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Intitulé

Calcul d'espace de phase pour la production de 2, 3 et 4 particules. Application sur l'intégration de l'espace de phase récursif

Soutenu le 04/05 /2017 devant le jury composé de:

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Calculation of the phase space of 2, 3 and 4 particles. Integration of the recursive phase space.

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Introduction

The High Energy Physics field is the branch of physics that try to understand how the universe works at its most fundamental level by exploring the elementary constituents of matter, probing the interactions between them, and discovering the basic nature of space and time. These interactions analyzed and described mathematically using Quantum field theory. In our work we will study one of these quantum field theory which called Quantum ChromoDynamics (QCD), that describe the strong interactions.

The study of the strong interactions was transformed with the advent of accelerators in higher-GeV energy range, where the first direct evidence for point-like constituents in the nucleons came from the discovery of scaling phenomenon in lepton-nucleon inclusive Deep-Inelastic Scattering (DIS) experiments at SLAC .

In late 1960s Bjorken and Feynman introduced the parton model to explain the experimental results obtained during DIS processes of electrons from protons, where the proton is shattered and a system with a large number of hadrons is produced, giving at the same time a justification of the mathematical structure of the quark model, and which led to the introduction of the fundamental field theory of strong interaction known as Quantum Chromodynamics (QCD).

Quantum Chromodynamics (QCD) is indeed a type of quantum field theory called a no-abelian gauge theory with symmetry group SU(3). It is the theory of the strong interaction, one of the four fundamental forces in nature. It describes the interactions between quarks and gluons, and in particular how they bind together to form hadrons. QCD has two particular phenomenon, called asymptotic freedom and color confinement.

In our work we are aiming to calculate the Drell-yan cross section, and study this mechanism properties using the perturbative quantum chromodynamic approach.

The Drell-Yan massive lepton-pair production in hadronic collisions provides a unique tool complementary to the Deep-Inelastic Scattering for probing the partonic substructures in hadron.

The main content of our work is roughly divided to three chapters. In the first chapter we attempt to follow chronological order trying to study the parton model and quantum chromodynamics; by describing the SLAC experiments results, and Bjorken scaling, after that the Feynman interpretation for the parton model, followed by a description of probability distribution functions (PDFs), and Evolution equation (DGLAP). After that we study QCD as a theory for this partons model.

In the second chapter we study the Drell -Yan process in the Leading order (LO). We calculate the cross section for the hardprocess (partonic cross section) between quark and antiquark, then use it in the calculation of the softprocess (hadronic cross section) between two protons using factorization theorem.

Finally, in the third chapter we study the Drell-yan process in next-to leading order NLO. We calculate the two-body phase space, three-body phase space, four-body phase space, in order to build recursive relation to calculate the total cross section at higher order of QCD radiation.

Chapter I

The parton model and QCD

I.1 Introduction

Since the mid-1970, most of physists have thought and agreed that the elementary particles that make up matter are a set of fermions, interacting primarily through the exchange of vector bosons.

These elementary particles include the leptons and the quarks. The Quark model which is developed by Murray-Gell-Mann allows to understand the hadrons spectroscopy, but not to describe hadrons interactions in high energy collisions. Until SLAC experiments came and probed new facts in high energy field!

I.2 SLAC experiment

The Stanford Linear Accelerator is 3.2 km long and has the capability of colliding electron and positron beams. SLAC also use to probe nucleons by scattering extremely short wavelength electrons from them. This produced the first convincing evidence of a quark structure inside nucleons in an experiment analogous to those performed by Rutherford long ago.

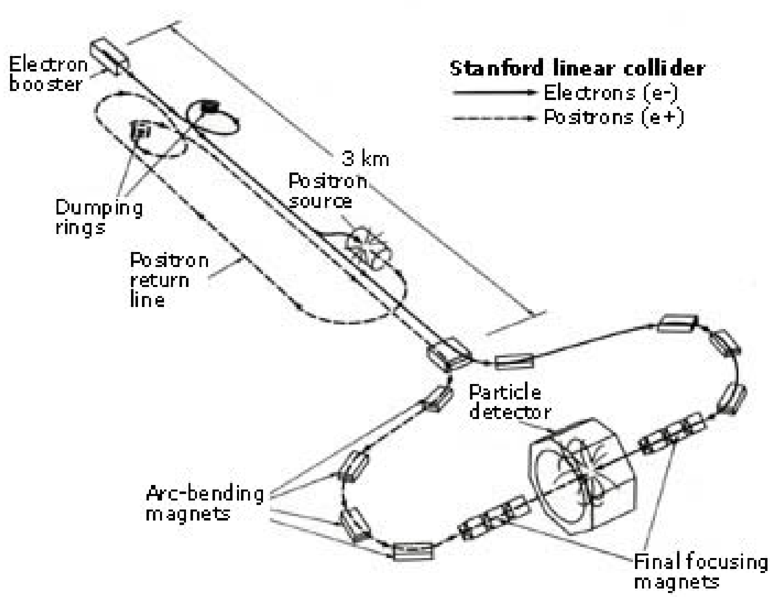


Figure I.1: Stanford Linear Accelerator Center (SLAC)

In the late 1960, in the SLAC(Stanford Linear Accelerator Center) experiments saw a substantial rate for hard scattering of electrons from protons. The total reaction rate was comparable to what would have been expected if the proton were an elementary particle scattering according to the example expectations from QED .

So, they used two levels of energies, caused two types of scattering: elastic scattering (ES), and deep inelastic scattering (DIS).

I.2.1 Elastic scattering cross section for e p scattering

The cross section for elastic scattering for $Q^2 < 1$Gev is given in form: [7]

$$|\frac{d\sigma}{d\Omega}\bigg|_{lab} = \frac{\alpha^2}{4E^2\sin\frac{\theta}{2}}\frac{E'}{E}\left(\frac{G_E^2+\tau G_M^2}{1+\tau}\cos\frac{\theta}{2} + 2\tau G_M^2\sin^2\frac{\theta}{2}\right) \quad \text{(I.1)}$$

Where, E, E' are the incoming electron energy and outgoing electron energy , respectively .

And

$$Q^2 = 4EE'\sin\frac{\theta}{2}$$

$$d\Omega = d\cos\theta d\varphi$$

G_E and G_M are the electric and magnetic factors respectively.

I.2.2 Deep inelastic scattering cross section for e p scattering

This process is for energies $Q^2 > 1Gev$

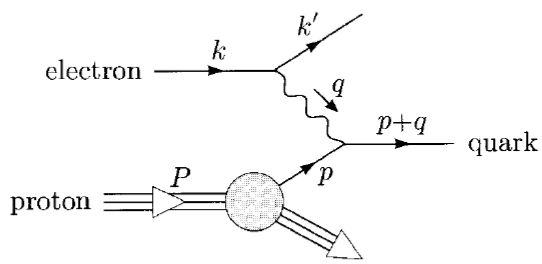


Figure I.2: Deep inelastic scatterig ep [3]

Kinematic variables

The four-momenta of the incoming and outgoing lepton are k and k' respectively, the momentum of the proton is p, and the momentum transferred by virtual photon $q = k - k'$.

The squared partonic center of mass(C.M) energy: $S = (p + p')^2$

Bjorken variable $x = \frac{Q^2}{2p.q} = \frac{Q^2}{2Mv}$, it is a dimensionless variable with :

	Proton	**parton**
Energy	E	xE
Momentum	P	xP
Mass	M	$m = \sqrt{x^2E^2 - x^2p^2} = xM$

The fraction of incident electron that is transferred to the hadronic system:

$$y = \frac{p.q}{p.k} = \frac{1 - E'}{E}$$

The invariant mass of hadronic system , produced by absorption of the virtual photon by the hadron :$W^2 = (p+q)^2 = Q^2(\frac{1}{x} - 1) + M$

Energy transferred by virtual photon: $v = \frac{p.q}{M}$

And M is the proton mass .

The square of the invariant amplitude for the inelastic scattering of an electron off a hadron is given by:

$$\overline{|M|}^2 = \frac{e^2}{Q^4} L_{\mu\upsilon} W^{\mu\upsilon} \tag{I.2}$$

Where $Q^2 = -q^2$ is the square of the virtual photon 4-momentum , and where the spin averaging factors have been absorbed into the tensors L and W , which describe the structure of the leptonic and hadronic vertices , respectively .

Calculation of the leptonic tensor $L_{\mu\upsilon}$

The amplitude for a virtual photon to "decay" into a e^+e^- pair is given by the Feynman rules by :

$$A_\mu(k', s', k, s) = -ie\overline{u}(k', s')\gamma_\mu v(k, s) \tag{I.3}$$

$$\begin{aligned}
L_{\mu\nu}(k', k) &= \frac{1}{2}\sum_{ss'} |A_\mu(k', s', k, s)|^2 \\
&= \frac{1}{2}\sum_{ss'} \left[-ie\overline{u}(k', s')\gamma_\mu v(k, s)\right]\left[iev^+(k, s)\gamma_\nu^+\overline{u}^+(k', s')\right] \\
&= \frac{1}{2}\sum_{ss'} e^2(\overline{u}(k', s')\gamma_\mu v(k, s))(v^+(k, s)\gamma_\nu^+\gamma^{0+}u(k', s')) \\
&= \frac{e^2}{2}\sum_{ss'}(\overline{u}(k', s')\gamma_\mu v(k, s))(v^+(k, s)\gamma^0\gamma_\nu u(k', s')) \\
&= \frac{e^2}{2}\sum_{ss'}\overline{u}(k', s')(\gamma_\mu)v(k, s)\overline{v}(k, s)(\gamma_\nu)u(k', s') \\
&= \frac{e^2}{2}k'^\alpha\gamma_\alpha(\gamma_\mu)k^\beta\gamma_\beta(\gamma_\nu) \\
&= \frac{e^2}{2}Tr(k'^\alpha\gamma_\alpha\gamma_\mu k^\beta\gamma_\beta\gamma_\nu) \\
&= 2e^2\left[k'^\alpha g_{\alpha\mu}k^\beta g_{\beta\nu} - k'^\alpha g_{\alpha\beta}k^\beta g_{\mu\nu} + k'^\alpha g_{\alpha\nu}k^\beta g_{\beta\mu}\right]
\end{aligned}$$

finaly the leptonic tensor is given by :

$$L_{\mu\nu}(k',k) = 2e^2\left[k'_\mu k_\nu + k'_\nu k_\mu - g_{\mu\nu}k'k\right] \tag{I.4}$$

where we have $m_e <<$

and

$$(\overline{u})^+ = (u^+\gamma^0)^+ = \gamma^{0+}u\ ,\ \gamma_\nu^+\gamma^{0+} = \gamma^0\gamma_\nu$$

The normalized Dirac spinors and antispinors completeness :

$$\sum_{s'} u_d(k',s')\overline{u}_a(k',s') = (k'^\alpha\gamma_\alpha)_{da} + m_e \tag{I.5}$$

$$\sum_{s} v_b(k,s)\overline{v}_c(k,s) = (k^\beta\gamma_\beta)_{bc} + m_e \tag{I.6}$$

$$Tr(\gamma_\alpha\gamma_\mu\gamma_\beta\gamma_\nu) = 4\left[g_{\alpha\mu}g_{\beta\nu} - g_{\alpha\beta}g_{\mu\nu} + g_{\alpha\nu}g_{\beta\mu}\right] \tag{I.7}$$

Calculation of The hadronic tensor $W^{\mu\upsilon}$

The hadronic tensor is given in the form :

$$W^{\mu\upsilon} = -W_1g^{\mu\nu} + \frac{W_2}{M^2}p^\mu p^\nu + \frac{W_3}{M^2}q^\mu q^\nu + \frac{W_4}{M^2}(p^\mu q^\nu + q^\mu p^\nu) \tag{I.8}$$

The current conservation at the hadronic vertex requires

$$q_\mu W^{\mu\nu} = q_\nu W^{\mu\nu} = 0 \tag{I.9}$$

Which results in

$$W_3 = \left(\frac{p.q}{q^2}\right)^2 W_2 + \frac{M^2}{q^2}W_1 \tag{I.10}$$

$$W_4 = -\frac{p.q}{q^2}W_2 \tag{I.11}$$

So

$$\begin{aligned} W^{\mu\nu} &= -W_1g^{\mu\nu} + \frac{W_2}{M^2}p^\mu p^\nu + \left[\left(\frac{p.q}{q^2}\right)^2 W_2 + \frac{M^2}{q^2}W_1\right]\frac{q^\mu q^\nu}{M^2} + \frac{1}{M^2}\left(-\frac{p.q}{q^2}W_2\right)(p^\mu q^\nu + q^\mu p^\nu) \\ &= W_1\left[-g^{\mu\nu} + \frac{q^\mu q^\nu}{q^2}\right] + \frac{W_2}{M^2}\left[\left(\frac{p.q}{q^2}\right)^2 q^\mu q^\nu + p^\mu p^\nu - \frac{p.q}{q^2}(p^\mu q^\nu + q^\mu p^\nu)\right] \end{aligned} \tag{I.12}$$

we have :

$$(a-b)^2 = a^2 + b^2 - (ab+ba) \tag{I.13}$$

We can make this variables changes

$$a = p^\mu \ , \ p^\nu \tag{I.14}$$

$$b = (\frac{p.q}{q^2})q^\mu \ , \ (\frac{p.q}{q^2})q^\nu \tag{I.15}$$

so we can rewrite (I.12)

$$\frac{W_2}{M^2}\left[\left(\frac{p.q}{q^2}\right)^2 q^\mu q^\nu + p^\mu p^\nu - \frac{p.q}{q^2}(p^\mu q^\nu + q^\nu p^\nu)\right] = \frac{W_2}{M^2}\left[\left(\frac{p.q}{q^2}\right) q^\mu \left(\frac{p.q}{q^2}\right) q^\nu + p^\mu p^\nu - p^\mu(\frac{p.q}{q^2}q^\nu) - (\frac{p.q}{q^2}q^\mu)p^\nu\right]$$

$$= \frac{W_2}{M^2}(p^\mu - \frac{p.q}{q^2}q^\mu)(p^\nu - \frac{p.q}{q^2}q^\nu)$$

finaly the hadronic tensor is given by :

$$W^{\mu\nu} = W_1\left[-g^{\mu\nu} + \frac{q^\mu q^\nu}{q^2}\right] + \frac{W_2}{M^2}(p^\mu - \frac{p.q}{q^2}q^\mu)(p^\nu - \frac{p.q}{q^2}q^\nu) \tag{I.16}$$

The procuct of the two tensors (leptonic and hadronic) results :

$$\begin{aligned}
L_{\mu\nu}W^{\mu\nu} &= L_{\mu\nu}\left[W_1\left[-g^{\mu\nu} + \frac{q^\mu q^\nu}{q^2}\right] + \frac{W_2}{M^2}(p^\mu - \frac{p.q}{q^2}q^\mu)(p^\nu - \frac{p.q}{q^2}q^\nu)\right] \\
&= -L_{\mu\nu}W_1 g^{\mu\nu} + L_{\mu\nu}\frac{W_2}{M^2}\left[p^\mu p^\nu\right] \\
&= -W_1 g^{\mu\nu} 2e^2\left[k'_\mu k_\nu + k'_\nu k_\mu - g_{\mu\nu}k'k\right] + \frac{W_2}{M^2}2e^2\left[k'_\mu k_\nu + k'_\nu k_\mu - g_{\mu\nu}k'k\right]\left[p^\mu p^\nu\right] \\
&= -W_1 2e^2\left[k'^\nu k_\nu + k'_\nu k^\nu - 4k'k\right] + \frac{W_2}{M^2}2e^2\left[(k'_\mu p^\mu)(k_\nu p^\nu) + (k'_\nu p^\nu)(k_\mu p^\mu) - k'k(p_\nu p^\nu)\right] \\
&= -W_1 2e^2\left[2k'k - 4k'k\right] + \frac{W_2}{M^2}2e^2\left[(k'.p)(k.p) + (k'.p)(k.p) - k'kM^2\right]
\end{aligned}$$

$$L^{\mu\nu}W^{\mu\nu} = e^2 4W_1 k'.k + \frac{W_2}{M^2}2e^2\left[2(k'.p)(k.p) - k'kM^2\right] \tag{I.17}$$

where

$$q_\mu L_{\mu\nu} = q_\nu L_{\mu\nu} = 0 \ , \ g^{\mu\nu}g_{\mu\nu} = 4 \ and \ p^\mu = \begin{pmatrix} M \\ 0 \\ 0 \\ 0 \end{pmatrix} \Longrightarrow p^\mu p_\mu = M^2$$

In the laboratory fram we have : $p.k = m_p E$; $p.k' = m_p E'$; $k.k' = EE'(1-\cos\theta)$ and $m_p = M$

The equation I.17 becomes :

$$\begin{aligned} L^{\mu\nu}W^{\mu\nu} &= e^2 4W_1 EE'(1-\cos\theta) + \frac{W_2}{M^2}2e^2\left[2M^2EE' - EE'(1-\cos\theta)M^2\right] \\ &= 4EE'e^2\left[W_1(1-\cos\theta) + W_2(1-\frac{1-\cos\theta}{2})\right] \\ &= 4EE'e^2\left[W_1(1-\cos\theta) + W_2(\frac{1+\cos\theta}{2})\right] \\ &= 4EE'e^2\left[2W_1(v,q^2)sin^2(\frac{\theta}{2}) + W_2(v,q^2)\cos^2(\frac{\theta}{2})\right] \end{aligned} \tag{I.18}$$

The cross section of DIS is the product of the phase space and squared impitude , and it is given in the form :

$$d\sigma = \frac{1}{4(p.k)}|\overline{M}|^2\frac{d^3k'}{2E_e(2\pi)^3}\frac{d^3p'}{2E_P(2\pi)^3}(2\pi)^4\delta^4((p+k)-(p'+k')) \tag{I.19}$$

$$= \frac{1}{16\pi^2}|\overline{M}|^2\frac{E'}{E}dE'd\Omega \tag{I.20}$$

where

$$|\overline{M}|^2 = \frac{e^2}{Q^4}4EE'e^2\left[2W_1(v,q^2)sin^2(\frac{\theta}{2}) + W_2(v,q^2)\cos^2(\frac{\theta}{2})\right] \tag{I.21}$$

so

$$\begin{aligned} \frac{d\sigma}{dE'd\Omega} &= \frac{1}{16\pi^2}\frac{E'}{E}\frac{e^2}{Q^4}4EE'e^2\left[2W_1(v,q^2)sin^2(\frac{\theta}{2}) + W_2(v,q^2)\cos^2(\frac{\theta}{2})\right] \\ &= \frac{1}{4\pi^2}\frac{e^4E'^2}{Q^4}\left[2W_1(v,q^2)sin^2(\frac{\theta}{2}) + W_2(v,q^2)\cos^2(\frac{\theta}{2})\right] \\ &= \frac{1}{4\pi^2}\frac{16\pi^2\alpha^2E'^2}{Q^4}\left[2W_1(v,q^2)sin^2(\frac{\theta}{2}) + W_2(v,q^2)\cos^2(\frac{\theta}{2})\right] \\ &= \frac{4\alpha^2E'^2}{Q^4}\left[2W_1(v,q^2)sin^2(\frac{\theta}{2}) + W_2(v,q^2)\cos^2(\frac{\theta}{2})\right] \\ &= \frac{\alpha^2}{E^2\sin^4(\frac{\theta}{2})}\left[2W_1(v,q^2)sin^2(\frac{\theta}{2}) + W_2(v,q^2)\cos^2(\frac{\theta}{2})\right] \end{aligned} \tag{I.22}$$

where $\alpha = \frac{e^2}{4\pi} \Longrightarrow e^4 = 16\pi^2\alpha^2$, $Q^2 = 2EE'\sin^4\frac{\theta}{2}$

$$\frac{d\sigma}{d\Omega} = \frac{\alpha^2}{4E^2\sin^4(\frac{\theta}{2})}\frac{E'}{E}\left[2W_1(v,q^2)sin^2(\frac{\theta}{2}) + W_2(v,q^2)\cos^2(\frac{\theta}{2})\right] \tag{I.23}$$

$$\frac{d\sigma}{d\Omega} = \sigma_{Mott}\frac{E'}{E}\left[2W_1(v,q^2)\tan^2(\frac{\theta}{2}) + W_2(v,q^2)\right] \tag{I.24}$$

where

$$\sigma_{Mott} = \left(\frac{\alpha\cos\frac{\theta}{2}}{2E\sin^2\frac{\theta}{2}}\right)^2 \tag{I.25}$$

$W_1(q^2, v)$ *and* $W_2(q^2, v)$ are called structure functions

By comparing (I.24) with csoss section of elastic scattering , and by consideration that the proton starts behaving like a free Dirac particle ($G_M = G_E = 1$) the proton structure functions thus become simply :

$$2W_1 = \frac{Q^2}{2M^2}\delta(v - \frac{Q^2}{2M}) \tag{I.26}$$

$$W_2 = \delta(v - \frac{Q^2}{2M}) \tag{I.27}$$

and by using the identity $\delta(\frac{x}{a}) = a\delta(x)$ we deduce the dimensionless structure functions :

$$2MW_1(v,Q^2) = \frac{Q^2}{2Mv}\delta(1 - \frac{Q^2}{2Mv}) \tag{I.28}$$

$$W_2(v,Q^2) = \delta(1 - \frac{Q^2}{2Mv}) \tag{I.29}$$

we can always write $W_{1,2}(q^2, v) \equiv W_{1,2}(q^2, x)$ as function of the Bjorking variable :

$$x = \frac{-q^2}{2Mv} \; and \; q^2$$

It was a great surprise in 1968 when , for the first time , the SLAC experiment showed that at large q^2 , the DIS cross section apeared much larger than expected . the structure function $W_{1,2}(x, q^2)$ extracted from data are found to behave very differently from the form factors squared $G^2_{E,M}(q^2)$. [1]

I.3 Parton Model

I.3.1 Bjorken scaling

This surprising behavior was in fact already anticipated by Bjorken in 1966 , from consideration based on the Gell-Mann quark model current algebra commentators, he discovered the scaling law according to which in the limit

$$-q^2 = Q^2 \longrightarrow \infty \; , \; v \longrightarrow \infty \; , \; with \; \frac{Q^2}{2Mv} = x \; fixed \tag{I.30}$$

The structure functions depend only on x :

$$W_1(q^2, v) \longrightarrow_{q^2,v\longrightarrow\infty} F_1(x)\ ,\ \frac{v}{M}W_2(q^2, v) \longrightarrow_{q^2,v\longrightarrow\infty} F_2(x) \tag{I.31}$$

The physical content of the Bjorking scaling law lies essentially in the finite limit of the structure function $F_1(x)$ *and* $F_2(x)$, since one can always write $W_1(q^2, v) = F_1(x, q^2)$ and $\frac{v}{M}W_2(q^2, v) = F_2(x, q^2)$. for each fixed value of x, when $-q^2 \longrightarrow \infty$, the limits of $F_1(x, q^2)$ *and* $F_2(x, q^2)$ can depend only on x. In principle, they may tend to infinity or zero .[1]

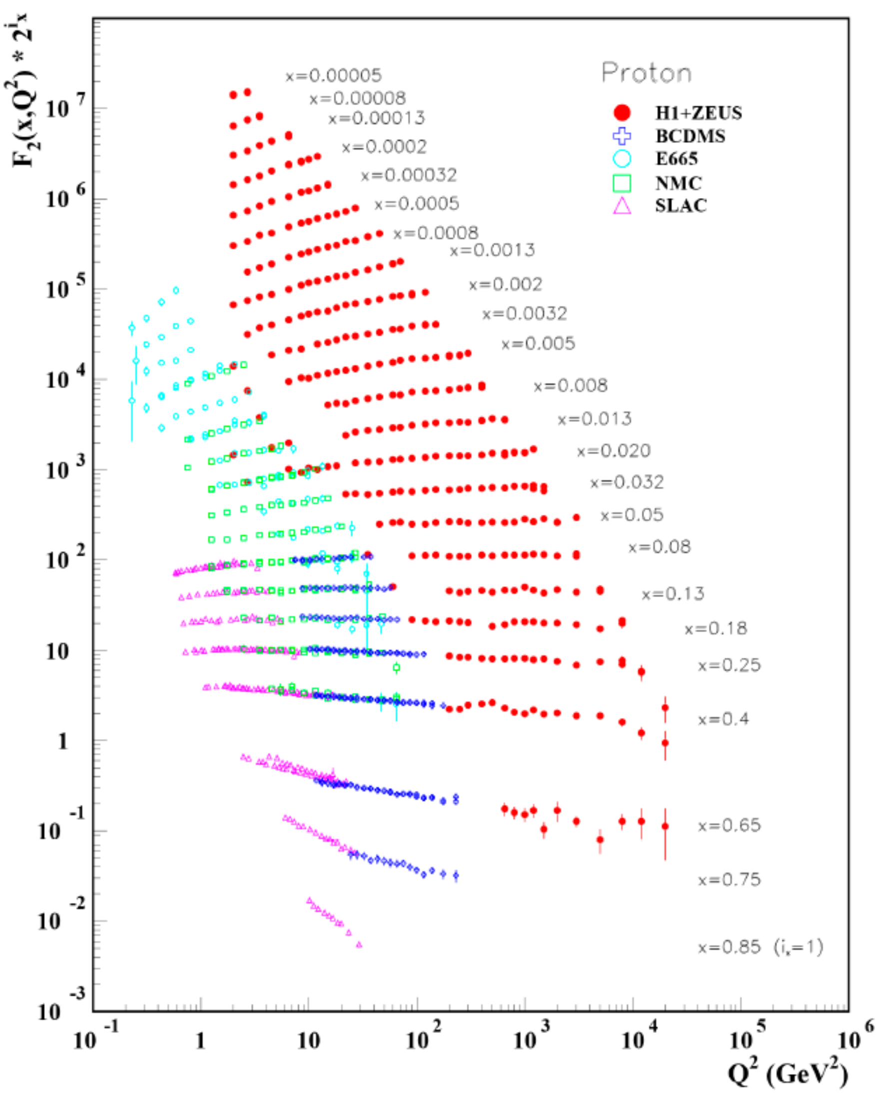


Figure I.3: Deviation from scaling, with increasing Q^2, the structure function $f_2(x, Q^2)$ increases at smal x and decreases at large x. [7]

I.3.2 Naïve parton model

Feynman came and interpreted the Bjorken scaling as the point-like nature of the nucleon's constituents when they were scattered by the incident electron.

Feynman named the point-like constituents partons. This is the parton model. Feyman left open the possibility that the partons need not be the quarks.

However, theorists quickly identify the partons with quarks (in the late 1960s and early 1970s QCD did not exist, and so gluons did not enter the picture).

A nucleon consists of three "valence" quarks which carry the nucleon's quantum numbers and a "sea" of quark-antiquark pairs.

The parton model accomplishes two things:

First, Bjorken scaling follows from the point-like constituents of the nucleon and the scattering from the incident electron.

second, parton model identifies the structure functions as the fractional longitudinal momentum distribution functions of the partons inside the nucleon. [4]

I.3.3 probability distribution functions

PDFs describe the way the momentum of an incoming high energy nucleon is partitioned among its constituent partons. But there is a problem! That we don't know how hadrons are constructed from partons because of the color confinement!

The problem simplified by consideration that partons could be free inside hadrons, and only distribution (PDF) of one dimensional momentum of the interacting partons in the hadron-hadron collision is necessary.

These PDFs are basic ingredients for calculating essentially all processes at hadron colliders, and they are obtained from experimental data.

Indeed, the sum of average mumentum fraction of each parton must be equal to the unite :

$$\sum_i \int xF_i(x)dx = 1 \tag{I.32}$$

Where $i \longrightarrow$ partons

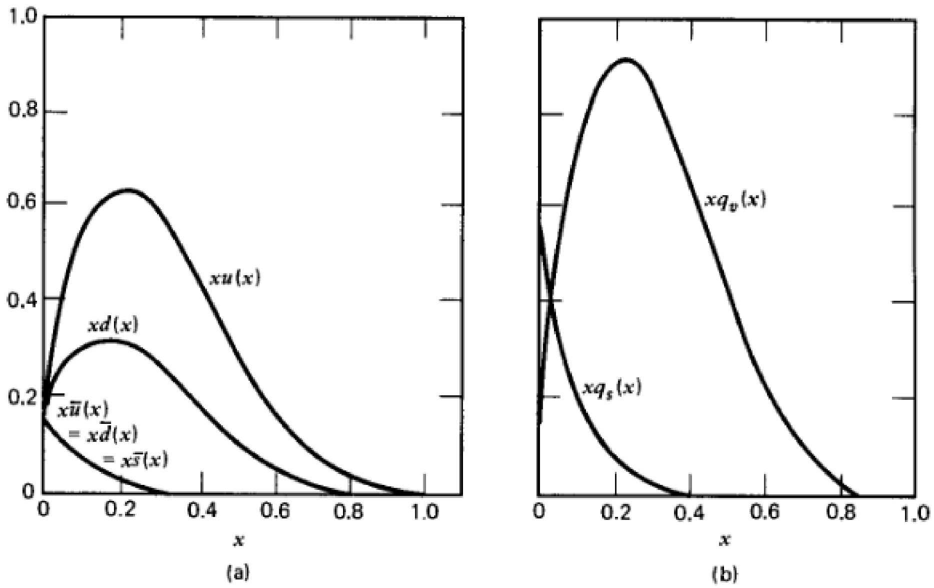


Figure I.4: The quark strusture functions extracted fron an analysis of DIS data. Figure (b) shows the total valence and sea quark contributions to the structure of the proton. [7]

The proton consists of three valence quarks(uud) which carry its electric charge and barym quantum numbers, and an infinite sea of $q\hat{q}$ pairs. In scale Q we have the sum rules

$$\int_0^1 dx u_v(x) = \int_0^1 dx(f_u(x) - f_{\hat{u}}(x)) = 2 \;\; i.e.two \;\; u - valence \;\; quarks.$$

$$\int_0^1 dx d_v(x) = \int_0^1 dx(f_d(x) - f_{\hat{d}}(x)) = 1 \;\; i.e.two \;\; d - valence \;\; quarks.$$

And, momentum has to be conserved

$$\int_0^1 dx \;\; x \left[f_g(x) + \sum_q (f_q(x) + f_{\hat{q}}(x)) \right] = 1$$

Experimentally the quarks only carry about 50% of the proton's momentum. The rest is carried by gluon constituents.

I.3.4 Evolution equation (DGLAP)

Also known as the Altarelli-Parisi equations (after development called DGLAP). They describe the coupled evolution of partons distributions $F_f(z, Q), F_{\overline{f}}(x, Q)$ for each flavor of quark and antiquark that can be treated as massless at the scale Q , together with the parton distribution of gluons, $F_g(x, Q)$. Explicitly :

$$\frac{dF_g(x,Q)}{d\log(Q)} = \frac{\alpha_s(Q^2)}{\pi}\int_x^1 \frac{dz}{z}\left(P_{g\longleftarrow q}(z)\sum_f\left[F_f(\frac{x}{z},Q)+F_{\overline{f}}(\frac{x}{z},Q)\right]+P_{g\longleftarrow g}(z)F_g(\frac{x}{z},Q)\right) \tag{I.33}$$

$$\frac{dF_f(x,Q)}{d\log(Q)} = \frac{\alpha_s(Q^2)}{\pi}\int_x^1 \frac{dz}{z}\left(P_{q\longleftarrow q}(z)F_f(\frac{x}{z},Q)+P_{q\longleftarrow g}(z)F_g(\frac{x}{z},Q)\right) \tag{I.34}$$

$$\frac{dF_{\overline{f}}(x,Q)}{d\log(Q)} = \frac{\alpha_s(Q^2)}{\pi}\int_x^1 \frac{dz}{z}\left(P_{q\longleftarrow q}(z)F_{\overline{f}}(\frac{x}{z},Q)+P_{q\longleftarrow g}(z)F_g(\frac{x}{z},Q)\right) \tag{I.35}$$

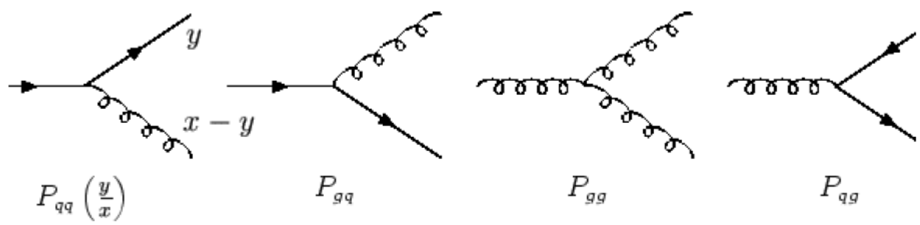


Figure I.5: The processes related to the lowest order QCD splitting functions. Each splitting function $P_{pp'}(\frac{y}{x})$ gives the probability that a parton of type p converts into a parton of type p', carrying a fraction $\frac{y}{x}$ of the momentum of parton p.

The splitting function $P_{i\longrightarrow j}(z)$ are given by :

$$P_{q\longleftarrow q}(z) = \frac{4}{3}\left[\frac{1+z^2}{1-z}+\frac{3}{2}\delta(1-z)\right] \tag{I.36}$$

$$P_{g\longleftarrow q}(z) = \frac{4}{3}\left[\frac{1+(1-z^2)}{z}\right] \tag{I.37}$$

$$P_{q\longleftarrow g}(z) = \frac{1}{2}\left[z^2+(1-z)^2\right] \tag{I.38}$$

Where splitting functions are probability that a parton (quark or gluon) emits up parton (quark or gluon) with momentum fraction $\xi = \frac{x}{z}$ of the parent parton . [3]

I.4 Quantum chromodynamics

The best natural candidate for a model of the strong interactions is the no-Abelian gouge theory with gouge group SU(3) , coupled to fermions (quarks) in the fundamental representation .

The fundamental representation is a triplet . The three color charges of a quark B,G and R form the fundamental representation of an SU(3) symmetry group .

Color is exchanged by eight bicolored gluons, where number of generators of this group is equal to eight.

Gluons themselves carry color charge , and so they can interact with other gluons . [7]

I.4.1 Feynman rules and Diagrams for QCD

the QCD Lagrangian is given by :

$$\Im_{QCD}(x) = -\frac{1}{4}F^{\alpha}_{\mu\upsilon}(x)F^{\mu\upsilon}_{\alpha}(x) + \overline{\psi_j}(x)(i(D^{\alpha}\gamma_{\alpha})_{ji} - m)\psi_i(x) - \frac{1}{2\xi}(\partial_{\mu}A^{\mu}_{\alpha}(x))^2 + \partial_{\mu}\eta^{\alpha^+}(D^{\mu}_{ab}\eta^b) \tag{I.39}$$

with

$$D^{\mu}_{ij} = \partial^{\mu}\delta_{ij} + ig(T^c)_{ij}A^{c\mu} \tag{I.40}$$

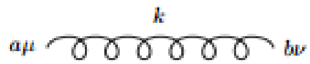
$$D^{\mu}_{ab} = \partial^{\mu}\delta_{ab} + ig(F^c)_{ab}A^{c\mu} \tag{I.41}$$

The matrices T are the generators of SU (3)group in the fundamental representation, and the matrices F are the generators in adjoint representation.

Quark propagator:

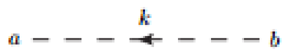
$$\delta_{ij}\frac{i}{\not{p}-m}$$

Gluon propagator:



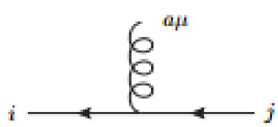
$$\delta_{ab}\frac{-i}{k^2}\left(g_{\mu\nu}-(1-\xi)\frac{k_\mu k_\nu}{k^2}\right)$$

Ghost propagator:



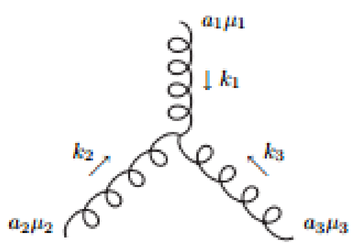
$$\delta_{ab}\frac{i}{k^2}$$

Quark-gluon vertex:



$$ig_s\gamma_\mu T^a_{ij}$$

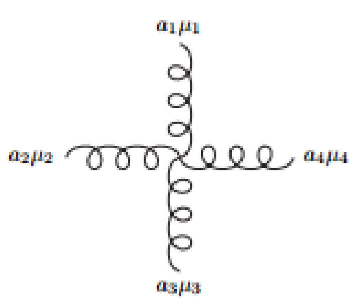
Three-gluon vertex:



$$g_s f^{a_1a_2a_3}V_{\mu_1\mu_2\mu_3}(k_1,k_2,k_3)$$

$$\begin{aligned}V_{\mu_1\mu_2\mu_3}(k_1,k_2,k_3) &= (k_1-k_2)_{\mu_3}\,g_{\mu_1\mu_2}+(k_2-k_3)_{\mu_1}\,g_{\mu_2\mu_3}\\ &\quad +(k_3-k_1)_{\mu_2}\,g_{\mu_3\mu_1}\\ &\quad \text{(all momenta incoming)}\end{aligned}$$

Four-gluon vertex:



$$-ig_s^2 W^{a_1a_2a_3a_4}_{\mu_1\mu_2\mu_3\mu_4}$$

$$\begin{aligned}W^{a_1a_2a_3a_4}_{\mu_1\mu_2\mu_3\mu_4} &= \left(f^{13,24}-f^{14,32}\right)g_{\mu_1\mu_2}g_{\mu_3\mu_4}\\ &\quad +\left(f^{12,34}-f^{14,23}\right)g_{\mu_1\mu_3}g_{\mu_2\mu_4}\\ &\quad +\left(f^{13,42}-f^{12,34}\right)g_{\mu_1\mu_4}g_{\mu_3\mu_2}\\ f^{ij,kl} &= f^{a_ia_ja}f^{a_ka_la}\end{aligned}$$

Ghost-gluon vertex:

$$g_s f^{abc}k_\mu$$

Figure I.6: Feynman Diagrams for QCD

I.4.2 Renormalization

Quantum field theory must be finite or can be made finite (renormalized) by introducing a finite number of counterterms into the original Lagrangian without changing its basic form.

In QCD we have a problem in Feynman diagrams with increasing of scale Q^2, that the quantity e (charge electric in QED) which appears in the lowest-order Feynman amplitudes, is changed by higher-order interactions. Therefore, that causes in appearance of loops, thus accur divergence in our quantum theory!

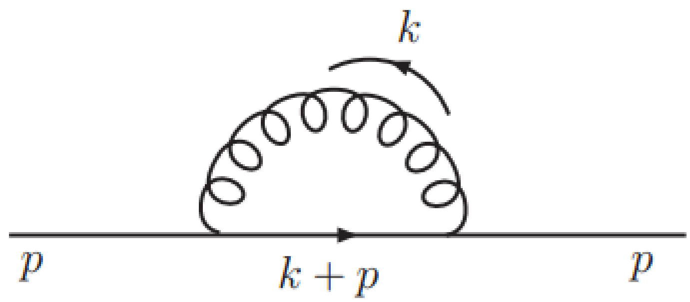


Figure I.7: The quark self-energy Feynman diagram

To remove divergences, we need to reparametrize (renormalise) new parameters: the "bare" charge e_0 and a second mass scale $\mu = Q^2$. where the ralation between e and e_0 is given by

$$e = e_0 \left[1 - \frac{1}{2} I(q^2 = -q^2) + O(e_0^4) \right]$$

this relation is the most experimental acceptibale terms, with the invariant amplitude verifies

$$M(e) = M(e_0)$$

It deos not matter that a theory is formulated in terms in finite quantities as long as abservable quantities are finite. Thus $|M|^2$ is an observable and must be independant of the value of μ, so the μ independence of M may be expressed by :

$$\mu^2 \frac{d}{d\mu^2} M(\frac{Q^2}{\mu^2}, \alpha_s) = \left[\mu^2 \frac{\partial}{\partial \mu^2} + \mu^2 \frac{\partial \alpha_s}{\partial \mu^2} \frac{\partial}{\partial \alpha_s} \right] M = 0 \tag{I.42}$$

which called normalization group equation[5]

By introducing these notations :

$$t = ln \left(\frac{Q^2}{\mu^2} \right) \ , \ \beta(\alpha_s) = \mu^2 \frac{\partial \alpha_s}{\partial \mu^2} \tag{I.43}$$

So, the derivation of the coupling in the definithion of β function is performed at fixed bare coupling. Then previous equation became

$$\left[-\frac{\partial}{\partial t} + \beta(\alpha_s)\frac{\partial}{\partial \alpha_s}\right] R(e^t, \alpha_s) = 0 \tag{I.44}$$

which solved by implicitly defining a new function, the running coupling $\alpha_s(Q^2)$:

$$t = \int_{\alpha_s}^{\alpha_s(q^2)} \frac{dx}{\beta(x)} \tag{I.45}$$

where $\alpha_s(\mu^2) = \alpha_s$

and by differentiating it, we obtian :

$$\frac{\partial \alpha_s(Q^2)}{\partial t} = \beta(\alpha_s(Q^2)) \tag{I.46}$$

$$\frac{\partial \alpha_s(Q^2)}{\partial \alpha_s} = \frac{\beta(\alpha_s(Q^2))}{\beta(\alpha_s)} \tag{I.47}$$

I.4.3 The Beta function

In QCD, the β function has the perturbative expansion

$$\beta(\alpha_s) = -b\alpha_s^2(1 + b'\alpha_s + b''\alpha_s^2 + O(\alpha_s^2)) \tag{I.48}$$

where b, b', b" are given on function of, n_f, which is the number of active light flavours. the alternative notation for β is

$$\beta(\alpha_s) = -\alpha_s \sum_{n=0}^{\infty} \beta_n(\frac{\alpha_s}{4\pi})^{n+1} = -\beta_0(\frac{\alpha_s^2}{4\pi}) - \beta_1(\frac{\alpha_s^3}{4\pi^2}) - \beta_2(\frac{\alpha_s^4}{4\pi^3}) + ... \tag{I.49}$$

$$\beta_0 = 11 - \frac{2}{3}n_f \tag{I.50}$$

I.4.4 Asymptotic freedom and color confinement

According to what we got before, we obtain the running coupling constant, which takes the following form

$$\alpha_s(Q^2) = \frac{\alpha_s(\mu^2)}{1 + \alpha(\mu^2)bt} \tag{I.51}$$

with :

$$t = \ln \frac{Q^2}{\mu^2} \ , \ b = \frac{1}{12\pi}(33 - 2n_f) \tag{I.52}$$

Hence, the running coupling constant QCD corresponds the anti-screening, where the virtual gluons surrounding the quarks to dilute the color charge, more Q^2 increase the coupling constant α_s decrease. At high Q^2 the coupling α_s tends to 0, which means that the quarks almost not interacting, this is the asymptotic freedom phenomenon .

Conversly, when two quarks separate and move away, the coupling constant increases, this is color confinement phenomenon. from ordre $\approx$ fermi, the interacting energy is sufficient to create new pair of $q\overline{q}$, therefore, the quarks are assembled and confinned into a neutral color hadron.

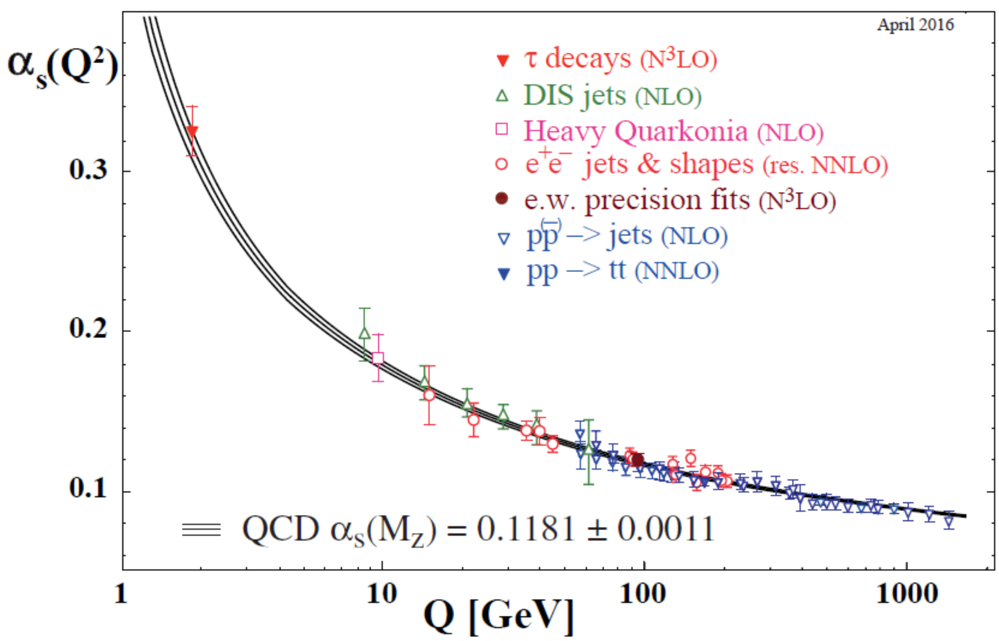


Figure I.8: Synthesis of different measurments of α_s in function of Q^2 [11]

QCD as the theory for strong interactions led to try improve it using Drell-Yan process, therefore the success of this mechanism has played an important role in validating it, and that what we will see next chapters.

Chapter II

Lower-ordre Drell-Yan process

II.1 Introduction to Drell-Yan process

The Drell-Yan process, proposed over years ago by Sid Drell and Tung-Mow Yan to describe high-mass lepton-pair production in hadron-hadron collision, has played an important role in validating QCD as the correct theory for strong interaction.

This process is considered also a powerful accurate tool for testing the Standard model (SM) at hadron colliders because of its many characteristics, like a large cross section, a clean experimental signature, and it is very sensitive to the proprties of the gauge bosons.

Furthermore, it has become the golden channel for probing the partonic structures of hadrons, by extracting information on PDFs. [6, 8]

II.2 The Large Hardon Collider at CERN

The superconducting Large Hadron Collider (LHC) is currently the highest energy collider and with a circumference of $\sim 26.7km$ also the biggest particle accelerator in the world. It is situated at CERN (Conseil Europeen pour la Recherche Nucleaire) in Switzerland and France and is home to 7 experiments: the two general purpose experiments ATLAS and CMS, ALICE studying heavy ion collisions, LHCb which is dedicated to B physics, LHCf focusing on forward production of neutral particles, MoEDAL, the monopole and exotics physics search detector at the LHC and fnally, TOTEM which measures total and diraction pp cross sections. [12]

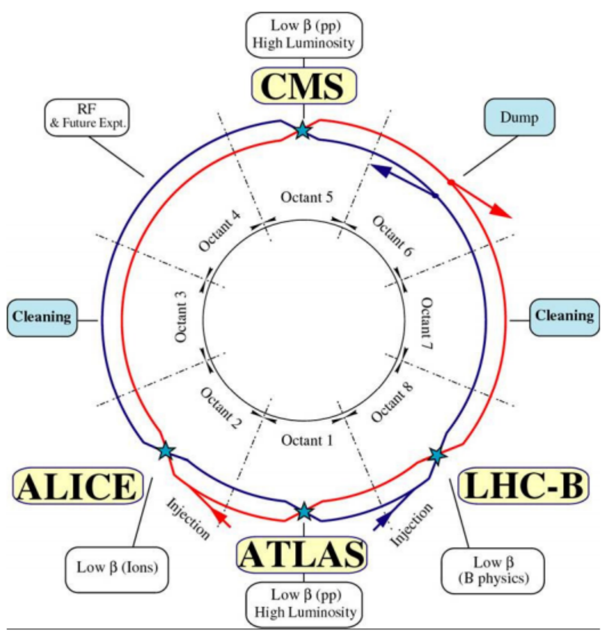


Figure II.1: Schematic of the LHC ring and the 4 large experiments.

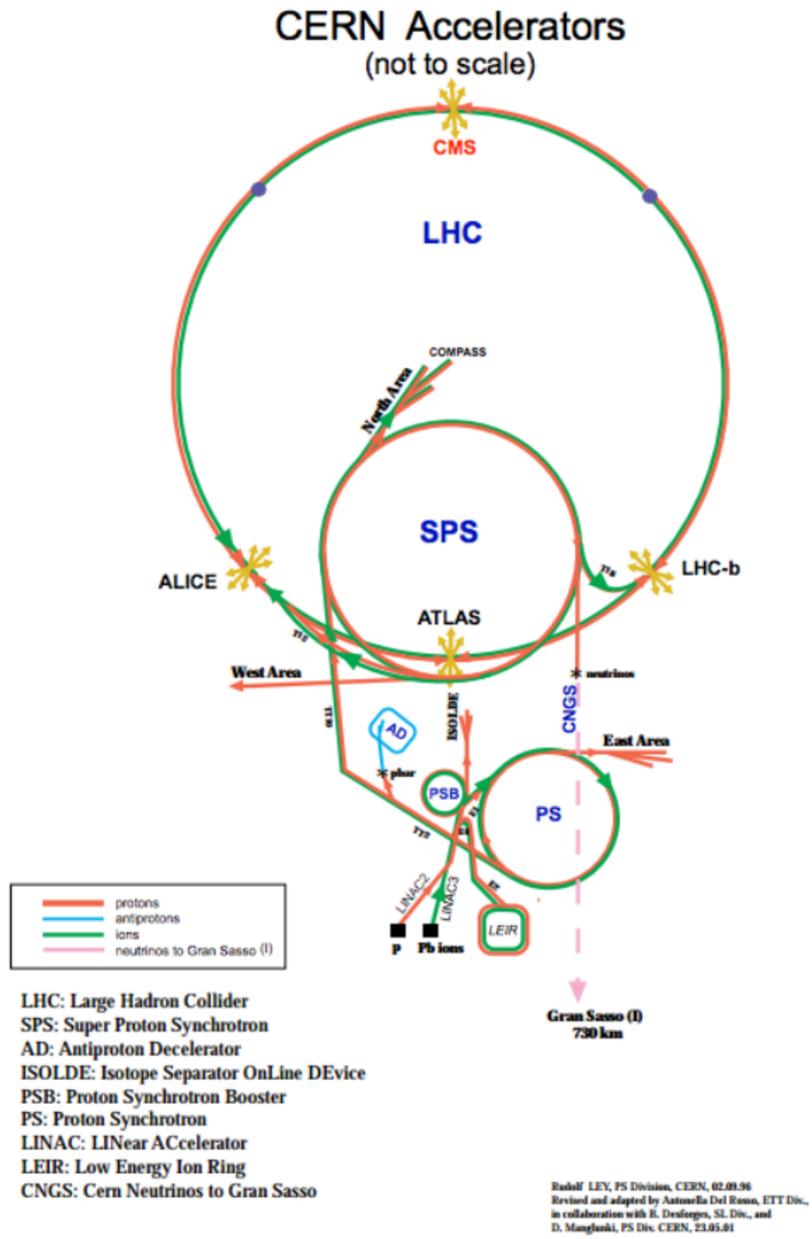


Figure II.2: The proton lifecycle: the protons are produced and pre-accelerated before injection into the LHC and then collided at the centre of the experiments.

II.3 Leading ordre Drell-Yan mechanism

The leading order Drell-Yan process as shown on (II.3) a high-mass lepton pair l^+l^- emerges from $q\overline{q}$ annihilation in a proton-proton collision. The quark carries a fraction x_1 of the first hadron's longitudinal momentum, and a fraction x_2 of the other hadron's longitudinal momentum is carried by the antiquark.

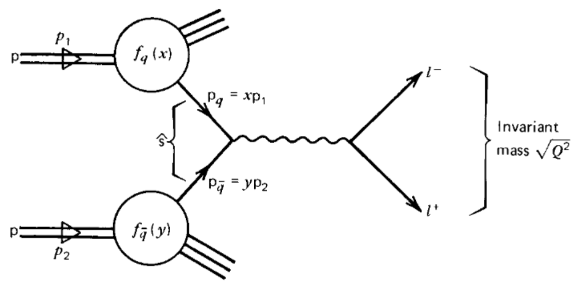


Figure II.3: The Drell-Yan process $p\ p \longrightarrow l\ l^+$ [7]

II.4 Factorization theorem

The connection between the physies world-of hadrons- and the patron world -quarks and qluons- is made possible by the crucial concept of " factorisation "!

Drell-Yan cross section in QCD is a combination of short and long-distance behavior, and it is not computable directly in perturbation theory, the factorization theorem allows us to derive predictions for these cross sections.

At the Born approximation the short-distance cross section is identical to the normal parton scattering cross section. At higher orders, the short-distance cross section is derived from the parton scattering cross section by separating the long-distance pieces, and factoring them into the parton distribution functions (PDF). The short-distance cross section is process dependent and is calculable in perturbation theorem. [8]

Consider the process ($p\ +\ p \longrightarrow l^+\ + l^- +\ X$) the factorization theorem is contained in the following expression at any order in QCD:

$$\sigma(p\ p \longrightarrow l^-\ l^+) = \int\int dx_1 dx_2 \sum_a F_a(x_1) F_{\overline{a}}(x_2)\ \widehat{\sigma}(q_a(x_1 p_1) + \overline{q}_{\overline{a}}(x_2 p_2) \longrightarrow l^-\ l^+ + X) \tag{II.1}$$

where the sum runs over all species of quarks and antiquarks. X denotes any hadronic final state.

II.5 The hard scattering partonic cross section

According to the feynman rules, the amplitude may be writen as

$$M = \overline{v}(p_2) ie Q_i \gamma_\nu u(p_1) \left(\frac{-ig^{\mu\nu}}{q^2 + i\xi} \right) \overline{u}(p_3) ie \gamma_\mu v(p_4)$$

by neglecting ξ we obtain :

$$M = iQ_i \frac{e^2}{q^2} \overline{v}(p_2) \gamma_\mu u(p_1) \overline{u}(p_3) \gamma_\nu v(p_4) \tag{II.2}$$

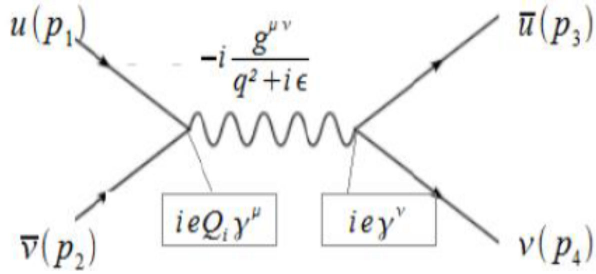


Figure II.4: Quark-antiquark interaction diagram

$$|M|^2 = M\ M^+$$

we have

$$\begin{aligned}[\overline{v}(p_2)\gamma_\nu u(p_1)]^+ &= u^+(p_1)\gamma_\nu^+(\overline{v}(p_2))^+ \\ &= u^+(p_1)\gamma_\nu^+(v^+(p_2)\gamma^0)^+ \\ &= u^+(p_1)\gamma_\nu^+\gamma^{0+}v(p_2) \\ &= u^+(p_1)\gamma^0\gamma_\nu v(p_2) \\ &= \overline{u}(p_1)\gamma_\nu v(p_2)\end{aligned} \tag{II.3}$$

also

$$[\overline{u}(p_3)ie\gamma_\mu v(p_4)]^+ = \overline{v}(p_4)\gamma_\mu u(p_3) \tag{II.4}$$

so

$$M^+ = -iQ_i\frac{e^2}{q^2}(\overline{u}(p_1)\gamma_\nu v(p_2))((\overline{v}(p_4)\gamma_\mu u(p_3))$$

then

$$|M|^2 = Q_i^2\frac{e^4}{q^4}\left[\overline{v}(p_2)\gamma_\nu u(p_1)\overline{u}(p3)ie\gamma_\mu v(p_4)\right]\left[\overline{u}(p_1)\gamma_\nu v(p_2)(\overline{v}(p_4)\gamma_\mu u(p_3)\right] \tag{II.5}$$

In the other hand we have the normalized Dirac spinors and antispinors completeness :

$$\sum_{spin} u_(p)\overline{u}(p) = p^\alpha\gamma_\alpha + m \tag{II.6}$$

$$\sum_{spin} v(p)\overline{v}(p) = p^\alpha\gamma_\alpha - m \tag{II.7}$$

with $me <<$, $m <<$
so

$$|M|^2 = Q_i^2\frac{e^4}{q^4}\left[(p_2^\alpha\gamma_\alpha - m)\gamma_\mu(p_1^\beta\gamma_\beta + m)\gamma_\nu\right]\left[(p_3^\alpha\gamma_\alpha + m)\gamma_\nu(p_4^\beta\gamma_\beta - m)\gamma_\mu\right] \tag{II.8}$$

$$|M|^2 = Q_i^2\frac{e^4}{q^4}Tr(p_2^\alpha\gamma_\alpha\gamma_\mu p_1^\beta\gamma_\beta\gamma_\nu)Tr(p_3^\alpha\gamma_\alpha\gamma_\nu p_4^\beta\gamma_\beta\gamma_\mu) \tag{II.9}$$

by using traces properies we obtain

$$
\begin{aligned}
Tr(p_2^\alpha\gamma_\alpha\gamma_\mu p_1^\beta\gamma_\beta\gamma_\nu)Tr(p_3^\alpha\gamma_\alpha\gamma_\nu p_4^\beta\gamma_\beta\gamma_\mu) &= p_2^\alpha p_1^\beta Tr(\gamma_\alpha\gamma_\mu\gamma_\beta\gamma_\nu)p_3^\alpha p_4^\beta Tr(\gamma_\alpha\gamma_\nu\gamma_\beta\gamma_\mu \\
&= p_2^\alpha p_1^\beta(g_{\alpha\mu}g_{\beta\nu} - g_{\alpha\beta}g_{\mu\nu} + g_{\alpha\nu}g_{\beta\mu})p_3^\alpha p_4^\beta(g_{\alpha\nu}g_{\beta\mu} - g_{\alpha\beta}g_{\nu\mu} + g_{\alpha\mu}g_{\beta\nu}) \\
&= 4(p_{2\mu}p_{1\nu} - p_{2\beta}p_2^\beta g_{\mu\nu} + p_{2\nu}p_{1\mu})4(p_{3\nu}p_{4\mu} - p_{3\beta}p_4^\beta g_{\mu\nu} + p_{3\mu}p_{4\nu}) \\
&= 16\,[(p_1p_3)(p_2p_4) + (p_2p_3)(p_1p_4) + (p_2p_3)(p_1p_4) + (p_2p_4)(p_1p_3)] \\
&= 16\,[2(p_1p_3)(p_2p_4) + 2(p_2p_3)(p_1p_4)] \\
&= 32\,[(p_1p_3)(p_2p_4) + (p_2p_3)(p_1p_4)]
\end{aligned}
$$

so

$$Tr(p_2^\alpha\gamma_\alpha\gamma_\mu p_1^\beta\gamma_\beta\gamma_\nu)Tr(p_3^\alpha\gamma_\alpha\gamma_\nu p_4^\beta\gamma_\beta\gamma_\mu) = 32\,[(p_1p_3)(p_2p_4) + (p_2p_3)(p_1p_4)] \tag{II.10}$$

we have Mandelstam variables (experimentally observable)

$$\widehat{s} = (p_1 + p_2)^2 = (p_3 + p_4)^2 \simeq 2p_1.p_2 \simeq 2p_3.p_4 \tag{II.11}$$

$$\widehat{t} = (p_1 + p_3)^2 = (p_2 - p_4)^2 \simeq -2p_1.p_3 \simeq -2p_2.p_4 \tag{II.12}$$

$$\widehat{u} = (p_1 + p_4)^2 = (p_2 - p_3)^2 \simeq -2p_1.p_4 \simeq -2p_3.p_2 \tag{II.13}$$

then

$$32\,[(p_1p_3)(p_2p_4) + (p_2p_3)(p_1p_4)] = 8(\widehat{t}^2 + \widehat{u}^2) \tag{II.14}$$

so

$$|M|^2 = Q_i^2\frac{e^4}{q^4}8(\widehat{t}^2 + \widehat{u}^2) \tag{II.15}$$

$$\alpha = \frac{e^2}{4\pi} \Longrightarrow e^4 = \alpha^2 16\pi^2 \tag{II.16}$$

$$q = p_1 + p_2 = p_3 + p_4 = \sqrt{\widehat{s}} \Longrightarrow q^4 = \widehat{s}^2 \tag{II.17}$$

then

$$|M|^2 = Q_i^2\alpha^2\pi^2 128(\frac{\widehat{t}^2 + \widehat{u}^2}{\widehat{s}^2}) \tag{II.18}$$

$$|\overline{M}|^2 = (\frac{1}{2})^2 3(\frac{1}{3})^2 \sum_{coulor}\sum_{spin}|M|^2 \tag{II.19}$$

where the sum is over quark flavors. The factors of $\frac{1}{3}$ average over the initial quark and antiquark colors, and the factor 3 sums over the $q\overline{q}$ color combinations .

$$|\overline{M}|^2 = Q_i^2 \frac{\alpha^2\pi^2 32}{3}(\frac{\widehat{t}^2+\widehat{u}^2}{\widehat{s}^2}) \tag{II.20}$$

and in the C.M fram we have

$$p_1 = \frac{\sqrt{\widehat{s}}}{2}(1,0,0,1) \tag{II.21}$$

$$p_2 = \frac{\sqrt{\widehat{s}}}{2}(1,0,0,-1) \tag{II.22}$$

$$p_3 = \frac{\sqrt{\widehat{s}}}{2}(1,+\sin\theta,0,\cos\theta) \tag{II.23}$$

$$p_4 = \frac{\sqrt{\widehat{s}}}{2}(1,-\sin\theta,0,-cos\theta) \tag{II.24}$$

$$\widehat{t} = \frac{-\widehat{s}}{2}(1-\cos\theta)\ ;\ \widehat{u} = \frac{-\widehat{s}}{2}(1+\cos\theta) \tag{II.25}$$

by replacing each term

$$\begin{aligned}\widehat{t}^2+\widehat{u}^2 &= \frac{\widehat{s}^2}{4}\left[(1-\cos\theta)^2+(1+\cos\theta)^2\right]\\ &= \frac{\widehat{s}^2}{4}\left[1+\cos^2\theta-2\cos\theta+1+\cos^2\theta+2\cos\theta\right]\\ &= \frac{\widehat{s}^2}{2}\left[1+\cos^2\theta\right]\end{aligned} \tag{II.26}$$

the amplitude becomes

$$|\overline{M}|^2 = Q_i^2\alpha^2\frac{16\pi^2}{3}(1+\cos^2\theta) \tag{II.27}$$

The cross section is the product of phase space and amplitude

$$d\sigma = (2\pi)\delta^{(4)}(p_i-p_f)\frac{1}{2E_1 2E_2|v_{rel}|}\left(\prod_f \frac{d^3p_f}{(2\pi)^3 2E_f}\right)|\overline{M}|^2 \tag{II.28}$$

$$= \frac{1}{4E_1E_2}(2\pi)^4\delta^{(4)}((p_1+p_2)-(p_3+p_4))\frac{d^3p_3}{(2\pi)^3 2E_3}\frac{d^3p_4}{(2\pi)^3 2E_4}|\overline{M}|^2 \tag{II.29}$$

with

$$E_1E_2|v_{rel}| = \sqrt{p_1p_2-m_1^2m_2^2} \tag{II.30}$$

$$E_1^2E_2^2 = p_1^2p_2^2-m_1^4m_2^4 \implies E_1E_2 = p_1p_2 = \frac{\widehat{s}}{2} \tag{II.31}$$

and

$$d\sigma = \frac{1}{4(p_1 p_2)} |\overline{M}|^2 dL_{ips} \tag{II.32}$$

$$d\sigma = \frac{|\overline{M}|^2}{2\widehat{s}} dL_{ips} \tag{II.33}$$

where L_{ips} is the two particles lorentz-invariant phase space

$$\begin{aligned} L_{ips} &= \frac{1}{4E_1E_2}(2\pi)^4 \delta^{(4)}((p_1+p_2)-(p_3+p_4))\frac{d^3p_3}{(2\pi)^3 2E_3}\frac{d^3p_4}{(2\pi)^3 2E_4} && \text{(II.34)} \\ &= \frac{d\cos\theta}{16\pi} && \text{(II.35)} \end{aligned}$$

$$d\widehat{\sigma} = \frac{Q_i^2 \alpha^2 \pi}{6\widehat{s}}(1+\cos^2\theta) d\cos\theta \tag{II.36}$$

$$\widehat{\sigma} = Q_i^2 \frac{\alpha^2 \pi}{6\widehat{s}} \int_{-1}^{1} (1+\cos^2\theta) d\cos\theta \tag{II.37}$$

by integrating

$$\widehat{\sigma} = Q_i^2 \frac{\alpha^2 \pi}{6\widehat{s}} \left[\cos\theta + \frac{1}{3}\cos^3\theta\right]_{-1}^{1} \tag{II.38}$$

Finaly the partonic cross section for hard process in n = 4 space time dimensions is given by

$$\widehat{\sigma} = Q_i^2 \frac{\alpha^2 4\pi}{9\widehat{s}} \tag{II.39}$$

II.6 The soft scattering hadronic cross section

The hadronic cross section can now be obtained using the structure functions $F_i(x)$

$$\sigma(p\ p \longrightarrow e^-\ e^+) = \int\int dx_1 dx_2 F_{p\longleftrightarrow q}(x_1) F_{p\longleftrightarrow \overline{q}}(x_2) \widehat{\sigma}(q+\overline{q} \longleftrightarrow e^-\ e^+) \tag{II.40}$$

where $F_{p\longleftrightarrow q}(x_1)$ *and* $F_{p\longleftrightarrow \overline{q}}(x_2)$ are the probability of finding a quark and an antiquark with momentum, respectively :

$$p_q = x_1 p_1\ ,\ p_{\overline{q}} = x_1 p_1 \tag{II.41}$$

and where p_1 *and* p_2 are the momentum of the initial two protons

it is convenient to define the dimensionless variables

$$\tau = \frac{Q^2}{s} \ , \ \widehat{\tau} = \frac{Q^2}{\widehat{s}} \tag{II.42}$$

where Q^2 is the mass of electrons pair and where s is the external proton-proton center of mass energy squared

$$s = (p_1 + p_2)^2 = 2p_{cm}^2 \tag{II.43}$$

$$p_{cm} = \frac{1}{2}\sqrt{s} \tag{II.44}$$

and $\widehat{s}$ is the internal parton parton center of mass energy squared

$$\widehat{s} = (p_q + p_{\overline{q}})^2 = 2p_q p_{\overline{q}} \tag{II.45}$$

Equation(II.41) imply :

$$\widehat{s} = x_1 x_2 s \tag{II.46}$$

$$\tau = x_1 x_2 \widehat{\tau} \tag{II.47}$$

the longitudinal momentum of the electron-positron pair is

$$P_L = p_q - p_{\overline{q}} \tag{II.48}$$

and , if we assume that the incoming partons are parallel to the incident protons then the total energy is

$$E^2 = P_L^2 + Q^2 \tag{II.49}$$

since in this case the electron pair has no transverse momentum
equation (II.48) implies

$$x_L = x_1 - x_2 \tag{II.50}$$

where

$$x_L \equiv \frac{2P_L}{\sqrt{s}} \tag{II.51}$$

and (II.49) gives

$$x_E^2 = x_L^2 + 4\tau^2 \tag{II.52}$$

where

$$x_E \equiv \frac{2E}{\sqrt{s}} \tag{II.53}$$

The total cross section for a quark and antiquark to annihilate into pair of an electron and a positron , is given by

$$\widehat{\sigma}(q\ \overline{q} \longleftrightarrow e^-\ e^+) = Q_i^2 \frac{\alpha^2 4\pi}{9\widehat{s}} \tag{II.54}$$

in another hand we have

$$\widehat{s} = Q^2\ ,\ \widehat{\tau} = 1 \tag{II.55}$$

from (II.47) and (II.50) we see that x_1 *and* x_2 are completely specified in terms of τ *and* x_L according to

$$x_1 x_2 = \tau \tag{II.56}$$

$$x_1 - x_2 = x_L \tag{II.57}$$

and (II.40) gives

$$\frac{d\sigma}{d\tau dx_L} = \frac{4\pi\alpha^2}{9Q^2}\frac{1}{x_1 + x_2} P_{q\overline{q}}(x_1, x_2) \tag{II.58}$$

with the joint $q\overline{q}$ probability function given by

$$P_{q\overline{q}}(x_1, x_2) = \sum_{i=1}^{n_f} Q_i^2 \left[F_{p\longleftrightarrow q_i}(x_1) F_{p\longleftrightarrow \overline{q}_i}(x_2) + F_{p\longleftrightarrow \overline{q_i}}(x_1) F_{p\longleftrightarrow q_i}(x_2)\right] \tag{II.59}$$

where n_f mentions quark flavors
Equation (II.56) and (II.57) implies

$$x_1 = \frac{1}{2}(x_E + x_L) = \sqrt{\tau}\exp(y) \tag{II.60}$$

$$x_2 = \frac{1}{2}(x_E - x_L) = \sqrt{\tau}\exp(-y) \tag{II.61}$$

where x_E is given in (II.52) and y is the rapidity of the electron-positron defined by

$$y \equiv \frac{1}{2}\log\left(\frac{E + p_L}{E - p_L}\right) \tag{II.62}$$

in this case

$$y = \frac{1}{2}\log\left(\frac{x_1}{x_2}\right) \tag{II.63}$$

we can verify that

$$\frac{d\sigma}{dy} = x_E \frac{d\sigma}{dx_E} = (x_1 + x_2)\frac{d\sigma}{dx_L} \tag{II.64}$$

so that

$$\frac{d\sigma}{d\tau dy} = \frac{4\pi\alpha^2}{9Q^2} P_{q\bar{q}}(x_1, x_2) \tag{II.65}$$

integrating (II.58) over x_L or (II.65) over y gives

$$\frac{d\sigma}{d\tau} = \frac{4\pi\alpha^2}{9Q^2} \int_{\tau}^{1} \frac{dx_1}{x_1} P_{q\bar{q}}(x_1, \frac{\tau}{x_1}) \tag{II.66}$$

where $P_{q\bar{q}}(x_1, x_2)$ is given in (II.59)

So $\frac{d\sigma}{d\tau}$ is a function of both s and Q^2, but (II.66) shows that in the naive parton model

$$Q^2 \frac{d\sigma}{d\tau} = F(\tau) \tag{II.67}$$

is function only of the dimensionless variable $\tau = \frac{Q^2}{s}$.

In this chapeter we ignored the dynamical role of gluons as the carriers of the strong force associated with colored quarks.In next chapeter we will take in consideration the fact that quark can radiate qluons. Thus studying real emissions to drell-Yan processes at the NLO.

Chapter III

Higher-order Drell-Yan process

III.1 Introduction

In evry process that contains coulored or charged particles (gluons or photons) in initial or final stat that caused radiation, may give large corrections to the overall system. Starting from a basic $2 \longrightarrow 2$, $2 \longrightarrow 3$, $2 \longrightarrow 4$, and so on. In this chapter we will study these corrections.

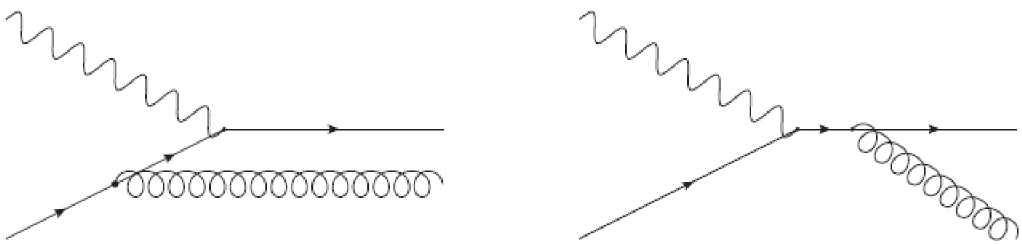


Figure III.1: $O(\alpha_s)$ corrections to the naive parton model through real gluon emission.[13]

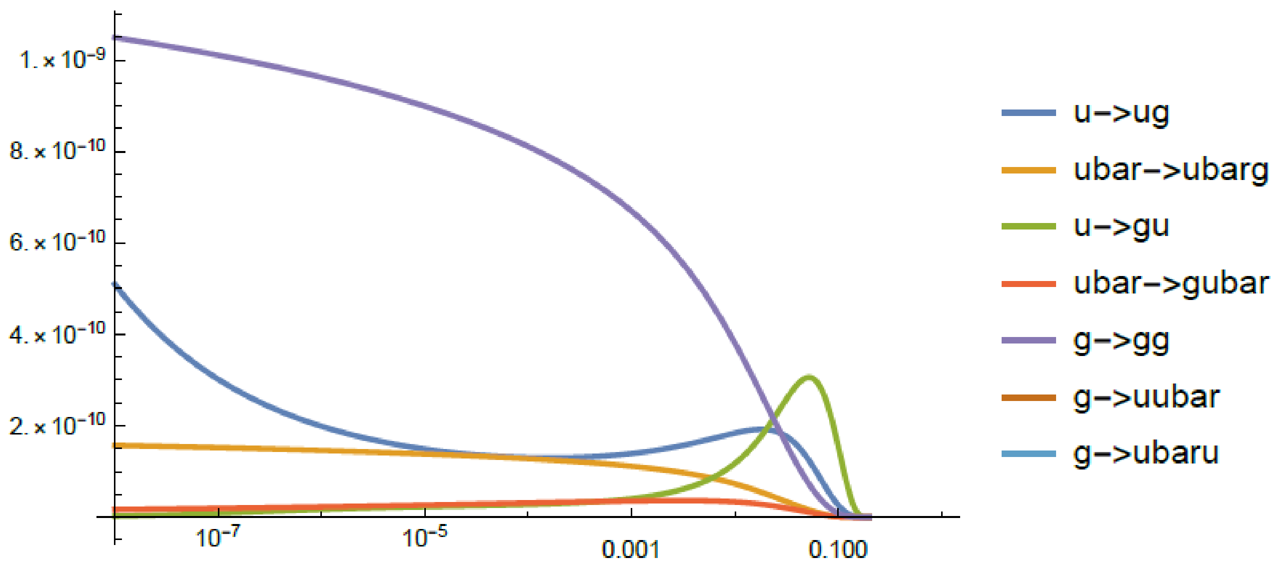


Figure III.2: Plott of splitting function $f(t, z, x)$ and x at $z = 0.2$ *and* $t = \frac{14000^2}{2}(1 - x^2)$ (see **appendix A**)

The calculation of NLO Drell-Yan cross section from a technical point of view would involve the calculation of a complicated Feynman diagrams. Therefore theorist are using the parton shower merging and matching approach, where the complication of the calculation is far reduced by calculating separately the matrix elements for n-particles and n-body phase space using numerical calculation such as MC.

In this work we are aiming to apply this approach. We start first by the calculation of n-body phase space, for that we need to build a recursive relation for Monte Carlo .

The hard cross section in this case is given by

$$\sigma_n = \int \frac{d\sigma_n}{d\phi_n} d\phi_n$$

III.2 Phase space

The general form of phase space is :

$$d\phi_n = (2\pi)^4 \;\; \delta^{(4)}(P - \sum_{i=1}^{n} p_i) \;\; \prod_{i=1}^{n} \;\; \frac{d^3 p_i}{(2\pi)^3 \;\; 2E_i} \tag{III.1}$$

By using (III.1) we can calculate the phase space to produce two, three and four particles, and finally find the recursive phase space volum $d\phi_n$.

III.2.1 Two-Body Phase Space

The two-body phase is the basis of computing higher body phase spaces. We compute it in the rest frame of the two-body system, $P = p_1 + p_2$.
With $P = (E_{cm}, \vec{0})$, $p_1 = (E_1, \vec{p_1})$, $p_2 = (E_2, \vec{p_2})$.

and

$$E_1^2 = p_1^2 + m_1^2$$
$$E_2^2 = p_2^2 + m_2^2$$

$$\begin{aligned} d\phi_2(P; p_1, p_2) &= (2\pi)^4 \delta^{(4)}(P - p_1 - p_2)) \frac{d^3 p_1}{(2\pi)^3 2E_1} \frac{d^3 p_2}{(2\pi)^3 2E_2} \\ &= \frac{\delta^{(4)}(P - p_1 - p_2)}{16\pi^2 E_1 E_2} d^3 p_1 d^3 p_2 \end{aligned} \tag{III.2}$$

By using the Delta function property we can write

$$\begin{aligned} \delta^{(4)}(P - p_1 - p_2) &= \delta(E_{cm} - E_1 - E_2) \delta^{(3)}(\vec{0} - (\vec{p_1} + \vec{p_2})) \\ &= \delta(E_{cm} - E_1 - E_2) \delta^{(3)}(\vec{p_1} + \vec{p_2}) \end{aligned} \tag{III.3}$$

$$\begin{aligned} d\phi_2(P; p_1, p_2) &= \frac{\delta(E_{cm} - E_1 - E_2) \delta^{(3)}(\vec{p_1} + \vec{p_2})}{16\pi^2 E_1 E_2} d^3 p_1 d^3 p_2 \\ &= \frac{d^3 p}{16\pi^2 E_1 E_2} \delta(E_{cm} - E_1 - E_2) \\ &= \frac{p^2 dp d\Omega}{16\pi^2 E_1 E_2} \delta \left(E_{cm} - \sqrt{p^2 + m_1^2} - \sqrt{p^2 + m_2^2} \right) \end{aligned} \tag{III.4}$$

where

$$\begin{aligned} d^3p &= p^2 dp d\Omega \\ &= p^2 dp \sin\theta d\theta d\varphi \\ &= p^2 dp \quad d\cos\theta d\varphi \end{aligned} \tag{III.5}$$

We have from the Delta Dirac function properties :

$$\delta[g(x)] = \sum_{x_i \ , \ g(x_i)=0} \frac{\delta(x - x_i)}{|g'(x_i)|} \tag{III.6}$$

Where the relation (III.6) holds for any function $g(x)$ with one zero, $x = x_i$, in the interval of interest. (see **appendix B**)

By putting $x \equiv p \ and \ x_i \equiv p'$. We obtain

$$\delta[g(p)] = \sum_{p' \ , \ g(p')=0} \frac{\delta(p - p')}{|g'(p')|} \tag{III.7}$$

With

$$g(p) = E_{cm} - \sqrt{p^2 + m_1^2} - \sqrt{p^2 + m_2^2} \tag{III.8}$$

Now we have to calculate the value of p' where function g is zero

$$g(p') = 0 \quad \Leftrightarrow \quad E_{cm} - \sqrt{m_1^2 + p'^2} - \sqrt{m_2^2 + p'^2} = 0$$

We simplify the equality

$$E_{cm}^2 = m_1^2 + p'^2 + m_2^2 + p'^2 + 2\sqrt{(m_1^2 + p'^2)(m_2^2 + p'^2)}$$

$$[(E_{cm}^2 - 2p'^2) - (m_1^2 + m_2^2)]^2 = (2\sqrt{(m_1^2 + p'^2)(m_2^2 + p'^2)})^2$$

$$(E_{cm}^2 - 2p'^2)^2 + (m_1^2 + m_2^2)^2 - 2(E_{cm}^2 - 2p'^2)(m_1^2 + m_2^2) = 4(m_1^2 + p'^2)(m_2^2 + p'^2)$$

$$-4E_{cm}^2 p'^2 = -E_{cm}^4 - m_1^4 - m_2^4 - 2m_1^2 m_2^2 + 2E_{cm}^2(m_1^2 + m_2^2) + 4m_1^2 m_2^2$$

$$-4E_{cm}^2 p'^2 = -E_{cm}^4 - (m_1^2 - m_2^2)^2 + 2E_{cm}^2(m_1^2 + m_2^2)$$

$$4p'^2 = E_{cm}^2 + \frac{(m_1^2 - m_2^2)^2}{E_{cm}^2} - 2(m_1^2 + m_2^2)$$

$$p' = \frac{1}{2}\sqrt{E_{cm}^2 + \frac{(m_1^2 - m_2^2)^2}{E_{cm}^2} - 2(m_1^2 + m_2^2)}$$

$$p' = \frac{E_{cm}}{2}\sqrt{1 + \frac{(m_1^2 - m_2^2)^2}{E^4} - \frac{2(m_1^2 + m_2^2)}{E^2}}$$

$$p' = \frac{E_{cm}}{2}\beta_{12} \tag{III.9}$$

With

$$\beta_{12} = \sqrt{1 + \frac{(m_1^2 - m_2^2)^2}{E_{cm}^4} - \frac{2(m_1^2 + m_2^2)}{E_{cm}^2}} \tag{III.10}$$

if $m_1 = m_2$, as in our work

$$\beta_{12} = \sqrt{1 - \frac{4m^2}{E^2}}$$

The derivation of g in p'

$$\begin{aligned} g'(p') &= -\frac{p'}{\sqrt{m_1^2 + p'^2}} - \frac{p'}{\sqrt{m_2^2 + p'^2}} \\ &= -(\frac{p'}{E_1} + \frac{p'}{E_2}) \\ &= -(\frac{p'(E_1 + E_2)}{E_1 E_2}) \end{aligned} \tag{III.11}$$

Therefore the two body phase space becomes

$$\begin{aligned} d\phi_2(P; p_1, p_2) &= \frac{p^2 dp d\Omega}{16\pi^2 E_1 E_2} \frac{E_1 E_2 \delta(p - p')}{p'(E_1 + E_2)} \\ &= \frac{p'^2}{16\pi^2 p'(E_1 + E_2)} d\Omega \\ &= \frac{p'}{16\pi^2 E_{cm}} d\Omega \\ &= \frac{\beta_{12}}{32\pi^2} d\cos\theta d\varphi \end{aligned} \tag{III.12}$$

III.2.2 Three-body phase space

From the general phase space form we can also write $d\phi_3$

$$d\phi_3(P;p_1,p_2,p_3) = (2\pi)^4\delta^{(4)}(P-p_1-p_2-p_3)\frac{d^3p_1}{(2\pi)^3 2E_1}\frac{d^3p_2}{(2\pi)^3 2E_2}\frac{d^3p_3}{(2\pi)^3 2E_3} \tag{III.13}$$

by adding $(\delta^{(4)}(p_{12}-p_1-p_2)d^4p_{12}$) and $(\delta(p_{12}^2-m_{12}^2)dm_{12}^2$) in the first and fourth steps, respectively to (III.13)

$$\begin{aligned}
d\phi_3(P;p_1,p_2,p_3) &= (2\pi)^4\delta^{(4)}(P-p_1-p_2-p_3)\frac{d^3p_1}{(2\pi)^3 2E_1}\frac{d^3p_2}{(2\pi)^3 2E_2}\frac{d^3p_3}{(2\pi)^3 2E_3}\delta^{(4)}(p_{12}-p_1-p_2)d^4p_{12} \\
&= \delta^{(4)}(P-p_{12}-p_3)d^4p_{12}\frac{d^3p_3}{(2\pi)^3 2E_3}\left[\delta^{(4)}(p_{12}-p_1-p_2)(2\pi)^4\frac{d^3p_1}{(2\pi)^3 2E_1}\frac{d^3p_2}{(2\pi)^3 2E_2}\right] \\
&= \delta^{(4)}(P-p_{12}-p_3)d^4p_{12}\frac{d^3p_3}{(2\pi)^3 2E_3}d\phi_2(p_{12};p_1,p_2) \\
&= \delta^{(4)}(P-p_{12}-p_3)d^4p_{12}\frac{d^3p_3}{(2\pi)^3 2E_3}d\phi_2(p_{12};p_1,p_2)\delta(p_{12}^2-m_{12}^2)dm_{12}^2 \\
&= dm_{12}^2 d\phi_2(p_{12};p_1,p_2)\frac{1}{2\pi}\left[\delta^{(4)}(P-p_{12}-p_3)(2\pi)^4\frac{d^3p_{12}}{(2\pi)^3 2E_{12}}\frac{d^3p_3}{(2\pi)^3 2E_3}\right] \\
&= \frac{dm_{12}^2}{2\pi}d\phi_2(P;p_{12},p_3)d\phi_2(p_{12};p_1,p_2)
\end{aligned} \tag{III.14}$$

With Lorentz covariant

$$\begin{aligned}
d^4p\delta(p^2-m^2)\theta(E) &= d^4p\delta(E^2-(p^2+m^2))\theta(E) \\
&= \frac{d^3p}{2E}
\end{aligned} \tag{III.15}$$

Finally we obtain the forme of three-body phase space

$$d\phi_3(P;p_1,p_2,p_3) = \frac{dm_{12}^2}{2\pi}\frac{\beta_{(12)3}}{32\pi^2}d\cos\theta d\varphi\frac{\beta_{12}}{32\pi^2}d\cos\theta_{12}d\varphi_{12} \tag{III.16}$$

It is important to specify that

$$\beta_{(12)3} = \sqrt{1+\frac{(m_{12}^2-m_3^2)^2}{E_{cm}^4}-\frac{2(m_{12}^2+m_3^3)}{E_{cm}^2}} \quad ,with \;\; m_{12}=m_1+m_2$$

III.2.3 Four-body phase space

By following the same steps we can obtain the four-body phase space

$$d\phi_4(P;p_1,p_2,p_3,p_4) = (2\pi)^4\delta^4(P-p_1-p_2-p_3-p_4)\frac{d^3p_1}{(2\pi)^3 2E_1}\frac{d^3p_2}{(2\pi)^3 2E_2}\frac{d^3p_3}{(2\pi)^3 2E_3}\frac{d^3p_4}{(2\pi)^3 2E_4} \tag{III.17}$$

$$d\phi_4 = (2\pi)^4\delta^4(P-p_1-p_2-p_3-p_4)\frac{d^3p_1}{(2\pi)^3 2E_1}\frac{d^3p_2}{(2\pi)^3 2E_2}\frac{d^3p_3}{(2\pi)^3 2E_3}\frac{d^3p_4}{(2\pi)^3 2E_4}\delta^{(4)}(p_{123}-p_{12}-p_3)d^4p_{123}$$
$$= \delta^4(P-p_{123}-p_4)d^4p_{123}\frac{d^3p_4}{(2\pi)^3 2E_4}\left[\delta^{(4)}(p_{123}-p_{12}-p_3)(2\pi)^4\frac{d^3p_1}{(2\pi)^3 2E_1}\frac{d^3p_2}{(2\pi)^3 2E_2}\frac{d^3p_3}{(2\pi)^3 2E_3}\right]$$
$$= \delta^4(P-p_{123}-p_4)d^4p_{123}\frac{d^3p_4}{(2\pi)^3 2E_4}d\phi_3(P;p_1,p_2,p_3)$$
$$= \delta^4(P-p_{123}-p_4)d^4p_{123}\frac{d^3p_4}{(2\pi)^3 2E_4}d\phi_3(P;p_1,p_2,p_3)\delta^{(4)}(p_{123}^2-m_{123}^2)dm_{123}^2$$
$$= dm_{123}^2 d\phi_3(P;p_1,p_2,p_3)\frac{1}{2\pi}\left[(2\pi)^4\delta^4(P-p_{123}-p_4)\frac{d^3p_{123}}{(2\pi)^3 2E_{123}}\frac{d^3p_4}{(2\pi)^3 2E_4}\right]$$
$$= \frac{dm_{123}^2}{2\pi}d\phi_3(P;p_1,p_2,p_3)d\phi_2(P;p_{123},p_4)$$

by replacing $d\phi_3$ by its relation we obtain

$$d\phi_4(P;p_1,p_2,p_3,p_4) = \frac{dm_{12}^2}{2\pi}\frac{dm_{123}^2}{2\pi}d\phi_2(p_{12};p_1,p_2)d\phi_2(p_{123};p_{12},p_3)d\phi_2(P;p_{123},p_4) \quad \text{(III.18)}$$

finally we obtain the phase space of four particles

$$d\phi_4(P;p_1,p_2,p_3,p_4) = \frac{dm_{12}^2}{2\pi}\frac{dm_{123}^2}{2\pi}\frac{\beta_{12}}{32\pi^2}d\cos\theta_{12}d\varphi_{12}\frac{\beta_{(12)3}}{32\pi^2}d\cos\theta_{(12)3}d\varphi_{(12)3}\frac{\beta_{(12)(34)}}{32\pi^2}d\cos\theta d\varphi \quad \text{(III.19)}$$

With

$$\beta_{(12)(34)} = \sqrt{1+\frac{(m_{12}^2-m_{34}^2)^2}{E_{cm}^4}-\frac{2(m_{12}^2+m_{34}^2)}{E_{cm}^2}} \quad ,with \quad m_{34}=m_3+m_4$$

III.2.4 Recursive phase spase

From what we got before we can build the recursive relation of phase space

$$d\phi_3 = \tfrac{dm_{12}^2}{2\pi}d\phi_2(P;p_{12},p_3)d\phi_2(P;p_1,p_2)$$

$$d\phi_4 = \tfrac{dm_{123}^2}{2\pi}d\phi_2(P;p_{123},p_4)d\phi_3(P;p_1,p_2,p_3)$$

$$d\phi_5 = \tfrac{dm_{1234}^2}{2\pi}d\phi_2(P;p_{1234},p_5)d\phi_4(P;p_1,p_2,p_3,p_4)$$

.

.

.

$$d\phi_n = \tfrac{dm_{12...(n-1)}^2}{2\pi}d\phi_2(P;p_{12...(n-1)},p_n)d\phi_{(n-1)}(P;p_1,p_2,....,p_{n-1})$$

So a recursive phase space relation is obtain and given by

$$d\phi_n = \frac{dm^2_{12...(n-1)}}{2\pi} d\phi_2(P; p_{12...(n-1)}, p_n) d\phi_{(n-1)}(P; p_1, p_2, ..., p_{n-1}) \tag{III.20}$$

III.3 Parton branching

Parton Branching is the common name of QCD radiations, where the soft and collinear emissions are the singularities and the limits of the parton shower.
The parton branching typically happens in the hard interaction of incoming partons, which can be calculated using pQCD, where all incoming partons branch giving rise to the so called parton showers.
A branching can be seen as a (a $\longrightarrow$ bc) process, where a is called the mother and b is the daughter and c is the emission. Each particle is free to branch as well, so that a shower-like structure can evolve.

The branching kinematics can be described by two variables, z and t. We define z as the fraction of energy carried by daughter b. The variable t has the dimensions of squared mass. It may defined as the absolute value of the mass squared of the daughter.
the differential probability that one branching occurs in terms of t and z, is given by

$$dP_\alpha = \sum_{b,c} \frac{\alpha_s}{2\pi} \; P_{a\longrightarrow bc} \; \frac{dt}{t} \; dz \tag{III.21}$$

Where the sum runs over all branchings the parton is allowed to make, e.g. where a (q $\longrightarrow$ qg) branching is followed by further branchings of the daughters. That way a whole shower develops. The functions $P_{a\longrightarrow bc}(z)$ are the so called splitting kernels.

The probability for branching at a time t needs to take into account the probability that the parton has not branched at earlier times $t0 < t$ brings us a new concept, which is called the Sudakov factor [5, 14, 15]

$$P(branching \;\; happens) = 1 - P(no - branching \;\; happens) \tag{III.22}$$

giving rise to the actual probability that a branching occurs at time t:

$$dP_\alpha = \left(\sum_{b,c} \frac{dt}{t} \; \int \; \frac{\alpha_s}{2\pi} \; P_{a\longrightarrow bc} \;\; dz \right) \exp\left[- \sum_{b,c} \int_{t_0}^{t} \frac{dt'}{t'} \; \int \frac{\alpha_s}{2\pi} \; P_{a\longrightarrow bc} \; dz' \right] \tag{III.23}$$

where the exponent is the Sudakov factor.

Conclusion

We were motivated to study the behavior of quarks which cannot be released by neither the QED nor the quark model developed by Murray-Gell-Mann to understand the hadrons spectroscopy. For that we needed to establish Quantum Chromodynamic theory to explain partons behavior.

In this work we were motivated in studying the Drell-yan process in Leading and higher order. Our aim was to calculate the n-body phase space and a better understanding of parton shower cross section calculation. We started by a phenomenological study of SLAC experiments results by studying the DIS between electrons and protons, which gives rise to the parton model, the Bjorken scaling and Feynman interpretation of scaling violation. That drove us to talk about the PDFs, to describe the probability distribution of a giving parton carries a specific fraction of the proton's momentum.

This led us to study a quantum field theory model for the QCD. We discussed the lagrangian. We extracted Feynman rules and diagrams.We gave a brief definition of renormalization, running coupling, color confinement and the asymptotic freedom.

The Drell-yan process in proton-proton collision is the hard annihilation of quark-antiquark. We calculate in detail the cross section of hard process, the two body phase space and the matrix element at leading order. This process is very important in high energy particle physics due to its importance and relevance in the calculation of PDFs and the Z boson production, this process has a large cross section and a clean experimental signature.

we did not ignore the dynamical role of gluons as the carriers of the strong force associated with colored quarks. we took in consideration the fact that quark can radiate qluons. We studied the radiation effect in drell-Yan processes at the NLO. We found that the computation of NLO Drell-Yan cross section is becaming more complicated because of the QCD radiations due to gluon-gluon interactions.
The best method to approach this calculation is the parton shower merging and matching method by calculating separately the matrix element of n-particle and n-body phase space.

We discussed the parton branching and we extracted the differential probability distribution in function of the splitting kernels and the Sudakov form factor. We built a recursive relation of phase space, based on the calculation of two, three and four particles, in order to implement it in Monte Carlo phase space integration.

Appendix A

Mathematica program for the splitting function

```
parton[-6] = "t-bar quark";
parton[-5] = "b-bar quark";
parton[-4] = "c-bar quark";
parton[-3] = "s-bar quark";
parton[-2] = "d-bar quark";
parton[-1] = "u-bar quark";
parton[0] = "gluon";
parton[1] = "u quark";
parton[2] = "d quark";
parton[3] = "s quark";
parton[4] = "c quark";
parton[5] = "b quark";
parton[6] = "t quark";
```

```
αs = .118; CF = 4/3; CA = 3; TR = .5; Q = 14000;
```

$$f[a\_, b\_, c\_][t\_, z\_, x\_] := \frac{\alpha s}{2\pi}\frac{1}{t}P[a, b, c][z]\,\frac{1}{z}\,\frac{\text{cteq5pdf}[3, a, x/z, t]}{\text{cteq5pdf}[3, b, x, t]};$$

The splitting functions

functions splitting The

```
P[a_ /; a > 0, a_ /; a > 0, 0][z_] = CF (1 + z^2)/(1 - z);
P[a_ /; a < 0, a_ /; a < 0, 0][z_] = CF (1 + z^2)/(1 - z);
P[a_ /; a > 0, 0, a_ /; a > 0][z_] = CF (1 + (1 - z)^2)/z;
P[a_ /; a < 0, 0, a_ /; a < 0][z_] = CF (1 + (1 - z)^2)/z;
P[0, 0, 0][z_] = CA (z/(1 - z) + (1 - z)/z + z (1 - z));
P[0, a_ /; a ≠ 0, -a_ /; a ≠ 0][z_] = TR (z^2 + (1 - z)^2);
P[0, -a_ /; a ≠ 0, a_ /; a ≠ 0][z_] = TR (z^2 + (1 - z)^2);
```

In[306]:= PDF Fits with X variable

Out[306]= Fits PDF variable with *X*

```
In[307]:= LogLinearPlot[{f[1, 1, 0][14000^2/2 (1 - x^2), 0.2, x],
   f[-1, -1, 0][14000^2/2 (1 - x^2), 0.2, x], f[1, 0, 1][14000^2/2 (1 - x^2), 0.2, x],
   f[-1, 0, -1][14000^2/2 (1 - x^2), 0.2, x], f[0, 0, 0][14000^2/2 (1 - x^2), 0.2, x],
   f[0, 1, -1][14000^2/2 (1 - x^2), 0.2, x], f[0, -1, 1][14000^2/2 (1 - x^2), 0.2, x]},
  {x, 10^-8, 1}, PlotLegends → {"u->ug", "ubar->ubarg", "u->gu",
    "ubar->gubar", "g->gg", "g->uubar", "g->ubaru"}, PlotStyle → Thick]
```

Appendix B

The Dirac Delta Function

Mathematically, the delta function is not a function, because it is too singular. Instead, it is said to be a "distribution." It is a generalized idea of functions, but can be used only inside integrals.
The Dirac delta function, $\delta(x)$ is an infinitely high, infinitesimally narrow spike at the origin , with area 1, (III.3). Specifically :

$$\delta(x) = \begin{pmatrix} 0 & forx \neq 0 \\ \infty, & forx = 0 \end{pmatrix} \; and \; \int_{-\infty}^{\infty} \delta(x)dx = 1$$

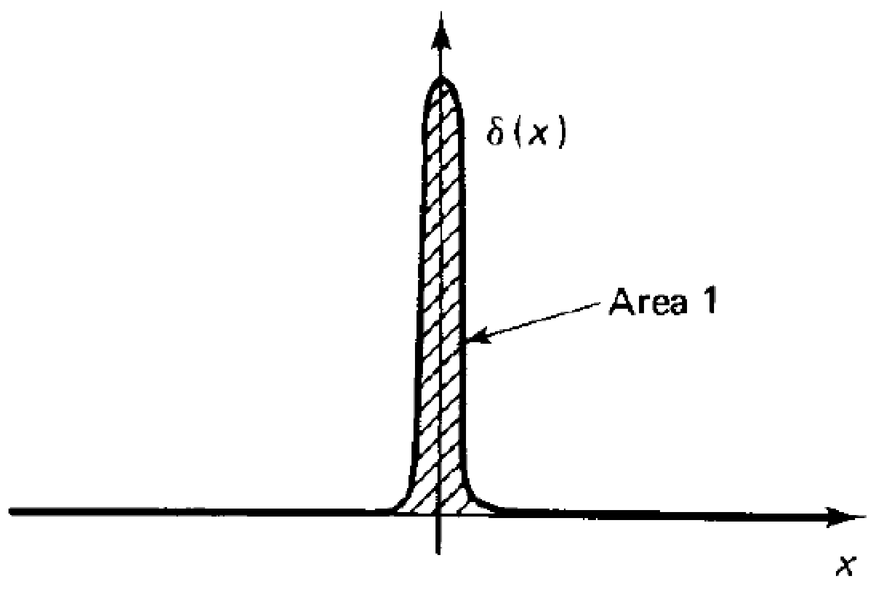


Figure III.3: The Dirac delta function [10]

The Delta function has some fundamentals properties :

$$\delta(-x) = \delta(x)$$

$$x\delta(x) = 0$$

for $x \neq a$

$$\delta(ax) = \frac{1}{|a|}\delta(x)$$

$$\delta(x^2 - a^2) = \frac{1}{2|a|}\left[\delta(x+a) + \delta(x-a)\right]$$

More generally, the Delta function of a function of x is given by :

$$\delta[g(x)] = \sum_i \frac{\delta(x - x_i)}{|g'(x_i)|}$$

where the x_i are the roots of g. The fondamental equation that defines derivatives of the Delta function $\delta(x)$ is :

$$\int f(x)\delta^{(n)}(x)dx = -\int \frac{\partial f}{\partial x}\delta^(n-1)(x)dx$$

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Abstract

This work is dedicated to expose the perturbative QCD as a quantum field theory. We devoted the first part to study the parton model in detail according to chronological order by describing the SLAC experiments results. We computed the DIS cross section for electron-proton collision. Further, descriptions of probability distribution functions (PDFs) and Evolution equation (DGLAP) were discussed .

The second part is dedicated to the Drell-Yan process in LO. We computed the partonic cross section between quark and antiquark. We used the latter in the computation of hadronic cross section between two protons using Factorization theorem.

In the last part we discussed the Drell-Yan process in NLO and its challenging calculation at higher order of QCD radiation. We computed two-body phase space, three-body phase space , four-body phase space. Finally we built a recursive relation of n-body phase space in order to implement it in Monte Carlo phase space integration.

Key Words : perturbative QCD , Parton model , Bjorken scaling, PDFs, DGLAP, Collider physics, Drell-Yan process cross section , LO, NLO, Phase space .

ملخـــص

في هذه العمل تناولنا في دراستنا الكروموديناميك الكمي الاضطرابي كنظرية من نظريات الحقول الكمية .
الفصل الاول , خصص لدراسة نموذج البارتون بشكل مفصل تباعًا لترتيب زمني من خلال وصف نتائج تجارب SLAC . ثم دراسة التصادم المرن بين الالكترون والبروتون, حيث قمنا بحساب المقطع الفعال لهدا التصادم. ناقشنا بنية البروتون من خلال دوال التوزيع الاحتمالي للجسيمات داخل البروتون و كذا معادلة التطور (DGLAP).

تطرقنا في الفصل الثاني لدراسة طريقة دريل-يان في LO, حيث قمنا بحساب المقطع الفعال بين كوارك و ضديد الكوارك, و استعملناه في حساب المقطع الفعال الاجمالي لتفاعل بروتونين باستعمال نظرية التجزئة .

في الفصل الأخير , ناقشنا طريقة دريل-يان في NLO و التحديات التي تواجهها . قمنا بحساب فضاء الاطوار في انتاج جسيمين , ثلاث جسيمات ثم أربع جسيمات . ثم توصلنا اخيراً إلى علاقة تراجعية مختصرة تصف فضاء الأطوارمن اجل استعمالها في محاكات المونتي كارلو .

كلمات مفتاحية : الكروموديناميك الكمي الاضطرابي, نموذج البارتون, سلم بجوركن, دوال التوزيع البارتوني, معادلة التطور, فيزياء المصادمات, المقطع الفعال لطريقة دريل-يان, فضاء الاطوار.

Résumé

Ce travail s’articule autour d’exposer la QCD perturbative comme une théorie des champs quantique.

La première partie est consacrée à étudier en détail le modèle de parton suivant un ordre chronologique en décrivant les résultats expérimental du SLAC, puis étudier la diffusion inélastique profonde (DIS). On a calculé la section efficace pour la collision électron-proton. On a discuté l’échelle de Bjorken, l’interprétation de Feynman pour le modèle de parton. Suivé par un description des fonctions de distribution des partons (PDFs) et l’équation d’évolution (DGLAP).

La seconde partie est dédiée à le processus Drell-Yan en LO. On a calculé la section efficace partonique entre quark et antiquark. Puis utiliser ce dernière dans le calcul de la section efficace hadronique entre deux protons par l’utilisation de la théorème de factorisation.

La dernière partie , on a discuté le processus Drell-Yan en NLO et les défis en ordre supérieur de radiation de QCD. On a calculé l’espace du phase pour deux, trois et quatre particules. Finalement, on a construite la relation récursive d’espace du phase pour n-corps pour l'implémenter dans l'intégration d’espace de phase de Monte Carlo.

Mots clés : QCD perturbative, Modèle de parton , L’échelle de Bjorken, Fonctions de distribution des partons, DGLAP, Physique des collisionneurs, Section efficace du processus Drell-Yan , LO, NLO, Espace du phase .