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Study of a fluid flow problem in front of a circular cylinder

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Thanks

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Dedication

How beautiful it is for a person to give what is most precious to him, and how beautiful it is for him to give what is most precious to him

The precious is the fruit of my effort that I harvest today, and it is a gift to which I dedicate myself:

To the one I prefer to myself, to the source of tenderness،

To those whose feet describe paradise, to my beloved mother, may God prolong their lives

“ Mom” 'Fellak Malika'

To the one who illuminated my path with the light of his candles which do not melt for a future built on humility and knowledge, my dearest creature in the world.

“ My father” 'Fellak Rabeh'

To my brothers and sisters: Kheirddine, Bahaeddine, Lamia, Anissa and Sabrina.

Upper arm and forearm, my brothers and sisters, I offer you the gift of love, dignity and exaltation.

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Support, even with a smile, to all those who did not find their name in this gift.

Alaeddine

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Symboles

T	Surface tension.
g	Severity.
$\vec{V}$	Speed vector.
U	Speed to infinity.
$u;v$	Components of the velocity vector.
ξ	Conjugate complex speed.
f	Complex potential function.
ϕ	Velocity potential function.
ψ	Current.
t	Time.
ρ	Fluid density.
P_0	Atmospheric pressure.
P	Fluid pressure.
$\overline{\overline{\sigma}}$	Stress tensor.
μ,η	Viscosity coefficients.
K	Curvature of the free surface.
R	Radius of curvature.
$\vec{\gamma}$	Acceleration.
θ	Speed slope angle.
div	Divergence.
Δ	Laplacian.
rot	Rotational.
$\overrightarrow{grad}$	Gradient.
$ds\,,dv$	Surface element, volume element.
C_c	The cavitation number.
$\vec{F}$	The force per unit volume.
$\vec{n}$	The normal vector to ds.
$\alpha_m,\ \beta_m,\ \theta_m$	Arbitrary constants.
p_c	The constant pressure in the cavity.
α	Weber number

General introduction

Many applications in fluid mechanics involve streams in areas defined by hard walls on the one hand, and interfaces (free surfaces) on the other. Classic examples are streams, rivers, canals, etc. Or the boat's movement on a water surface can be absorbed into the free surface flow.

Studying fluid flow around different objects is an important topic in mechanical engineering, chemical engineering and many other engineering disciplines. Flux around a circular cylinder is one of the classic cases studied to understand physical and mathematical phenomena associated with fluid movement. This type of study focuses on understanding how fluid moves around a circular cylinder and the effect of various factors such as fluid speed, surface characteristics, viscosity, changes in pressure and heat, this study also includes the use of many mathematical and physical concepts such as Navier-Stokes equations, thermodynamics, and liquid mechanics principles.

The real problems, as mentioned in previous examples, are 3D. But large simplifications can be offered if the flow recognizes symmetry. More specifically, if the flow characteristics are fixed in a vertical direction at a constant level, the problem becomes two-dimensional with respect to the decent variables of the constant level considered. Another version is cylindrical symmetry. Flow characteristics are constant with respect to the θ angle of the cylindrical coordinates referred to as a fixed axis (symmetry axis). In the case where the flow is two-dimensional, we determine the level of the dictate variables $\left(x, y\right)$ of the flow with the level of the compound variable z .

Flow is two-dimensional, constant, and non-rotational. The study is therefore linked to the search for a miromorphic function with well-defined vocabulary according to the problem in question. We address the problem of fluid flow study around a circular cylinder.

We will solve this problem using a series-cutting method as well as conformal transformations and streamlines, and study fluid flow around a circular cylinder that includes the use of many mathematical and physical concepts. Analysis of these phenomena can help develop better designs of objects that interact with fluid and understand natural phenomena related to flow.

Our study is based on three main chapters as follows:

In chapter one , we will explore some basic ideas and concepts related to fluid mechanics as well as different interpretations of complex changing functions.

Chapter two addresses the impact of circular cylinder circulation on its flow field, focusing on the production of side force, ignoring gravity and surface forces by relying on streamlining and conformal transformations.

Chapter three focuses on the study of pure free capillary surface flows, where surface tension is taken into account but gravity is ignored. The liquid is non-sticky, non-compressable and non-rotational. The basic model used is the flow of the cavity through a circular cylinder based on the series carving method and conformal transformations.

CHAPTER 1

Preliminary notes on fluid mechanics

1.1 Introduction

In this chapter we will discuss some initial concepts related to fluid mechanics and some definitions of complex variable functions.

1.2 Momentum equations

1.2.1 Navier-stokes equation

We consider the fluid domain D, delimited by the surface S. The forces acting on a fluid particle are:

1. Volume forces that act on the mass of the particle, for example gravity forces.
They are given by the formula:

$$\vec{F_v} = \int_D \rho \vec{F} dv \qquad \textbf{(1.2.1)}$$

Where $\vec{F}$ is the force per unit volume.

2. Surface forces that act on particle surfaces, e.g. pressure forces, viscosity forces.
They are given by:

$$\vec{F_s} = \int_S f\,(x,n)ds \qquad \textbf{(1.2.2)}$$

Where $f\,(x,n)$ is the force per unit area. There exists a stress tensor $\overline{\overline{\sigma}}$ such as :

$f\,(x,n) = \overline{\overline{\sigma}}.\vec{n}$: $\vec{n}$ being the normal vector to ds

Case of incompressible fluids

In the case of an incompressible fluid, we have $div\vec{V} = 0$.

SO : $$\rho\frac{d\vec{V}}{dt} = -\rho\overrightarrow{grad}U - \overrightarrow{grad}\,p + \mu\Delta\vec{V}$$

This equation is known as the “Navier-Stokes equation”.

1.2.2 Euler's equation

If we neglect viscosity, the Navier-Stokes equation becomes:

$$\rho\frac{d\vec{V}}{dt} = \rho\vec{F} - \overrightarrow{grad}\,p$$

This equation is developed by Euler and named after him.
The Euler's equation can be written in another form:

$$\frac{\partial\vec{V}}{\partial t} + \overrightarrow{grad}\left(\frac{V^2}{2}\right) + \frac{1}{\rho}\overrightarrow{grad}\,p = \vec{F} - \left(rot\vec{V}\right)\wedge\vec{V} \qquad \textbf{(1.2.3)}$$

1.2.3 Bernoulli's equation

From Euler's equation for an incompressible and perfect fluid and when the volume forces derive from potential, equation (1.2.3) is written:

$$\frac{\partial \vec{V}}{\partial t}+\overrightarrow{grad}\left(\frac{V^2}{2}\right)+\left(rot\vec{V}\right)\wedge\vec{V}=-\frac{1}{\rho}\overrightarrow{grad}\,p-\overrightarrow{grad}U$$

We then have:

$$\frac{\partial \vec{V}}{\partial t}+\overrightarrow{grad}\left(\frac{V^2}{2}+\frac{p}{\rho}+U\right)+\left(\overrightarrow{grad}\wedge\vec{V}\right)\wedge\vec{V}=0$$

And in the case of a permanent (stationary) flow, we can write:

$$\overrightarrow{grad}\left(\frac{V^2}{2}+\frac{p}{\rho}+U\right)+\left(\overrightarrow{grad}\wedge\vec{V}\right)\wedge\vec{V}=0$$

Thus, we will have:

$$\frac{\partial}{\partial s}\left(\frac{V^2}{2}+\frac{p}{\rho}+U\right)=0$$

The volume forces are most often reduced to the force of gravity alone, in this case we have:

$$\frac{V^2}{2}+\frac{p}{\rho}+U=const \qquad \textbf{(1.2.4)}$$

In general, the constant changes with the current line.

1.3 Potential flows

1.3.1 Properties of velocity potentials

One of the important properties of perfect fluid flows is that if a volume of fluid is irrotational $\left(\omega=\nabla\wedge u=0\right)$, it remains so indefinitely. If for example, the fluid is initially at rest and if it is set in motion by the application of a pressure gradient, the flow created will be potential. This property is not verified exactly in real fluids. However, under certain conditions, particularly on profiled bodies, viscous effects are only present in the boundary layers and, outside the boundary layers, flow is potential.

If, in addition, the fluid is incompressible, the irrotational character : $u=\nabla\phi$ coupled with the incompressibility condition leads to: $\nabla u=\nabla.\left(\nabla\phi\right)=\Delta\phi=0$

1.3.2 Uniform flow

The velocity field is given by : $u_x=U\ ,u_y=U$. The corresponding speed potential is: $\phi=U_x$ and the current function is $\psi=U_y$.

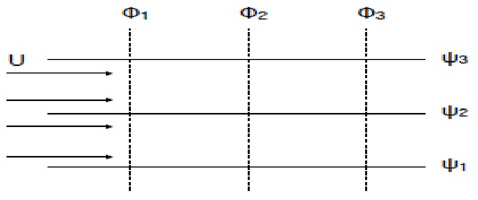


Figure 1 - Equipotential and streamlines for uniform flow.

1.3.3 Flow around a cylinder

The search for the velocity potential for the flow around an obstacle must only satisfy two conditions: far from the obstacle, we must find a uniform flow with a potential $\phi=Ux$ and on the obstacle, the speed normal to the wall must be zero, that is the wall must be a streamline.(Be careful, we neglect the effects of viscosity here, the fluid can then slide along the wall. This is obviously not the case with a real fluid. The cylinder considered here would be a virtual cylinder located above the boundary layer). The obstacle constituting a disturbance of the uniform flow, one of the methods for finding the potential consists of adding to the uniform velocity field, the speed resulting from a multipolar development of the potential. We have seen above what the potential of a monopole (source) and a dipole are.

Let us consider here the addition of the potential of a dipole to a uniform flow making it possible to represent the flow around a cylinder. The axis of the dipole is parallel to the average flow to maintain axial symmetry and the potential is:

$$\phi = Ux - \frac{p\cos\theta}{2\pi r} = \left(Ur - \frac{p}{2\pi r}\right)\cos\theta$$

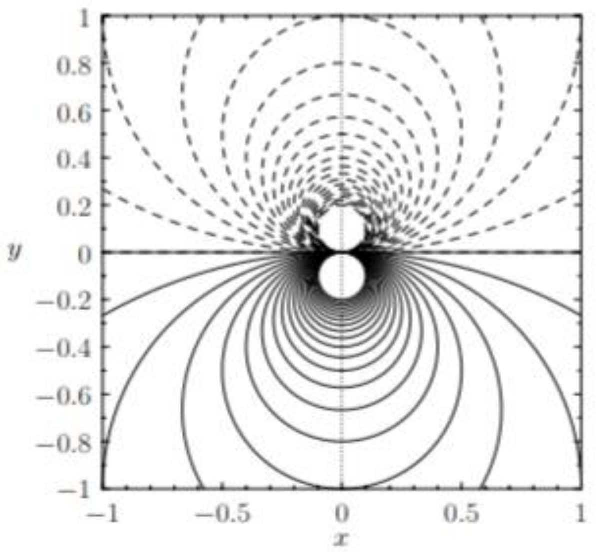


Figure 2 - Flow around cylinder shown with streamlines and isobaric lines normalized by pressure.

The corresponding velocity field is:

$$u_r = \left(U + \frac{p}{2\pi r^2}\right)\cos\theta \;,\; u_\theta = -\left(U - \frac{p}{2\pi r^2}\right)\sin\theta$$

It must satisfy the boundary conditions: $u = Ui$ to infinity and $u_r = 0$ in $r = a$ (on the circle) The first condition is verified whatever the strength of the dipole because the potential of the dipole decreases in $1/r$ and its contribution cancels out to infinity. The second condition imposes the strength of the dipole: $p = -2\pi r^2 U$

and the resulting potential is: $\phi = Ur\cos\theta\left(1 - \frac{a^2}{r^2}\right)$

and the velocity field is: $u_r = U\cos\theta\left(1 - \frac{a^2}{r^2}\right) \quad , \quad u_\theta = -U\sin\theta\left(1 + \frac{a^2}{r^2}\right)$

The current function is deduced by integration: $\psi = -Ur\sin\theta\left(1 + \frac{a^2}{r^2}\right)$

1.3.4 Irrotational flow

A flow is said to be irrotational if:

$$\overrightarrow{rot}\left(\vec{V}\right) = 0 \tag{1.3.1}$$

At any point in the fluid, or $\overrightarrow{rot}\left(\vec{V}\right)$ represents the rotation or vorticity vector.

1.3.5 Incompressible flow

A fluid is said to be incompressible if the volume of each fluid particle does not vary over time of movement is translated by the equation:

$$\vec{\nabla}\vec{u} = 0 \tag{1.3.2}$$

1.3.6 Perfect flow

A perfect flow is an approximation in the case where the viscous effects are negligible and a perfect flow having strictly zero viscosity there is only one implication.

Perfect fluid ⇒ Perfect flow

1.3.7 Permanent flow

We also say stationary if its speed components are independent of the time variable.

$$\frac{\partial \vec{V}}{\partial t} = 0 \tag{1.3.3}$$

1.3.8 Stationary flow

We call stationary flow or even permanent flow, a flow of which all the quantitative characteristics are independent of time, in particular for the speed $\frac{\partial v\left(x_1,x_2,x_3\right)}{\partial t} = 0$. This simply means that the streamlines do not change over time. It is easy to see that in a stationary flow the streamlines are the same as the trajectories.

1.4 Complex variable function

1.4.1 The Cauchy-Riemann Conditions

A necessary condition for $w = f\left(z\right) = u\left(x,y\right) + iv\left(x,y\right)$ to be analytic in an open Ω connection is that in Ω, u and v satisfy the Cauchy- Riemann conditions :

$$\frac{\partial u}{\partial x}\left(x,y\right) = \frac{\partial v}{\partial y}\left(x,y\right) \quad , \quad \frac{\partial u}{\partial y} = -\frac{\partial v}{\partial x} \tag{1.4.1}$$

If the partial derivatives are continuous in Ω , then the Cauchy-Riemann conditions are sufficient conditions for $f\left(z\right)$ to be analytic in Ω . The functions $u\left(x,y\right)$ and $v\left(x,y\right)$ are often called conjugate functions. If we enter one into it, we can determine the other (up to an additive constant) of such way that $f\left(z\right) = u + iv$ is analytical.

1.4.2 Harmonic function

If the second partial derivatives of u and v compared with x and to y existing is continuous, then we obtain:

$$\frac{\partial^2 u}{\partial^2 x}+\frac{\partial^2 u}{\partial^2 y}=0 \quad , \quad \frac{\partial^2 v}{\partial^2 x}+\frac{\partial^2 v}{\partial^2 y}=0 \qquad \textbf{(1.4.2)}$$

We deduce the real and imaginary parts of an analytical function verified by the Laplac equation:

$$\frac{\partial^2 \varphi}{\partial^2 x}+\frac{\partial^2 \varphi}{\partial^2 y}=0 \quad , \quad \nabla^2\varphi=0 \qquad \textbf{(1.4.3)}$$

Functions such as $u(x,y)$ and $v(x,y)$ which satisfy the Laplace equation in are called harmonic functions.

1.4.3 Analytical function

Either Ω is an open from $\mathbb{C}$ and $f:\Omega\to\mathbb{C}$. f is said to be holomorphic on Ω if f is derivable at every point from Ω.

Either Ω is an open from $\mathbb{C}$ et $f:\Omega\to\mathbb{C}$. f is said to be analytical if it can be developed into an entire series at any point of Ω.

Any analytical function on Ω is holomorphic on Ω.

1.5 The complex potential

We see that the speed potential is harmonic, i.e., verifies the Laplace equation:

$$\frac{\partial^2 \phi}{\partial^2 x}+\frac{\partial^2 \phi}{\partial^2 y}=0 \qquad \textbf{(1.5.1)}$$

We deduce that there exists a conjugate harmonic function $\psi(x,y)$ such as :

$$\Omega(z)=\phi(x,y)+i\psi(x,y) \qquad \textbf{(1.5.2)}$$

Or analytical. We according to (1.5.2), by derivation:

$$\frac{d\Omega}{dz}=\Omega'(z)=\frac{\partial\phi}{\partial x}+i\frac{\partial\psi}{\partial x}=\frac{\partial\phi}{\partial x}-i\frac{\partial\phi}{\partial y}=v_x-iv_y \qquad \textbf{(1.5.3)}$$

The speed (sometimes called complex speed) is therefore given by:

$$\vec{V}=V_x-iV_y=\overline{d\Omega/dz}=\overline{\Omega'(z)}$$

Is a per module:

$$V=\left|\vec{V}\right|=\sqrt{V_x^{\,2}+V_y^{\,2}}=\left|\overline{\Omega'(z)}\right|=\left|\Omega'(z)\right|$$

The points for which the speed is zero, ie. $\Omega'(z)=0$, are called breakpoints, the function $\Omega(z)$ whose importance is fundamental in the characterization of a flow is called the complex potential.

1.5.1 The velocity vector derives from a potential

If v_x and v_y designate the components of the fluid velocity in (x, y) along the x-axes and y there exists a function φ called speed potential such that:

$$V_x = \frac{\partial \varphi}{\partial x} \quad , \quad V_y = \frac{\partial \varphi}{\partial y} \tag{1.5.4}$$

1.5.2 Potential function

We recall that for an irrational flow: $\vec{\nabla} \wedge \vec{V} = 0$ Can always be represented by the gradient of a scalar function: $\vec{V} \wedge \vec{\nabla}\phi = 0$.

The function ϕ is called potential function. We can therefore write that:

$$u = \frac{\partial \phi}{\partial x} \quad , \quad v = \frac{\partial \phi}{\partial y} \tag{1.5.5}$$

If in addition the fluid is incompressible the function ϕ verifies the Laplace equation.

1.6 Transformations

Definition:

The equation set: $$u = u(x, y) \quad , \quad v = v(x, y) \tag{1.6.1}$$

Defines, in general, a transformation which establishes a correspondence between the points of the plane. equations (1.6.1) are called transformation equation, if at each point of the plane uv corresponds to a single point on the plane xy, and conversely we speak of a one-to-one transformation.

In such a case, a set of points in the plane (such as a curve or region) is mapped into a set of points in the plane uv (curve or region) and vice versa. The corresponding sets of points in the two planes are often called images of each other.

1.6.1 Complex form of a transformation

It is particularly interesting to consider the case where u and v designate the real part and the imaginary part of an analytical function of the complex variable : $z = x + iy$,i.e.

$w = u + iv = f(z) = f(x + iy)$. In such a case of Jacobian transformation:

$$\frac{\partial(u,v)}{\partial(x,y)} = |f'(z)|^2 \tag{1.6.2}$$

We deduce from this that the transformation is consistent in the domains where $f'(z) \neq 0$. The points for which $f'(z) = 0$ are called critical points.

1.6.2 Some transformations

In the following examples α, β, γ are complex constants, a, θ_0, being real constants.

Rotation $w = e^{i\theta_0} z$

By this transformation the figures of the plane z undergo an angle rotation θ_0.

Homothety $w = az$

By this transformation the figures are dilated (or contracted) if a > 1 (if 0 < a < 1). We consider contraction as a special case of expansion.

Translation $w = z + \beta$

By this transformation the figures of the plan z are moved or translated in the direction of the vector β.

Inversion $w = 1/z$

For z non-zero transforms the circles into circles/straight lines, the lines into straight lines/circles depending on whether the object passes through the origin or not.

Joukowski transformation

The joukowski transformation is defined by: $w = \frac{\pi z}{\alpha}$

$J(z) = z + \frac{1}{z}; \forall z \neq 0$ This application is holomorphic on $C - 0$ because :

$\forall z \neq 0, J'(z) = 1 - \frac{1}{z^2}$. Besides $\forall z \notin \{-1,0,1\}, J'(z) \neq 0$, therefore this transformation is an application compliant on all open areas of a complex plan not including neither 0 nor 1 nor -1.

Transformation linéaire $w = \alpha z + \beta$

If α and β are complex constants is called a linear transformation. Given that we can write $w = \alpha z + \beta$ by means of successive transformations $w = \zeta + \beta, \zeta = e^{i\theta_0}, \tau = \alpha z \, \alpha = \alpha e^{i\theta_0}$ we see that the most general linear transformation is expressed in the form of the product of the transformation such as translation, rotation, scale.

Fractional linear transformation

The transformation $w = \frac{az + b}{cz + d}$

With a, b, c, d complex. Transformed the circles into straight lines and respectively.

1.7 Fluid lines

1.7.1 The trajectories

We call the trajectory of the particle the set of positions occupied by the particle over time.

1/ Lagrange's description directly gives the trajectory, in fact: $X = \mathrm{L}(X, t)$

is the equation parameterized by t of the trajectory of the particle identified by X.

2/ If the movement is described by the Euler method, the knowledge of the trajectories comes back to the Lagrange description according to the equivalence method. The trajectories are then solution of the differential system:

$$\frac{dX}{dt} = V$$

SO :

$$\frac{dx_1}{dv_1} = \frac{dx_2}{dv_2} = \frac{dx_3}{dv_3} = dt$$

1.7.2 Power lines

The Eulerian discription also leads to a representation of the velocity field at a time t, in the form of a family of lines tangent at each point to the velocity vector which we call streamlines. The equation of streamlines is deduced directly from this definition by writing that : a small displacement $\overrightarrow{dx}$ on the current line is collinear with the velocity vector :

$$\vec{V} \wedge \overrightarrow{dx} = 0 \quad \text{either} \quad \varepsilon_{jk} v_j dx_k = 0$$

By developing this relationship, we obtain:

$$\begin{cases} v_2 dx_3 - v_3 dx_2 = 0 \\ v_3 dx_1 - v_1 dx_3 = 0 \\ v_1 dx_2 - v_2 dx_1 = 0 \end{cases}$$

The current lines are therefore the integral of the differential system:

$$\frac{dx_1}{v_1(X,t)} = \frac{dx_2}{v_2(X,t)} = \frac{dx_3}{v_3(X,t)} \tag{1.7.1}$$

In which t to the fixed value. Power lines cannot be cut.

1.8 Current function

In a domain D we call planar (or two-dimensional) flow if at any point in this domain at time *t*. The velocity vector $\vec{V}$ is parallel to a plane:

$$div\left(\vec{V}\right) = 0 \tag{1.8.1}$$

For all points in this area:

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0 \rightarrow \frac{\partial u}{\partial x} = -\frac{\partial v}{\partial y} \tag{1.8.2}$$

This implies that the differential form $udy - vdx$ is *t*, fixed the total differential of a certain

function ψ : $$\exists \psi(x,y,t), d(\psi) = udy - vdx \tag{1.8.3}$$

We therefore immediately:

$$u = \frac{\partial \psi}{\partial y} \qquad v = -\frac{\partial \psi}{\partial x} \tag{1.8.4}$$

ψ is called the current function.

$\psi(x,y,t) = c(t)$ are the current lines.

The property of irrational flow for plane flow leads to:

$$\vec{\nabla} \wedge \vec{V} = \vec{0} = \begin{pmatrix} \frac{\partial}{\partial x} \\ \frac{\partial}{\partial y} \end{pmatrix} \wedge \begin{pmatrix} u = \partial\psi / \partial y \\ v = -\partial\psi / \partial x \end{pmatrix} = -\partial^2\psi / \partial^2 x - \partial^2\psi / \partial^2 y = 0 \tag{1.8.5}$$

$\Rightarrow \Delta\psi = 0$, ψ also checks the Laplace equation.

1.9 Surface tension

Often, flowing fluids are not homogeneous. In stable laminar flows the fluids separate such that the heavy fluid is placed below the light fluid. The separating surface is called “interface”. A fluid particle inside a fluid is subject to balanced internal forces resulting in a scalar called pressure. If the particle is on the interface, the forces on the particle are out of balance due to the discontinuity of the density on either side of the interface (the two fluids are of different densities). To keep the particle on the interface in balance, tangential forces at the interface are created to fill the deficit in the resultant surface forces acting on the particle. This force known only on the interfaces is called "surface tension". It is this surface tension that allows a mosquito to walk on a water surface without drowning.

This work introduces the theory of lines of free current and their applications, focusing on complex speed, in free surface interfaces with negligible fluid density. Surface tension is the pressure difference between two faces.

CHAPTER 2

Two-dimensional rotating flow through a moving cylinder with circulation

2.1 Introduction

We begin with a discussion of the simple and fundamental case of a circular cylinder, for which formulae describing the flow field are already known. One of the purposes of this section is to examine the effect of the circulation round the cylinder on the flow field. We saw in that one important effect of the circulation, in combination with translation of the cylinder, is to produce a side-force on the cylinder.

2.2 Two-Dimensional Irrotational Flow in Cylindrical Coordinates

In a two-dimensional flow pattern, we can automatically satisfy the incompressibility constraint, $\nabla v = 0$, by expressing the pattern in terms of a stream function. Suppose, however, that, in addition to being incompressible, the flow pattern is also irrotational. In this case, yields:

$$\nabla^2\psi = 0 \quad \textbf{(2.2.1)}$$

In cylindrical coordinates, because $\psi = \psi(r,\theta)$, this expression implies that :

$$\frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial\psi}{\partial r}\right)+\frac{1}{r^2}\frac{\partial^2\psi}{\partial\theta^2}=0 \quad \textbf{(2.2.2)}$$

Let us explore the possibility of finding a solution that can be separated for Equation (2.2.2)in the following form:

$$\psi(r,\theta)=R(r)\Theta(\theta) \quad \textbf{(2.2.3)}$$

It is easily seen that:

$$\frac{r}{R}\frac{d}{dr}\left(r\frac{dR}{dr}\right)=-\frac{1}{\Theta}\frac{d^2\Theta}{d\theta^2} \quad \textbf{(2.2.4)}$$

which can only be satisfied if:

$$r\frac{d}{dr}\left(r\frac{dR}{dr}\right)=m^2R \quad \textbf{(2.2.5)}$$

$$\frac{d^2\Theta}{d\theta^2}=-m^2R \quad \textbf{(2.2.6)}$$

Where m^2 is an arbitrary (positive) constant. The general solution of Equation (2.2.6) is a linear combination of exp $(im\theta)$ and exp $(-im\theta)$ factors. However, assuming that the flow extends over all θ values, the function $\Theta(\theta)$ must be single-valued in v , otherwise $\nabla\psi$ and, hence, v would not be be single-valued (which is un-physical).

It follows that m can only take integer values (and that m^2 must be a positive, rather than a negative, constant). The general solution of Equation (2.2.5) is a linear combination of r^m and r^{-m} factors ,except for the special case $m = 0$,when it is a linear combination of r^0 and $\ln r$ factors. Thus, the general stream function for steady two-dimensional irrotational flow (that extends over all values of θ) takes the form :

$$\phi(r,\theta)=\alpha_0-\beta_0\theta+\sum_{m>0}\left(\alpha_m r^m-\beta_m r^{-m}\right)\cos\left[m\left(\theta-\theta_0\right)\right] \quad \textbf{(2.2.7)}$$

where α_m, β_m, and θ_m are arbitrary constants.

We can recognize the first few terms on the right-hand side of the previous expression. The constant term α_0 has zero gradient, and therefore does not give rise to any flow. The term $\beta_0 \ln r$ is the flow pattern generated by a vortex filament of intensity $2\pi\beta_0$, coincident with the z-axis. The term $\alpha_1 r\sin(\theta-\theta_1)$ corresponds to uniform flow of speed α_1 whose direction subtends a (counter-clockwise) angle θ_1 with the minus x-axis. Finally the term $\beta_1\sin(\theta-\theta_1)/\mathrm{r}$ corresponds to a dipole flow pattern.

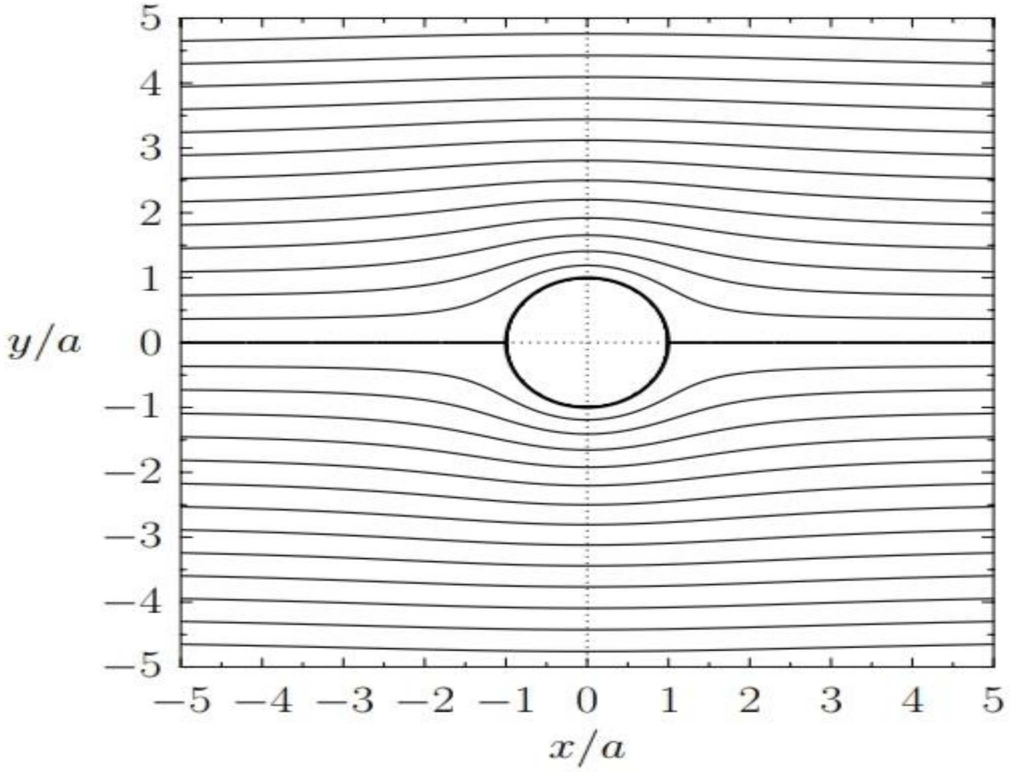


Figure -1 The normalized circulation of a cylindrical obstacle in a uniform flow field, with its axis running along the z-axis, is characterized by its streamlines.

The velocity potential associated with the irrotational stream function (2.2.7) satisfies .

$$\frac{\partial\phi}{\partial r}=\frac{1}{r}\frac{\partial\psi}{\partial\theta} \quad \textbf{(2.2.8)}$$

$$\frac{1}{r}\frac{\partial\phi}{\partial\theta}=-\frac{\partial\psi}{\partial r} \quad \textbf{(2.2.9)}$$

It follows that :

$$\phi(r,\theta)=\alpha_0-\beta_0\theta+\sum_{m>0}\left(\alpha_m r^m-\beta_m r^{-m}\right)\cos\left[m\left(\theta-\theta_0\right)\right] \quad \textbf{(2.2.10)}$$

2.3 Applications of the theory of irrotational flow in a circular cylinder

The instantaneous motion of a rigid cylinder is completely determined by its central velocity and the angular velocity of the cylinder; the latter motion has no effect on thin liquids and is ignored. We'll look at the shape of streamlines. It's easiest to see what streamlines mean when the motion is related to an axis fixed in a cylinder, mainly because the cylinder surface itself is a streamline. Therefore, the required velocity potential and flow functions are those that describe the rotation-free flow due to a cylinder of radius a maintained in a uniform velocity $\left(-U,-V\right)$ flow away from the cylinder and circulating κ around the cylinder.

The corresponding complex potential is known to be:

$$\omega(z)=-(U-iV)z-(U+iV)\frac{a^2}{z}-\frac{i\kappa}{2\pi}\log\frac{z}{a} \qquad \textbf{(2.3.1)}$$

In the case of this object with circular symmetry, allowing V to be non-zero does not give us greater generality in terms of instantaneous motion, so we assume that the cylinder and the fluid are at infinity The relative motion is parallel to the x-be axis. Then the velocity potential land current functions are:

$$\begin{aligned}\phi&=-U\left(r+\frac{a^2}{r}\right)\cos\theta+\frac{\kappa\theta}{2\pi}\\ \psi&=-U\left(r-\frac{a^2}{r}\right)\sin\theta-\frac{\kappa}{2\pi}\log\frac{r}{a}\end{aligned} \qquad \textbf{(2.3.2)}$$

Where $r^2=x^2+y^2$ and $\theta=\tan^{-1}y/x$ there is a simple infinite family, corresponding to different flow fields with different values of κ/aU . The case's $\kappa/aU=0$ streamlined pattern is the only one that is symmetrical about the x-axis.

We gain insight into the effect of changing this value of κ/aU by noting that the fluid velocity at the surface of the cylinder is as follows:

$$\left(\frac{1}{r}\frac{\partial\phi}{\partial\theta}\right)_{r=a}=2U\sin\theta+\frac{\kappa}{2\pi a} \qquad \textbf{(2.3.3)}$$

and disappear at two points:

$$\sin\theta=-\frac{\kappa}{4\pi aU}$$

The two stationary points located at the front and rear of the cylinder $\kappa=0$ move downward as κ/aU the motion increases and merge at $\theta=-\frac{\pi}{2}$ when $\kappa/aU=4\pi$.

Simplify the cases $0<\kappa/aU<4\pi$ and the processes for the corresponding cases $\kappa/aU=4\pi$, for value of κ/aU greater than 4π , the velocity is non-zero and increases at all points on the cylinder surface .

The stationary point here has moved along a line $\theta = -\dfrac{\pi}{2}$ away from the cylinder and is at a radial position given by one of the roots (the other referring to a motion inside the cylinder)

$$\left(\frac{1}{r}\frac{\partial\phi}{\partial\theta}\right)_{\theta=-\frac{\pi}{2}} = -U\left(\mathrm{I}+\frac{a^2}{r^2}\right)+\frac{\kappa}{2\pi r}=0, \quad \text{since } \partial\phi/\partial r = 0$$

On this line pass symmetry. The streamlines show that some fluid simply circulates around the cylinder and remains near the cylinder.

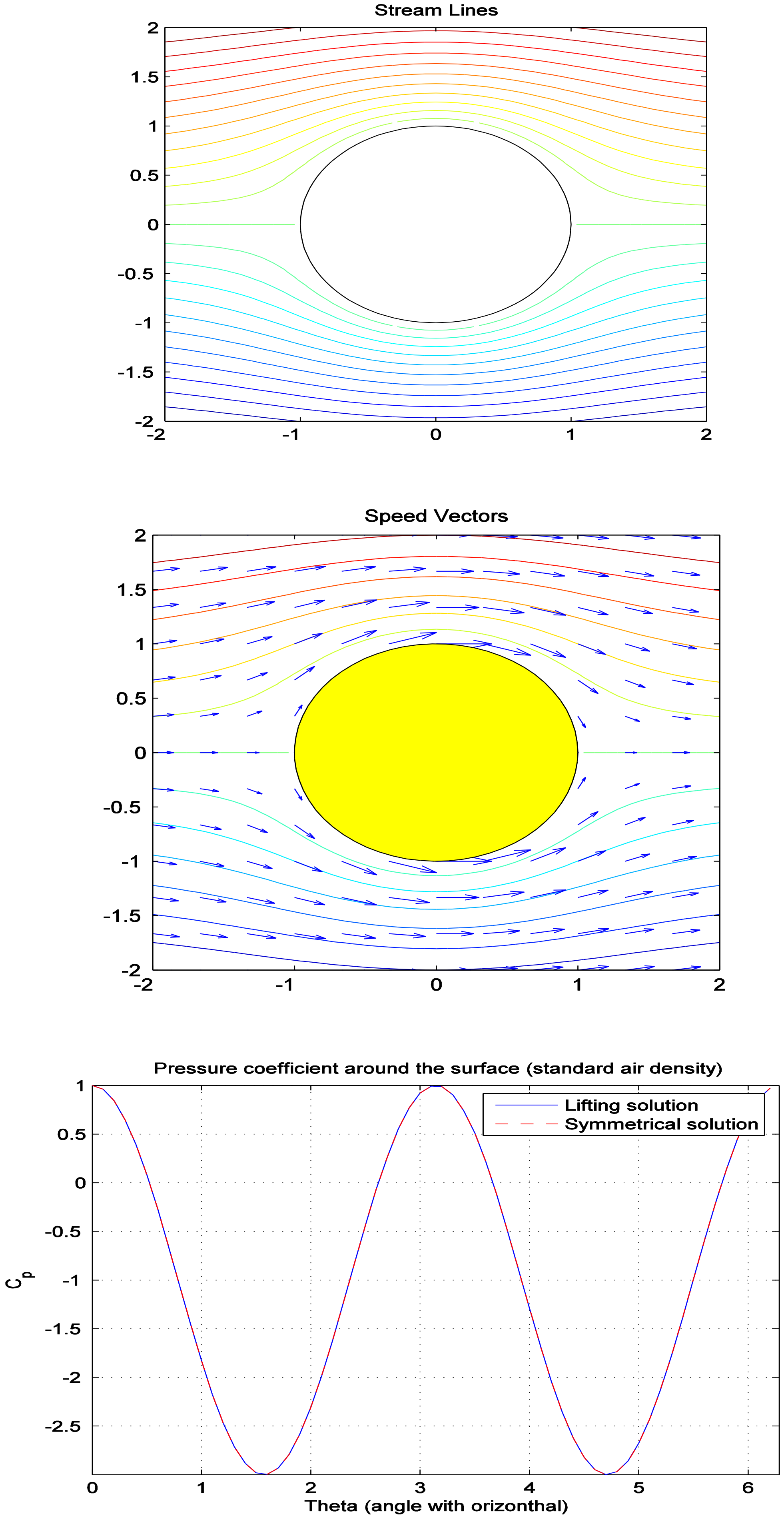
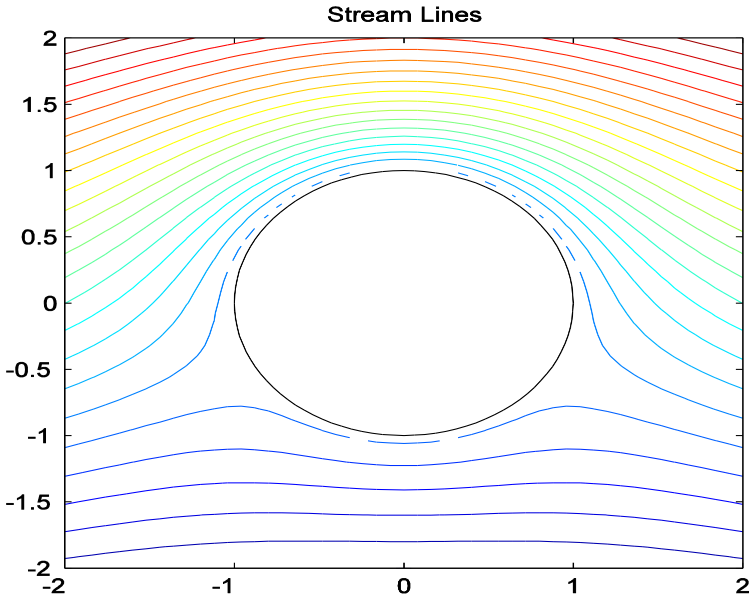


Figure 2 - Streamlines for flow around a circular cylinder and speed vectors in case $\kappa = 0$.



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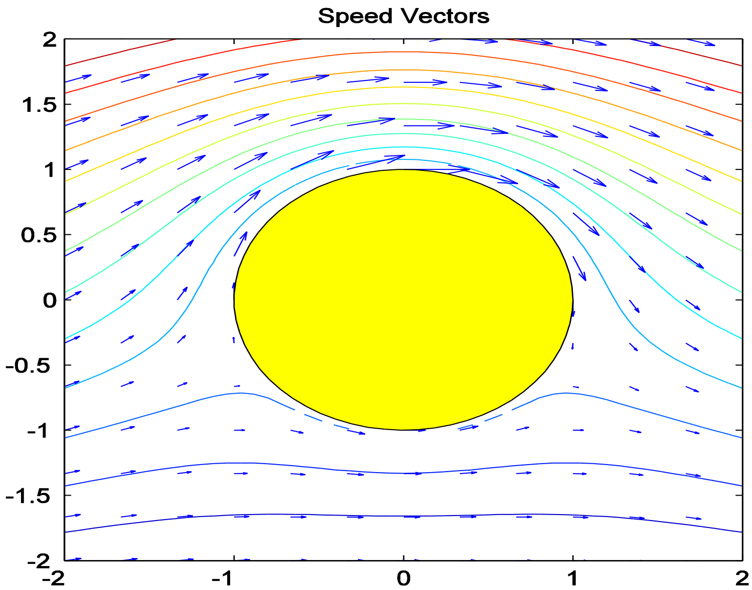

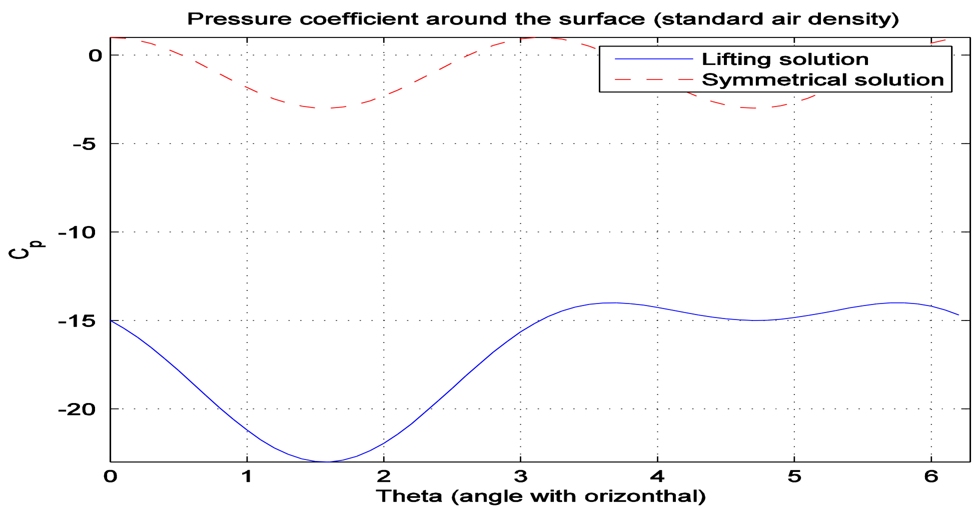

Figure 3 - Streamlines for flow around a circular cylinder and speed vectors in case $\kappa / aU = 4\pi$.

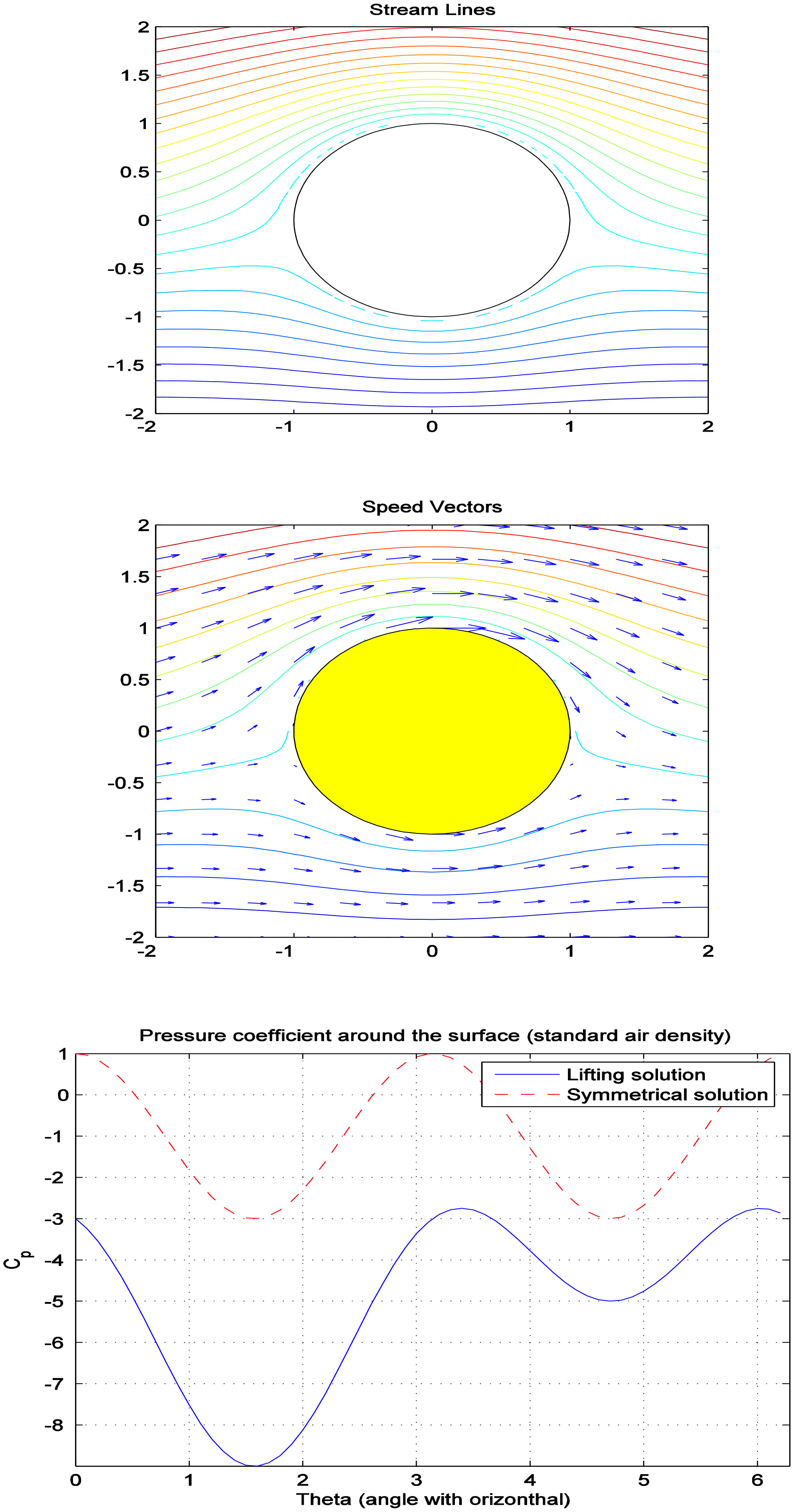

Figure 4 - Streamlines for flow around a circular cylinder and speed vectors in case $\kappa / aU = 2\pi$ **.**

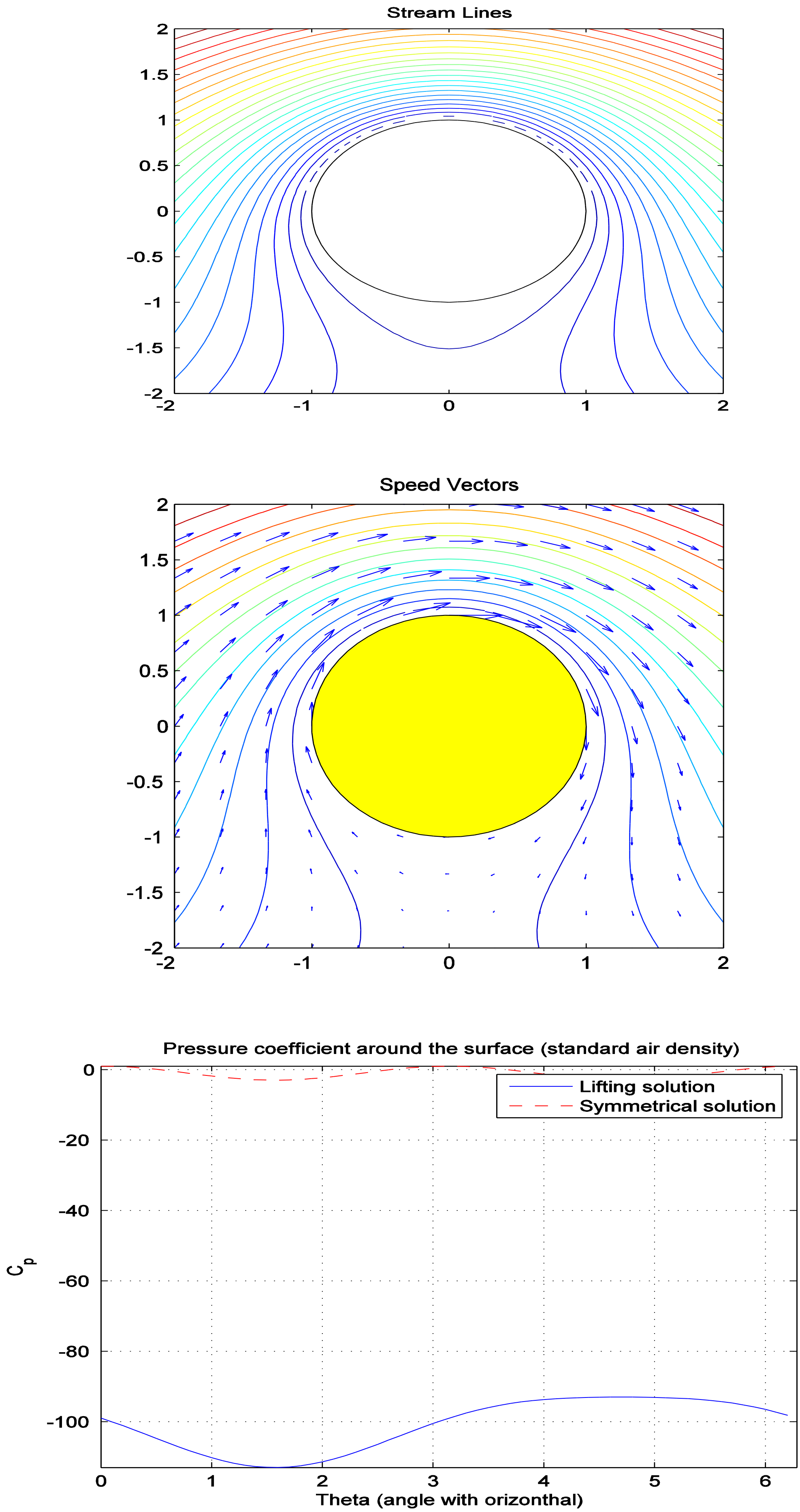

Figure 5 - Streamlines for flow around a circular cylinder and speed vectors in case $\kappa / aU = 8\pi$.

2.4 Flow About a Cylinder without Circulation

Inviscid-incompressible flow around a cylinder in uniform flow is the combination of a uniform flow and a doublet, with the potential and stream function combining.

Potential Function :

$$\phi = U_0 x + \frac{\chi \cos\theta}{r}$$

Stream function :

$$\psi = U_0 y - \frac{\chi \sin\theta}{r}$$

Streamlines

In two dimensional flow, a streamline may be interpreted as:

A streamline is the tangential velocity vector on a surface with no flow normal to the surface, forming the contour of an impervious two-dimensional body.

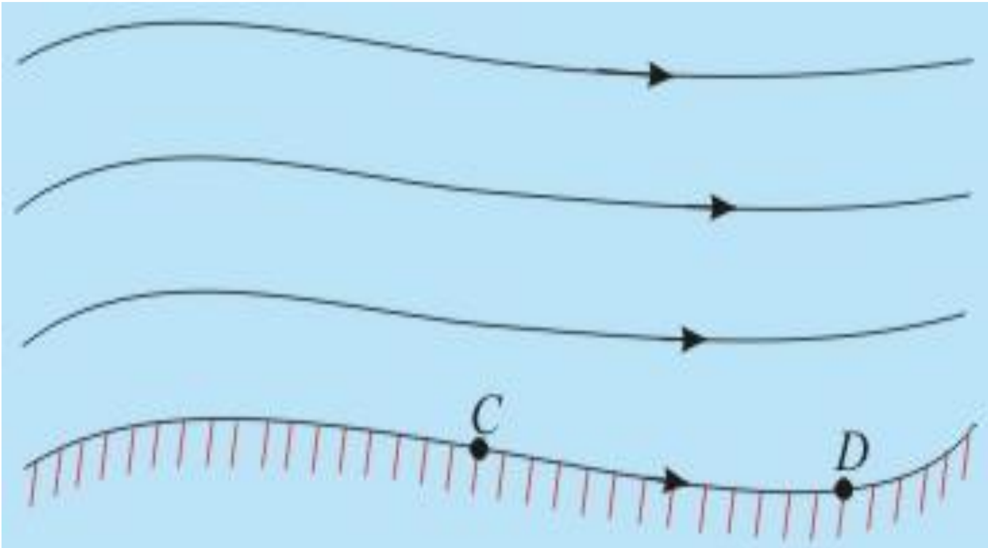

Figure 6 - Surface Streamline.

1. The streamline C-D may be considered as the edge of a two-dimensional body .
2. Other streamlines form the flow about the boundary.

To create a flow around a body, a streamline encloses a crucial shape in fluid flow, while the remaining streamlines outside this solid region form the flow.If we look for the streamline whose value is zero, we will obtain:

$$U_0 y - \frac{\chi \sin\theta}{r} = 0 \qquad \textbf{(2.4.1)}$$

Replacing y by $r\sin\theta$, we have:

$$\sin\theta\left(U_0 r - \frac{\chi}{r}\right) = 0 \qquad \textbf{(2.4.2)}$$

So the solution of this equation is :

If $\theta = 0$ or $\theta = \pi$, the equation is satisfied. This indicates that the x-axis is a part of the streamline $\psi = 0$.

When the quantity in the parentheses is zero, the equation is identically satisfied . Hence it follows that :

$$r = \left(\frac{\chi}{U_0}\right)^{1/2} \qquad \textbf{(2.4.3)}$$

Interpretation of the solution

There is a circle of radius $r = \left(\frac{\chi}{U_0}\right)^{1/2}$ which is an intrinsic part of the streamline $\psi = 0$.

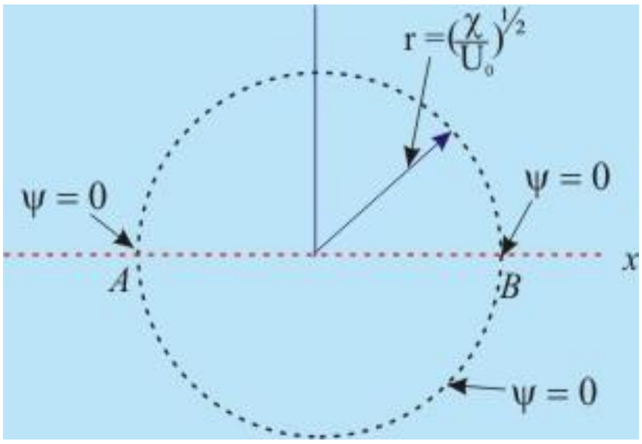

Figure 7 - The streamline $\psi = 0$ is characterized by stagnation points in a superimposed flow of doublet and uniform streams.

The velocity at specific points is determined by calculating partial derivatives of the velocity potential in two orthogonal directions and adjusting the appropriate coordinate values.

$$\phi = U_0 r \cos\theta + \frac{\chi \cos\theta}{r}$$

Since :

$$v_r = \frac{\partial \phi}{\partial r} = U_0 \cos\theta - \frac{\chi \cos\theta}{r^2}$$

$$v_\theta = \frac{1}{r}\frac{\partial \phi}{\partial r} = \left[-U_0 \, \text{r}\sin\theta - \frac{\chi \sin\theta}{r}\right] = -U_0 \, \text{r}\sin\theta - \frac{\chi \sin\theta}{r^2}$$

Point A:

$$\left[\theta = \pi, r = \left(\frac{\chi}{U_0}\right)^{1/2}\right]$$

$$v_r = 0, v_\theta = 0$$

Point B:

$$\left[\theta = 0, r = \left(\frac{\chi}{U_0}\right)^{1/2}\right]$$

$$v_r = 0, v_\theta = 0$$

The points A and B are the stagnation points through which the flow divides and subsequently reunites forming a zone of circular bluff body.

The solid cylinder in an inviscid flow can be visualized as the circular region enclosed by a portion of the streamline $\psi = 0$. In a cross-flow configuration, the flow moves uniformly at a significant distance from the cylinder.

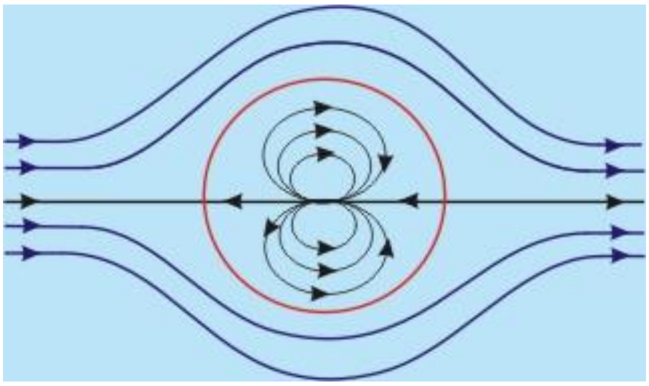

Figure 8 - Flow past a Cylinder without Viscosity.

1. The streamlines outside the circle describe the flow pattern of the inviscid irrotational flow across a cylinder.
2. The streamlines inside the circle may be disregarded since this region is considered as a solid obstacle.

CHAPTER 3

Study of fluid flow around the surface of a circular cylinder

3.1 Introduction

This chapter focuses on pure capillary free surface flows, where surface tension is considered but gravity is ignored. The fluid is non-viscous, incompressible, and non-rotational. Crapper found the exact solution for second-order free surface flows, which included nonlinear periodic waves traveling at a constant speed on the liquid's surface. Other branches of solutions were found by Blythe Crowdy and Vanden Broeck . The basic model used is the cavitation flow through a circular cylinder. Classical free rheological solutions are described, along with results related to open cavitis and the series truncation method.

Vanden Broeck's method for calculating curved cavities is also presented. Curved cavities are calculated analytically by Lighthill Special Geometry and numerically by Southwell and Vaisey. The integral boundary equation method generalizes Lighthill's work for constraints of arbitrary shapes.

The effect of surface tension on open cavities is addressed, with a specific contact angle defined at separation points. The position of separation points on the circular cylinder is uniquely defined by determining the values of T. Similar results can be obtained when a free surface intersects a smooth solid surface. Solutions with waves appearing in the far field are also discussed.

3.2 Problem formulation

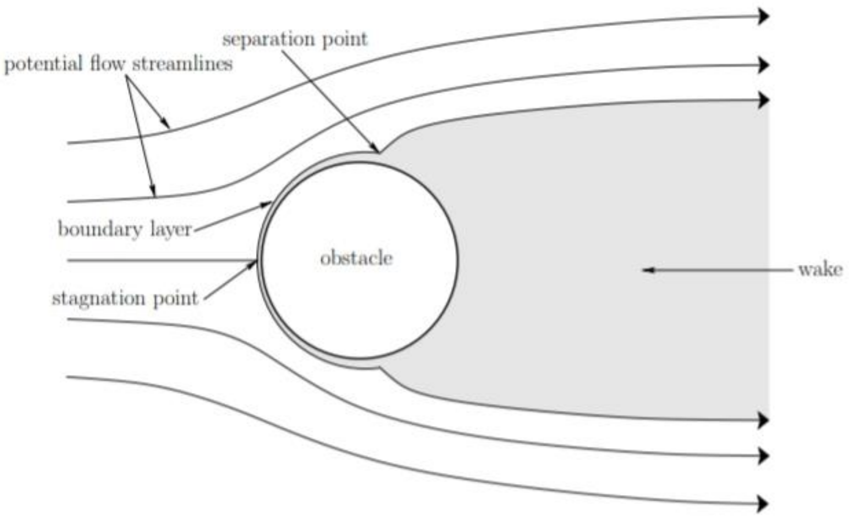

Figure 1 - Sketch of the flow past a circular cylinder with boundary layer separation.

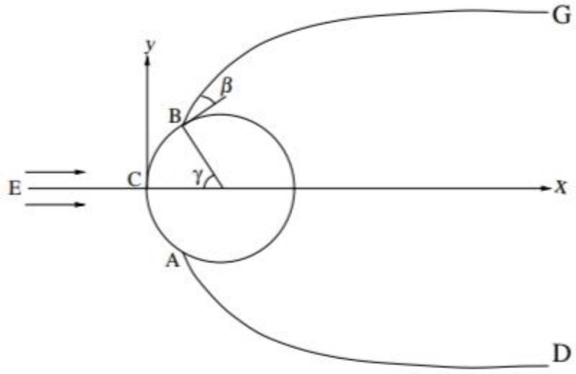

Figure 2 - Flow configuration and coordinates of surface tension T and angle β in cylinder.

In this section we review classical properties of free-streamline solutions, which will be useful in the remaining part of the paper. We consider the cavitating flow past a circle sketched in Figure 1.

We denote by R the radius of the cylinder and by U the constant velocity of the flow at infinity. We assume that the fluid is inviscid and incompressible and that the flow is irrotational. We introduce Cartesian coordinates x and y and assume that the flow is symmetric with respect to the $x-axis$. The cavity is bounded by the streamlines BG and AD and by the portion BA of the cylinder. The cavity is open in the sense that it is unbounded as $x \to \infty$.

We define dimensionless variables by using the radius R of the cylinder as the reference length and the velocity U as the reference velocity.

We introduce the potential function $b\phi$, the stream function $b\psi$ and the complex potential $f = \phi + i\psi$. Without loss of generality we choose t at the point C and $\psi = 0$ on the streamlines ECAD and ECBG. The constant b is defined so that $\varphi = 1$ at the separation points A and B. The flow configuration in the complex potential plane is illustrated in the next Figure.

We introduce the complex velocity $u - iv$ and define the function $\tau - i\theta$ by the relation:

$$u - iv = e^{\tau - i\theta} \quad \textbf{(3.2.1)}$$

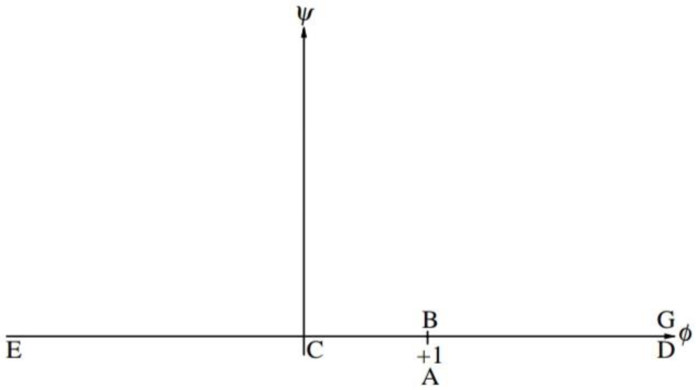

Figure 3 - The flow in the plane of the complex potential $f = \phi + i\psi$

Points	z -plan	f –plan
A	$x = -1,\ y = d$	$x = +1,\ y = 0$
B	$x = 1,\ y = -d$	$x = +1,\ y = 0$
C	$x = 0,\ y = 0$	$x = 0,\ y = 0$
D	$x = -\infty,\ y = -\infty$	$x = +\infty,\ y = 0$
G	$x = +\infty,\ y = +\infty$	$x = +\infty,\ y = 0$

Table 1 - Transformation from the z -plan to the f –plan

We will look for $\tau - i\theta$ as an analytical function of $f = \phi + i\psi$ in the half-plane $\psi < 0$. The complex potential plane is sketched in Figure 2 . To infinity we need the velocity Figure 1 schematic diagram of a cavity flow problem be a unit in the x direction so that the function $\tau - i\theta$ at infinity disappears in view A at the surface of the cavity , the Bernoulli's equation and the pressure created by it are present from (3.2.1).

We denote by p_∞ the constant pressure in the flow at infinity and by p_c the constant pressure in the cavity and we define the cavitation number C_c by the relation:

$$C_c = \frac{p_\infty - p_c}{\frac{1}{2}\rho U^2} \qquad \textbf{(3.2.2)}$$

Bernoulli's equation implies that :

$$\frac{1}{2}q^2 + \frac{p}{\rho} = \frac{1}{2}U^2 + \frac{p_\infty}{\rho} \qquad \textbf{(3.2.3)}$$

Everywhere in the fluid. Here q is the magnitude of the velocity and p is the pressure in th fluid. In the absence of surface tension, $p = p_c$ on the surface of the cavity. It follows from Equations (3.2.2) and (3.2.3) that :

$$q^2 = U^2(1 + C_c) \qquad \textbf{(3.2.4)}$$

On the surface of the cavity. Therefore the velocity q is constant on the free surfaces *BC* and *AD*. Such free surfaces are referred to as free streamlines.

Since the value of q on the free surface approaches U as $x \to \infty$, Equation (3.2.3) implies $C_c = 0$ and on BG and AD :

$$q = U \tag{3.2.5}$$

Solutions with $C_c = 0$ will be discussed in the next section.

Here u and v are the horizontal and vertical components of the velocity. The definition (3.2.5) has been used by many previous investigators. One of its advantage is that the curvature K of a streamline is given by the simple formula:

$$K = \frac{e^{\tau}}{b}\frac{\partial\theta}{\partial\phi} \tag{3.2.6}$$

We shall seek $\tau - i\theta$ as an analytic function of $\phi + i\psi$ in the half plane $\psi < 0$. The solution in $\psi > 0$ can then be obtained by symmetry. The boundary conditions on $\psi = 0$ are then given by:

$$\theta = 0 \quad \text{on} \quad \psi = 0 \quad -\infty < \phi < 0 \tag{3.2.7}$$

$$\frac{e^{\tau}}{b}\frac{\partial\theta}{\partial\phi} = 1 \text{ on } \psi = 0 \quad 0 < \phi < 1 \tag{3.2.8}$$

$$\tau = 0 \quad \text{on} \quad \psi = 0 \quad 1 < \phi < \infty \tag{3.2.9}$$

This completes the formulation of the problem. We seek $\tau - i\theta$ as an analytic function of $\phi + i\psi$ in $\psi = 0$ satisfying (3.2.7–3.2.9). We solve the problem by following the series truncation method.

We consider the open-wake model with the effect of the surface tension T included in the dynamic boundary condition. The condition $P = P_c$ on the surface AD of the cavity is then replaced by:

$$P = P_c - \frac{T}{\rho}K \tag{3.2.10}$$

Where K is the curvature of the free surface. Proceeding as we seek $\tau - i\theta$ as an analytic function of $\phi + i\psi$ in the lower half plane $\psi < 0$ of Figure 2,and:

$$-\frac{e^{\tau}}{b}\frac{\partial\theta}{\partial\phi} = \frac{\alpha}{2}\left(e^{2\tau} - 1\right) \quad \text{on} \quad \psi = 0 \ , \ 1 < \phi < \infty \tag{3.2.11}$$

Here α is Weber number defined by: $\alpha = \dfrac{\rho U^2 R}{T}$

The symmetry of the problem and the kinematic condition on the cylinder give:

$$\theta(\phi) = 0 \qquad (\psi = 0, \phi < 0) \tag{3.2.12}$$

$$F\left[x(\phi), y(\phi)\right] = 0 \quad (\psi = 0, 0 < \phi < 1) \tag{3.2.13}$$

3.3 Numerical procedure

First we have Kirchhof complex function Ω defined by:

$$\Omega = \log\left(\frac{u}{q}\right) + i\theta \tag{3.3.1}$$

$$= \log\left(U\frac{dz}{df}\right) = -\tau + i\theta$$

$$\frac{\partial z}{\partial f} = e^{-\tau + i\theta}$$

$$\Rightarrow \frac{\partial f}{\partial z} = e^{\tau + i\theta} = u - iv$$

We find the analytical solution of the problem:
When the surface tension is neglected, reduces to the free line condition $\tau = 0$. Then we put $f = \phi + i\psi$. We define the new variable t by the transformation:

$$f^{\frac{1}{2}} = \left(t - \frac{1}{t}\right)\frac{1}{2i} \tag{3.3.2}$$

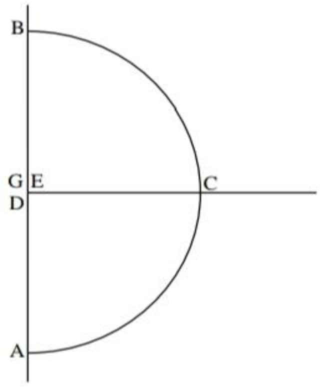

Figure 4 - Sketch of the flow of Figure 1 in the complex t-plan.

Points	f - plan	t –plan
A	$x = +1,\ y = 0$	$x = 0,\ y = -\frac{\pi}{2}$
B	$x = +1,\ y = 0$	$x = 0,\ y = \frac{\pi}{2}$
C	$x = 0,\ y = 0$	$x = 1,\ y = 0$
D	$x = +\infty,\ y = 0$	$x = 0,\ y = 0$
G	$x = +\infty,\ y = 0$	$x = 0,\ y = 0$

Table 2 - Transformation from the f - plan to the t -plan

$$\theta = 0 \qquad \text{on} \quad 0 < \operatorname{Re}(t) < 1$$

$$\frac{e^{\tau}}{b}\frac{\partial\theta}{\partial\phi} = 1 \quad \text{on} \quad -\pi/2 < \sigma < 0$$

$$\tau = 0 \qquad -1 < \operatorname{Im}(t) < 0$$

For a symmetrical curved object, we are led to write as an analytical function of t by the relation:

$$\Omega(t) = -\log\left(\frac{1+t}{1-t}\right) + \sum_{n=1}^{\infty} A_n t^{2n-1} \tag{*}$$

We assume here that A_n is real, and that the series has only an odd power of t. This is subject to the following 2 conditions :

- On AO $(\theta = 0)$ is real $\Rightarrow t$ is real .

  $\log\frac{1+t}{1-t} \Rightarrow \sum_{n=1}^{\infty}\xi$ either must be real .

- On the free surface $u^2 + v^2 = e^{2\tau}$ since $u - iv = e^{\tau - i\theta}$ we have $u = e^{\tau}\cos\theta$ and $v = e^{\tau}\sin\theta$.

We determine the coefficients A_n . This is done numerically by series truncation and collocation thus we truncate the infinite series, and we have:

$$-\tau + i\theta = -\log\left(\frac{1+e^{i\sigma}}{1-e^{i\sigma}}\right) + \sum_{n=1}^{\infty} A_n e^{i(2n-1)\sigma} \qquad \textbf{(3.3.3)}$$

We have $N+1$ unknowns, i.e. $A_1, A_2, \ldots\ldots, A_N$. On the unit circle we write $t = e^{i\sigma}$ where $-\pi/2 < \sigma < \pi/2$. let's remember that:

$$\frac{\partial z}{\partial f} = \frac{\partial x}{\partial \varphi} + i\,\frac{\partial y}{\partial \varphi} = e^{-\tau + i\theta}$$

$$e^{-\tau}\left(\cos\theta + i\,\sin\theta\right) = \frac{\partial x}{\partial \varphi} + i\,\frac{\partial y}{\partial \varphi}$$

$$e^{-\tau}\cos\theta + ie^{-\tau}\sin\theta = \frac{\partial x}{\partial \varphi} + i\,\frac{\partial y}{\partial \varphi}$$

$$\Rightarrow \begin{cases} \dfrac{\partial x}{\partial \varphi} = e^{-\tau}\cos\theta \\ \dfrac{\partial y}{\partial \varphi} = e^{-\tau}\sin\theta \end{cases} \qquad \textbf{(3.3.4)}$$

And we have: $f^{\frac{1}{2}} = \left(t - \frac{1}{t}\right)\frac{1}{2i}$

And in the other hand : $t = e^{i\sigma}$

Hence:

$$f^{\frac{1}{2}} = \left(e^{i\sigma} - e^{-i\sigma}\right).\frac{1}{2i}$$

$$= \left(\cos\theta + i\,\sin\theta - \cos\theta + i\,\sin\theta\right).\frac{1}{2i}$$

$$= \sin\theta$$

And on the free surface : $f = \phi + i\psi \quad , \psi = 0$

So: $\phi^{\frac{1}{2}} = \sin\sigma$

$$\phi = \sin^2\sigma$$

$$\Rightarrow d\varphi = 2\cos\sigma\sin\sigma d\sigma$$

This relationship is used in (3.3.3), we try this:

$$dx = e^{-\tau}\cos\theta\sin 2\sigma d\sigma$$

$$dy = e^{-\tau}\sin\theta\sin 2\sigma d\sigma$$

$$\Rightarrow \frac{\partial x}{\partial \sigma} = e^{-\tau}\cos\theta\sin 2\sigma$$

$$\Rightarrow \frac{\partial y}{\partial \sigma} = e^{-\tau}\sin\theta\sin 2\sigma$$

By replacing $t = e^{i\sigma}$ in equation (3.3.3), we obtain:

$$\tau(\sigma)=\log\left(\frac{-\sin\sigma}{1-\cos\sigma}\right)-\sum_{n=1}^{N}A_n\cos(2n-1)\sigma \qquad (**)$$

On the other hand, we have:

$$1+e^{i\sigma}=1+\cos\sigma+i\sin\sigma$$

And:

$$1-e^{i\sigma}=1-\cos\sigma-i\sin\sigma$$

So:
$$\frac{1+e^{i\sigma}}{1-e^{i\sigma}}=\frac{(1+\cos\sigma+i\sin\sigma)(1-\cos\sigma+i\sin\sigma)}{(1-\cos\sigma-\sin\sigma)(1-\cos\sigma+i\sin\sigma)}$$

$$=\frac{i\sin\sigma}{1-\cos\sigma}$$

We substitute in the relation (**) we find:

$$-\tau+i\theta=-\left[\log\left(\frac{|\sin\sigma|}{1-\cos\sigma}\right)-\frac{i\pi}{2}\right]+\sum_{n=1}^{\infty}\left[A_n\left(\cos(2n-1)\sigma+i\sin(2n-1)\sigma\right)\right]$$

$$=-\log\left(\frac{|\sin\sigma|}{1-\cos\sigma}\right)+\sum_{n=1}^{N}A_n\cos(2n-1)\sigma+i\left(\frac{\pi}{2}+\sum_{n=1}^{N}A_n\sin(2n-1)\sigma\right)$$

Hence:

$$\theta(\sigma)=\left(\frac{\pi}{2}-\sum_{n=1}^{N}A_n\sin(2n-1)\sigma\right) \qquad \textbf{(3.3.5)}$$

So:
$$\Rightarrow\frac{\partial\theta}{\partial\sigma}=\sum_{n=1}^{N}(2n-1)A_n\cos(2n-1)\sigma \qquad \textbf{(3.3.6)}$$

$$\tau(\sigma)=\log\left(\frac{-\sin\sigma}{1-\cos\sigma}\right)-\sum_{n=1}^{N}A_n\cos(2n-1)\sigma \qquad \textbf{(3.3.7)}$$

Finally, we substitute both equations (3.3.5) and (3.3.7) into the Bernoulli equation.

For $-\frac{\pi}{2}<\sigma<0$ by symmetry of the problem We substitute the relation (3.3.5) and (3.3.6) into the relation (3.3.4) and integrate to obtain x and y .

We solve the problem numerically. First we define the mesh points on the circumference of the unit circle by: $\sigma_I=-\frac{\pi}{2M}I \quad , \quad I=1,2,.......,M$

And M intermediate point of the mesh $\sigma_I^M=-(\pi/2M)\left(1-\frac{1}{2}\right)$ $,I=1,2,........,M$ From these we can calculate $(\partial x/\partial\sigma)_{\sigma=\sigma_I^M}$ and $(\partial y/\partial\sigma)_{\sigma=\sigma_I^M}$ Furthermore we obtain $x(\sigma_I)$ and $y(\sigma_I)$ from then :

$$\begin{cases}\dfrac{\partial x}{\partial\sigma}=\dfrac{x(\sigma_{I-1}^M)-x(\sigma_I^M)}{\sigma_{I-1}^M-\sigma_I^M}\\[2ex]\dfrac{\partial y}{\partial\sigma}=\dfrac{y(\sigma_{I-1}^M)-y(\sigma_I^M)}{\sigma_{I-1}^M-\sigma_I^M}\end{cases}$$

$$x(\sigma_I^M)=x(\sigma_{I-1}^M)-\left[e^{-\tau}\cos\theta\sin 2\sigma_I^M\right]\frac{\pi}{2M}$$

$$y\left(\sigma_I^M\right)=y\left(\sigma_{I-1}^M\right)-\left[e^{-\tau}\sin\theta\sin 2\sigma_I^M\right]\frac{\pi}{2M}$$

The equation $F\left(x,y\right)=0$, provides M algebraic equation:

$$F\left(x\left(\sigma_I\right),y\left(\sigma_I\right)\right)=0\ ,\ I=1,2,\ldots\ldots,M \qquad \textbf{(3.3.8)}$$

An additional equation is obtained by specifying the separation point A then:

$$x\left(\frac{-\pi}{2}\right)=\text{given quantity} \qquad \textbf{(3.3.9)}$$

The system (3.3.8), (3.3..9) is easily solved by Newton's method .

3.4. Discussion of results

Cavitation flow around a circular object is being analyzed. The reference length L is defined as the radius of the circle, leading to the following expression:

$$F\left(x,y\right)=\left(x-1\right)^2+y^2-1$$

We used the numerical scheme from section 3 to calculate solutions for different values of α . The coefficients A_n and b were found to decrease rapidly as n increases. For each value $\alpha\neq+\infty$, there exists a unique solution which leaves the obstacle tangentially. Typical profiles of the free surface for $\alpha=0$, $\alpha=2$ and $\alpha=+\infty$ are shown in Figure 4 .Each of these profiles is characterized by a different angular position γ of the separation points. The values of γ as a function of $\frac{1}{\alpha}$ are shown in Figure 5.

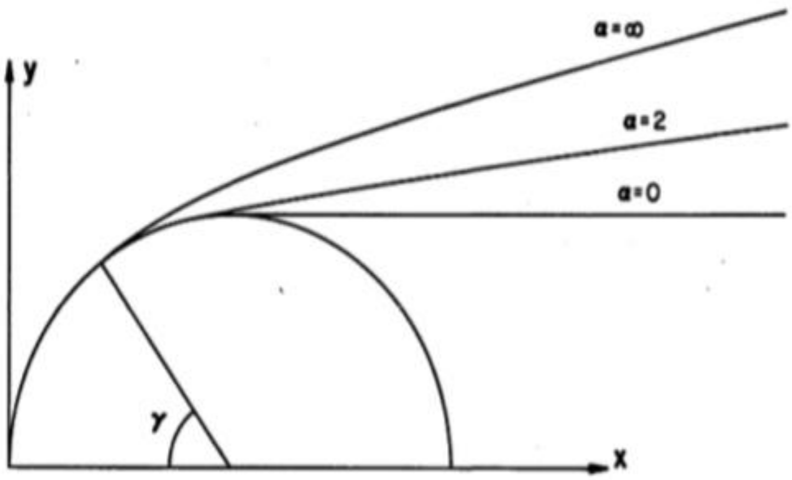

Figure 5 - Cavities in a stationary two-dimensional flow passing around a circle for different Weber number α values.

As $\alpha\to\infty$, our solution approaches the classical free-line solution that satisfies the Brillouin -Villat condition . This particular solution is defined by $\gamma=\gamma^*=55^\circ$. Consequently, the degeneracy of the problem at $T=0$ is resolved by solving the problem at $T\neq0$ and subsequently taking the limit as $T\to0$.

As $\alpha\to0$ and $\gamma\to\frac{\pi}{2}$, the profile of the free surface converges to two horizontal lines that are parallel to the direction of the velocity at infinity.

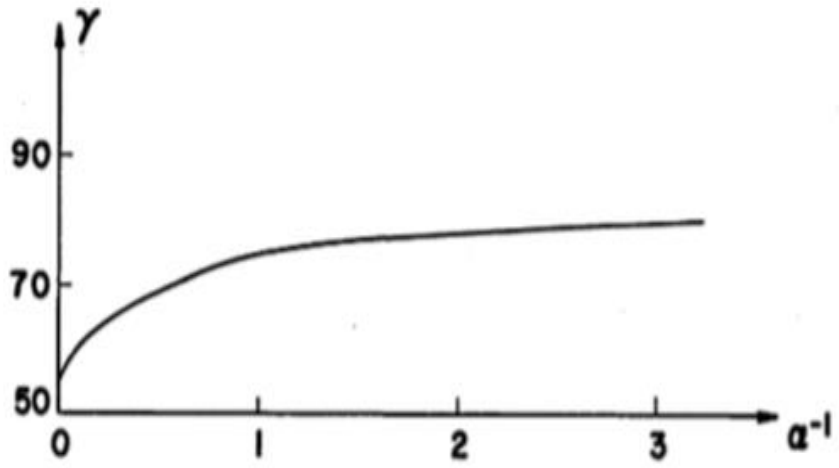

Figure 6 - Values of the angular position γ of the separation points for flow with cavitation around a circular cylinder as a function of $\frac{1}{\alpha}$

General conclusion

In this work we study the problem of flow of a fluid around a rotating cylinder where the fluids are incompressible and non-viscous. When force effects occur, surface tension and gravity are neglected. The solution to the problem is obtained explicitly and the shape of the free surface is determined parametrically using the theory of free streamlines, conformal transformations and the series truncation method. In this case, the surface tension is taken into account or the gravitational force is non-zero. The problem can only be solved through a numerical approach due to the nonlinear term in the case. At the edge of the free surface.

We are interested in finding the analytical and approximate solution to a two-dimensional problem related to the flow of fluid around a cylinder. We consider that the fluid is incompressible and invisible, and that the effects of gravity are neglected. This problem is characterized by the nonlinearity provided by the Bernoulli equation at the free surface, as well as the unknown shape of this flow. When the effect of surface tension is neglected, the exact solution can be obtained. But if we introduce the effect of surface tension, it becomes difficult to find the exact solution to this problem, so we resort to using the approximate method to find the approximate solution, and both methods depend on the theory of conformal transformations in addition to relying on the method String segmentation and analysis of the results with a Weber number α . These two methods show that the ease of their implementation gives very important simplifications to this type of problem. In both cases, the difficulty of the problem is reduced, and it is transformed from a two-dimensional problem to a one-dimensional problem and solve it .

ملخص

فـــي هذه المذكرة تطرقنا لموضوعين أساسيين ، أولهما : تدفق مائع حول أسطوانة دائرية مع اهمال تأثير الجاذبية و القوى السطحية ، حيث اعتمدنا في ذلك على التحويل المحافظ على الزوايا و خطوط التيار . أما الثاني : تدفق المائع حول أسطوانة دائرية مع الأخذ بعين الاعتبار التدفق السطحي وهذا باستخدام التحويل المحافظ على الزوايا و طريقة اقتطاع السلسلة .

الكلمات المفتاحية : السطح الحر، الجهد، خطوط التيار ، اقتطاع السلسلة ،التوتر السطحي ، عدد ويبر α .

Abstract

In this memory, we have discussed two fundamental subjects. The initial topic revolves around the fluid flow around a circular cylinder without considering the impact of gravity and surface forces. To analyze this, we have employed conformal transformations and current lines. The second topic focuses on the fluid flow around a circular cylinder, considering the surface flow. To examine this, we have utilized conformal transformations and the series truncation method.

Keywords: Free surface, potential, current lines, series truncation, surface tension, Weber number α .

Résumé

Dans ce mémoire, nous avons abordé deux sujets fondamentaux. Le sujet initial tourne autour de l'écoulement d'un fluide autour d'un cylindre circulaire sans tenir compte de l'impact de la gravité et des forces de surface. Pour analyser cela, nous avons utilisé des transformations conformes et des lignes de courant. Le deuxième sujet se concentre sur l'écoulement du fluide autour d'un cylindre circulaire, en considérant l'écoulement superficiel. Pour examiner cela, nous avons utilisé des transformations conformes et la méthode de troncature de serie.

Mots clés : Surface libre, potentiel, lignes de courant, troncature de série, tension superficielle, numéro Weber α .

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