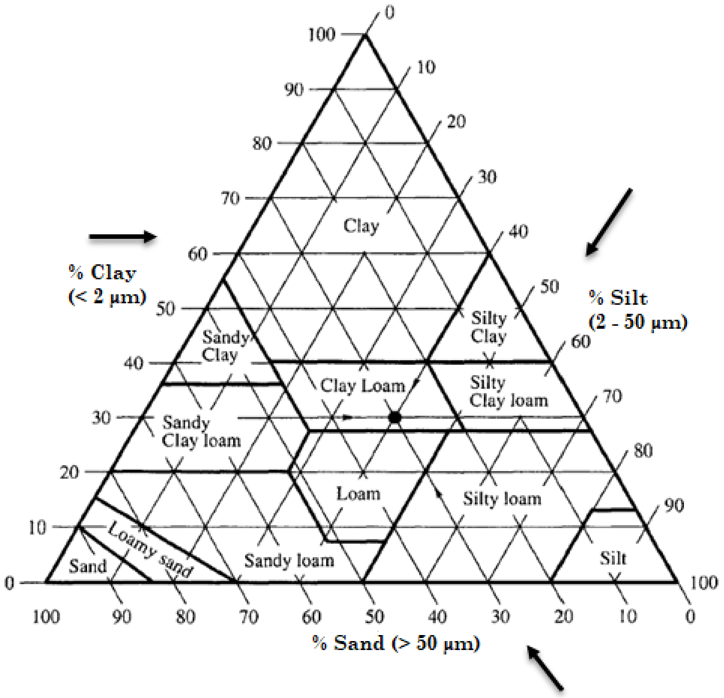


Figure 2.6 Triangular classification of fine soils (containing less than 30% of particles larger than 2 mm)

2.4.3 Classification (LPC/USCS) of soils

The LPC classification (1965) is based on the results of standard soil identification tests, including:

- Particle-size criteria:
  - percentages of gravel, sand, and fine particles determined using the 2 mm and 0.08 mm sieves;
  - shape of the particle-size distribution curve:
    - ✓ uniformity coefficient C_u,
    - ✓ coefficient of curvature C_c;
- Plasticity characteristics:
  - liquid limit (LL),
  - plasticity index (PI),
  - A-line equation (Casagrande relationship): $PI = 0.73(LL - 20)$

- Organic matter content.

Classification may also be carried out by visual examination of the soil and by simple field identification tests. However, applying this field method correctly requires considerable experience.

a) Soil groups

The LPC classification system distinguishes 15 soil types, each identified by a two-letter symbol selected from the following three groups, as shown in Table 2.3.

Table 2.3 Soil groups

Soil element	**granularity of soil**	**plasticity of soil**
G: Gravel (gravel is the main fraction) **S**: Sand (sand is the main fraction) **L**: Silt or Silty **A**: Clay or clayey **T**: Peat **O**: Organic soil (contains organic matter)	**b**: Well graded **m**: Poorly graded	**t**: Highly plastic **p**: Slightly plastic

b) Classification procedure

The overall LPC classification is described in Appendix 1. Figure 2.7 presents the classification of fine-grained soils, and Table 2.4 presents the classification of coarse-grained soils.

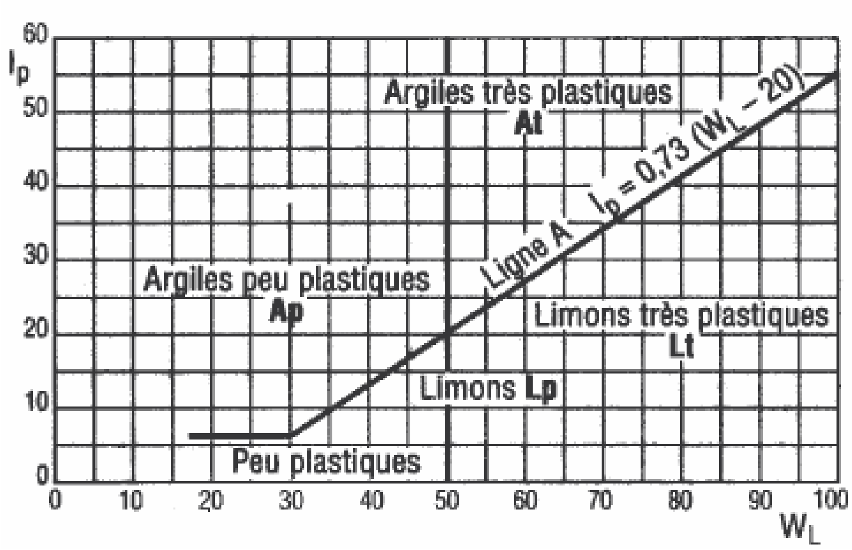


Figure 2.7 LPC classification of fine soils in the laboratory (plasticity chart)

Table 2.4 Classification of coarse-grained soils (more than 50% of particles larger than 0.08 mm)

Soil type	Conditions		LPC (USCS)	Criteria	Name
Gravel	More than 50% of particles > 0.08 mm have a diameter > 2mm	Fines < 5%	Gb (GW)	$C_u > 4$ and $1 < C_c < 3$	Well-graded clean gravel
			Gm (GP)	Criteria not satisfied	Poorly graded clean gravel
		Fines > 12%	GL (GM)	Atterberg limits below A-line	Silty gravel
			GA (GC)	Atterberg limits above A-line	Clayey Gravel
Sand	More than 50% of particles > 0.08 mm have a diameter < 2 mm	Fines < 5%	Sb (SW)	$C_u > 6$ and $1 < C_c < 3$	Well-graded clean sand
			Sm (SP)	Criteria not satisfied	Poorly graded clean sand
		Fines > 12%	SL (SM)	Atterberg limits below A-line	Silty sand
			SA (SC)	Atterberg limits above A-line	Clayey sand
Additional rule When the percentage of fines (particles < 0.08 mm) is between **5% and 12%**, a duale symbol is used (e.g., Sb-SL, SW-SM).					

2.4.4 Modified LPC classification

The modifications introduced in the LPC/USCS classification concern mainly the description of organic soils, particularly when the organic matter content exceeds 3%. Soils containing less than 10% organic matter are still classified as fine-grained soils. For higher organic contents, greater emphasis is placed on the degree of decomposition (degree of humification) of the organic material, which is evaluated using the von Post test. The modified classification distinguishes twenty-two soil types, grouped as follows:

- Coarse-grained soils:
  Gb, **Gm**, **GA**, **GL**, **Sb**, **Sm**, **SA**, **SL**
- Fine-grained soils:
  At, **Ap**, **Lt**, **Lp**
- Weakly organic soils:
  fO-At, **fO-Ap**, **fO-Lt**, **fO-Lp**
- Moderately organic soils:
  mO-a, **mO-sf**, **mO-f**
- Highly organic soils:
  tO-a, **tO-sf**, **tO-f**

The symbols used for coarse-grained and fine-grained soils remain the same as in the LPC/USCS classification. For organic soils, the suffixes:

- **a**: amorphous organic matter
- **sf**: semi-fibrous organic matter
- **f**: fibrous organic matter

describe the nature and structure of the organic material.

Appendix 2 presents the general scheme of the modified LPC classification (1974/1980).

2.5 Exercises

Exercise 01

1) Knowing the porosity of a soil, determine its void ratio and justify your answer.
2) Knowing the void ratio of a soil, determine its porosity and justify your answer.

Solution

we have:

$$n = \frac{Vv}{V} \qquad \text{and} \qquad e = \frac{Vv}{Vs}$$

were:

V_v: volume of voids

V_s: volume of solids

V: total volume

Also, $V = Vv + Vs$

1) Expression of e as a function of n

From the definition of porosity:

$$n = \frac{Vv}{V}$$

Since $V = Vv + Vs$, then:

$$1 = \frac{Vv}{V} + \frac{Vs}{V} = n + \frac{Vs}{V}$$

thus,

$$\frac{Vs}{V} = 1 - n$$

Now,

$$e = \frac{Vv}{Vs}$$

Divide numerator and denominator by V:

$$e = \frac{Vv/V}{Vs/V} = \frac{n}{1-n}$$

Therefore,

$$e = \frac{n}{1-n}$$

2) Expression of n as a function of e

Starting from:

$$e = \frac{Vv}{Vs}$$

we write:

$$V_v = eV_S$$

Since:

$$V = Vv + Vs = Vs + eV_S = Vs(1+e)$$

then:

$$n = \frac{Vv}{V} = \frac{eV_S}{Vs(1+e)}$$

Hence,

$$n = \frac{e}{1+e}$$

Exercise 02

A saturated clay sample has a mass of 1526 g. After oven drying, its mass becomes 1053 g. The specific gravity of the soil solids is $G_s = 2.7$.

Determine:

1) the water content,
2) the void ratio,
3) the porosity,

4) the wet unit weight of the soil,
5) the wet density of the soil.

Take:

$g = 9.81$ m/s², $\gamma_w = 9.81$ kN/m^3

Solution

1) Mass and weight of water

The mass of water is:

$$m_w = 1526 - 1053 = 473 \text{ g}$$

The weight of the saturated sample is:

$$W_{sat} = m_{sat}\, g = 1.526 \text{ x } 9.81 = 14.97 \text{ N}$$

The weight of the dry soil solids is:

$$W_s = m_s\, g = 1.053 \text{ x } 9.81 = 10.33 \text{ N}$$

The weight of water is therefore:

$$W_w = W_{sat} - W_s = 14.97 - 10.33 = 4.64 \text{ N}$$

2) Water content:

The water content is:

$$w = \frac{W_w}{W_s} = \frac{4.64}{10.33} = 0.499$$

Hence:

$$w = 0.449 \approx 45\ \%$$

3) Volume of water

Since the sample is saturated, the volume of voids is equal to the volume of water:

$$V_V = V_w$$

The volume of water is:

$$V_w = \frac{W_w}{\gamma_w} = \frac{4.64}{9.81 \times 10^3} = 0{,}473 \times 10^{-3}\text{m}^3 = \ 473 \text{ cm}^3$$

$$V_w = 473 \text{ cm}^3$$

4) Volume of solids

The unit weight of solids is:

$$\gamma s = G_s \gamma_w = 2.7 \times 9.81 = 26.49\ kN/m^3$$

or

$$\gamma s \approx 26.5 \times 10^3\ N/m^{\ 3}$$

The volume of solids is:

$$Vs = \frac{W_s}{\gamma_s} = \frac{10.33}{26.5 \times 10^3} = 0.390 \times 10^{-3} m^{\,3} = 390\ cm^3$$

$$Vs = 390\ cm^3$$

5) Void ratio

The void ratio is:

$$e = \frac{V_v}{V_s}$$

Since $V_V = V_w$

$$e = \frac{473}{390} = 1.21$$

Thus:

$$e = 1.21$$

6) Porosity

The porosity is:

$$n = \frac{V_v}{V} = \frac{V_v}{V_v + V_s}$$

$$n = \frac{473}{473 + 390} = \frac{473}{863} = 0.548$$

Hence:

$$n = 0.548 = 54.8\%$$

7) Wet unit weight

The wet unit weight is:

$$\gamma = \frac{W_{sat}}{V} = \frac{14.97}{0.863 \times 10^{-3}} = 17.35 \times 10^3\ N/m^3$$

Therefore:

$$\gamma = 17.35\ kN/m^3$$

8) Wet density

The wet density is:

$$\rho = \frac{m_{sat}}{V} = \frac{1.526}{0.863 \times 10^{-3}} = 1768\ kg/m^3$$

Thus:

$$\rho = 1768\ kg/m^3$$

or equivalently:

$$\rho = 1.77\ \mathrm{Mg/m^3}$$

Exercise 3

A soil sample has a mass of 129.1 g and a total volume of 56.4 cm³. The dry mass of the soil solids is 121.5 g. The specific gravity of the soil solids is $G_s = 2.7$

Determine:

1) the water content,
2) the void ratio,
3) the degree of saturation.

Take:

$g = 9.81$ m/s², $\gamma_w = 9.81$ kN/m³

Solution

1) Weight of the sample, solids, and water

The total weight of the sample is:

$$W = 0.1291 \times 9.81 = 1.2665 \text{ N}$$

The weight of the dry soil solids is:

$$W_s = 0.1215 \times 9.81 = 1.1919 \text{ N}$$

The weight of water is obtained by difference:

$$W_w = W - W_s = 1.2665 - 1.1919 = 0.0746 \text{ N}$$

2) Water content:

The water content is:

$$w = \frac{W_w}{W_s} = \frac{0.0746}{1.1919} = 0.0626$$

Hence:

$$w = 6.26\% \approx 6.3\%$$

3) Volume of solids

The unit weight of solids is:

$$\gamma s = G_s \gamma_w = 2.7 \times 9.81 = 26.49\ kN/m^3$$

or

$$\gamma s \approx 26.5 \times 10^3\ N/m^3$$

The volume of solids is:

$$Vs = \frac{W_s}{\gamma_s} = \frac{1.1919}{26.5 \times 10^3} = 4.50 \times 10^{-5}\ m^3$$

$$Vs = 45\ cm^3$$

4) Volume of voids

The total volume is given: $V = 56.4$ cm³

Hence, the volume of voids is:

$$V_v = V - V_s = 56.4 - 45 = 11.4 \text{ cm}^3$$

5) Void ratio

The void ratio is:

$$e = \frac{V_v}{V_s} = \frac{11.4}{45} = 0.253$$

Thus:

$$e = 0.253 \approx 0.25$$

6) Volume of water

The volume of water is:

$$V_w = \frac{W_w}{\gamma_w} = \frac{0.0746}{9.81 \times 10^3} = 7.60 \times 10^{-6} \text{ m}^3 = \ 473 \text{ cm}^3$$

$$V_w = 7.6 \text{ cm}^3$$

7) Degree of saturation

The degree of saturation is:

$$S = \frac{V_w}{V_v} = \frac{7.6}{11.4} = 0.667$$

Therefore:

$$S = 66.7\% \approx 67\%$$

Exercise 4

A quartz sand has a dry unit weight of γ_d =15.4 kN/m^3

Determine its saturated unit weight and saturated density.

Take:

G_s = 2.66 , g = 9.81 m/s^2 , γ_w = 9.81 kN/m^3

Solution

The unit weight of the solid particles is:

$$\gamma s = G_s \gamma_w = 2.66 \times 9.81 = 26.1 \, k\text{N/m}^3$$

For 1 m^3 of dry sand, the weight of solids is:

$$W_s = \gamma_d \times 1 = 15.4 \text{ kN}$$

Hence, the volume of solids is:

$$s = \frac{W_s}{\gamma_s} = \frac{15.40}{26.10} = 0.59 \, m^3$$

Therefore, the volume of voids is:

$$Vv = 1 - \ 0.59 \ = \ 0.41 \text{ m}^3$$

When the sand is saturated, the voids are completely filled with water. Thus, the weight of water is:

$$W_w = V_v\gamma_w = 0.41 \times \ 9.81 \ = \ 4.02 \text{ kN}$$

The total weight of 1 m^3 of saturated sand is:

$$W_{sat} = W_s + W_w = 15.4 + 4.02 \ = \ 19.42 \text{ kN}$$

Hence, the saturated unit weight is:

$$\gamma_{sat} = \frac{W_{sat}}{1} = 19.42 \text{ kN/m}^3$$

The saturated density is:

$$\rho_{sat} = \frac{\gamma_{sat}}{g} = \frac{19.42 \times 10^3}{9.81} = 1979.6 \text{ kg/m}^3$$

Thus:

$$\rho \approx 1980 \text{ kg/m}^3$$

or

$$\rho = 1.98 \text{ Mg/m}^3$$

Exercise 5

A clay sample is placed in a glass container. The total mass of the wet sample and the container is: A = 72.49 g

After oven drying, this mass becomes: B = 61.28 g

The mass of the empty container is: C = 32.54 g

The specific gravity of the soil solids is: G_s = 2.69

1) Assuming the sample is saturated, determine:
   - the water content,
   - the porosity,
   - the void ratio,
   - the wet density,
   - the dry density,
   - the submerged (buoyant) density.
2) Before oven drying, the volume of the sample is measured by immersion in mercury and found to be: V = 22.31 cm^3. Determine:
   - the actual degree of saturation,

– the corrected values of the densities.

Solution

1) Mass relationships

Mass of water:

$$m_w = A - B = 72.49 - 61.28 = 11.21\ \text{g}$$

Mass of dry soil solids:

$$m_s = B - C = 61.28 - 32.54 = 28.74\ \text{g}$$

2) Water content

$$w = \frac{m_w}{m_s} = \frac{11.21}{28.74} = 0.39$$

$$w = 39\%$$

3) Porosity (assuming saturation)

For a saturated sample:

$$V_v = V_w$$

Since $\rho_w = 1\ \text{g/cm}^3$:

$$V_w = 11.21\ \text{cm}^3$$

Volume of solids:

$$V_s = \frac{m_s}{\rho_s} = \frac{28.74}{2.69} = 10.68\ \text{cm}^3$$

Total volume:

$$V = V_v + V_s = 11.21 + 10.68 = 21.89\ \text{cm}^3$$

Porosity:

$$n = \frac{V_v}{V} = \frac{11.21}{21.89} = 0.512$$

$$n = 0.51\ (51\%)$$

4) Void ratio

$$e = \frac{V_v}{V_s} = \frac{11.21}{10.68} = 1.05$$

5) Wet density

Mass of wet sample:

$$m = A - C = 72.49 - 32.54 = 39.95\ \text{g}$$

$$\rho = \frac{m}{V} = \frac{39.95}{21.89} = 1.83\ \text{g/cm}^3$$

$$\rho = 1830\ \text{kg/cm}^3$$

6) Dry density

$$\rho_d = \frac{m_s}{V} = \frac{28.74}{21.89} = 1.31 \text{ g/cm}^3$$

$$\rho_d = 1310 \text{ kg/m}^3$$

7) Submerged (buoyant) density

The apparent (submerged) mass of solids is:

$$m' = m_s - V_s = 28.74 - 10.68 = 18.06 \text{ g}$$

Thus:

$$\rho' = \frac{18.06}{21.89} = 0.83 \text{ g/cm}^3$$

$$\rho' = 830 \text{ kg/m}^3$$

Alternatively:

$$\rho' = \rho_{sat} - \rho_w$$

Part (b): Actual state (measured volume)

Measured total volume: $V = 22.31 \text{ cm}^3$

Since the theoretical saturated volume was 21.89 cm^3, the sample is not saturated.

Volume of air:

$$V_a = 22.31 - 21.89 = 0.42 \text{ cm}^3$$

Total volume of voids:

$$V_v = V_w + V_a = 11.21 - 0.42 = 11.63 \text{ cm}^3$$

8) Degree of saturation

$$S = \frac{V_w}{V_v} = \frac{11.21}{11.63} = 0.963$$

$$S = 96.3\% \approx 96\%$$

9) Corrected densities

Wet density:

$$\rho = \frac{39.95}{22.31} = 1.79 \text{ g/cm}^3$$

$$\rho = 1790 \text{ kg/m}^3$$

Dry density:

$$\rho_d = \frac{28.74}{22.31} = 1.29 \text{ g/cm}^3$$

$$\rho_d = 1290 \text{ kg/m}^3$$

Remark: The submerged density is only defined for saturated soils, so it is not meaningful to compute it in this case.

Exercise 6

The water content and the bulk (wet) unit weight of a clay sample are: w = 25%, γ = 19 kN/m^3

After saturation, the water content becomes: w_{sat} = 28.7%. Determine:

- the specific gravity of the soil solids,
- the void ratio,
- the porosity,
- the degree of saturation (in the initial state).

Solution

1) Dry unit weight

The dry unit weight is given by:

$$\gamma_d = \frac{\gamma}{1+w} = \frac{19}{1+0.25} = 15.2\ \mathrm{kN/m^3}$$

$$\gamma_d = 15.2\ \mathrm{kN/m^3}$$

2) Relationship between unit weights

We use:

$$\gamma_d = \frac{\gamma_s}{1+e}$$

With:

$$\gamma_s = G_S\gamma_W$$

Thus:

$$\gamma_d = \frac{G_S\gamma_W}{1+e}$$

$$\Rightarrow (1+e)\gamma_d = G_S\gamma_W$$

$$\Rightarrow 15.2(1+e) = G_S 9.81 \qquad (1)$$

3) Saturation condition

The general relation is:

$$Se = wG_S$$

At saturation:

$$S = 1, \qquad w = w_{sat}$$

Thus:

$$e = w_{sat}G_S = 0.287G_S \qquad (2)$$

4) Determination of G_s

Substitute (2) into (1): $15.2(1+0.287G_S) = G_S 9.81$

$$15.2 + 4.36G_S = G_S 9.81$$

$$15.2 = 5.45G_S$$

$$G_S = 2.79 \approx 2.8$$

5) Void ratio

$$e = 0.287 \times 2.8 = 0.8$$

6) Porosity

$$n = \frac{e}{1+e} = \frac{0.8}{1+0.8} = 0.444$$

$$n = 44.4\%$$

7) Degree of saturation (initial state)

$$Se = wG_S$$

$$S = \frac{wG_S}{e} = \frac{0.25 \times 2.8}{0.8} = 0.87$$

$$S \approx 87\%$$

Exercise 7

A soil sample has a natural water content of 13% and a porosity of 38%. Its liquid limit and plastic limit are 22% and 9%, respectively. The specific gravity of the soil solids is $G_s = 2.65$. Determine the plasticity index, the water content at saturation, the degree of saturation, and the consistency of the soil in the natural and saturated states.

Solution

1) Plasticity index

The plasticity index is:

$$PI = LL - PL = 22 - 9 = 13\%$$

Thus:

$$PI = 13\%$$

2) Void ratio

Since the porosity is given as:

$$n = 38\% = 0.38$$

The void ratio is:

$$e = \frac{n}{1-n} = \frac{0.38}{1-0.38} = 0.613$$

Thus:

$$e = 0.613$$

3) Water content at saturation

The general relationship is:

$$w = \frac{Se}{G_s}$$

At saturation: $S = 1$

Therefore:

$$w_{sat} = \frac{e}{G_s} = \frac{0.613}{2.65} = 0.231$$

Hence:

$$w_{sat} = 23.1\% \approx 23\%$$

4) Degree of saturation in the natural state

Using:

$$S = \frac{wG_s}{e} = \frac{0.13 \times 2.65}{0.613} = 0.562$$

Thus:

$$S = 56.2\% \approx 56\%$$

5) Consistency in the natural state

The natural water content is: $w = 13\%$

Since:

$$LP < w < LL$$

That is:

$$9\% < 13\% < 22\%$$

the soil is in the plastic state.

6) Consistency at saturation

At saturation:

$$w_{sat} = 23.2\%$$

Since:

$$w_{sat} > LL$$

That is:

$$23.2\% > 22\%$$

the soil would be in the liquid state according to the Atterberg limits.

Exercise 8

A dry sieve analysis is performed on a 500 g sand sample. A stack of six sieves is used, with mesh openings (from top to bottom) of: 2; 1; 0.4; 0.25; 0.10; 0.08 mm. The mass retained (partial retained) on each sieve (from top to bottom) is: 12.5; 321.1; 133.3; 18.1; 14.1; 0.3 g. The mass collected in the pan is: 0 g.

1) Construct the particle-size distribution curve and determine:
    - the effective diameter D_{10},
    - the coefficient of uniformity C_u,
    - the coefficient of curvature C_c.
2) Determine the LCPC classification of this sand.

Solution

The objective is to construct the grain-size distribution curve, which represents the percentage passing (by weight) versus the sieve size on semi-logarithmic coordinates.

Key relations:

- Cumulative percentage passing:

$$\%\text{passing} = 100 - \%\text{retained (cumulative)}$$

- Cumulative percentage retained:

$$\%\text{retained} = \frac{\text{cumulative retained mass}}{\text{total sample mass}} \times 100$$

- Cumulative retained mass:

$$R_n = R_{n-1} + r_n$$

where r_n is the partial retained mass on sieve n.

Grain-size analysis Table

Seive (mm)	Partial retained (g)	Cumuative retained (g)	% Retained	% Passing
2	12.5	12.5	2.5	97.5
1	321.1	333.6	66.8	33.2
0.4	133.3	466.9	93.5	6.5
0.25	18.1	485.0	97.1	2.9
0.1	14.1	499.1	99.9	0.1
0.08	0.3	499.4	100	0
Pan	0			

Characteristic diameters

From the particle-size distribution curve (semi-log plot):

$D_{10} \approx 0.55$ mm; $D_{30} \approx 0.95$ mm; $D_{60} \approx 1.5$ mm

Coefficient of uniformity

$$C_u = \frac{D_{60}}{D_{10}} = \frac{1.5}{0.55} = 2.72$$

Coefficient of curvature

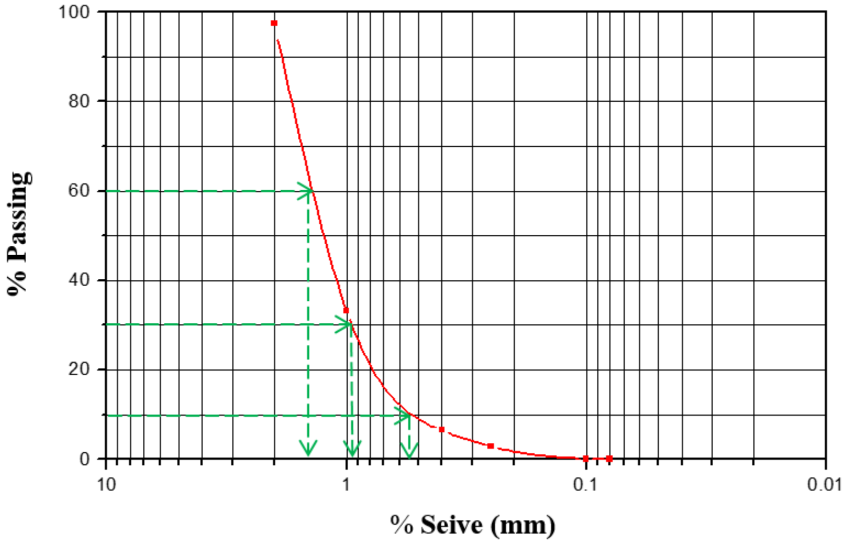
$$C_c = \frac{(D_{30})^2}{D_{10} \cdot D_{60}} = \frac{0.95^2}{0.55 \times 1.5} = 1.09$$

Classification (LCPC)

- $C_u < 6$ → poorly graded
- $1 < C_c < 3$ → acceptable curvature

Thus, according to the LCPC classification, the sand is:

Clean, poorly graded sand (**Sm**)



Chapter 3
Soil compaction

3.1 Introduction

The in-situ soil may sometimes be highly compressible, highly permeable, and of low shear strength. When selecting another construction site is not possible, soil stabilization may be required in order to improve the engineering properties of the soil.

Several methods can be used for soil improvement:

a) Chemical stabilization:

By mixing or injecting chemical agents into the soil, such as Portland cement, lime, asphalt, calcium chloride, sodium chloride, or industrial by-products (paper and pulp residues).

b) Thermal treatment:

By heating the soil to modify its physical properties.

c) Electrical treatment:

By applying an electric current to the soil (electro-osmosis).

d) Mechanical treatment:

Primarily through compaction and densification.

e) Other improvement methods:

By lowering the groundwater table to reduce pore water pressures, or by preloading and temporary surcharge loading to reduce long-term settlements.

3.2 Definitions and benefits

3.2.1 Definition of key terms

- **Compaction** consists of mechanical operations that apply energy to the soil in order to increase its density by reducing the volume of air voids.
- **Maximum dry unit weight ($\gamma_{d,max}$)** is the maximum dry unit weight that a soil can achieve under a given compaction energy.
- **Optimum water content (w_{opt})** is the water content at which a soil reaches its maximum dry unit weight for a given compaction effort.
- **Degree of compaction (D_c),** also called relative compaction, is the ratio of the field dry unit weight to the maximum dry unit weight obtained from the Proctor test.

3.2.2 Benefits of soil compaction

Proper soil compaction provides several engineering benefits:

1. Increased soil strength.
2. Increased load-bearing capacity.
3. Reduction of settlement (lower compressibility).
4. Reduction of soil permeability and water seepage.
5. Reduction of swelling and collapse potential.
6. Increased soil stability.
7. Reduction of frost damage.

Consequences of Improper Compaction

Improper compaction may lead to:

1. Structural distress caused by excessive total or differential settlements.
2. Cracking of pavements, floors, and foundations.
3. Structural damage to buried structures, water and sewer pipes, and utility conduits.
4. Soil erosion.

3.3 Compaction theory (Proctor theory)

Proctor demonstrated that soil compaction depends mainly on four parameters: the water content, the compaction energy, the type of soil (grain size distribution, mineralogy, etc.), and the compaction method.

When the water content is low, water acts as a lubricant between soil particles, facilitating their rearrangement, and the dry density increases with increasing water content (Figure 3.1). When the water content becomes too high, water occupies the voids and prevents further densification of the soil. As a result, part of the compaction energy is dissipated and the dry density decreases.

Compaction curves vary according to the nature of the soil (Figure 3.2). They are generally flatter for sandy soils, whose compaction is therefore only slightly influenced by water content. Such materials are considered the most suitable for embankment construction.

When the compaction energy increases, the maximum dry density increases while the optimum water content decreases (Figure 3.3).

Compaction curves are bounded by an envelope called the zero air voids curve, which represents the theoretical condition of complete soil saturation (Figure 3.4).

The equation of this curve is:

$$\frac{\gamma_d}{\gamma_w} = \frac{\gamma_s}{\gamma_s . w + \gamma_w}$$

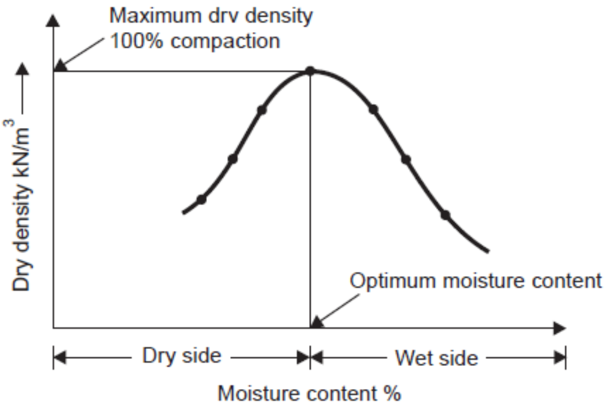


Figure 3.1 Compaction curve

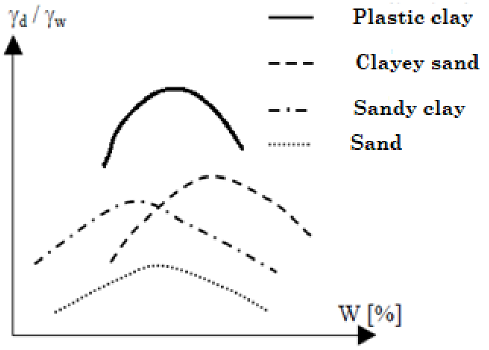


Figure 3.2 Influence of soil type

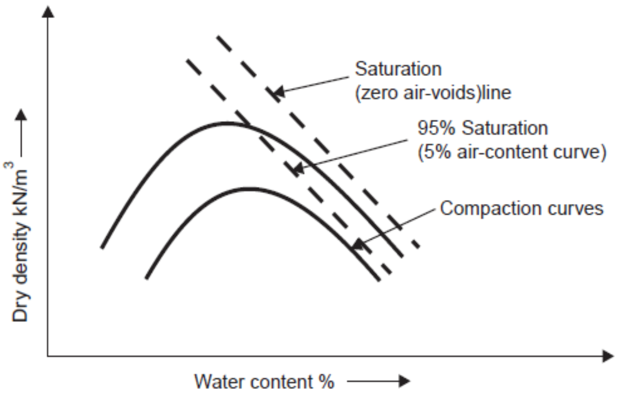


Figure 3.4 Saturation curve

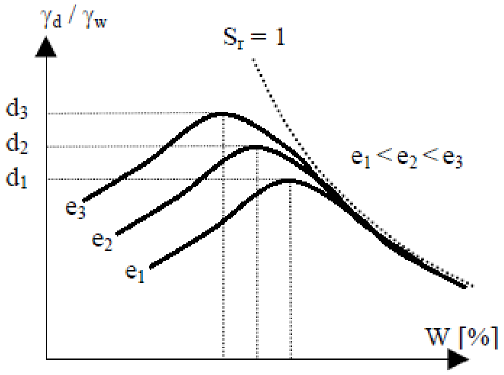


Figure 3.3 Influence of the compaction energy

3.4 Laboratory compaction tests (Proctor and CBR tests)

3.4.1 Aim and method of the Proctor test

a) Purpose

The Proctor test aims to determine the optimum water content (w_{opt}) corresponding to the maximum dry unit weight (γ_{dmax}) for a given soil under specified compaction conditions.

b) Procedure

The test consists of compacting a soil sample in a standardized mold using a standard hammer, according to a well-defined procedure. After compaction, the water content and the dry unit weight of the soil are determined.

The test is repeated several times using soil samples prepared at different water contents. Several points of the curve $\gamma_d = f(w)$ are thus obtained.

The resulting curve, called the compaction curve, exhibits a maximum. The abscissa of this maximum corresponds to the optimum water content (w_{opt}) and the ordinate corresponds to the maximum dry unit weight (γ_{dmax}) (Figure 3.5).

3.4.2 Types of molds

Two types of molds with different dimensions are used for these tests:

a) Proctor mold (Φ = 101.6 mm / H = 117 mm)

Used when the soil contains no particles larger than 5 mm.

b) CBR mold (California Bearing Ratio mold) (Φ = 152 mm / H = 152 mm)

Used for soils containing particles between 5 mm and 20 mm.

With each of these molds, it is possible to perform:

- the ***Standard Proctor test***, generally used for earthworks and dam construction,
- the ***Modified Proctor test***, mainly used for road construction and pavement layers.

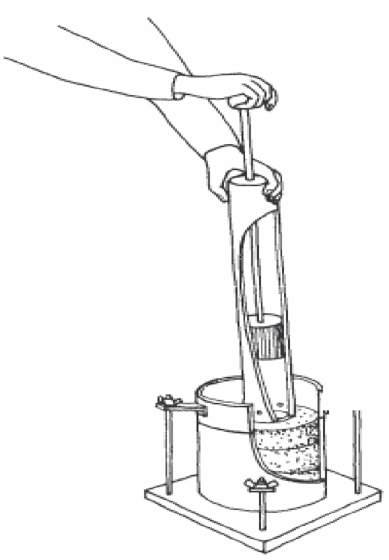


Figure 3.5 Proctor test

Table 3.1 Normal and modified Proctor test conditions

Proctor Test	Hammer Mass (Kg)	Drop height (cm)	Blows per layer	Number of layers	Compaction energy (KJ/dm^3)
Standard Proctor	2.49	30.5	25 (Proctor mold)	3	0.59
			55 (CBR mold)	3	0.53
Modified Proctor	4.54	45.7	25 (Proctor mold)	5	2.71
			55 (CBR mold)	5	2.41

3.5 In-situ compaction equipment and methods

3.5.1 Common compaction methods

In common compaction processes, the following equipment is used:

a) Vibratory compaction:

For granular or cohesionless soils, effective compaction is achieved using vibration with equipment such as:

- vibratory plates
- self-propelled vibratory rollers
- pneumatic-tired rollers
- heavy tamping devices

b) Pneumatic rammers:

Used for compacting thin soil layers, especially in confined areas.

c) Tamping rammers (jumping jacks):

Suitable for cohesive or mixed soils and small surface areas.

d) Heavy tampers mounted on cranes:

Masses of 2 to 3 tonnes dropped repeatedly to compact soil. These are used for various soil types but are generally limited to *small areas*.

e) Smooth drum rollers:

Used mainly for granular soils and non-plastic soils.

f) Pneumatic-tired rollers:

Used for irregular surfaces and mixed soils, providing kneading action.

g) Sheepsfoot rollers:

Particularly effective for cohesive soils, especially clayey soils.

h) Vibratory rollers and compactors:

Used mainly for coarse-grained soils such as sand and gravel.

3.5.2 Special compaction methods

For very thick soil layers, special compaction methods may be used.

a) Explosive compaction

This method consists of detonating explosives within the soil to generate shock waves that densify loose soils.

b) Vibroflotation (vibrocompaction)

This technique generates alternating stresses and vibrations, causing rearrangement and densification of soil particles.

Applications include:

- vibrating probes (vibroflots) used in very permeable soils
- stone columns, constructed using compacted granular materials, commonly used in cohesive soils

c) Dynamic compaction

Dynamic compaction can be applied to many soil types.

It consists of repeatedly dropping a heavy mass onto the ground surface to transmit high-energy impacts.

Typical parameters:

- falling mass: 10 to 30 tonnes (sometimes up to 140 tonnes)
- drop height: 15 to 30 m

The depth of improvement can be estimated using the expression proposed by Léonard et al. (1980):

$$D = \frac{1}{2}\sqrt{wh}$$

where:

w: falling mass (metric tonnes)

h: drop height (m)

D: depth of influence (m)

3.6 Compaction specifications and control

Despite the wide variety of compaction equipment available, several key factors influence the effectiveness of a compactor for a given soil.

These include factors related to the soil properties (soil type, water content, etc.) as well as factors related to the compaction equipment and operating conditions such as:

- number of passes
- compactor speed
- contact pressure
- vibration frequency and amplitude

Regardless of the type of equipment used, field compaction must generally be carried out in thin layers:

- 20 to 30 cm for road earthworks
- 10 to 15 cm for building earthworks

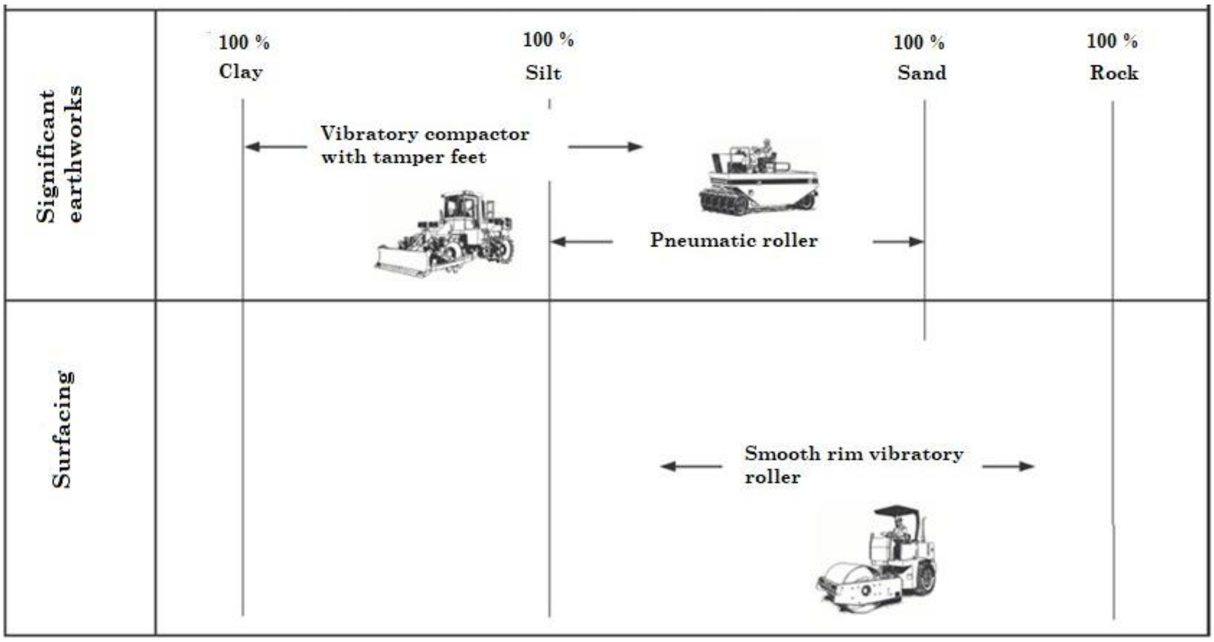


Figure 3.6 Field of application of the main compaction machines in road earthworks

3.6.1 Trial compaction section

A large part of the difficulties encountered in earthworks projects arises from the fact that soil placement (for example, in backfills) often depends on the atmospheric conditions during construction.

A trial compaction section is therefore carried out before the start of the earthworks in order to determine the appropriate compaction parameters related to:

- the compaction equipment used
- the soil properties at the time of construction (water content, etc.)
- machine operating conditions (speed, number of passes, etc.)

This procedure ensures that the required degree of compaction specified in the project is achieved.

3.6.2 Influence of compactor speed

For a given compaction machine and specified compaction requirements, there is an optimal compactor speed that depends on:

- the thickness of the compacted layer
- the nature of the soil

This speed allows the maximum dry density to be achieved. The stricter the compaction requirements, the lower the optimal compactor speed must be.

In practice, the speed of most compactors is generally limited to about 8 km/h. For vibratory compactors, the optimal speed is approximately 5 km/h so that the vibrations can effectively influence the entire thickness of the soil layer.

3.6.3 Influence of the number of passes

For a given machine and specified compaction requirements, there is also an optimal number of passes, which depends on:

- compactor speed
- layer thickness
- soil type

In general, 3 to 8 passes are required to compact a 30 cm soil layer. However, this number may increase to 12 passes depending on the soil type, water content, and compactor weight.

If the required degree of compaction is not achieved after 12 passes under optimal water content conditions, it can be concluded that the compaction equipment used is not suitable for the soil being compacted.

In practice, the water content close to the optimum water content determined from the Proctor test is obtained by properly conditioning the soil moisture.

For each compactor speed, the number of passes required to achieve the specified compaction is determined. The relationship between compactor speed and number of passes can then be represented graphically to determine the optimal operating parameters for the compaction equipment.

A comparison of different types of field compaction equipment and the soil types for which they are most suitable is presented in Tables 3.2 and 3.3.

Table 3.2 Comparison of field compactors for different soil types

Soil type	**Maximum lift thickness (mm)**	**Static pressure**	**Kneading**	**Vibration**	**Impact**	**Ease of Compaction**
Gravel	300 ±	Not applicable	Very Good	Good	Poor	Very easy
Sand	250 ±	Not applicable	Good	Excellent	Poor	Easy
Silt	150 ±	Good	Excellent	Poor	Good	Difficult
Clay	150 ±	Very Good	Good	Not suitable	Excellent	Very difficult

Table 3.3 Typical equipment associated with each mechanism

Compaction mechanism	Typical equipment
Static pressure	Smooth drum roller
Kneading	Pneumatic-tired roller, sheepsfoot roller
Vibration	Vibratory roller, vibrating plate compactor
Impact	Tamping rammer, drop hammer

3.6.4 Degree of compaction

The effectiveness of compaction in the field is evaluated by comparing the dry unit weight of the soil in situ $\gamma_{d,site}$ with the maximum dry unit weight obtained from the Proctor test $\gamma_{d,max}$. The degree of compaction D_c, also called the percentage of compaction, is defined by the following relation:

$$D_c = \frac{\gamma_{d,site}}{\gamma_{d,max}}$$

The degree of compaction is one of the main criteria used to accept or reject compaction work. It is expressed as a percentage and approaches 100% when the in-situ dry unit weight approaches the maximum dry unit weight obtained from the Proctor test.

In practice, construction specifications generally require:

- $D_c \geq 95\%$ for standard earthworks
- $D_c \geq 98\%$ for road construction

The higher the value of D_c, the greater the degree of soil compaction and the more effective the compaction process.

3.6.5 Membrane densitometer

a) Purpose

The membrane densitometer is used to determine the in-situ unit weight of soils, either:

- wet unit weight γ
- dry unit weight γ_d

after excavation or compaction of the soil.

b) Procedure

The test consists of excavating a small cavity in the soil and collecting all the excavated material in order to determine its mass. The volume of the cavity is then measured using a membrane densitometer. The device contains a flexible waterproof membrane connected to a water reservoir. Under the action of a piston, water is forced into the membrane, which expands and fills the cavity.

A graduated scale allows the volume of the cavity to be read directly, making it possible to determine the in-situ density of the soil (Figure 3.7).

Figure 3.7 Membrane densitometer

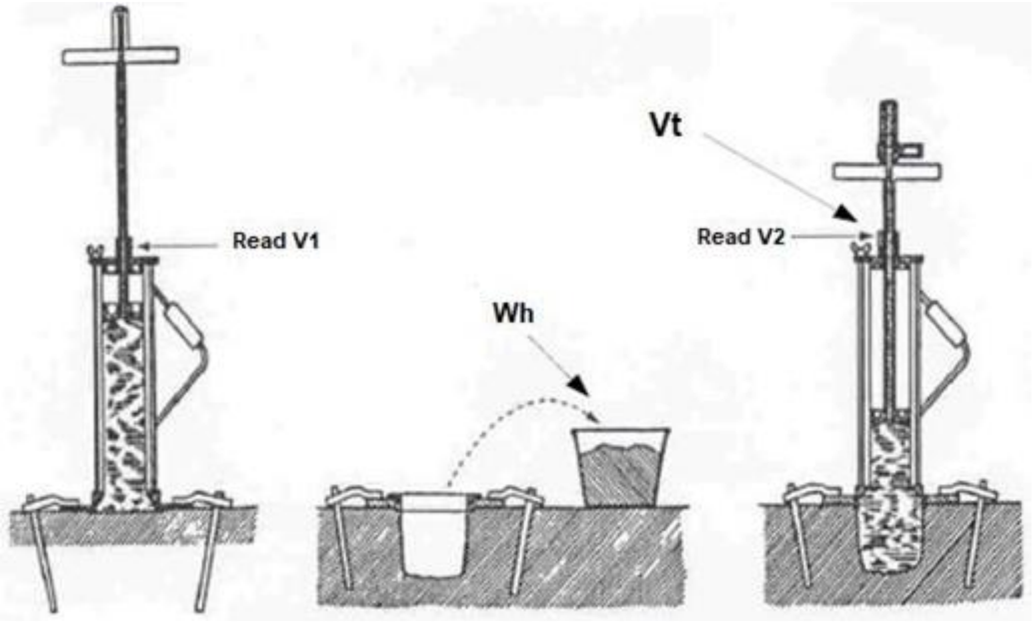


Figure 3.8 Membrane densitometer - Operating mode

3.7 Exercises

Exercise 1

The wet mass of a sample used in a Standard Proctor test is 1806 g, and its water content is 8%. The volume of the sample is 9.44×10^{-4} m^3.

Determine:

1) the wet unit weight
2) the dry unit weight

Solution

1) Determine the wet unit weight

$$m = 1806\text{ g} = 1.806\text{ kg}$$

$$W = mg = 1.806 \times 9.81 = 17.72\text{ N}$$

$$\gamma = \frac{W}{V} = \frac{17.72 \times 10^{-3}}{9.44 \times 10^{-4}} = 18.77\text{ kN/m}^3$$

2) Determine the wet unit weight

$$\gamma_d = \frac{\gamma}{1+w} = \frac{18.77}{1.08} = 17.38\text{ kN/m}^3$$

Exercise 2

Two samples (Sample 1 and Sample 2) of the same soil were compacted to the same dry unit weight, γ_d =19.6 kN/m^3, but at different water contents:

- w_1 = 4% for Sample 1
- w_2 = 12% for Sample 2

The unit weight of solids is γ_s = 27 kN/m^3.

Assume the unit weight of water is γ_w = 10 kN/m^3.

Determine:

1) the points corresponding to the two samples on the compaction curve (γ_d , w)
2) for each sample, the degree of saturation and the wet unit weight
3) if Sample 1 is brought to full saturation without any change in volume, and its total volume is 243 cm^3, determine the volume of water to be added
4) plot the zero-air-voids curve (or saturation curve)

Solution

1) Compaction curve

Only two points can be plotted on the compaction diagram:

- Sample 1 (w_1 = 4%, γ_d =19.6 kN/m^3)

- Sample 1 (w_2 = 12%, γ_d =19.6 kN/m^3)

2) Degree of saturation and wet unit weight

Using:

$$S = \frac{w}{\left(\frac{\gamma_w}{\gamma_d} - \frac{\gamma_w}{\gamma_s}\right)}$$

For sample 1:

$$S_1 = \frac{0.04}{\left(\frac{10}{19.6} - \frac{10}{27}\right)} = 28.5\%$$

$$\gamma_1 = \gamma_d(1 + w_1) = 19.6(1 + 0.04) = 20.4\ \mathrm{kN/m^3}$$

For sample 2:

$$S_1 = \frac{0.12}{\left(\frac{10}{19.6} - \frac{10}{27}\right)} = 85.4\%$$

$$\gamma_1 = \gamma_d(1 + w_1) = 19.6(1 + 0.12) = 22\ \mathrm{kN/m^3}$$

3) Water volume required to bring Sample 1 to saturation

$$w_{sat} = \left(\frac{\gamma_s}{\gamma_d} - 1\right)\frac{\gamma_w}{\gamma_s}$$

$$w_{sat} = \left(\frac{27}{19.6} - 1\right)\frac{10}{27} = 0.1398 = 13.98\%$$

$$\Delta w = w_{sat} - w_1 = 13.98\% - 4\% = 9.98\% = 0.0998$$

$$W_s = \gamma_d V = 19.6 \times 243 \times 10^{-6} = 4.7628 \times 10^{-3}\ \mathrm{kN}$$

$$\Delta W_w = W_s \Delta w = 4.7628 \times 10^{-3} \times 0.0998 = 4.753 \times 10^{-4}\ \mathrm{kN}$$

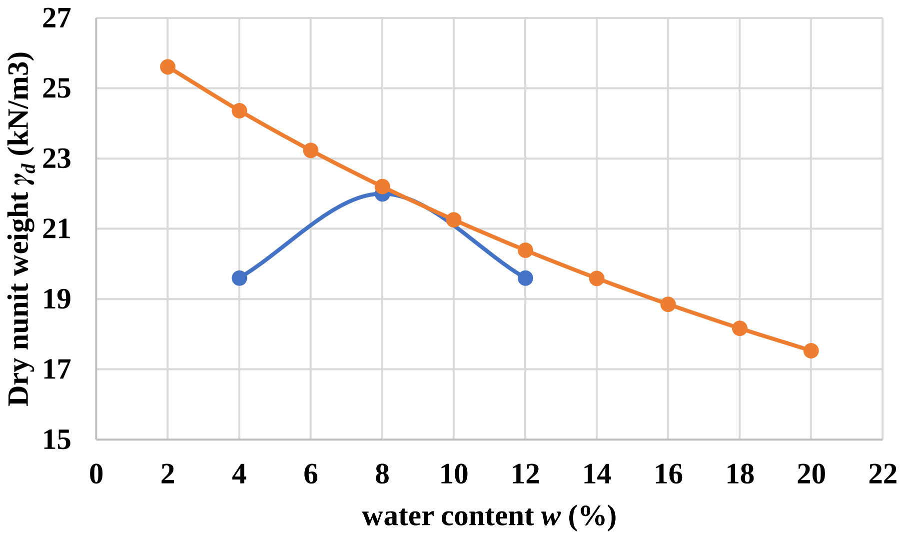
$$\Delta V_w = \frac{\Delta W_w}{\gamma_w} = \frac{4.753 \times 10^{-4}}{10} = 4.753 \times 10^{-5}\ \mathrm{m^3}$$

$$\Delta V_w = 47.53\ \mathrm{cm^3}$$

4) Saturation curve

The saturation or zero-air-voids curve may be plotted using the equation:

$$\frac{\gamma_d}{\gamma_w} = \frac{\gamma_s}{\gamma_s w + \gamma_w}$$

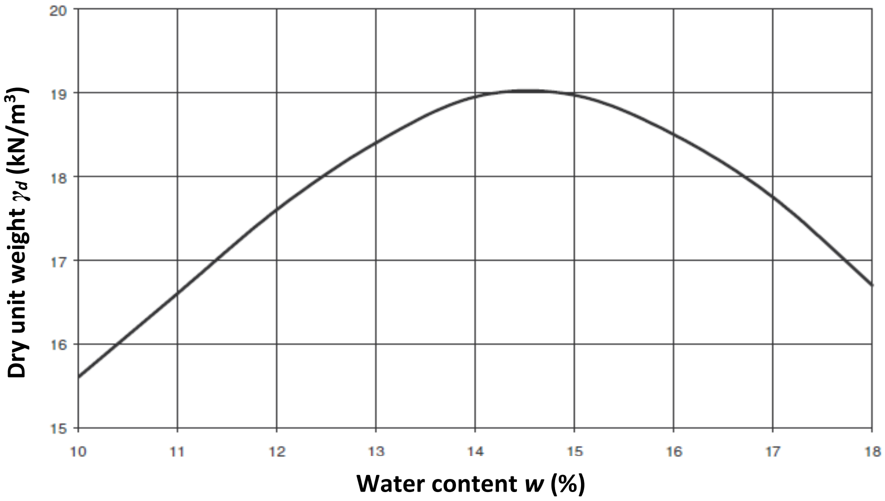


Exercise 3

To determine the compaction conditions of a sandy clay intended for use in a road embankment, Standard Proctor tests were carried out, making it possible to establish the compaction curve $\gamma_d = f(w)$.

Determine:

1) the optimum water content to be adopted for compaction.
2) the material has a total unit weight γ = 18.7 kN/m^3 and a dry unit weight γ_d =17 kN/m^3. Determine the volume of water to be added per cubic meter of material in order to reach the Standard Proctor optimum condition.



Solution

1) Optimum Proctor values

From the Proctor curve:

$$w_{opt} = 14.5\% \qquad ; \qquad \gamma_{dmax} = 19\ \text{kN/m}^3$$

2) Water volume to be added per cubic meter

Initial water content:

$$w = \frac{\gamma}{\gamma_d} - 1 = \frac{18.7}{17} - 1 = 0.10 = 10\%$$

For 1 m^3 of material:

$$W_s = \gamma_d \times 1 = 17 \text{ kN}$$

Additional water content required:

$$\Delta w = w_{opt} - w = 14.5\% - 10\% = 4.5\% = 0.045$$

$$\Delta W_w = W_s \Delta w = 17 \times 0.045 = 0.765 \text{ kN}$$

$$\Delta V_w = \frac{\Delta W_w}{\gamma_w} = \frac{0.765}{10} = 0.0765 \text{ m}^3$$

$$\Delta V_w = 76.5 \text{ L/m}^3$$

Chapter 4
Water in the soils

4.1 Introduction – Definitions

4.1.1 Groundwater

Groundwater is the water occupying the voids of soils and rocks in the saturated zone below the groundwater table. In geotechnical engineering and hydrogeology, the following terms are commonly used:

- **Aquifer:** a permeable geological formation capable of storing and transmitting appreciable quantities of groundwater.
- **Aquitard:** a low-permeability formation that transmits groundwater very slowly.
- **Water table (groundwater table):** the upper surface of the saturated zone in an unconfined aquifer.
- **Unconfined aquifer:** an aquifer whose upper boundary is the water table; at this surface, pore water pressure is atmospheric, that is, gauge pore pressure is zero.
- **Confined aquifer:** an aquifer bounded above, and usually below, by low-permeability layers.
- **Artesian condition:** a condition in a confined aquifer where the piezometric level rises above the top of the aquifer; if it rises above ground level, the aquifer is said to be flowing artesian.

4.1.2 Soil hydraulics

Water in soil may exist in several forms:

- **Constitutional (or structural) water:** water that is part of the chemical structure of the minerals.
- **Adsorbed water:** water held at the surface of soil particles in the form of thin films, especially in fine-grained soils.
- **Free water:** water occupying the void spaces between particles and capable of moving through the soil under hydraulic gradients.

In this chapter, soil hydraulics is limited to the study of:

- free water in soils,
- flow through fully saturated soils,

- mainly steady-state seepage conditions.

To analyze water flow through saturated soils, the following assumptions are commonly adopted:

- the pore water is incompressible,
- the solid particles are incompressible,
- the law of conservation of mass applies.

For a control volume of saturated soil under steady flow conditions, the rate of inflow is equal to the rate of outflow; therefore, no accumulation of water occurs within the volume. For an incompressible fluid, this continuity condition may be written as:

$$\nabla . \vec{v} = 0$$

or

$$\frac{\partial v_x}{\partial x} + \frac{\partial v_y}{\partial y} + \frac{\partial v_z}{\partial z} = 0$$

where $\vec{v} = \left(v_x\,,\ v_y,\ v_z\right)$ is the seepage velocity field (Appendix 3).

Figure 4.1 Continuity condition

The total stress σ, effective stress σ', and pore water pressure u are related by Terzaghi's effective stress principle:

$$\sigma = \sigma^{'} + u$$

for normal stresses, while the shear stress is unaffected directly by pore water pressure, so that:

$$\tau = \tau^{'}$$

4.2 Flow of water through soils (Darcy flow)

Consider a cylindrical soil sample of cross-sectional area *S*, subjected to water flow from point M to point N (Figure 4.2).

4.2.1 Velocity of water in soil

Let *Q* be the discharge through the sample.

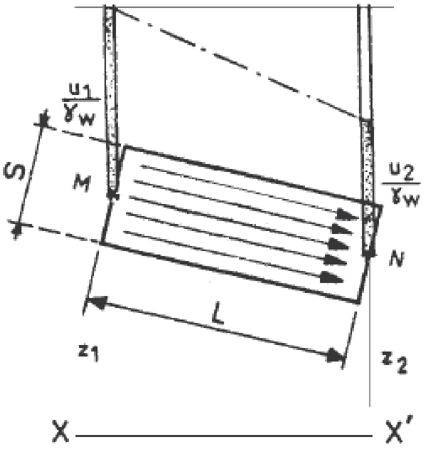
The Darcy velocity (or discharge velocity) is defined as:

$$v = \frac{Q}{S}$$

This velocity is a fictitious average velocity since water flows only through the void space of area *nS*, where n is the porosity.

The seepage velocity (actual velocity through pores) is:

$$v_s = \frac{Q}{nS}$$



Notations

u_M : pore water pressure at point M

u_N : pore water pressure at point N

z_M : elevation of point M

zN: elevation of point N

v : water velocity

L : length of the soil sample between M and N

Figure 4.2 Flow in a tube

4.2.2 Hydraulic head

The hydraulic head at a point M is defined as:

$$h_M = \frac{u_M}{\gamma_w} + z_M + \frac{v^2}{2g}$$

where:

u_M: pore water pressure

z_M: elevation

γ_w: unit weight of water

In soils, flow velocities are low; therefore, the kinetic term is negligible:

$$h_M = \frac{u_M}{\gamma_w} + z_M$$

4.2.3 Hydraulic gradient

The hydraulic gradient between two points A and B is:

$$i = \frac{h_A - h_B}{L}$$

In differential form:

$$i = -\frac{dh}{dl}$$

In vector form:

$$\vec{i} = -\nabla h$$

If $\vec{i} = 0$, the system is in hydrostatic equilibrium.

The hydraulic gradient represents the loss of head per unit length due to friction between flowing water and the soil skeleton.

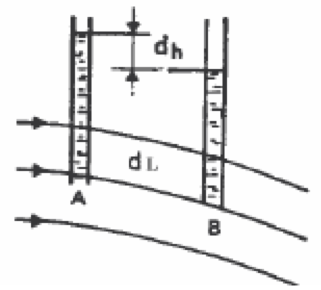


Figure 4.3 Definition of the hydraulic gradient

4.2.4 Seepage force

The hydraulic gradient induces a seepage force per unit volume:

$$f = i\gamma_w$$

This force acts in the direction of flow and may lead to engineering problems such as piping, uplift, and slope instability.

4.2.5 Darcy's law

Darcy's law is the fundamental law governing the flow of water through saturated soils. First established experimentally by Darcy in 1856, it expresses the proportionality between flow velocity and hydraulic gradient.

$$\vec{v} = K.\vec{i}$$

where:

$\vec{v}$: Darcy (discharge) velocity

$\vec{i}$: hydraulic gradient

K: coefficient of permeability (hydraulic conductivity)

Darcy's law is empirical and is valid provided that the flow remains laminar, which is generally the case for water flow through soils.

The coefficient of permeability K depends on the nature of the soil and the properties of the fluid. It is expressed in m/s (or cm/s in geotechnical practice).

The discharge through a soil sample of cross-sectional area S is given by:

$$Q = KiS$$

4.2.6 Equipotential surfaces

In the case of hydrostatic conditions (no flow), the hydraulic head is the same at every point in the soil.

When flow occurs, water moves along flow lines. The surfaces connecting points having the same hydraulic head are called *equipotential surfaces*.

Equipotential surfaces have the following properties:

- they represent surfaces of constant hydraulic head,
- they are everywhere perpendicular to the flow lines,
- no flow occurs along an equipotential surface.

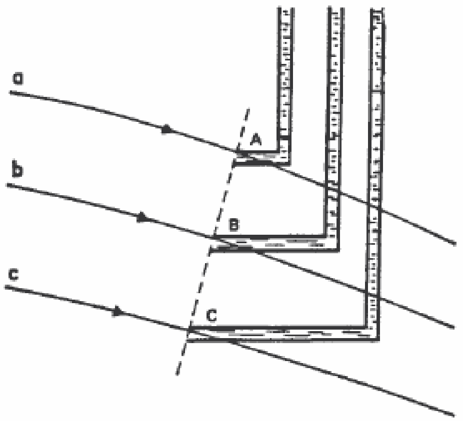


Fig. 4.4. Seepage flow with flow lines and equipotential surfaces

4.3 Soil permeability

Consider a cylindrical soil sample of length L and cross-sectional area S, subjected to a head difference $\Delta h = h_1 - h_2$.

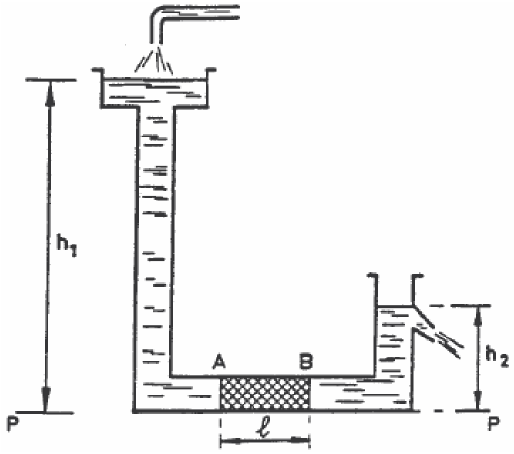


Figure 4.5 One-dimensional flow through a cylindrical soil sample

Experimental observations show that the discharge Q is given by:

$$Q = KS\frac{h_1 - h_2}{L} = KS\frac{\Delta h}{L}$$

where:

S: cross-sectional area of the sample

L: length of the sample

h_1, h_2: hydraulic heads at points A and B

Δh: head loss

K: coefficient of permeability

4.4 Laboratory measurement of permeability

The coefficient of permeability K of a saturated soil depends on its grain size distribution, structure, void ratio, and temperature. The most common laboratory device used to determine permeability is the permeameter. It consists of a cylindrical soil sample of length L and cross-sectional area S, connected at both ends to water reservoirs through porous stones.

Two methods based on Darcy's law are commonly used:

- Constant head test (for coarse-grained soils)
- Falling head test (for fine-grained soils)

4.4.1 Constant head permeameter

The water level in the reservoir is maintained constant (Figure 4.6). The test consists of measuring the quantity of water Q that flows through the soil sample during a time interval t.

Taking the reference plane at the outlet level:

- At point A : $h_A = \frac{u_A}{\gamma_w} + z_A = H - L = h$
- At point B : $h_B = \frac{u_B}{\gamma_w} + z_B = 0$
- Head loss : $h_A - h_B = h$
- Hydraulic gradient : $i = \frac{h}{L}$

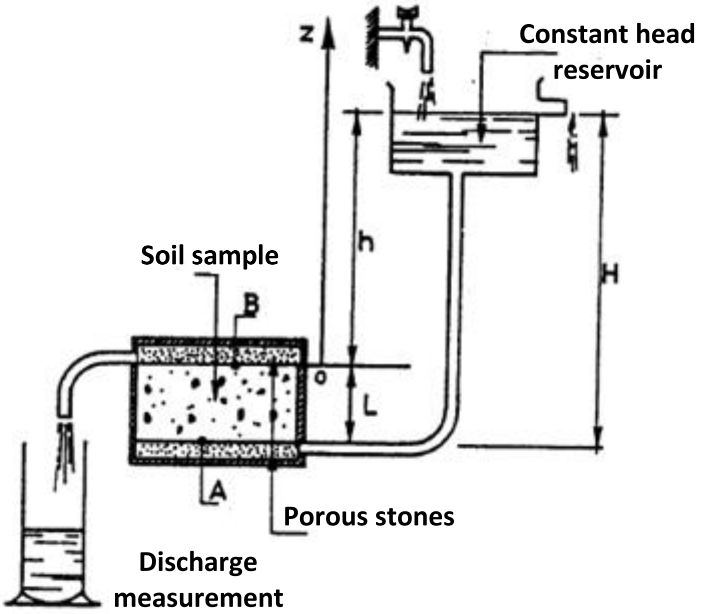


Figure 4.6 Constant head permeameter

According to Darcy’s law:

$$Q = KiSt = K\frac{h}{L}St$$

From which:

$$K = \frac{QL}{hSt}$$

4.4.2 Falling head permeameter

When the permeability is low, the constant head test becomes impractical because the flow rate is very small. In such cases, a falling head permeameter is used. Water flows from a standpipe of cross-sectional area s connected to the soil sample, and the water level in the tube drops from h_1 to h_2 during a measured time interval.

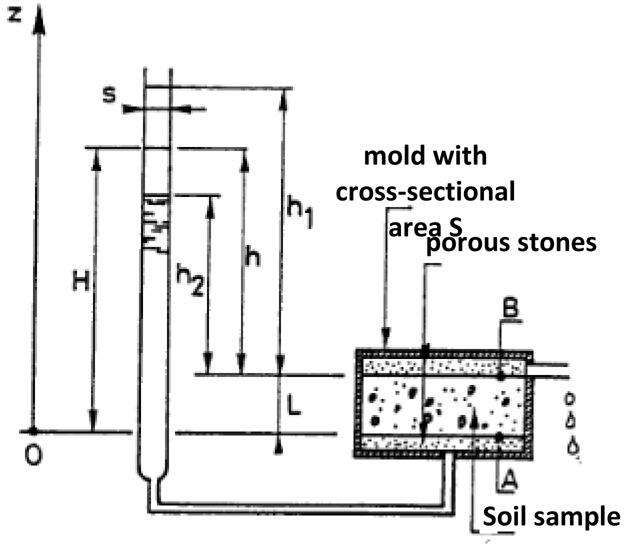


Figure 4.7 Falling head permeameter

Taking the reference plane at the inlet level:

- At point A: $h_A = \frac{u_A}{\gamma_w} + z_A = H + 0$
- At point B: $h_B = \frac{u_B}{\gamma_w} + z_B = 0 + L$
- Head loss: $h_A - h_B = H - L = h$
- Hydraulic gradient : $i = \frac{h}{L}$

At time $t = t_1$, the water height in the tube is h_1.

At time $t = t_2$, the height becomes h_2.

During a small time interval *dt*, the quantity of water that flows is:

$$Q = -sdh$$

According to Darcy's law, this same quantity is:

$$Q = vSdt = KiSdt$$

Hence:

$$Q = KSdt\frac{h}{L}$$

Equating the two expressions gives:

$$K.dt = -\frac{s.L}{S}.\frac{dh}{h}$$

Integrating between t_1 and t_2, and h_1 and h_2:

$$K\int_{t_1}^{t_2} dt = -\frac{s}{S}L\int_{h_1}^{h_2}\frac{dh}{h}$$

Which gives:

$$K = \frac{sL}{S}\frac{\ln(h_1/h_2)}{t_2 - t_1}$$

or

$$K = 2.3\frac{sL}{S}\frac{log(h_1/h_2)}{t_2 - t_1}$$

The table below provides some characteristics corresponding to various values of *K*.

Soil type	**Gravel**	**Sand**	**Silt**	**Clay**
K (m/s)	10^{0} 10^{-3}	10^{-6}	10^{-9}	10^{-11}
Laboratory measurement method	Constant head permeameter		Falling head permeameter	

Note:

- 10^{-6} cm/s corresponds to a flow velocity of approximately 30 cm/year.
- The permeability of uniformly graded sands, where $C_u \leq 5 \text{ to } 10$ can be estimated using Hazen's empirical formula:

$$K = 100\,D_{10}^2$$

- As mentioned earlier, for both types of permeameters, the total head loss across the sample is equal to the difference in hydraulic head between the inlet and the outlet.

4.5 In situ measurement

Laboratory measurements of permeability may not fully represent field conditions due to sample disturbance, small sample size, and soil heterogeneity. In many cases, permeability values measured in the laboratory are lower than those obtained in situ. Several field methods exist for measuring permeability, including the Lefranc test and other methods based on Dupuit assumptions.

4.6 Permeability of stratified soils

Many sedimentary soils are composed of superimposed layers with different grain sizes and, consequently, different permeabilities. Permeability is one of the soil properties most sensitive to anisotropy.

Consider a stratified soil deposit of total thickness H, composed of n horizontal layers having thicknesses H_i and permeabilities K_i. An equivalent homogeneous soil can be defined such that, under the same head loss conditions, it would allow the same flow rate.

4.6.1 Case of flow parallel to the stratification plane

Let K_h be the permeability coefficient of the equivalent homogeneous soil.

By expressing that:

- the head loss is the same across all layers, and therefore the hydraulic gradient i is the same in each layer,
- the total flow rate is equal to the sum of the flow rates through each layer,

it can be shown that:

$$K_h = \frac{1}{H} \cdot \sum_{i=1}^{i=n} K_i \cdot H_i$$

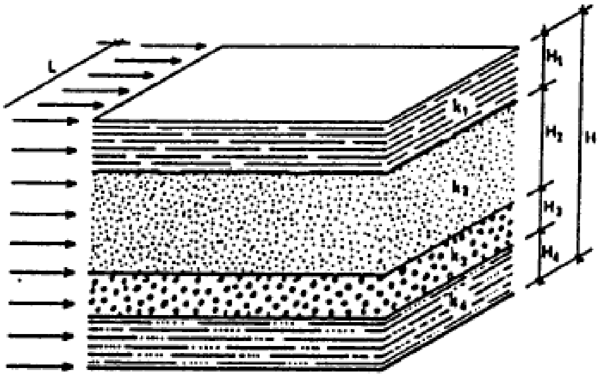


Figure 4.8 Average horizontal permeability of stratified soils

4.6.2 Case of flow perpendicular to the stratification plane

Let K_v be the permeability coefficient of the equivalent homogeneous soil.

By expressing that:

- the total head loss is equal to the sum of the head losses across each layer,
- the flow rate is the same through all layers, and therefore the discharge velocity is the same,

it can be shown that:

$$\frac{1}{K_v} = \frac{1}{H} \cdot \sum_{i=1}^{i=n} \frac{H_i}{K_i}$$

or, equivalently

$$K_v = \frac{H}{\sum_{i=1}^{i=n} \frac{H_i}{K_i}}$$

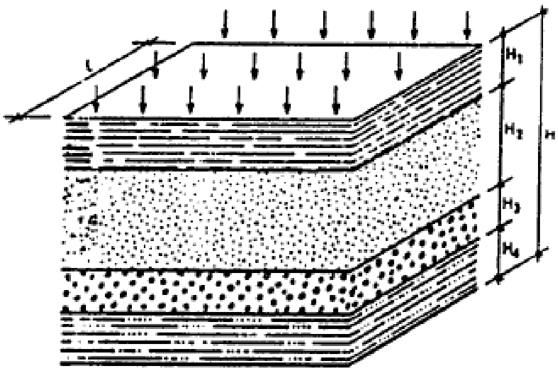


Figure 4.9 Average vertical permeability of stratified soils

4.6.3 Equivalent permeability coefficient

In the case of two-dimensional flow involving both horizontal and vertical seepage, the following equivalent permeability may be defined:

$$K_{equivalent} = \sqrt{K_h K_v}$$

Note:

- The permeability of the equivalent homogeneous soil is generally greater in the direction parallel to the layers than in the direction perpendicular to them. In stratified soils, permeability is therefore usually higher parallel to the stratification than perpendicular to it.
- The direction of flow in sedimentary soils is important. Since successive deposits are generally horizontal, water usually flows more easily horizontally than vertically.

4.7 Seepage under civil engineering structures

4.7.1 Laplace equation

Consider a saturated soil subjected to steady flow. By combining the continuity equation and Darcy's law, we obtain:

$$\frac{\partial V_x}{\partial x}+\frac{\partial V_y}{\partial y}+\frac{\partial V_z}{\partial z}=0$$

$$V_x = -K\frac{\partial h}{\partial x}, \quad V_y = -K\frac{\partial h}{\partial y}, \quad V_z = -K\frac{\partial h}{\partial z}$$

Combining these relations leads to the Laplace equation:

$$\frac{\partial^2 h}{\partial x^2}+\frac{\partial^2 h}{\partial y^2}+\frac{\partial^2 h}{\partial z^2}=0$$

or

$$\nabla^2 h = 0$$

Note:

This equation governs the distribution of hydraulic head in the soil. It is valid under the assumptions of steady flow in a saturated, homogeneous, and isotropic medium.

4.7.2 Two-dimensional flow in homogeneous and isotropic soil

Many practical seepage problems can be approximated as two-dimensional.

In this case, the Laplace equation becomes:

$$\frac{\partial^2 h}{\partial x^2}+\frac{\partial^2 h}{\partial y^2}=0$$

or

$$\nabla^2 h = 0$$

Several methods can be used to solve this equation:

- numerical method,
- analog method (electrical analogy),
- graphical method (flow net).

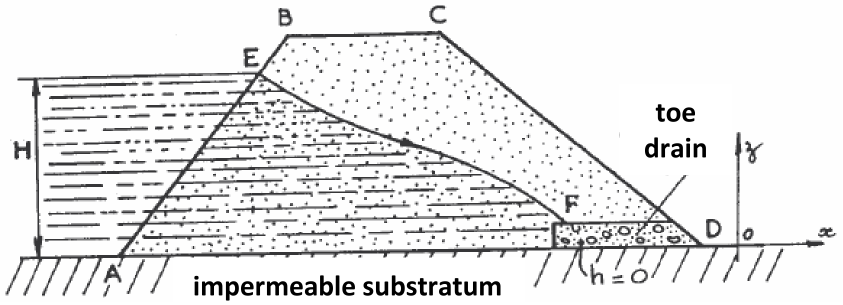


Figure 4.10 Two-dimensional seepage flow beneath a dam

Definitions

- **Streamline:** a curve that is everywhere tangent to the velocity vector of the flow.
- **Equipotential line:** a curve connecting points of equal hydraulic head, defined by:
  h = constant
- **Orthogonality:** in isotropic soils, streamlines and equipotential lines intersect at right angles.
- **Flow tube:** the region bounded by two adjacent streamlines.

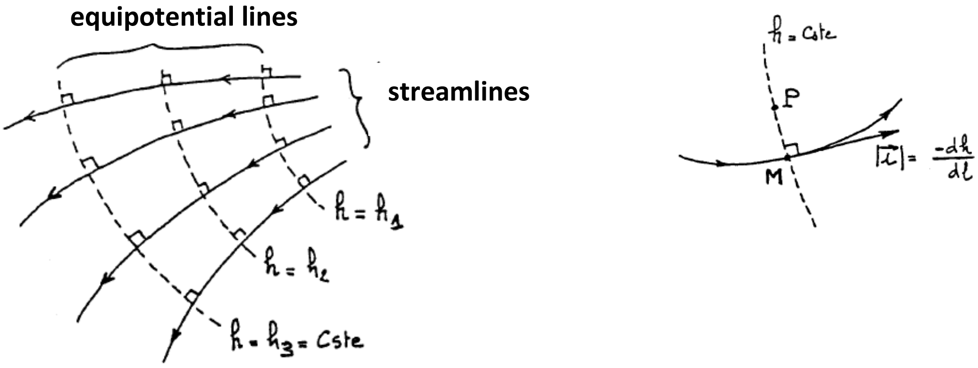


Figure 4.11 Streamlines and equipotential lines in two-dimensional flow

4.7.3 Graphical construction of a two-dimensional flow net

Flow nets are used to solve two common practical problems in soil mechanics:

- calculation of seepage discharge (e.g., dams, excavation dewatering),
- evaluation of pore water pressures for stability analysis (slopes, earth dams, retaining walls, sheet piles, etc.).

Consider the flow net under a sheet pile wall (Figure 4.12).

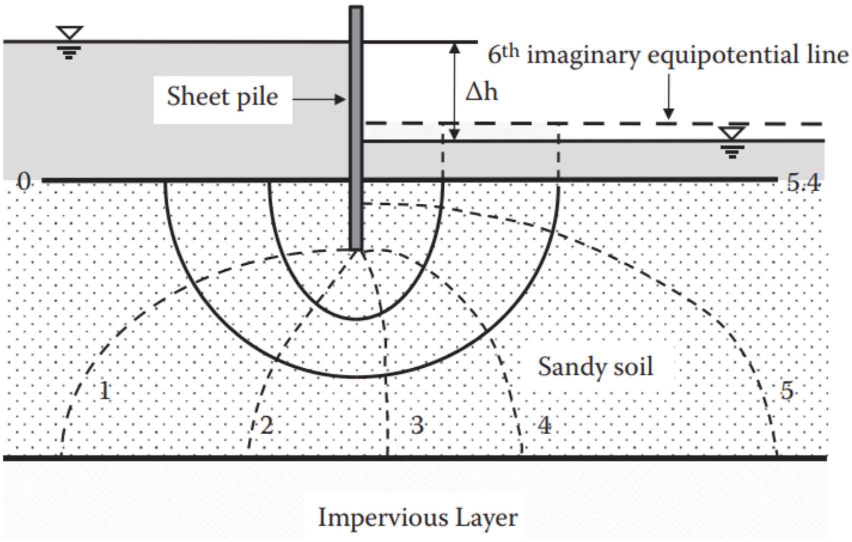


Figure 4.12 Two-dimensional flow net under a sheet pile wall

To represent a solution of the Laplace equation graphically, the following conditions must be satisfied:

- flow lines and equipotential lines must intersect at right angles and form approximate curvilinear squares,
- the head loss between successive equipotential lines is constant.

Note

In practice, the curvilinear shapes are not perfect squares. However, they can be considered acceptable when the average width *b* is approximately equal to the average height *a*, or when a circle can be inscribed within the element.

Construction steps

1. The cross-section of the structure and soil is drawn to scale. The scale must be sufficiently large for accuracy but not excessively large (Figure 4.13a).
2. Boundary conditions are identified (Figure 4.13a):
    - impermeable boundaries act as flow lines,
    - constant-head boundaries act as equipotential lines.

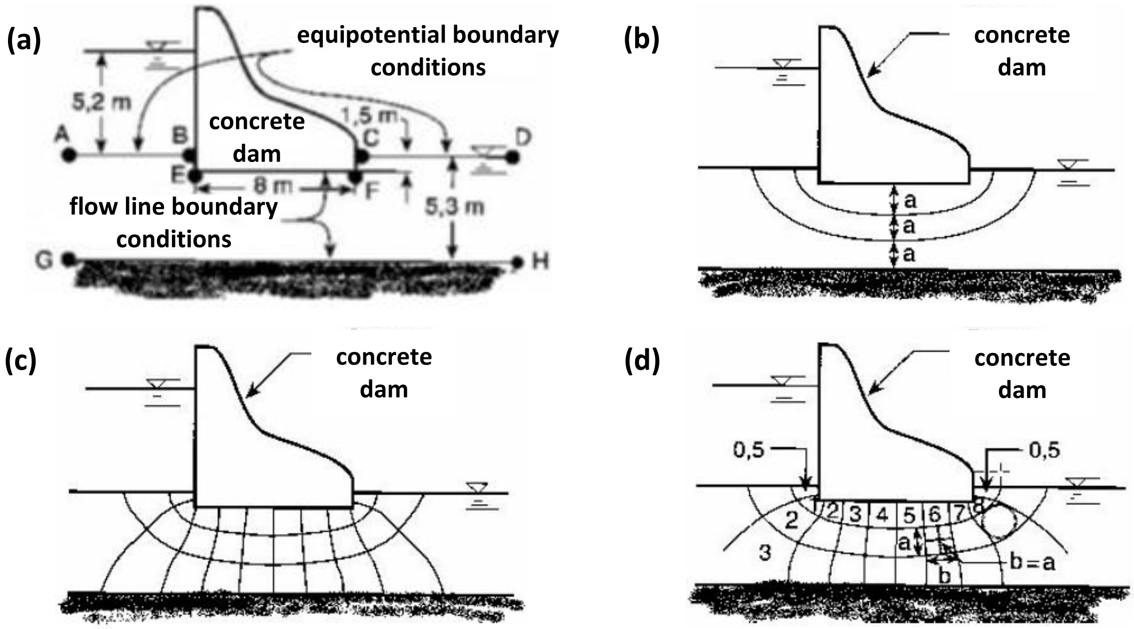


Figure 4.13 Two-dimensional flow net under a concrete dam

3. A sufficient number of flow lines are drawn to form typically three to six flow channels. In regions where flow is nearly one-dimensional, flow lines can be spaced uniformly (Figure 4.13b).
4. Equipotential lines are drawn so that they intersect flow lines at right angles, forming approximate curvilinear squares. Adjustments may be made to improve the overall flow net (Figure 4.13c).

5. The accuracy of the flow net is checked by verifying (Figure 4.13d):
    - orthogonality of lines,
    - approximate square shape of elements.

Once completed, the number of flow channels N_c and the number of potential drops N_h are determined.

4.7.4 Graphical solution using flow nets

Consider the flow net under a sheet pile wall (Figure 4.14).

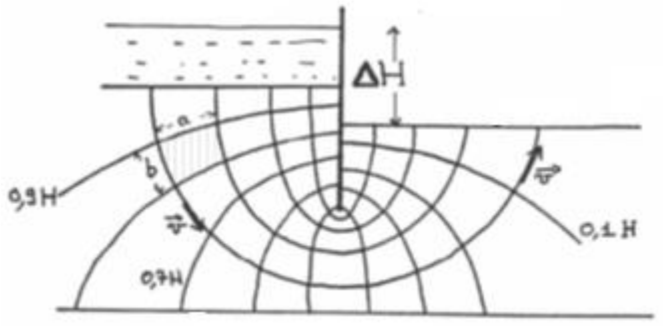


Figure 4.14 Seepage under a sheet pile wall

Note:

- Boundaries in contact with water (such as the excavation bottom and riverbed) act as equipotential boundaries (constant head),
- Impermeable boundaries (such as the sheet pile and impermeable base) act as flow lines,
- Flow lines are perpendicular to equipotential boundaries, and equipotential lines are perpendicular to impermeable boundaries,
- The head loss between successive equipotential lines is constant and equal to $\Delta h/N_h$.

Flow through an elementary cell

Consider a curvilinear quadrilateral of width a and length b. The discharge per unit thickness is:

$$\Delta q = V \cdot a = Ki \cdot a$$

with:

$$i = \frac{\Delta h}{b}$$

Thus:

$$\Delta q = K \Delta h \frac{a}{b}$$

For similar elements:

$$\frac{a}{b} = \frac{c}{d} = \text{constant}$$

Total discharge

Let:

- ΔH: total head difference,
- N_h: number of potential drops,
- N_c: number of flow channels.

$$\Delta h = \frac{\Delta H}{N_h}$$

$$\Delta q = K \frac{\Delta H}{N_h}$$

$$Q = N_c \cdot \Delta q$$

Hence:

$$Q = K \frac{N_c}{N_h} \Delta H$$

Pore water pressure

At a point M, with elevation z and hydraulic head h, the pore water pressure is:

$$u = \gamma_w (h - z)$$

4.8 Radial flow toward a well

Radial flows are encountered during pumping operations. The main applications are:

- water supply,
- lowering of the water table,
- in situ determination of the average permeability coefficient of a soil.

4.8.1 Steady-state pumping test: Dupuit formula

During pumping, a cone of depression develops in the water table around the well.

Under steady-state conditions, and assuming homogeneous, isotropic soil and horizontal flow (Dupuit assumptions), the discharge is (see Appendix 4):

Unconfined aquifer (free water table)

$$Q = \pi K \frac{H^2 - h^2}{ln\left(\frac{R}{r}\right)}$$

or

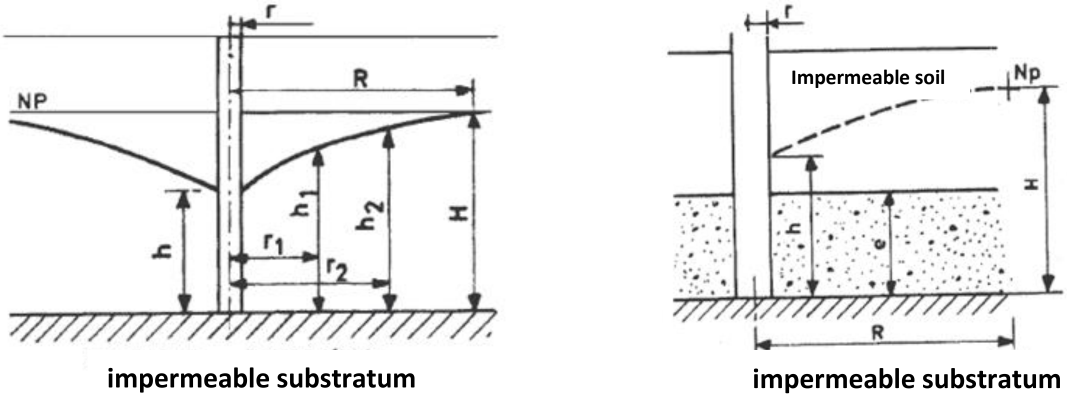
$$Q = 1.365 \cdot K \cdot \frac{H^2 - h^2}{log\left(\frac{R}{r}\right)}$$

Confined aquifer

$$Q = 2\pi Ke\frac{H-h}{ln\left(\frac{R}{r}\right)}$$

or

$$Q = 2.73Ke\frac{H-h}{log\left(\frac{R}{r}\right)}$$



(a) Unconfined aquifer **(b)** Confined aquifer

Figure 4.15 Pumping Test

4.8.2 Radius of influence and determination of *K*

The radius of influence *R* is the distance beyond which the drawdown becomes negligible. It is often estimated empirically:

$$100r < R < 300r$$

Sichardt proposed the empirical relation:

$$R = 3000(H-h)\sqrt{K}$$

where:

- K in m/s,
- R, $(H-h)$ in meters.

To determine *K* in practice, the drawdown curve is measured using piezometers installed around the well, and the data are fitted using:

Unconfined aquifer

$$K = \frac{Q}{\pi}\frac{ln\left(\frac{r_2}{r_1}\right)}{(h_2^2 - h_1^2)}$$

Confined aquifer

$$K = \frac{Q}{2\pi e}\frac{ln\left(\frac{r_2}{r_1}\right)}{(h_2 - h_1)}$$

4.8.3 Lefranc test

Pumping tests are relatively costly and time-consuming. However, permeability can be estimated during drilling operations using a simpler in situ method known as the Lefranc test (Figure 4.16).

A cavity is created at the bottom of the borehole. This cavity is typically cylindrical, with length L and diameter D.

Steady-state test

Under steady-state conditions, the discharge is given by:

$$Q = Ckh$$

where:

C: shape factor depending on cavity geometry

K: coefficient of permeability

h: head difference

For a cylindrical cavity:

$$C = \frac{2\pi L}{ln\left(\frac{2L}{D}\right)} \quad \text{if } L > 2D$$

If $L < 2D$, the cavity behaves approximately as a spherical source, and an empirical relation is used:

$$C = 2\pi D\sqrt{\frac{L}{D} + \frac{1}{4}}$$

For $L = D$, this gives:

$$Q = 2.24\pi D$$

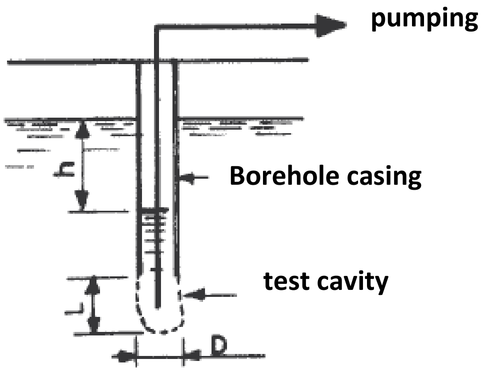


Figure 4.16 Lefranc Test

Variable head test

For low-permeability soils, the test can be performed under variable head conditions. The permeability coefficient is then:

$$k = \frac{S}{C} \cdot \frac{ln\left(\frac{h_0}{h_1}\right)}{(t_1 - t_0)}$$

Where:

- $S = \frac{\pi D^2}{4}$: cross-sectional area of the borehole,
- h_0, h_1: hydraulic heads at times t_0 , t_1, measured relative to the groundwater level

4.9 Formation of quicksand and piping

4.9.1 Quicksand

Consider two containers, R_1 and R_2, connected by a rubber tube (Figure 4.17). The first container, R_1, is kept filled with water, while the second container, R_2, contains saturated sand. Let us examine the upward flow of water in this system.

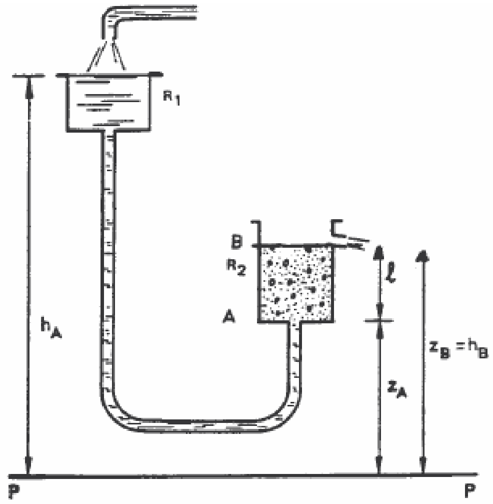


Figure 4.17 Quicksand phenomenon

Let h_A and h_B be the hydraulic heads at points A and B, respectively. The hydraulic gradient in the sand column of length l is:

$$i = \frac{h_A - h_B}{l}$$

If the upstream water level is progressively raised, the upward hydraulic gradient increases. The seepage force acting upward on the soil grains also increases.

At a critical value of the hydraulic gradient, called the critical gradient i_c, the upward seepage force becomes equal to the submerged weight of the saturated sand. The effective stress then becomes zero, and the sand loses its stability. The grains are lifted by the upward flow, and the soil enters a boiling or quick condition.

The critical gradient is given by:

$$i_c = \frac{\gamma'}{\gamma_w}$$

Since:

$$\gamma' = \frac{(G_s - 1)\gamma_w}{1 + e}$$

It follows that:

$$i_c = \frac{G_s - 1}{1 + e}$$

where:

G_s is the specific gravity of the soil particles,

e is the void ratio,

γ' is the submerged unit weight of the saturated soil,

γ_w is the unit weight of water.

For many sands, the critical gradient is close to 1.

The same phenomenon may occur in nature in the presence of upward seepage through fine sand. If the hydraulic gradient becomes sufficiently high, the sand grains are lifted and the soil loses its bearing capacity. This phenomenon is known as quicksand or boiling. It may occur, for example, at the bottom of excavations, near sheet pile structures, or downstream of hydraulic works.

4.9.2 Piping

Water infiltration beneath a structure (dam, foundation, sheet pile, etc.) may lead to a phenomenon known as piping, especially when the hydraulic gradient becomes sufficiently high.

Initially, the flow velocity increases, causing the progressive erosion and transport of fine soil particles. This process leads to the formation of preferential flow paths within the soil mass. As erosion develops, these channels enlarge, and the seepage flow may become uncontrolled, potentially leading to failure of the structure (dam failure, foundation instability, sheet pile collapse, etc.).

Piping is therefore an internal erosion process that can develop rapidly and cause severe damage to nearby structures.

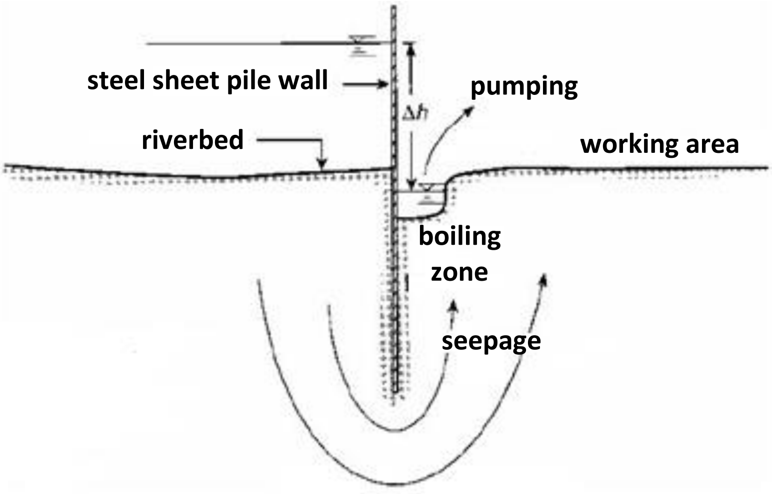


Figure 4.18 Sheet pile cofferdam and piping phenomenon

Note:

To prevent piping, it is essential that the hydraulic gradient i remains below the critical gradient i_c.

In practice, a safety factor is introduced:

$$F_s = \frac{i_c}{i} \geq 3$$

4.9.3 Construction measures to prevent piping

Various measures can be considered to reduce the risk of piping (internal erosion), including:

- **Reducing the hydraulic gradient**, for example by:
  - providing sheet piles with sufficient embedment depth,
  - installing an impermeable blanket on the upstream side of a structure.
- **Installing a filter layer** at the downstream exit of seepage, with carefully selected grain size to prevent the migration of fine soil particles. The following conditions must be satisfied:

Permeability condition:

$$\mathrm{d}_{15(\text{filter})} \geq 4 \text{ to } 5\ \mathrm{d}_{15(\text{soil})}$$

Retention condition:

$$\mathrm{d}_{15(\text{filter})} \leq 4 \text{ to } 5\ \mathrm{d}_{85(\text{soil})}$$

- **Installing relief wells or drainage systems** equipped with filters in areas where seepage emerges and piping may develop.

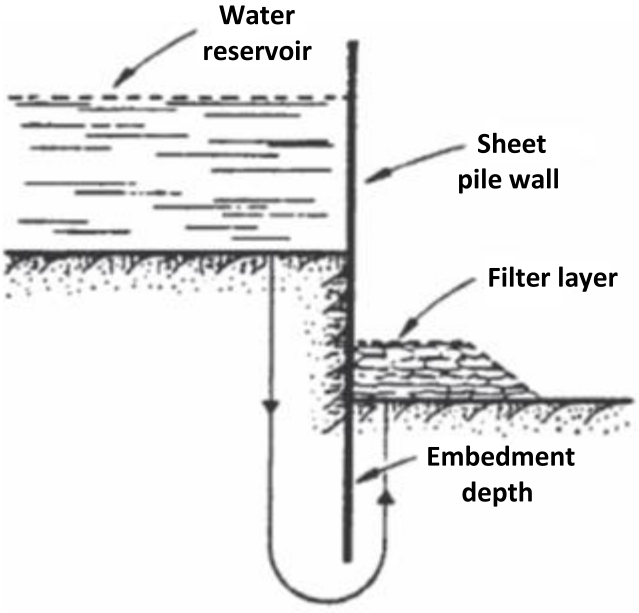


Figure 4.19 Example of anti-piping measure: sheet pile

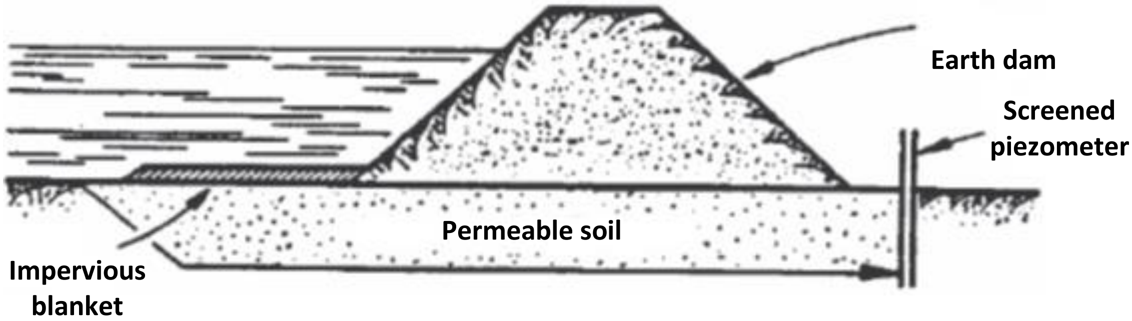


Figure 4.20 Example of anti-piping measure: earth dam

4.10 Capillary rise in unsaturated soils: Jurin's law

4.10.1 Description of the phenomenon

In a capillary tube, water rises to a height *h* such that the weight of the water column is balanced by the vertical component of the surface tension force (Figure 4.21).

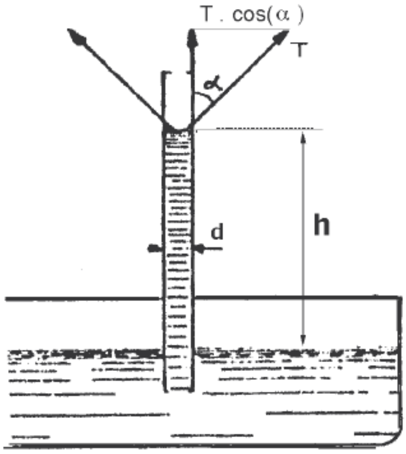


Figure 4.21 Jurin's law

The upward force due to surface tension is:

$$2\pi r T \cos\alpha$$

where:

r is the tube radius,

T is the surface tension of water,

α is the contact angle.

The weight of the water column is:

$$h\gamma_w \pi r^2$$

Equating the two gives:

$$h = \frac{2T\cos\alpha}{r\gamma_w}$$

This is Jurin's law.

Note:

- Jurin's law shows that capillary rise is inversely proportional to the tube radius.
- For water in contact with air, T is about 0.073 N/m at 20°C.
- Capillary rise becomes significant only when the pore radius is small.

4.10.2 Application to soils: practical consequences

A similar phenomenon occurs in soils, although the pore network is much more complex than a capillary tube. Above the groundwater table, water rises through the voids by capillary action, forming a capillary fringe.

The height of capillary rise depends mainly on pore size, and therefore on grain size and void ratio. In general, capillary rise increases as soil becomes finer.

An approximate empirical relation sometimes used is:

$$h = \frac{C}{ed_{10}}$$

where:

h_c is the capillary rise,

e is the void ratio,

d_{10} is the effective grain diameter,

C is an empirical constant.

In unsaturated soils, capillarity creates negative pore water pressure (matric suction), which may increase the apparent shear strength of the soil.

4.11 Exercises

Exercise 1

Using the soil cylinder and the piezometers shown in the figure below, determine the pressure head, elevation head, total head, and head loss at points B, C, D, and F, expressed in centimeters of water.

Then, plot the variation of the hydraulic head at each level.

Solution

The total head is equal to the sum of the pressure head and the elevation head:

$$h = h_p + z = \frac{u}{\gamma_w} + z$$

where:

h: total hydraulic head

$h_p = \frac{u}{\gamma_w}$: pressure head

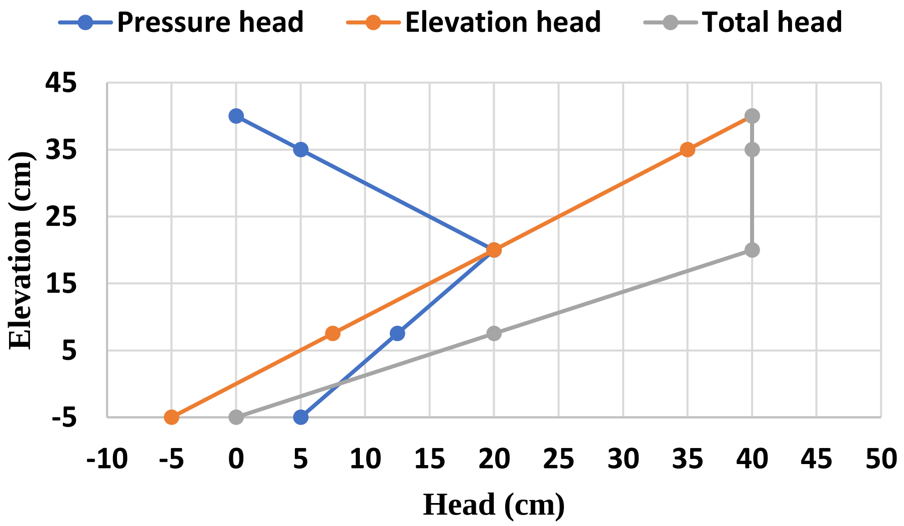
z: elevation head

u: pore water pressure

γ_w: unit weight of water

The unit of head is the centimeter of water (cm H_2O).

Point	Pressure head (cm)	Elevation head (cm)	Total head (cm)	Head loss (cm)
B	5	35	40	0
C	20	20	40	0
D	12.5	7.5	20	20
F	5	-5	0	40



Exercise 2

A cylindrical sand sample has a height of 10 cm and a diameter of 5.5 cm. It is placed in a constant head permeameter. Water flows through the sample under a hydraulic head of $h = 45$ cm. In 10 seconds, 50 g of water are collected.

1) Determine the coefficient of permeability K.
2) Determine the time required to collect 500 g of water.
3) If the sample is replaced by a clay sample of the same dimensions with a permeability coefficient $K' = 2.79\times10^{-7}$ cm/s, calculate the time required to collect 50 g of water at the outlet.

Solution

1) Determination of K

The flow is governed by Darcy's law:

$$K = \frac{QL}{hSt}$$

where:

Q: collected volume (cm³)

L: sample length (cm)

h: head difference (cm)

S: cross-sectional area (cm²)

t: time (s)

The cross-sectional area is:

$$S = \frac{\pi d^2}{4}$$

Thus :

$$K = \frac{4QL}{h\pi d^2 t} = \frac{4 \times 50 \times 10}{45 \times \pi \times (5.5)^2 \times 10}$$

$$K = 4.67 \times 10^{-2}\ \mathrm{cm/s}$$

2) Time required to collect 500 g of water

Using Darcy's law:

$$K = \frac{Q'L}{hSt'}$$

Thus :

$$t' = \frac{Q'L}{hSK} = \frac{4Q'L}{h\pi d^2 K} = \frac{4 \times 500 \times 10}{45 \times \pi \times (5.5)^2 \times 4.67 \times 10^{-2}}$$

$$t' = \frac{4 \times 500 \times 10}{45 \times \pi \times (5.5)^2 \times 4.67 \times 10^{-2}}$$

$$t' = 100\ s$$

3) Time required for 50 g of water (clay sample)

$$K' = \frac{QL}{hSt'}$$

Thus :

$$t' = \frac{QL}{hSK'} = \frac{4QL}{h\pi d^2 K'} = \frac{4 \times 500 \times 10}{45 \times \pi \times (5.5)^2 \times 2.79 \times 10^{-7}}$$

$$t' = \frac{4 \times 500 \times 10}{45 \times \pi \times (5.5)^2 \times 2.79 \times 10^{-7}}$$

$$t' = 19\ days, 9\ hours, 51\ minutes, 37\ seconds$$

Comment: The required time is extremely long. Therefore, it is not practical to use a constant head permeameter for clayey soils. In such cases, a *falling head permeameter* is more appropriate.

Exercise 3

A clay sample with a height of 2.5 cm and a diameter of 6.5 cm is placed in a falling head permeameter. Water flow is observed in a standpipe with a diameter of 1.7 mm.

The standpipe is graduated in centimeters (cm), from top to bottom. The zero mark of the scale is located 35 cm above the base of the mold, while the overflow level is 3 cm above the base.

At the beginning of the test, the water level in the standpipe is at the zero mark (0). After 6 minutes and 35 seconds, the water level has dropped to the mark (2).

1) Calculate the coefficient of permeability *K*.
2) Determine the water level in the standpipe after 17 minutes from the start of the test.
3) If the sample is replaced by a sand sample of the same dimensions with a permeability coefficient *K* = 4.68×10^{-2} cm/s, calculate the time required for the water level to drop to the graduation (30).

Solution

1) Determination of the coefficient of permeability *K*

For a falling head test:

$$K = 2.3 \frac{sL}{S} \frac{log(h_1/h_2)}{t_2 - t_1}$$

where:

s: cross-sectional area of standpipe

S: cross-sectional area of soil sample

L: sample length

Areas:

$$s = \frac{\pi d^2}{4} = \frac{\pi \times (0.17)^2}{4} = 2.27 \times 10^{-2}\ cm^2$$

$$s = \frac{\pi D^2}{4} = \frac{\pi \times (6.5)^2}{4} = 33.17\ cm^2$$

Heads:

$t_1 = 0$, $h_1 = 35-3 = 32$ cm

$t_2 = 395$ s , $h_2 = 32-2 = 30$ cm

Calculation

$$K = 2.3 \frac{2.27 \times 10^{-2} \times 2.5}{33.17} \frac{log(32/30)}{395-0}$$

$$K = 2.79 \times 10^{-7}\ cm/s$$

2) Water level after 17 minutes

$t_2 = 17$ min = 1020 s

Using:

$$log(h_1/h_2) = \frac{KS(t_2 - t_1)}{2.3sL} = \frac{2.79 \times 10^{-7} \times 33.17(1020-0)}{2.3 \times 2.27 \times 10^{-2} \times 2.5}$$

$$log(h_1/h_2) = \frac{2.79 \times 10^{-7} \times 33.17(1020-0)}{2.3 \times 2.27 \times 10^{-2} \times 2.5}$$

$$log(h_1/h_2) = 0.072$$

$$h_2 = 27.09 \text{ cm}$$

3) Time required to reach graduation (30)

$h_1 = 32$ cm, $h_2 = 32-30 = 2$ cm

Using:

$$t_2 = 2.3 \frac{sL}{S} \frac{log(h_1/h_2)}{K'} + t_1$$

$$t_2 = \frac{2.3 \times 2.27 \times 10^{-2} \times 2.5}{33.17} \cdot \frac{log(32/2)}{4.68 \times 10^{-2}} + 0$$

$$t_2 = 0.12\ s$$

Comment: The required time is extremely short. This confirms that a *falling head permeameter is not suitable for highly permeable soils such as sand*, for which a *constant head permeameter* is more appropriate.

Exercise 4

A dam of width 25 m, whose cross-section is shown in the figure below, rests on an alluvial layer with a unit weight γ = 21 kN/m^3 and thickness 20 m, underlain by an impermeable rock substratum. The upstream water level is at +7.5 m, and the downstream level is at –2 m, both measured with respect to the riverbed taken as the reference datum. The alluvial soil is assumed to be homogeneous and isotropic, with a coefficient of permeability $K = 4 \times 10^{-5}\ \mathrm{m/s}$.

Taking into account the boundary conditions, an approximate flow net is constructed using the graphical method.

1) Calculate the pore water pressure at point C.
2) Calculate the hydraulic gradient between points D and E (with DE = 2 m), and deduce the factor of safety against piping.
3) Calculate the seepage discharge through the foundation soil of the dam.

Solution

1) Pore water pressure at point C

The hydraulic head at point C is:

$$h_C = \frac{u_C}{\gamma_w} + z_C$$

Thus:

$$u_C = \gamma_w (h_C - z_C)$$

Given:

$$z_C = 2\ \mathrm{m}$$

From the flow net:

$$h_C = h_A - 5\Delta h$$

Where:

h_A = 7.5 m, $\Delta h = \frac{\Delta H}{N_h}$

ΔH = 7.5 m, N_h = 15,

$$\Delta h = \frac{7.5}{15} = 0.5\ m$$

$$h_C = 7.5 - 5 \times 0.5 = 5\ \mathrm{m}$$

Therefore:

$$u_C = 10 \times \big(5 - (-2)\big) = 70\ kN/m^2$$

2) Hydraulic gradient between D and E

$$i_{DE} = \frac{\Delta h}{DE} = \frac{\Delta H}{N_h \cdot DE} = \frac{7.5}{15 \times 2} = 0.25$$

Factor of safety against piping

The critical hydraulic gradient is:

$$i_C = \frac{\gamma'}{\gamma_w} = \frac{21 - 10}{10} = 1.1$$

Thus:

$$F_s = \frac{i_C}{i_{DF}} = \frac{1.1}{0.25} = 4.4 \geq 3$$

The foundation is safe against piping.

3) Seepage discharge through the foundation

Using the flow net relation:

$$Q = K\frac{N_c}{N_h}\Delta H$$

Given:

$N_c = 5$, $N_h = 15$, $\Delta H = 7.5$ m, $K = 4 \times 10^{-5}$ m/s

$$Q = 4 \times 10^{-5} \times \frac{5}{15} \times 7.5 = 1 \times 10^{-4}\ m^3/s$$

Exercise 5

In a sandy soil, a well of 0.4 m diameter is drilled, and water is pumped at a discharge rate of 30 m^3/h. The initial groundwater level is located 7 m above the impermeable layer, and it is lowered by 3.5 m in the well during pumping.The observed radius of influence is R = 200 m. Determine the coefficient of permeability K of the soil.

Solution

For steady-state pumping in an unconfined aquifer, the Dupuit equation is:

$$Q = 1.365K\frac{H^2 - h^2}{log\left(\frac{R}{r}\right)}$$

Thus:

$$K = \frac{Qlog\left(\frac{R}{r}\right)}{1.365(H^2 - h^2)}$$

Given data:

Q = 30 m^3/h, R = 200 m, r = 0.2 m

H = 7 m, h = 7 - 3.5 = 3.5 m

Calculation:

$$K = \frac{30 \times log\left(\frac{200}{0.2}\right)}{1.365(9^2 - 3.5^2)} = 1.8 \text{ m/h}$$

Final result:

$$K \approx 5 \times 10^{-4} \text{ m/s}$$

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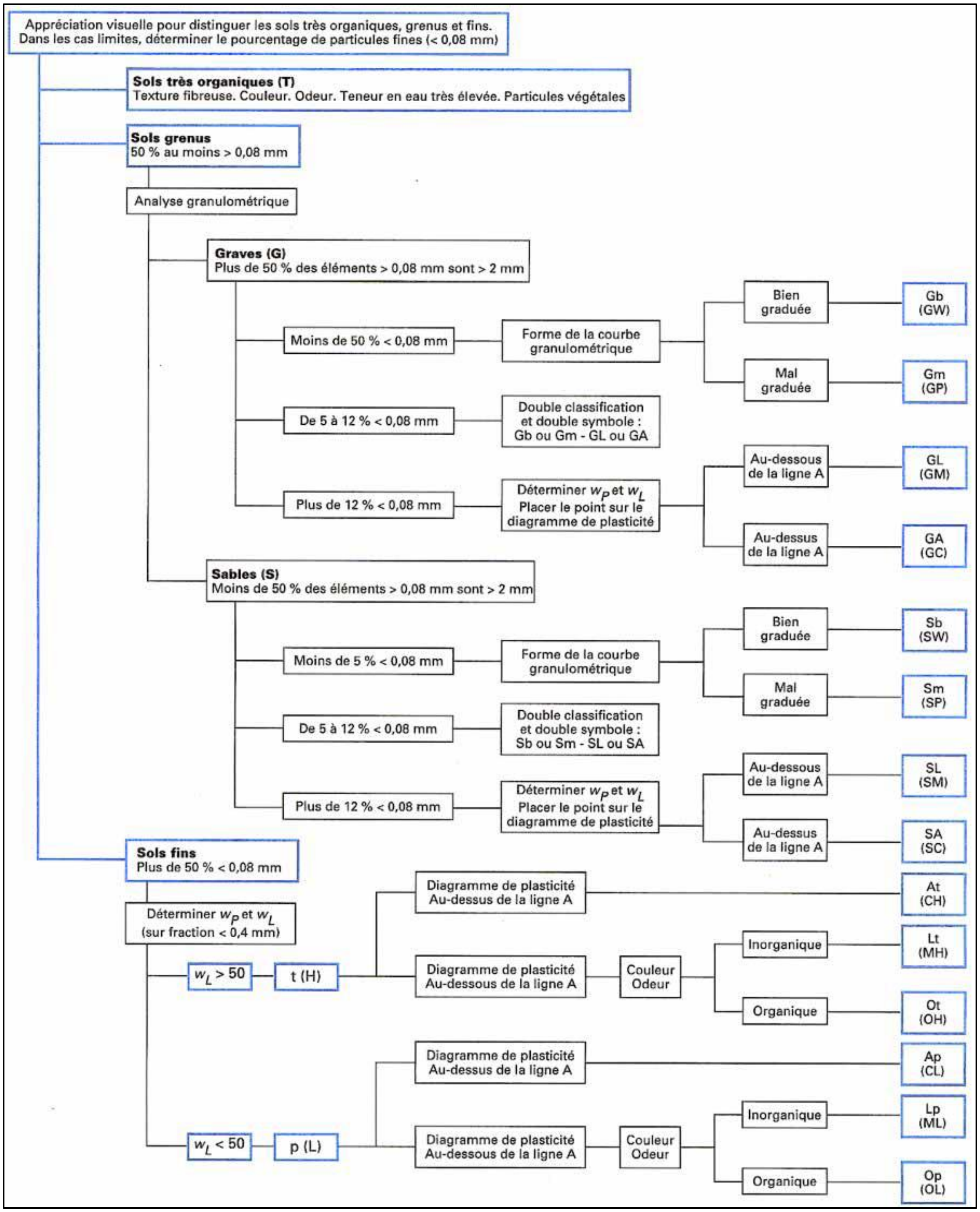
Appendices

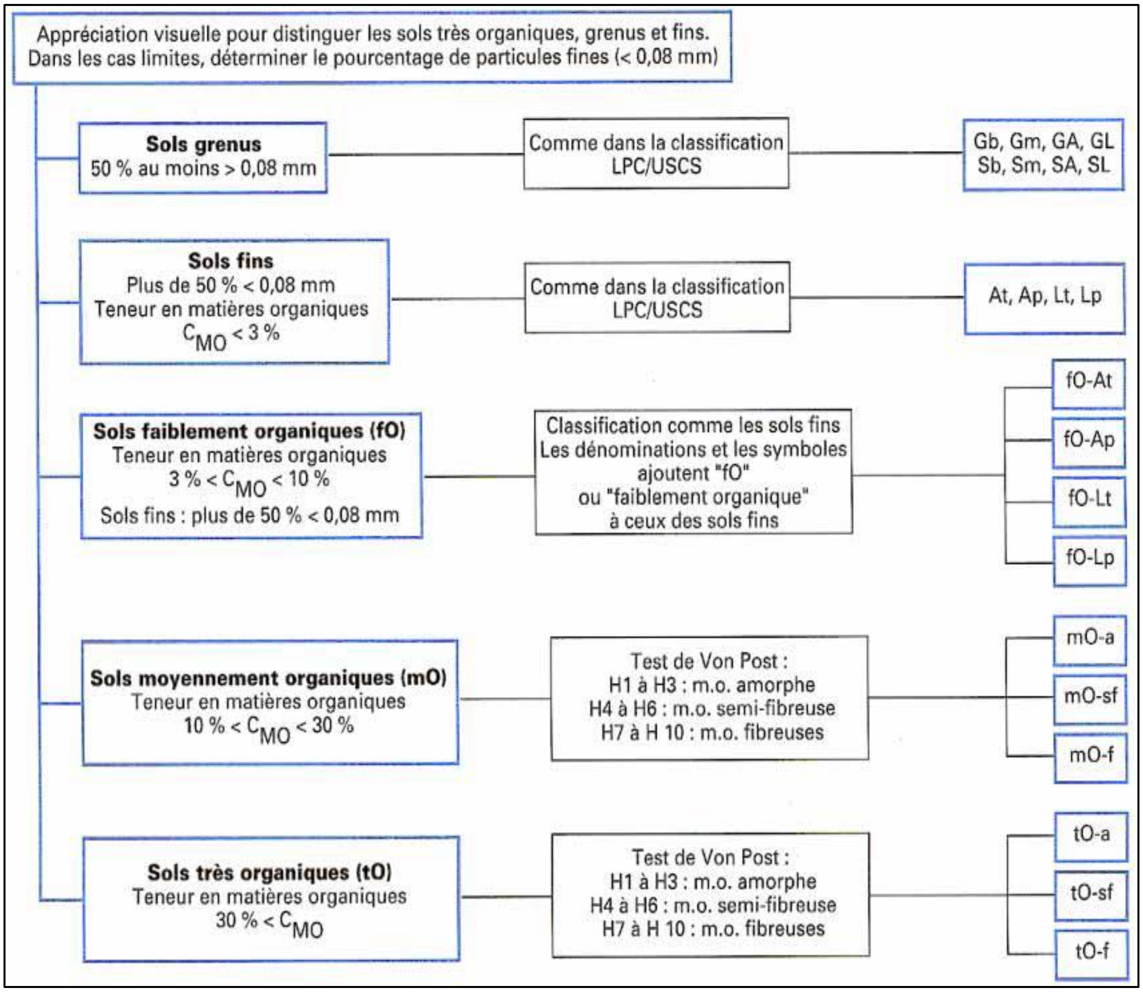
Appendix 1

LPC/USCS classification of laboratory soils



Appendix 2

Modified LPC soil classification (1974/1980)



Appendix 3

Continuity condition

Consider an arbitrary volume of saturated soil (V), bounded by a surface (S) and traversed by a flow (Fig. 1). During a given time interval dt, a volume of water dV_1 enters the surface (S) and a volume of water dV_2 exits.

Let $\vec{v}$ be the velocity of the water, with components v_x, v_y, and v_z, which are functions of the coordinates of the considered point.

The volume of water dV passing through the surface element dS, with an outgoing normal $\vec{n}$, during the time interval dt, is given by (Figure 1):

$$dV = \vec{v} \cdot \vec{n} \cdot dS \cdot dt$$

$dV < 0 \Leftrightarrow$ water enters the surface (S)

$dV > 0 \Leftrightarrow$ water exits from (S)

The continuity condition is written as:

$$dV_1 - dV_2 = 0 \ \Leftrightarrow dt \cdot \oiint \vec{v} \cdot \vec{n} \cdot dS = 0$$

The Ostrogradsky relation is written as:

$$\oiint \vec{v} \cdot \vec{n} \cdot dS = \oiiint div\, \vec{v} \cdot dV \qquad \text{(for any } V\text{)}$$

Hence:

$$\oiiint div\, \vec{v} \cdot dV = 0$$

Verified for any volume, and thus for any elementary volume $\Longrightarrow div\, \vec{v} \cdot dV = 0$

Therefore, after simplification, the continuity condition is:

$$div\, \vec{v} = 0$$

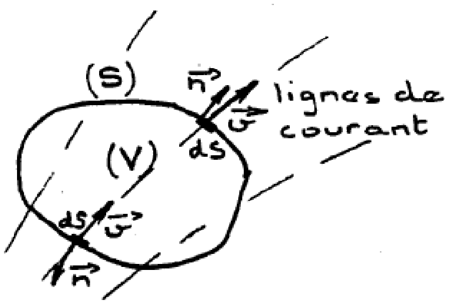


Figure 1

Appendix 4

Pumping Flow – TCHARNY's demonstration

The assumption of Dupuit regarding the slope of the free surface, which was assumed to be small, is no longer necessary.

Assumptions:

- Homogeneous and isotropic soil,
- Water and soil are incompressible,
- Laminar steady-state regime,
- Darcy's law is applicable,
- Radial flow,
- Pumped flow is taken from outside the influence zone of the pumping (supply through a cylinder of radius R, corresponding to the distance where the drawdown is zero),
- Existence of a resurgence zone in the well, with height $h' - h$ (which is not considered in Dupuit's demonstration).

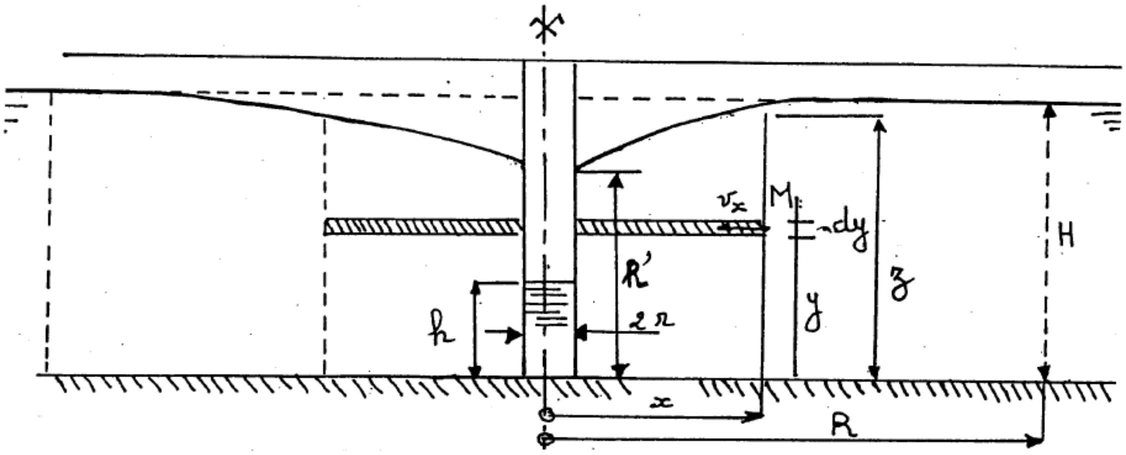


Figure 2

Let h^* be the hydraulic head[1] at a point M(x,y) of the flow, the velocity potential is given by:

$$\phi(x, y) = -K \cdot h^*$$

Let:

$$\phi(x, y) = -K\left(\frac{u}{\gamma_w} + y\right)$$

The flow velocity is:

$$\vec{v} = \overrightarrow{grad}\,(-K \cdot h^*)$$

and its horizontal component is:

$$v_x = \frac{\partial \phi}{\partial x}$$

The flow dq passing through an elemental cylinder of radius x and height dy is given by:

$dq = -2\pi \cdot x \cdot dy \cdot v_x = -2\pi \cdot x \cdot \frac{\partial \phi}{\partial x} \cdot dy$ (negative because x and v_x have opposite signs)

The total flow q passing through the cylinder of the same radius x and height z is therefore:

$$q = \int_0^z dq = -2\pi \cdot \int_0^z x \cdot \frac{\partial \phi}{\partial x} \cdot dy = -2 \cdot \pi \cdot \int_0^z \frac{\partial \phi}{\partial \ln x} dy$$

Using Leibniz's rule, we get:

$$\frac{d}{d \ln x} \int_0^z \phi(x,y) dy = \int_0^z \frac{\partial \phi}{\partial \ln x} dy + \phi(x,z) \frac{dz}{d \ln x}$$

$$\Rightarrow \int_0^z \frac{\partial \phi}{\partial \ln x} dy = \frac{d}{d \ln x} \int_0^z \phi(x,y) dy - \phi(x,z) \frac{dz}{d \ln x}$$

Hence:

$$q = -2 \cdot \pi \cdot \left[\frac{d}{d \ln x} \int_0^z \phi(x,y) dy - \phi(x,z) \frac{dz}{d \ln x} \right]$$

$\phi(x,z) = -K \cdot z$ (because at the free surface, $u = 0$)

Letting $I(x) = \int_0^z \phi(x,y) dy$, we get:

$q \cdot d \ln x = -2 \cdot \pi (dI + K \cdot z \cdot dz)$ (1)

I is unknown except for $x = r$ and $x = R$ as the boundary conditions give:

$$x = r \quad 0 \le y \le h \quad \phi(r,y) = -K \left[\frac{(h-y)\gamma_w}{\gamma_w} + y \right] = -Kh$$

$$x = r \quad h \le y \le h' \quad \phi(r,y) = -K[0 + y] = -Ky$$

$$x = R \quad \phi(R,y) = -K \left[\frac{(H-y)\gamma_w}{\gamma_w} + y \right] = -KH$$

By integrating the differential equation (1) between r and R, we get:

$$q \ln \frac{R}{r} = -2 \cdot \pi \left[I(R) - I(r) + \int_{h'}^{H} Kz dz \right]$$

$$I(R) = \int_{0}^{H} -KHdy = -KH^2$$

$$I(r) = \int_{0}^{h} -Khdy + \int_{h}^{h'} -Kydy = -Kh^2 - K\frac{h'^2 - h^2}{2}$$

---

[1] h^* because h represents the height of the water in the well.

[2] Leibniz's rule for differentiation under the integral sign.

Let the integral be $\phi(\alpha) = \int_{u_1}^{u_2} f(x,\alpha)dx$, where $a \leq \alpha \leq b$ and u_1 et u_2 may depend on the parameter α.

Then,

$$\frac{\partial\phi}{\partial\alpha} = \int_{u_1}^{u_2} \frac{\partial f}{\partial\alpha} dx + f(u_2,\alpha)\frac{\partial u_2}{\partial\alpha} - f(u_1,\alpha)\frac{\partial u_1}{\partial\alpha} \text{ for } a \leq \alpha \leq b$$

If $f(x,\alpha)$ and $\partial f/\partial\alpha$ are known in x and α, and if u_1 and u_2 are known and have continuous derivatives for $a \leq \alpha \leq b$.

If u_1 and u_2 are constants, the last two terms of the equation are zero. Hence:

$$q\ln\frac{R}{r} = -2\cdot\pi\left[-KH^2 + Kh^2 + K\frac{h'^2 - h^2}{2} + \frac{K}{2}\left((H^2 - h'^2)\right)\right]$$

$$q\ln\frac{R}{r} = -2\cdot\pi\left[-KH^2 + Kh^2 + K\frac{h'^2}{2} - K\frac{h^2}{2} + K\frac{H^2}{2} - K\frac{h'^2}{2}\right]$$

$$q\ln\frac{R}{r} = -2\cdot\pi\left[-K\frac{H^2}{2} + K\frac{h^2}{2}\right] = \pi\cdot K(H^2 - h^2)$$

$$q = \pi\cdot K\cdot\frac{H^2 - h^2}{\ln\frac{R}{r}}$$

This is indeed the Dupuit formula, but now h represents the water height in the well, while h', which represents the water height in the ground, does not appear.