

People's Democratic Republic of Algeria

وزارة التعليم العالي و البحث العلمي

Ministry of Higher Education and Scientific Research

Mohamed Boudiaf University of M'sila

Faculty of Technology

Department of Electrical Engineering



جامعة محمد بوضياف - المسيلة

كلية التكنولوجيا

قسم : الهندسة الكهربائية

Course

Analysis of three-phase networks in disturbed operating conditions using symmetrical components

3rd year Licence in Renewable Energies and Environment

Directed by:

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Academic year: **2023-2024**

Objectives of the Course

The dimensioning of an installation and the equipment to be used, the settings for the protection devices, and the analysis of electrical phenomena often require calculation of the currents and voltages in electrical networks

The purpose of this course is to set out a simple method of calculating all these parameters in three-phase networks subject to disturbance using the symmetrical components method, and to provide specific application examples.

Recommended prior knowledge:

mathematics and notions of electricity

Course outline:

Chapter I: Brief review of vector mathematics and Matrices

Chapter II: The per-unit (Pu) system

Chapter III: Symmetrical component analysis of three phases and unbalanced Vector

Chapter IV: Sequence Network

Chapter V: Fault calculations

V.1 Single line to ground fault (LG)

V.2 Line to line Faults

V.3 Line to line Faults

V.4 Double line to ground faults

V.5 Three phase faults

Recitation work

Laboratory experiments

Assessment Mode: Continuous assessment + Final exam

Chapter I
Brief review of vector mathematics and Matrices

I.1. Preview of vector mathematics:

I.1.1. Vector presentation:

A **Scalar** has magnitude only

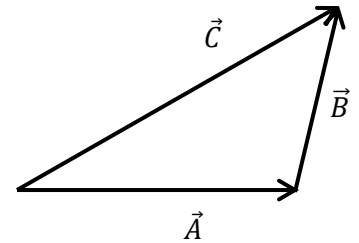
A **vector** has both magnitude and direction

the simple's vector is the displacement vector.

Notation \vec{A} , the magnitude A or $|A|$

Vector addition: $\vec{C} = \vec{A} + \vec{B}$

$$(\vec{A} + \vec{B}) + \vec{C} = \vec{A} + (\vec{B} + \vec{C})$$

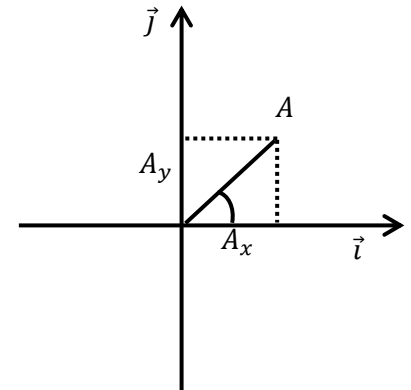


I.1.2. Components of Vector-numerical addition:

$$\vec{A} = \vec{A}_x + \vec{A}_y = A \cos\theta \vec{i} + A \sin\theta \vec{j}$$

Where: $|A_x| = A_x = A \cos \theta$

$$|A_y| = A_y = A \sin \theta$$



Then we can calculate:

$$A = \sqrt{A_x^2 + A_y^2} \text{ and}$$

$$\theta = \text{Arctong} \left(\frac{A_y}{A_x} \right)$$

I.1.3. Vector product:

$$\vec{A} \cdot \vec{B} = AB \cos\theta$$

$$|k \vec{A}| = k |\vec{A}|$$

$$K(\vec{A} + \vec{B}) = k\vec{A} + k\vec{B}$$

k: real number

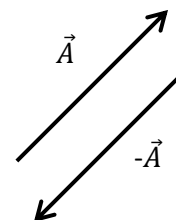
I.1.4. Inverse element for vector addition

$$(-1)\vec{A} = -\vec{A}$$

$$\vec{A} - \vec{A} = \vec{o}$$

$$|-\vec{A}| = |\vec{A}| = A$$

$$\vec{1A} = \vec{A}$$

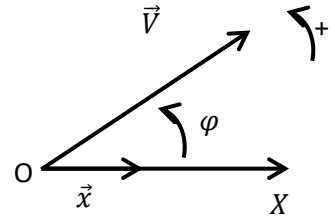


I.1.5. Basic definition:

Consider a sinusoidal vibrating electrical phenomenon represented by a rotating vector \vec{V}

A reference axis \vec{OX} of unit vector

$$\vec{x}: |\vec{x}| = 1$$



A direction conventionally defined as positive in the anti-clock wise direction+

An amplitude $|\vec{V}|$

A Phase $\varphi : (\vec{OX}, \vec{V})$

An Angular frequency ω (constant speed of rotation in radians per second).

At time t , the algebraic value of the projection is

$$V = V \cos(\omega t + \varphi), \omega = 2\pi f \text{ (1Hz=2}\pi \text{ rd/s)}$$

A three-phase system is a set of 3 vectors $\vec{V}_1, \vec{V}_2, \vec{V}_3$

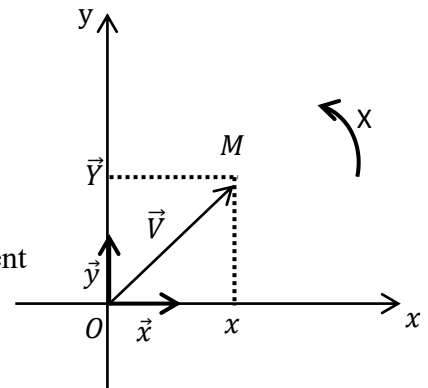
With the same origine, the same angular frequency and each with a constant amplitude.

An electrical system is linear when there is a proportion a proportionality in the relation of causes to effects.

I.1.6. Vector representation

The vector \vec{V} is traditionally represented in a system of rectangular coordinate axes.

$$\vec{V} = \vec{OM} = \vec{OX} + \vec{OY} = \overline{OX} \vec{x} + \overline{OY} \vec{y}$$



I.1.7. Operator "j"

To simplify operation on the vectors \vec{V} can be represented in an equivalent way by a complex number using the operator "j."

"j" is a vector operator which rotates the vector to which to

operation applied through $+\pi/2$, in others words

$$j\vec{x} = \vec{y}$$

Thus, we can write: $J^2 = -1$ (rotation of $2\frac{\pi}{2} = \pi$)

$$J^3 = -1 \text{ (rotation of } 3\frac{3\pi}{2} = 2\pi)$$

$$J^4 = +1 \text{ (rotation of } 4\frac{\pi}{2} = 2\pi)$$

Hence:

$$\vec{V} = \overline{OX} \cdot \vec{x} + \overline{OY} \cdot \vec{y} = \overline{OX} \cdot \vec{x} + \overline{OY} \cdot J\vec{x}$$

$$\vec{V} = \vec{x}(\overline{OX} + J\overline{OY})$$

$$\vec{V} = |V|\varphi$$

$$\vec{V} = |V|[\cos \varphi + J \sin \varphi]$$

I.1.8. Operator “a”

“a” is a vector operator which rotates the vector to which the operation is applied through $\frac{2\pi}{3}$

Thus, we can see that: $a=1 \angle 120^\circ = 1 \angle \frac{2\pi}{3}$

$$a^2 = 1 \angle 2 \cdot \frac{2\pi}{3} = 1 \angle \frac{4\pi}{3} = 1 \angle 240^\circ$$

$$a^3 = 1 \angle 3 \cdot \frac{2\pi}{3} = 1 \angle 2\pi = 1 \angle 360^\circ$$

$$a = 1 \cos\left(\frac{2\pi}{3}\right) + J \sin\left(\frac{2\pi}{3}\right)$$

$$= -0.5 + J \frac{\sqrt{3}}{2}$$

$$a^2 = -0.5 - J \frac{\sqrt{3}}{2}$$

Hence:

$$a^0 = a^3 = a^6 = \dots = 1$$

$$a = a^4 = a^7 = \dots$$

$$a^2 = a^{-2} = a^{-5} = \dots$$

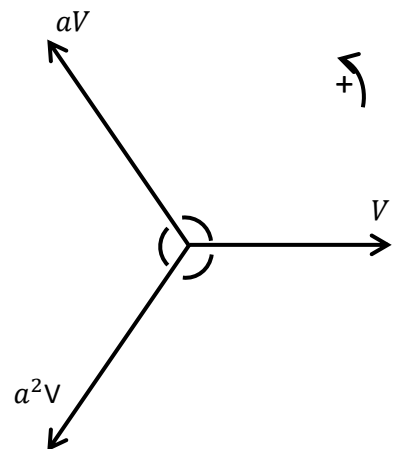
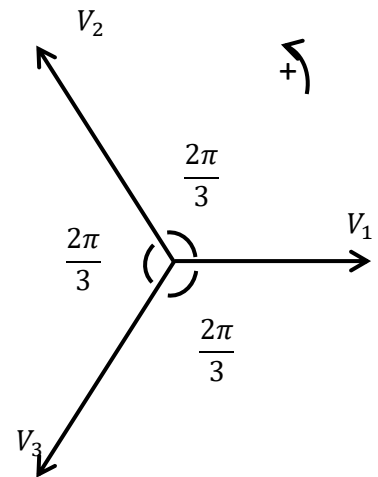
$$a - a^2 = J\sqrt{3}$$

$$1 + a + a^2 = 0$$

$$\vec{V} + a\vec{V} + a^2\vec{V}$$

$$\text{Where } \vec{V}(1 + a + a^2) = 0$$

$$\text{Hence } (1 + a + a^2) = 0$$



I.2. Matrices and Determinants

I.2.1. Introduction

In many applications analysis, variables are assumed to be related by sets of linear equations. Matrix of algebra provides a clear and concise notation for the formulation and solution of such problems, many of which would be complicated in convention of algebraic notation.

I.2.2. Matrix

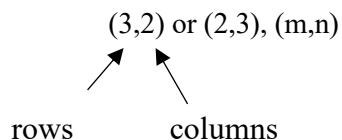
A set of mn numbers (real or complex) arranged in rectangular formation (array or table) having m rows and n columns and enclosed by a square bracket [] is called $m \times n$ matrix (read m by n matrix).

An $m \times n$ matrix is expressed as:

$$\begin{bmatrix} a_{11} & a_{12} & \cdots & a_{1n} \\ a_{21} & a_{22} & \cdots & a_{2n} \\ \vdots & \vdots & \ddots & \vdots \\ a_{m1} & a_{m2} & \cdots & a_{mn} \end{bmatrix}$$

Matrices are usually denoted by capital letter A,B,C and it's elements by small letters a,b,c...

1) Order of a Matrix:



the $\begin{bmatrix} 1 & 2 & 6 \\ 4 & 5 & 7 \end{bmatrix}$, $\begin{bmatrix} 4 \\ 5 \\ 9 \end{bmatrix}$ and $\begin{bmatrix} 1 & 2 & 3 & 4 \\ 5 & 6 & 7 & 5 \\ 4 & 2 & 1 & 0 \\ 3 & 1 & 0 & 6 \end{bmatrix}$

are matrices of orders (2×3) , (3×1) and (4×4) respectively.

2) Null or Zero Matrix

For example:

$$B = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} \text{ is a zero matrix of order } 3 \times 3$$

The Matrix B_{mn} has the property that for every matrix A_{mn} .

$$A + B = B + A = A$$

3) Square Matrix

A Matrix A having same numbers of rows and column is called a square matrix.

The $\begin{bmatrix} 3 & 5 \\ 1 & 2 \end{bmatrix}$ and $\begin{bmatrix} a & b & c \\ d & e & f \\ g & h & i \end{bmatrix}$ are square matrices of order 2 and 3.

4) Diagonal matrix

The diagonal of $\begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \\ 7 & 8 & 9 \end{bmatrix}$ consists of elements 1,5 and 9 in that order.

5) Scalar Matrix

$\begin{bmatrix} 3 & 0 \\ 0 & 3 \end{bmatrix}$ and $\begin{bmatrix} k & 0 & 0 \\ 0 & k & 0 \\ 0 & 0 & k \end{bmatrix}$ are scalar matrices.

6) Identity Matrix or Unit Matrix

Thus $I_1 = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$, $I_2 = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$ are the identity matrices of order 2 and 3.

7) Equal Matrices

$$A = \begin{bmatrix} x + 3 & y \\ z & a \end{bmatrix}, B = \begin{bmatrix} 2 & 5 \\ 3 & 4 \end{bmatrix}$$

$A = B$ if it has a same order and $a_{ij} = b_{ij}$

By the definition of equality of matrices, we have:

$$A = B \quad \begin{cases} x + 3 = 2 \\ y = 5 \\ z = 3 \\ a = 4 \end{cases}$$

8) The Negative of Matrix

$$\text{If } A_{3 \times 2} = \begin{bmatrix} 3 & -1 \\ 2 & -2 \\ -4 & 5 \end{bmatrix} \text{ then } -A_{3 \times 2} = \begin{bmatrix} -3 & 1 \\ -2 & 2 \\ 4 & -5 \end{bmatrix}$$

For every matrix $A_{m \times n}$, the matrix $-A_{m \times n}$ has the property that:

$$A + (-A) = (-A) + A = 0$$

I.2.3. Operation on Matrices

1) Multiplication of a matrix by a scalar

$$\text{If } A = \begin{bmatrix} 1 & 2 \\ 3 & 4 \\ 5 & 0 \end{bmatrix} \text{ then for a scalar } k, \quad kA = \begin{bmatrix} k & 2k \\ 3k & 4k \\ 5k & 0 \end{bmatrix}$$

$$\text{For example: } 5 \begin{bmatrix} -1 & 2 & 0 \\ 3 & 5 & 2 \\ 1 & 0 & 4 \end{bmatrix} = \begin{bmatrix} -5 & 10 & 0 \\ 15 & 25 & 10 \\ 5 & 0 & 20 \end{bmatrix}$$

2) Addition and subtraction of Matrices

A, B two matrices have a same order $m \times n$ ($A_{m \times n}, B_{m \times n}$)

$$A = \begin{bmatrix} 2 & 5 \\ 1 & 0 \end{bmatrix}, \quad B = \begin{bmatrix} 4 & -5 \\ 3 & 2 \end{bmatrix}$$

$$A + B = \begin{bmatrix} 2 + 4 & 5 + (-5) \\ 1 + 3 & 0 + 2 \end{bmatrix} = \begin{bmatrix} 6 & 0 \\ 4 & 2 \end{bmatrix}$$

$$A - B = \begin{bmatrix} 2 - 4 & 5 - (-5) \\ 1 - 3 & 0 - 2 \end{bmatrix} = \begin{bmatrix} -2 & 10 \\ -2 & -2 \end{bmatrix}$$

Note:

- 1) $A + B = B + A$
- 2) $(A + B) + C = A + (B + C)$
- 3) The same or difference of two matrices of different order is not defined.

3) Product of matrices

Two matrices A and B with $A_{m \times p}$, $B_{p \times n}$ (p) the columns equal the rows.

The product of $A_{m \times p}$ and $B_{p \times n}$ is the matrix $(AB)_{m \times n}$

$$A = \begin{bmatrix} 1 & 2 \\ -1 & 3 \end{bmatrix}, \quad B = \begin{bmatrix} 2 & 1 \\ 1 & 1 \end{bmatrix}$$

$$A \times B = \begin{bmatrix} 1 & 2 \\ -1 & 3 \end{bmatrix} \begin{bmatrix} 2 & 1 \\ 1 & 1 \end{bmatrix} = \begin{bmatrix} 1 * 2 + 2 * 1 & 1 * 1 + 2 * 1 \\ (-1 * 2) + 3 * 1 & (-1 * 1) + 3 * 1 \end{bmatrix} = \begin{bmatrix} 2 + 2 & 1 + 2 \\ -2 + 3 & -1 + 3 \end{bmatrix} = \begin{bmatrix} 4 & 3 \\ 1 & 2 \end{bmatrix}$$

$$B \times A = \begin{bmatrix} 2 & 1 \\ 1 & 1 \end{bmatrix} \begin{bmatrix} 1 & 2 \\ -1 & 3 \end{bmatrix} = \begin{bmatrix} (2 * 1) + (1 * -1) & (2 * 2) + (1 * 3) \\ (1 * 1) + (1 * -1) & (1 * 2) + (1 * 3) \end{bmatrix} = \begin{bmatrix} 2 - 1 & 4 + 3 \\ 1 - 1 & 2 + 3 \end{bmatrix} = \begin{bmatrix} 1 & 7 \\ 0 & 5 \end{bmatrix}$$

Note:

- $A \times B \neq B \times A$
- $A \times A = A^2, A \times A^2 = A^3, A^2 \times A^2 = A^4$

Example $A = \begin{bmatrix} 3 & 1 & 2 \\ 1 & 0 & 1 \end{bmatrix}, B = \begin{bmatrix} 1 & -1 \\ 2 & 1 \\ 3 & 1 \end{bmatrix}$, Find AB

Solution: $AB = \begin{bmatrix} 3 + 2 + 6 & -3 + 1 + 2 \\ 1 + 0 + 3 & -1 + 0 + 1 \end{bmatrix} = \begin{bmatrix} 11 & 0 \\ 4 & 0 \end{bmatrix}$

4) Determinants

- The determinant of a matrix is scalar (number)
- The determinant of a matrix is defined only for square matrix
- It is denoted by $(\det A)$ or $|A|$ for square matrix A

Example 01:

$$A = \begin{bmatrix} 3 & 1 \\ -2 & 3 \end{bmatrix}, \text{ Find } |A|.$$

$$A = \begin{bmatrix} 3 & 1 \\ -2 & 3 \end{bmatrix} = (3) \times (3) - (1) \times (-2) = 9 + 2 = 11$$

Example 02:

$$A = \begin{bmatrix} 3 & 2 & 1 \\ 0 & 1 & -2 \\ 1 & 3 & 4 \end{bmatrix}, \text{ Find } |A|.$$

$$|A| = \begin{vmatrix} 3 & 2 & 1 \\ 0 & 1 & -2 \\ 1 & 3 & 4 \end{vmatrix} = 3 \begin{vmatrix} 1 & -2 \\ 3 & 4 \end{vmatrix} - 2 \begin{vmatrix} 0 & -2 \\ 1 & 4 \end{vmatrix} + 1 \begin{vmatrix} 0 & 1 \\ 1 & 3 \end{vmatrix}$$

$$= 3(4+6) - 2(0+2) + 1(0-1)$$

$$= 30 - 4 - 1$$

$$= 25$$

$$\begin{aligned}
 |A| &= \begin{vmatrix} 3 & 2 & 1 \\ 0 & 1 & -2 \\ 1 & 3 & 4 \end{vmatrix} = 3 \begin{vmatrix} 1 & -2 \\ 3 & 4 \end{vmatrix} - 0 \begin{vmatrix} 2 & 1 \\ 3 & 4 \end{vmatrix} + 1 \begin{vmatrix} 2 & 1 \\ 1 & -2 \end{vmatrix} \\
 &= 3(4+6) - 0(6-3) + 1(-4-1) \\
 &= 30 + (-5) \\
 &= 25
 \end{aligned}$$

- If $|A| = 0$, the A is singular.

I.2.4. Solution of linear equations by determinants

- Use Cramer's rule
- Use matrices

1) Transpose of a matrix

$$\text{If } A = \begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \\ 7 & 8 & 9 \end{bmatrix} \text{ then } A^t = \begin{bmatrix} 1 & 4 & 7 \\ 2 & 5 & 8 \\ 3 & 6 & 9 \end{bmatrix}$$

2) Symmetric Matrix:

A square matrix A is called symmetric if $A = A^t$

$$A = \begin{bmatrix} a & b & c \\ b & d & e \\ c & e & f \end{bmatrix}, \quad A^t = \begin{bmatrix} a & b & c \\ b & d & e \\ c & e & f \end{bmatrix} = A$$

Thus, A is symmetric.

3) Adjoint of a Matrix:

$$A = \begin{bmatrix} 1 & 0 & -1 \\ 1 & 3 & 1 \\ 0 & 1 & 2 \end{bmatrix}$$

Cofactor of A are:

$$C = \begin{bmatrix} 5 & -2 & 1 \\ -1 & 2 & -1 \\ 3 & -2 & 3 \end{bmatrix}, \quad C^+ = \begin{bmatrix} 5 & -1 & 3 \\ -2 & 2 & -2 \\ 1 & -1 & 3 \end{bmatrix}$$

Hence $\text{adj}A = C^t$

Note: the adjoint of 2*2 matrix of $A = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$ is $\text{adj}A = \begin{bmatrix} d & -b \\ -c & a \end{bmatrix}$

4) Invers of a Matrix:

If A nonsingular square matrix, then $A^{-1} = \frac{\text{adj}A}{|A|}$
 $A = \begin{bmatrix} 3 & 4 \\ 1 & 2 \end{bmatrix}$

$$\text{adj}A = \begin{bmatrix} 2 & -4 \\ -1 & 3 \end{bmatrix}, \quad |A| = 6 - 4 = 2$$

hence $A^{-1} = \frac{1}{2} \begin{bmatrix} 2 & -4 \\ -1 & 3 \end{bmatrix}$

Note:

$$AB = BA = I, \quad A_{n \times n}, B_{n \times n}$$

$$B = A^{-1} \quad \text{or} \quad A = B^{-1}$$

Example 01:

$$A = \begin{bmatrix} 1 & -3 \\ -2 & 7 \end{bmatrix}, \quad B = \begin{bmatrix} 7 & 3 \\ 2 & 1 \end{bmatrix}$$

$$AB = \begin{bmatrix} 1 & -3 \\ -2 & 7 \end{bmatrix} \begin{bmatrix} 7 & 3 \\ 2 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

And

$$BA = \begin{bmatrix} 7 & 3 \\ 2 & 1 \end{bmatrix} \begin{bmatrix} 1 & -3 \\ -2 & 7 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

Hence $AB = BA = I$

And therefore $B = A^{-1} = \begin{bmatrix} 7 & 3 \\ 2 & 1 \end{bmatrix}$

Example 02: find the inverse, if it exists, of the matrix

$$A = \begin{bmatrix} 0 & -2 & -3 \\ 1 & 3 & 3 \\ -1 & -2 & -2 \end{bmatrix}$$

Solution:

$$|A| = 0 + 2(-2+3) - 3(-2+3) = 2 - 3 = -1$$

$|A| = -1$, so the solution exists.

Cofactor of A are

$$A_{11} = 0 \quad A_{12} = -1 \quad A_{13} = 1$$

$$\begin{array}{lll} A_{21} = 2 & A_{22} = -3 & A_{23} = 2 \\ A_{31} = 3 & A_{32} = -3 & A_{33} = 2 \end{array}$$

Matrix of transpose of the cofactors is

$$\text{adj}A=C=\begin{bmatrix} 0 & 2 & 3 \\ -1 & -3 & -3 \\ 1 & 2 & 2 \end{bmatrix}$$

$$\text{now, } A^{-1}=\frac{1}{|A|} \text{adj}A=\frac{1}{-1}\begin{bmatrix} 0 & 2 & 3 \\ 1 & -3 & -3 \\ 1 & 2 & 2 \end{bmatrix}=\begin{bmatrix} 1 & -2 & -3 \\ -1 & 3 & 3 \\ -1 & -2 & -2 \end{bmatrix}$$

Example 03: Use matrices to find the solution set of

$$x+y-2z=3$$

$$3x-y+z=5$$

$$3x+3y-6z=9$$

Solution :

$$\text{Let } A=\begin{bmatrix} 1 & 1 & -2 \\ 3 & -1 & 1 \\ 3 & 3 & 6 \end{bmatrix}$$

$$\text{Since } |A| = 3+21-24=0$$

Hence the solution of the given linear equations does not exist.

Example 04: use the matrices to find the solution set of

$$4x+8y+z=-6$$

$$2x-3y+2z=0$$

$$x+7y-3z=-8$$

$$\begin{bmatrix} 4 & 8 & 1 \\ 2 & -3 & 2 \\ 1 & 7 & -3 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} -6 \\ 0 \\ -8 \end{bmatrix}$$

$$\text{Let } A=\begin{bmatrix} 4 & 8 & 1 \\ 2 & -3 & 2 \\ 1 & 7 & -3 \end{bmatrix}, \quad |A| = -32+48+17= 61$$

Hence A^{-1} exists.

$$A^{-1}=\frac{1}{|A|} \text{adj}A=\frac{1}{61}\begin{bmatrix} -5 & 31 & 19 \\ 8 & -13 & -16 \\ 17 & -20 & -28 \end{bmatrix}$$

Now since $X=A^{-1}B$, we have

$$\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \frac{1}{61} \begin{bmatrix} -5 & 31 & 19 \\ 8 & -13 & -16 \\ 17 & -20 & -28 \end{bmatrix} \begin{bmatrix} -6 \\ 0 \\ -8 \end{bmatrix} = \frac{1}{61} \begin{bmatrix} 30 + 152 \\ -48 + 48 \\ -102 + 224 \end{bmatrix} = \begin{bmatrix} -2 \\ 0 \\ 2 \end{bmatrix}$$

the solution set $\{(x,y,z)\}=\{(-2,0,2)\}$.

Exercise 01

A. Find the inverse of matrix A:

$$A = \begin{bmatrix} + & - & + \\ -4 & -6 & 2 \\ 5 & -1 & 3 \\ -2 & 4 & -3 \end{bmatrix}$$

$$\begin{aligned} 1) \text{ Det} &= -4 \begin{vmatrix} -1 & 3 \\ 4 & -3 \end{vmatrix} - (-6) \begin{vmatrix} 5 & 3 \\ -2 & -1 \end{vmatrix} + 2 \begin{vmatrix} 5 & -1 \\ -2 & 4 \end{vmatrix} \\ &= -4(3-12+6(-15+6))+2(20-2) \\ &= -4(-9)+6(-9)+2(18) \\ &= 36-54+36 \end{aligned}$$

$$\text{det}=18$$

$$2) \text{ adj}A = ? \begin{bmatrix} + & - & + \\ - & + & - \\ + & - & + \end{bmatrix} = \begin{bmatrix} + \begin{bmatrix} -1 & 3 \\ 4 & -3 \end{bmatrix} & - \begin{bmatrix} 5 & 3 \\ -2 & -3 \end{bmatrix} & + \begin{bmatrix} 5 & -1 \\ -2 & 4 \end{bmatrix} \\ - \begin{bmatrix} -6 & 2 \\ 4 & -3 \end{bmatrix} & + \begin{bmatrix} -4 & 2 \\ -2 & -3 \end{bmatrix} & - \begin{bmatrix} -4 & -6 \\ -2 & 4 \end{bmatrix} \\ + \begin{bmatrix} -6 & 2 \\ -1 & 3 \end{bmatrix} & - \begin{bmatrix} -4 & 2 \\ 5 & 3 \end{bmatrix} & + \begin{bmatrix} -4 & -6 \\ 5 & -1 \end{bmatrix} \end{bmatrix} =$$

$$\begin{bmatrix} -9 & 9 & 18 \\ -10 & -8 & 4 \\ -16 & 22 & 34 \end{bmatrix}$$

$$\text{adj}A = \begin{bmatrix} -9 & -10 & -16 \\ 9 & -8 & 22 \\ 18 & 4 & 34 \end{bmatrix}$$

$$A^{-1} = \frac{1}{\text{det}A} \text{adj}A = \frac{1}{18} \begin{bmatrix} -9 & -10 & -16 \\ 9 & -8 & 22 \\ 18 & 4 & 34 \end{bmatrix}$$

$$A^{-1} = \begin{bmatrix} -\frac{1}{2} & -\frac{5}{9} & -\frac{8}{9} \\ \frac{1}{2} & -\frac{4}{9} & \frac{11}{9} \\ \frac{1}{1} & \frac{2}{9} & \frac{17}{9} \end{bmatrix}$$

B. We must find the inverse of the matrix A

$$A = \begin{bmatrix} 1 & -1 & 3 \\ 2 & 1 & 2 \\ -2 & -2 & 1 \end{bmatrix} \Rightarrow A^{-1} = \frac{1}{|A|} \text{adj}A$$

$$|A| = ?$$

$$\begin{vmatrix} 1 & -1 & 3 \\ 2 & 1 & 2 \\ -2 & -2 & 1 \end{vmatrix} \begin{vmatrix} 1 & -1 \\ 2 & 1 \\ -2 & -2 \end{vmatrix}$$

$$5 = -7 - (-12) = [(1) + (4) + (-12)] - [(-2) + (-4) + (-6)]$$

$$|A| = 1$$

adj A = ?

$$\begin{bmatrix} \begin{bmatrix} 1 & 2 \\ -2 & 1 \end{bmatrix} & \begin{bmatrix} 2 & 2 \\ -2 & 1 \end{bmatrix} & \begin{bmatrix} 2 & 1 \\ -2 & -2 \end{bmatrix} \\ \begin{bmatrix} -1 & 3 \\ -2 & 1 \end{bmatrix} & \begin{bmatrix} 1 & 3 \\ -2 & 1 \end{bmatrix} & \begin{bmatrix} 1 & -1 \\ -2 & -2 \end{bmatrix} \\ \begin{bmatrix} -1 & 3 \\ 1 & 2 \end{bmatrix} & \begin{bmatrix} 1 & 3 \\ 2 & 2 \end{bmatrix} & \begin{bmatrix} 1 & -1 \\ 2 & 1 \end{bmatrix} \end{bmatrix} = \begin{bmatrix} (1) + 4 & 2 + 4 & (-4) + 2 \\ -1 + 6 & 1 + 6 & -2 - 2 \\ -2 - 3 & 2 - 6 & 1 + 2 \end{bmatrix}$$

$$= \begin{bmatrix} 5 & -6 & -2 \\ -5 & 7 & +4 \\ -5 & +4 & 3 \end{bmatrix}$$

$$\text{adj} A = \begin{bmatrix} 5 & -5 & -5 \\ -6 & 7 & 4 \\ -2 & 4 & 3 \end{bmatrix}$$

$$A^{-1} = \frac{1}{5} \begin{bmatrix} 5 & -5 & -5 \\ -6 & 7 & 4 \\ +2 & 4 & 3 \end{bmatrix} = \begin{bmatrix} 1 & -1 & -1 \\ -\frac{6}{5} & \frac{7}{5} & \frac{4}{5} \\ -\frac{2}{5} & \frac{4}{5} & \frac{3}{5} \end{bmatrix}$$

C. Find the value of x given that:

$$[1 \quad 2 \quad 1] \begin{bmatrix} 1 & 2 & 0 \\ 2 & 0 & 1 \\ 1 & 0 & 2 \end{bmatrix} \begin{bmatrix} 0 \\ 2 \\ x \end{bmatrix} = 0$$

$$[1 \times 3][3 \times 3] = [1 \times 3]$$

$$[1 \quad 2 \quad 1] \begin{bmatrix} 1 & 2 & 0 \\ 2 & 0 & 1 \\ 1 & 0 & 2 \end{bmatrix} = [1 + 4 + 1 \quad 2 + 0 + 0 \quad 0 + 2 + 2]$$

$$= [6 \quad 2 \quad 4]$$

$$[6 \quad 2 \quad 4] \begin{bmatrix} 0 \\ 2 \\ x \end{bmatrix} = [0 + 4 + 4x] = 4 + 4x = 0$$

$$[1 \times 3][3 \times 1] = [1 \times 1]$$

$$4x = -4, \quad x = -1$$

Exercise 02

- 1) Solving a system of equation with matrix inverse method, the number of variables must equal the number of equations
both equal 3 here

$$x - y + 3z = 2$$

$$2x + y + 2z = 2$$

$$-x - 2y + z = 3$$

By the matrix inverse method in three distinct steps, as follows:

- 1- Write the given system (above) as a single matrix equation

$$\begin{bmatrix} 1 & -1 & 3 \\ 2 & 1 & 2 \\ -2 & -2 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 2 \\ 2 \\ 3 \end{bmatrix}$$

$$A \cdot X = B$$

- 2- Solve the matrix equation obtained in step [1] above, finding X
We solve for the matrix variable X by left multiplying both sides of the above matrix equation ($AX = B$) By A^{-1}

$$A^{-1} \cdot A \cdot X = A^{-1} \cdot B$$

$$I \cdot X = A^{-1} \cdot B$$

$$X = A^{-1} \cdot B \Rightarrow \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} -3 \\ \frac{14}{5} \\ \frac{13}{5} \end{bmatrix}$$

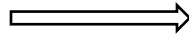
3- Using step [2] above, write the solution to the original system

Original system

$$x - y + 3z = 2$$

$$2x + y + 2z = 2$$

$$-x - 2y + z = 3$$



solution to original system

$$\left\{ \begin{array}{l} x = -3 \\ y = 2 \\ z = \frac{13}{5} \end{array} \right.$$

Chapter II
The per-unit (Pu) system

II.1. Introduction

Power system quantities such as current, voltage, impedance and power are often expressed in pu values. For examples: base voltage if 220Kv is specified. Then the voltage is 210 Kv, the value $210/220=0.954$ Pu is per unit value. The advantage of the per unit value is that by properly specifying base quantities, the equivalent impedance of transformer can be simplified. ($Z_p = Z_s$).

Another advantage of the per-unit system is that the comparison of the characteristics of the various electrical apparatus of different types and ratings is facilitated by expressing the impedance in per-unit based on their rating.

The different voltage levels disappear and power networks (generators, transformers, lines) reduces to a system of simple impedance.

$$\text{Per - unit quantity} = \frac{\text{Actual quantity}}{\text{base value of quantity}}$$

Let us define,

$$S_{pu} = \frac{S_{act}}{S_b}, V_{pu} = \frac{V_{act}}{V_b}, I_{pu} = \frac{I_{act}}{I_b} \text{ and } Z_{pu} = \frac{Z_{act}}{Z_b}$$

$S_b, V_b,$ and Z_b are always real numbers

Usually, the S_b (MVA) and line to line (V_b (Kv)) are selected

$$\begin{aligned} S &= \sqrt{3}VI \Rightarrow \\ I_b &= \frac{S(MVA)}{\sqrt{3}V_b(Kv)} \\ Z_b &= \frac{V_b^2}{S_b} \end{aligned}$$

Here $S_{pu} = V_{pu} \cdot I_{pu}^*$ (I^* : complexe conjugate of per - unit current I_{pu}).

$$Z_{pu} = |V_{pu}|^2 / S_{pu}^*$$

Furthermore, the impedance of generators, transformers and motors supplied by the manufacturer are generally given in P.U values on their own rating. For power system analysis all impedances must expressed in per-unit values on common base.

When base quantities are changed from $S_{b,old}$ to S_b^{new} and from V_b^{old} to V_b^{new} , the new per-unit impedance can be given by:

$$Z_{Pu}^{new} = Z_{Pu}^{old} \cdot \frac{S_b^{new}}{S_b^{old}} \cdot \left(\frac{V_b^{old}}{V_b^{new}} \right)^2$$

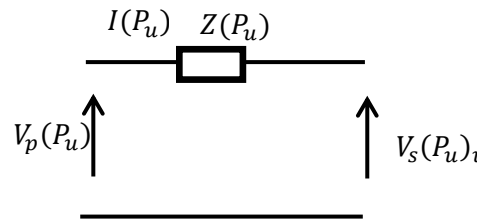
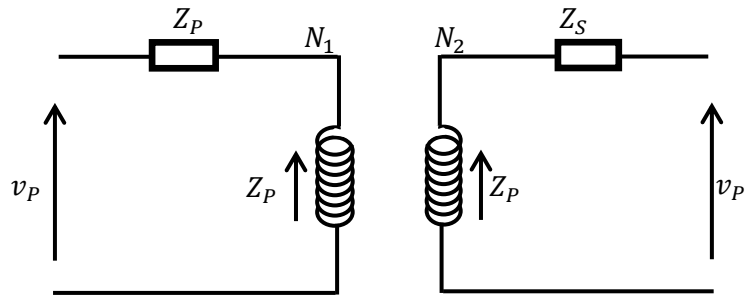
II.2. Per-unit of transformer

The per-unit of transformer of a transformer is the same whether computed from primary or secondary side

$$Z_1(P_u) = Z_2(P.U) = Z_{pu}$$

$$Z_1(P_u) = Z_p(P.U) + Z_s(P_u)$$

$$Z_2(P_u) = Z_s(P.U) + Z_p(P_u)$$



Per-unit equivalent circuit of single-phase transformer

$$\frac{V_1}{V_2} = \frac{N_1}{N_2} = \left(\frac{1}{\alpha} \right) \leftarrow \text{(transformer ratio)}$$

$$\frac{V_p}{V_s} = \frac{N_p}{N_s} = \frac{1}{\alpha}$$

II.3. Example 01

A single phase two winding transformer is rated 25KVA, 1100/440V, 50Hz. The equivalent leakage impedance of the transformer referred to the low voltage side is $0.06 \angle 78^\circ \Omega$.

Using transformer rating as base values, determine the pre-unit leakage impedance referred to low voltage winding and referred to high voltage winding.

Solution

$$S_b = 25Kva, V_{p,b} = 1.1KV, V_{s,b} = 0.44KV$$

Base impedance on the low side is

$$Z_{s,b} = \frac{V_{s,b}^2}{S_b} = \frac{(0.44)^2}{0.025} = 7.744\Omega$$

Per-unit leakage impedance referred to the low voltage side is

$$Z_{s,pu} = \frac{Z_{s,eq}}{Z_{sb}} = \frac{0.06\angle 78^\circ}{7.444} = 7.74 \cdot 10^{-3} \angle 78^\circ pu$$

If $Z_{p,eq}$ referred to primary winding (HV Side)

$$Z_{p,eq} = \alpha^2 \cdot Z_{s,eq} = \left(\frac{N_1}{N_2}\right)^2 \cdot Z_{s,eq} = \left(\frac{1.1}{0.44}\right)^2 \cdot 0.06\angle 78^\circ$$

$$Z_{p,eq} = 0.375\angle 78^\circ \Omega$$

Hence, base impedance in heigh voltage is 1.1Kv

$$Z_{p,b} = \frac{V_b^2}{S_b} = \frac{(1.1)^2}{0.025} = 48.4\Omega$$

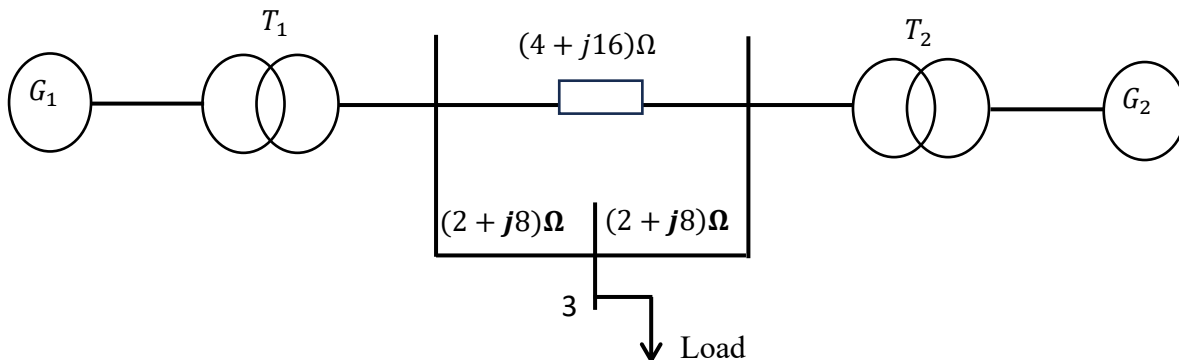
$$Z_{p,pu} = \frac{Z_{peq}}{Z_{pb}} = \frac{0.375\angle 78^\circ}{48.4} = 7.47 \cdot 10^{-3} \angle 78^\circ pu$$

Therefore, per-unit leading impedance remains unchanged and this has been achieved by specifying

$$\frac{V_{PB}}{V_{SB}} = \frac{1.1}{0.44} = 2.5$$

Example 02

Draw the per-unit impedance diagram of the system shown figure below, assumed base values are 100 MVA and 100KV.



$$G_1: 50 \text{ MVA}, 12,2 \text{KV}, \quad x_{g1} = 0.10 \text{ Pu.}$$

$$G_2: 20 \text{ MVA}, 13.8 \text{KV}, \quad x_{g2} = 0.10 \text{ Pu.}$$

$$T1: 80 \text{MVA}, 12,2/132 \text{KV}, \quad x_{t1} = 0.10 \text{Pu}$$

$$T2: 40 \text{MVA}, 13,8/132 \text{KV}, \quad x_{t2} = 0.10 \text{Pu}$$

Load: 50MVA, 0.80 PF lagging operating at 124 KV

Solution:

Base KV is 100 KV in transmission line

$$V_b(G_1) = V_b \frac{V_1}{V_2} = 100 \frac{12.2}{132} = 9.24 \text{KV}, \quad V_{pu} = \frac{12.2}{9.4} \left(\frac{V_1}{V_2} = \frac{N_1}{N_2} \right)$$

$$V_b(G_2) = V_b \frac{V_1}{V_2} = 100 \frac{13.8}{132} = 10.45 \text{KV}$$

Now for:

$$x_{g1}^{(new)} = x_{g1}^{old} \cdot \left(\frac{S_b^{new}}{S_b^{old}} \right) \left(\frac{V_b^{old}}{V_b^{new}} \right)^2 = 0.10 \left(\frac{100}{50} \right) \left(\frac{12.2}{9.24} \right)^2$$

$$x_{g1}^{new} = 0.3486 \text{Pu}$$

$$x_{g2}^{(new)} = x_{g2}^{old} \left(\frac{S_b^{new}}{S_b^{old}} \right) \left(\frac{V_b^{old}}{V_b^{new}} \right)^2 = 0.10 \left(\frac{100}{20} \right) \left(\frac{13.8}{10.45} \right)^2$$

$$x_g^{new} = 0.8719 \text{ Pu}$$

$$\text{For } T_1, x_{T1}^{(new)} = 0.10 \frac{100}{80} \cdot \left(\frac{132}{100} \right)^2 = 0.2179 \text{pu}$$

$$\text{For } T_2, x_{T2}^{new} = 0.10 \frac{100}{40} \cdot \left(\frac{132}{100} \right)^2 = 0.4359 \text{pu}$$

→ base impedance of transmission line:

$$Z_b(\text{Line}) = \left(\frac{V_b^2}{S_b} \right) = \frac{(100)^2}{100} = 100 \Omega$$

$$Z_{pu}^{(12)} = \frac{Z_{12}}{Z_b(\text{line})} = \frac{4 + j16}{100} = (0.04 + j0.16) \text{pu}$$

$$Z_{pu}(13) = Z_{pu}(23) = \frac{2 + j8}{100} = (0.02 + j0.08)pu$$

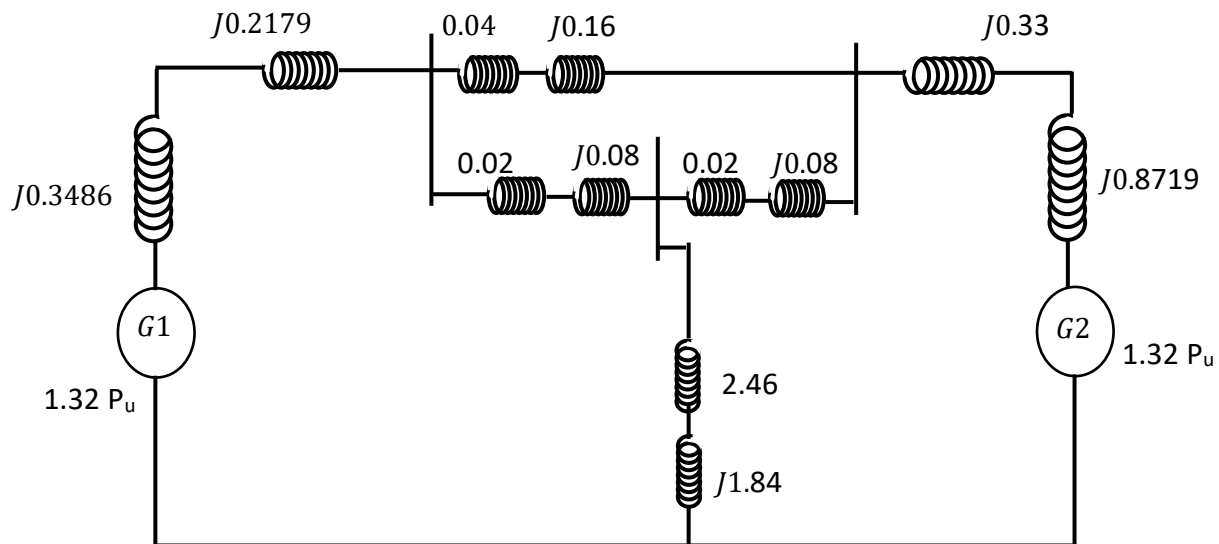
The load is specified as:

$$S_b = 50(0.8 + j0.6) = 40 + j30MVA \leftarrow S_{complex} = S(\cos \varphi + j \sin \varphi)$$

$$Z_{load} = \frac{V^2}{S} = \frac{(124)^2}{(40 + j30)} = 307.52 \angle -36.87^\circ \Omega$$

$$Z_{load}(P_u) = \frac{Z_{act}}{Z_{b(line)}} = \frac{307.52 \angle -36.78^\circ}{100} = (2.46 + j1.845)pu$$

$$R_{series} = 2.46P_u, X_{series} = 1.845P_u$$



Chapter III
**Symmetrical component analysis of three phases
and unbalanced Vector**

III.1. Introduction

Areas of power electrical engineering:

- Power electronics (Inverter, rectifier)
- Power transmission and distribution
- Electrical machines
- Power system protection.
- **Power system analysis**
- Renewable energy
- HVDC systems
- other planning reliability

In normal, balanced, symmetrical operation, the study of three phase networks can be reduced to the study of an equivalent single-phase network with voltage equal to the phase to neutral voltage of the network, currents equal to those of the networks and Impedances equal to those of the network, known as cyclic Impedance.

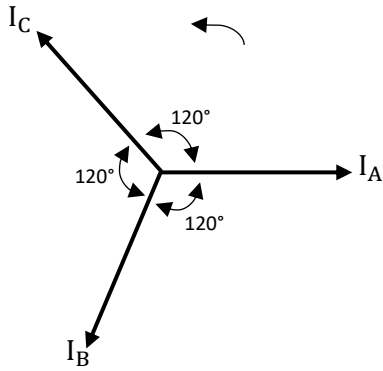
Asymmetrical operation can occur in a network if there is an unbalance in the voltage or impedance system of the electrical elements (due to a fault or by design)

If the asymmetry is significant, simplification is no longer possible because the relation in the various conductors cannot be determined by means of a cyclic impedance for each element of the network.

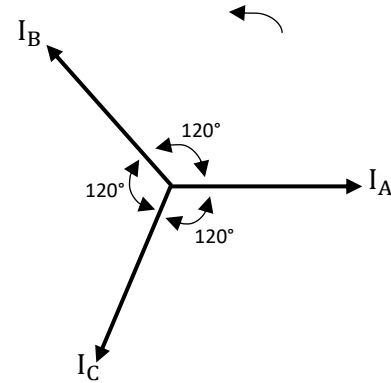
The general method based on ohms and Kirchhoff's laws is possible but it is complex and laborious (lourd).

The "symmetrical components" method described in this module and we can simplify the calculations and provides a much easier solution by reducing it to the superposition of three independent single-phase networks.

III.2. Phase rotation or phase sequence



ABC phase sequence

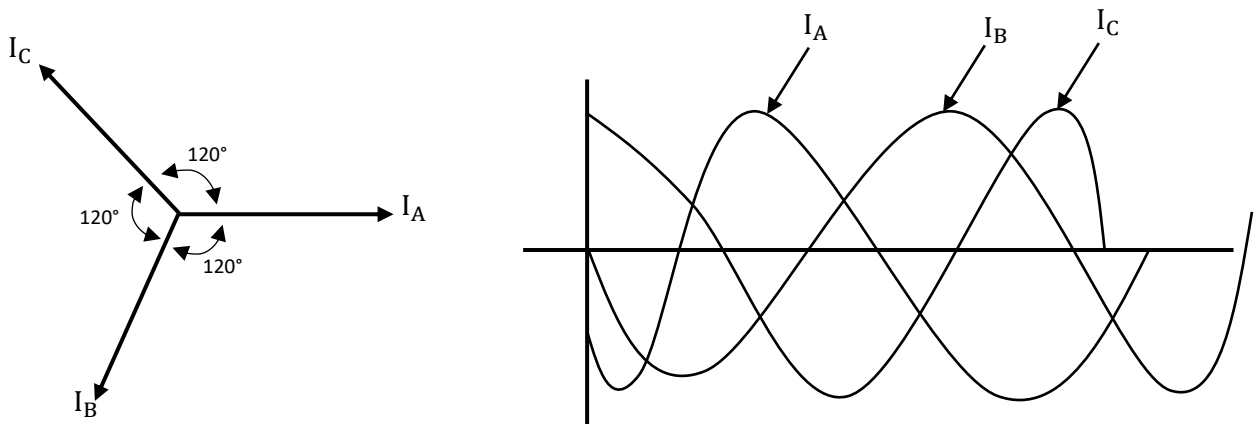


ACB phase sequence

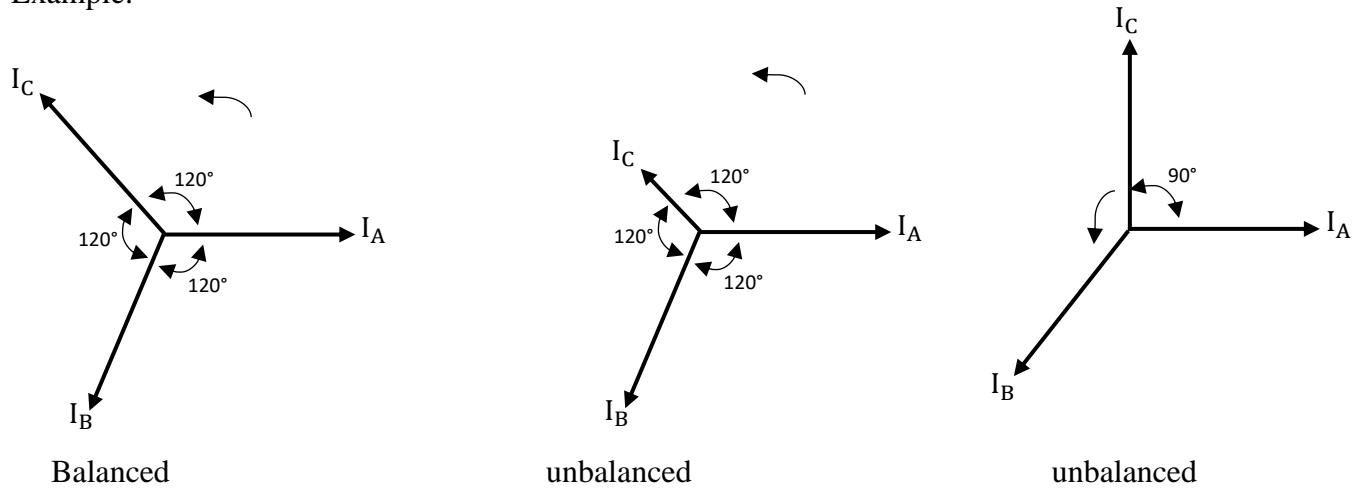
III.3. Balance symmetrical three phases Vector

The set of phases is balanced then it:

- Equal magnitude of I_A , I_B and I_C
- The phase shift between the phases is 120°



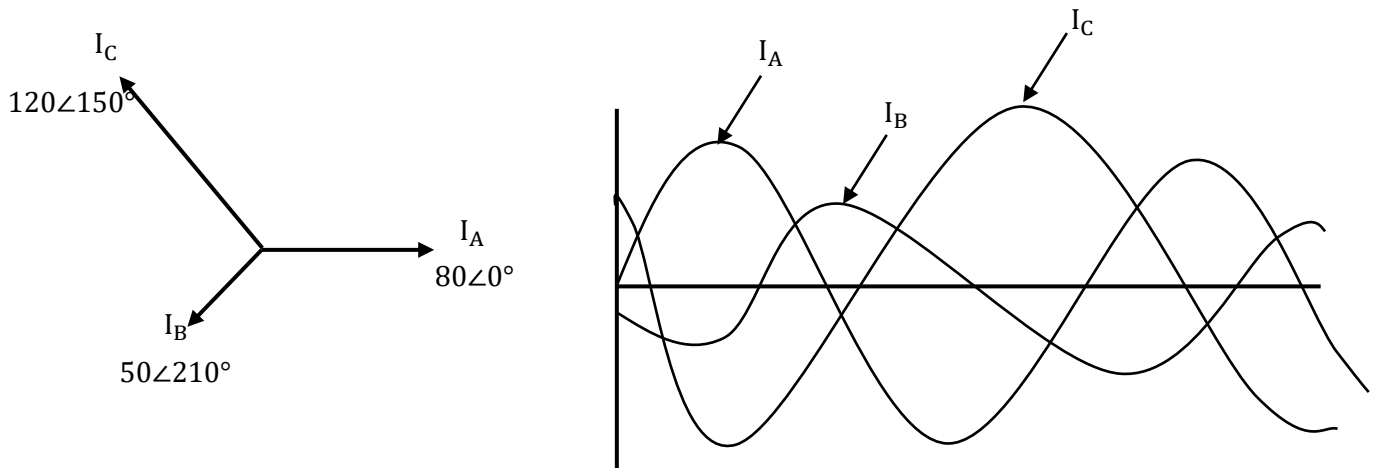
Example:



III.4. Unbalanced (asymmetrical) three phases Vector

The set of phases is balanced then it:

- The angle between the phases is not 120°
- The amplitude of all phases is not same



In 1918 Dr C.L.Fortrscue presented a method of symmetrical components deal such complex problem of unbalanced three phase systems

- he showed that any imbalanced (asymmetrical) three phase voltage or current Vector can be broken in to three components, which are balanced (symmetrical)

These three symmetrical components are called:

- Positive sequence components.
- negative sequence components
- Zero sequence components

Example:

	Positive sequence	Negative sequence	Zero sequence
$V_1 = V_1^{(1)} + V_1^{(2)} + V_1^{(0)}$ $V_2 = V_2^{(1)} + V_2^{(2)} + V_2^{(0)}$ $V_3 = V_3^{(1)} + V_3^{(2)} + V_3^{(0)}$	have the same amplitude are shifted by 120° sequence is same as the original system (123)	have the same amplitude are shifted by 120° sequence is opposite to the original system(132)	have the same amplitude no phase sequence

For all sequences we can define V_B and V_C in terms of V_A

$$a = 1 \angle 120^\circ$$

$$a^2 = 1 \angle 240^\circ$$

$V_A^{(1)} = V_A^{(1)}$ $V_B^{(1)} = a^2 V_A^{(1)}$ $V_C^{(1)} = a V_A^{(1)}$	$V_A^{(2)} = V_A^{(2)}$ $V_B^{(2)} = a V_A^{(2)}$ $V_C^{(2)} = a^2 V_A^{(2)}$	$V_A^{(0)} = V_B^{(0)} = V_C^{(0)}$

Therefore

$$V_A = V_A^{(0)} + V_A^{(1)} + V_A^{(2)}$$

$$V_B = V_A^{(0)} + a^2 V_A^{(1)} + a V_A^{(2)}$$

$$V_C = V_A^{(0)} + a V_A^{(1)} + a^2 V_A^{(2)}$$

$$\begin{bmatrix} V_A \\ V_B \\ V_C \end{bmatrix} = \begin{bmatrix} 1 & 1 & 1 \\ 1 & a^2 & a \\ 1 & a & a^2 \end{bmatrix} \begin{bmatrix} V_A^{(0)} \\ V_A^{(1)} \\ V_A^{(2)} \end{bmatrix}$$

Original unbalanced phases

symmetrical components phases A

Given a set of unbalanced voltage, we can calculate the symmetrical components as follows:

$$AX=B$$

$$X=A^{-1} B$$

$$\begin{bmatrix} V_A^{(0)} \\ V_A^{(1)} \\ V_A^{(2)} \end{bmatrix} = \frac{1}{3} \begin{bmatrix} 1 & 1 & 1 \\ 1 & a & a^2 \\ 1 & a^2 & a \end{bmatrix} \begin{bmatrix} V_A \\ V_B \\ V_C \end{bmatrix}$$

It is important to know the definition of all parameters, which are identified as follows:

I_A, I_B, I_C (original imbalanced phases).

$I_A^{(0)}, I_A^{(1)}, I_A^{(2)}$ (symmetrical component of phase A)

$I_B^{(0)}, I_B^{(1)}, I_B^{(2)}$ (symmetrical component of phase B)

$I_C^{(0)}, I_C^{(1)}, I_C^{(2)}$ (symmetrical component of phase C)

Example 01

given the symmetrical components of a set of unbalanced three phases are:

$$I_A^{(0)} = 0.53 \angle -57.9^\circ$$

$$I_A^{(1)} = 1.27 \angle 72.1^\circ$$

$$I_A^{(2)} = 0.64 \angle 44.1^\circ$$

- Obtain the original unbalanced phases (I_A, I_B, I_C) and draw the phases diagram of all parameters.

Answer:

$$I_B^{(1)} = a^2 I_A^{(1)}, \quad I_C^{(1)} = a I_A^{(1)} / \quad I_B^{(2)} = a I_A^{(2)}, \quad I_C^{(2)} = a^2 I_A^{(2)} \quad / \quad I_A^{(0)} = I_B^{(0)} = I_C^{(0)}.$$

We may use the Matrix to directly evaluate the values of the original unbalanced phases (I_A, I_B, I_C) as follows:

$$\begin{bmatrix} I_A \\ I_B \\ I_C \end{bmatrix} = \begin{bmatrix} 1 & 1 & 1 \\ 1 & a^2 & a \\ 1 & a & a^2 \end{bmatrix} \begin{bmatrix} I_A^{(0)} \\ I_A^{(1)} \\ I_A^{(2)} \end{bmatrix}$$

$$I_A = I_A^{(0)} + I_A^{(1)} + I_A^{(2)}$$

$$I_A = 0.53 \angle -57.9^\circ + 1.27 \angle 72.1^\circ + 0.64 \angle 44.1^\circ$$

$$I_A = 1.653 \angle 46.8^\circ$$

$$I_B = I_A^{(0)} + a^2 I_A^{(1)} + a I_A^{(2)}$$

$$= 0.53 \angle -57.9^\circ + 1.27 \angle (72.1 + 240)^\circ + 0.64 \angle (44.1 + 120)^\circ$$

$$= 1.322 \angle -66.94^\circ$$

$$I_C = I_A^{(0)} + a I_A^{(1)} + a^2 I_A^{(2)}$$

$$= 0.53 \angle -57.9^\circ + 1.27 \angle (72.1 + 120)^\circ + 0.64 \angle (44.1 + 240)^\circ$$

$$= 1.56 \angle -121.05^\circ$$

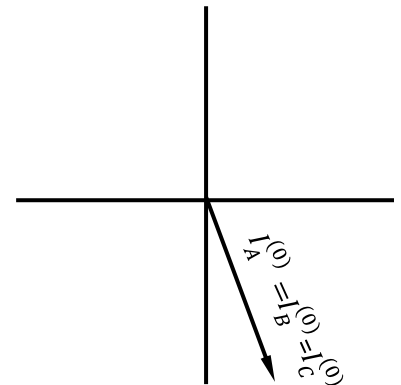
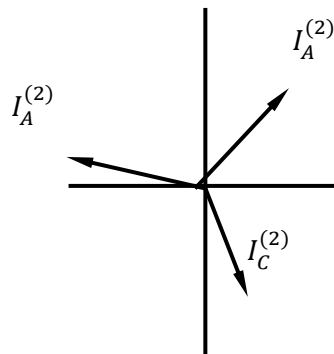
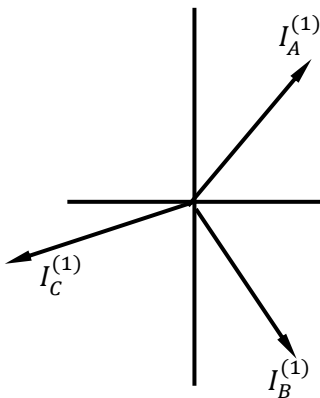
We may draw the phasor diagram of all parameters follows

$$I_A^{(1)} = 1.27 \angle 72.1^\circ$$

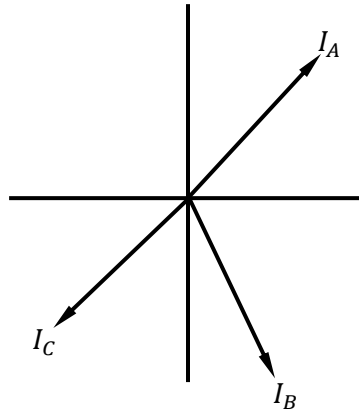
$$I_A^{(1)} = 0.3903 + j1.2085$$

$$I_A^{(2)} = 0.64 \angle 44.1^\circ$$

$$I_A^{(2)} = 0.4596 + j0.4454$$



for original imbalanced phasor



Example 02: Obtain the symmetrical components for the following set of unbalanced current:

$$I_A = 1.65 \angle 46,8^\circ$$

$$I_B = 1.32 \angle -67^\circ$$

$$I_C = 1.56 \angle -121^\circ$$

$$I_A^{(0)}, I_A^{(1)}, I_A^{(2)}?$$

Solution:

The Matrix is directly used here to evaluate the symmetrical components of phase A:

$$\begin{bmatrix} I_A^{(0)} \\ I_A^{(1)} \\ I_A^{(2)} \end{bmatrix} = \frac{1}{3} \begin{bmatrix} 1 & 1 & 1 \\ 1 & a^2 & a \\ 1 & a & a^2 \end{bmatrix} \begin{bmatrix} I_A \\ I_B \\ I_C \end{bmatrix}$$

$$I_A^{(0)} = \frac{1}{3} (I_A + I_B + I_C)$$

$$I_A^{(0)} = \frac{1}{3} (1.65 \angle 46,8^\circ + 1.32 \angle -67^\circ + 1.56 \angle -121^\circ) = 0.53 \angle -58.04^\circ$$

$$I_A^{(1)} = \frac{1}{3} (I_A + aI_B + a^2I_C)$$

$$I_A^{(1)} = \frac{1}{3} [1.65 \angle 46,8^\circ + 1.32 \angle (-67^\circ + 120^\circ) + 1.56 \angle (-121^\circ + 240^\circ) = 1.268 \angle 72.13^\circ$$

$$I_A^{(2)} = \frac{1}{3} (I_A + a^2 I_B + a I_C)$$

$$I_A^{(2)} = \frac{1}{3} [1.65 \angle 46,8^\circ + 1.32 \angle (-67^\circ + 240^\circ) + 1.56 \angle (-121^\circ + 120^\circ) = 0.64 \angle 44,10^\circ$$

these are the symmetrical components of phase A. How about the symmetrical components of phases B and C?

$$I_B^{(0)} = I_A^{(0)} = 0.53 \angle -58.04^\circ \text{ A}$$

$$I_B^{(1)} = a^2 I_A^{(1)} = 1.268 \angle (72.13^\circ + 240^\circ) = 1,268 \angle 312.13^\circ \text{ A}$$

$$I_B^{(2)} = a I_A^{(2)} = 0.64 \angle (44,10^\circ + 120^\circ) = 0.64 \angle 164.10^\circ \text{ A}$$

$$I_C^{(0)} = I_A^{(0)} = 0.53 \angle -58.04^\circ \text{ A}$$

$$I_C^{(1)} = a I_A^{(1)} = 1.268 \angle (72.13^\circ + 120^\circ) = 1,268 \angle 192.13^\circ \text{ A}$$

$$I_C^{(2)} = a^2 I_A^{(2)} = 0.64 \angle (44,10^\circ + 240^\circ) = 0.64 \angle 284.10^\circ \text{ A}$$

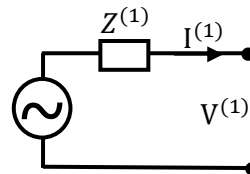
Chapter IV
Sequence Network

IV.1. Introduction

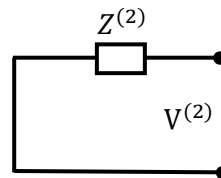
Sequence network is the representation of reactance diagram using positive negative and zero sequences. For the balance faults studied previously, we have one reactance diagram (positive sequence reactance diagram)

However, for unbalanced faults, three reactance diagrams are needed (network models):

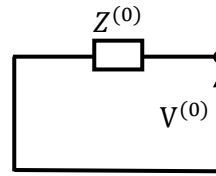
- 1- one reactance diagram for positive sequence



- 2- one reactance diagram for negative sequence



- 3- one reactance diagram for zero sequence



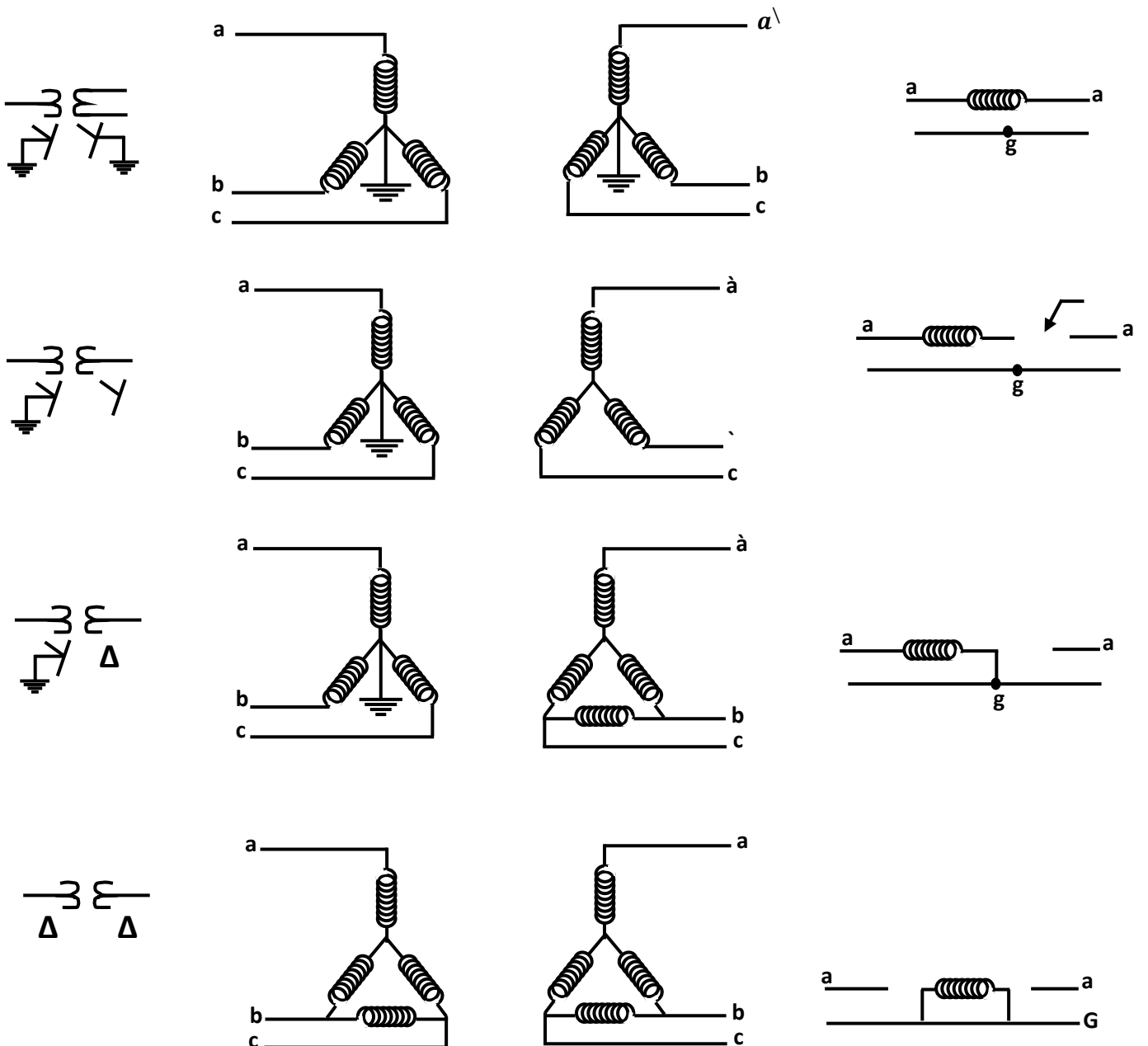
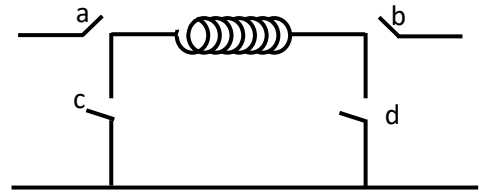
- In positive sequence reactance diagram: system is drownd as usual.
- In negative sequence reactance diagram, same as positive sequence but synchronous machines are shorted.
- For zero sequence reactance diagram, we can define two basic concepts:
 - 1- Sequence network for transformer.
 - 2- Sequence network for generator.

There are different configurations for the transformer and generators reactance's based on the connection type.

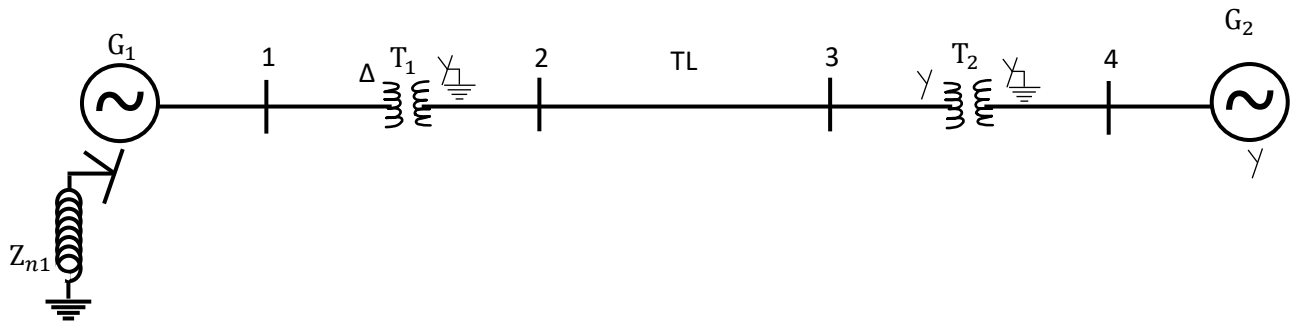
IV.2. Sequence impedance of transformer:

The transformer is modeled with:

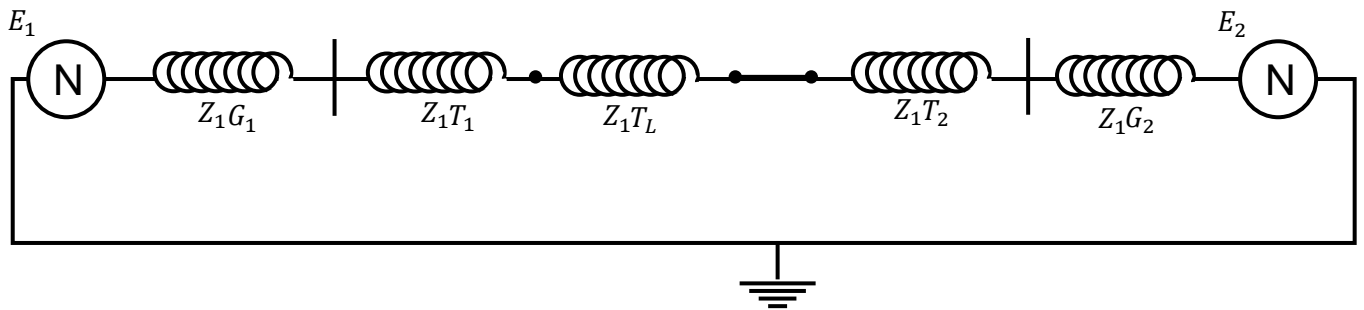
- ✓ rule 01 close a (stare grounding)
- ✓ rule 02 close b (stare grounding)
- ✓ rule 03 close c (Delta connection)
- ✓ rule 04 close d (Delta connection)



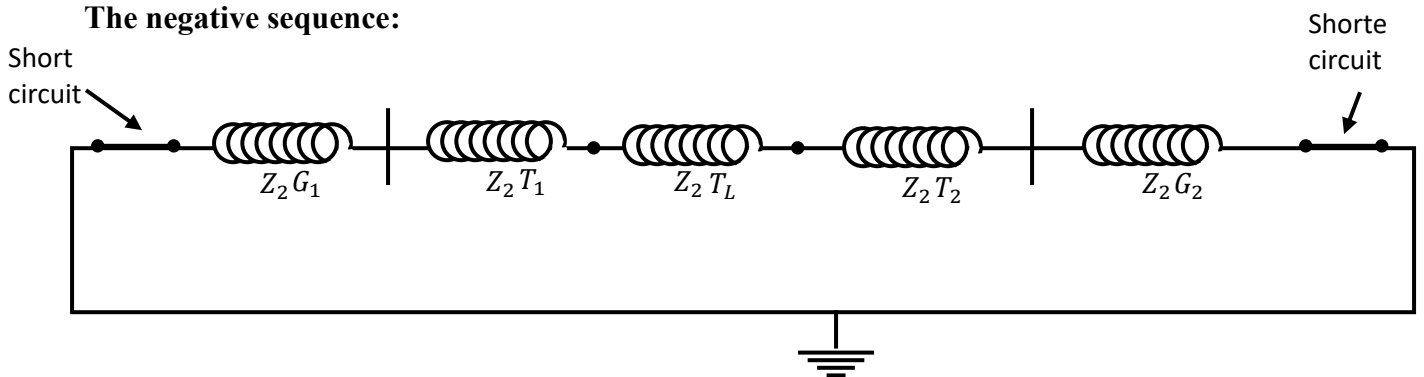
Example: Draw the Zero sequence, Positive sequence and Negative sequence of the system (SLD) of the power system shown below



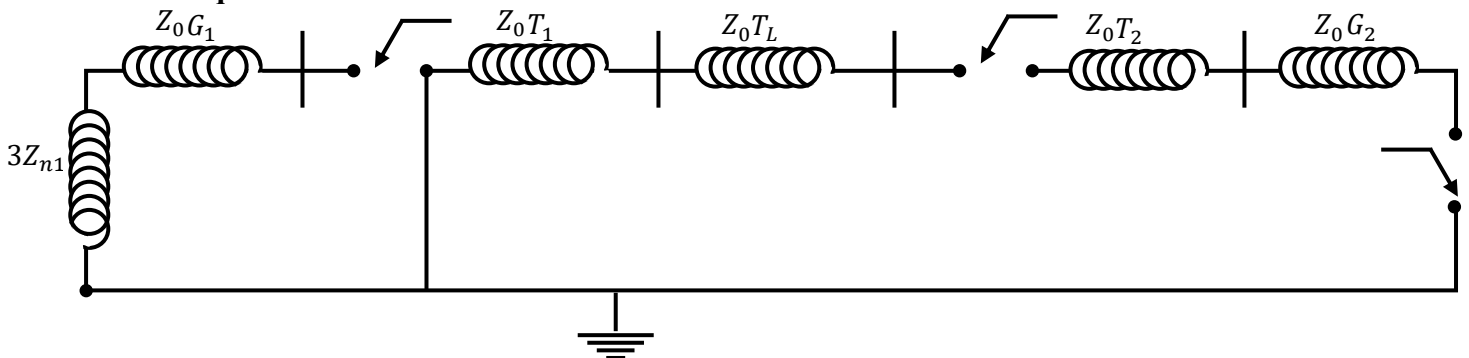
The positive sequence:



The negative sequence:



Zero sequence:



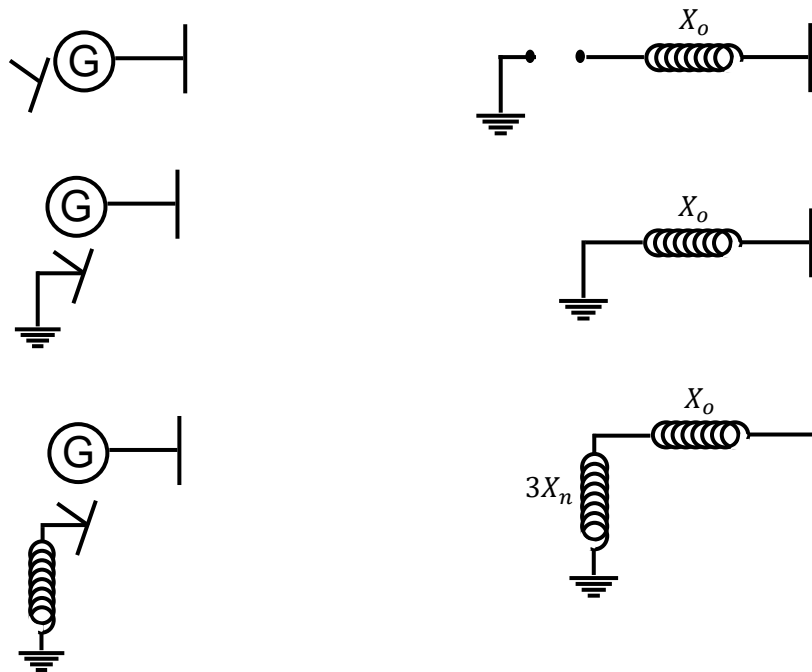
The positive, negative and zero sequence impedance of transformer are equal which correspond to the transformer leakage impedance.

IV.3. Sequence impedance of transmission lines:

- Positive and Negative impedances are equal.
- The zero-sequence impedance is more than (two to the six times) the positive or negative sequence impedance.

IV.4. Sequence impedance of synchronous machines:

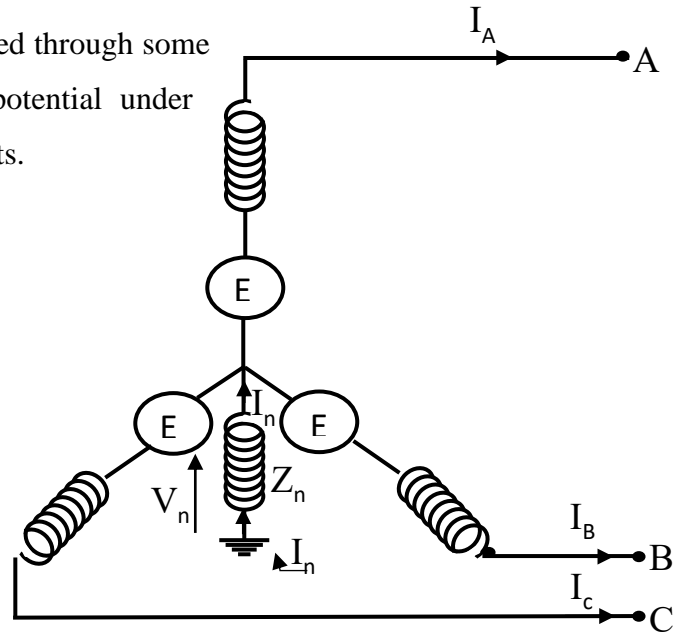
The negative sequence impedance is close to the positive sequence sub transient impedance, the zero sequence is represented as the figure in bellow:



IV.5. The voltage of the neutral:

The potential of the neutral when it is grounded through some impedance or isolated will not be at ground potential under unbalanced conditions such as unsymmetrical faults.

$$V_n = -I_n Z_n$$



Where:

Z_n is the neutral grounding impedance and I_n the neutral current

The negative sign is used as the current flows from the ground to the neutral of the system and the potential of the neutral is lower than the ground.

For a three-phase system:

$$I_n = I_A + I_B + I_C$$

$$I_A = I_A^{(1)} + I_A^{(2)} + I_A^{(0)}$$

$$I_B = I_A^{(0)} + a^2 I_A^{(1)} + a I_A^{(2)}$$

$$I_C = I_A^{(0)} + a I_A^{(1)} + a^2 I_A^{(2)}$$

$$I_n = (I_A^{(1)} + I_A^{(2)} + I_A^{(0)}) + (I_A^{(0)} + a^2 I_A^{(1)} + a I_A^{(2)}) + (I_A^{(0)} + a I_A^{(1)} + a^2 I_A^{(2)})$$

$$I_n = I_A^{(1)}(1 + a + a^2) + I_A^{(2)}(1 + a + a^2) + 3I_A^{(0)}$$

$$I_n = 3I_A^{(0)}$$

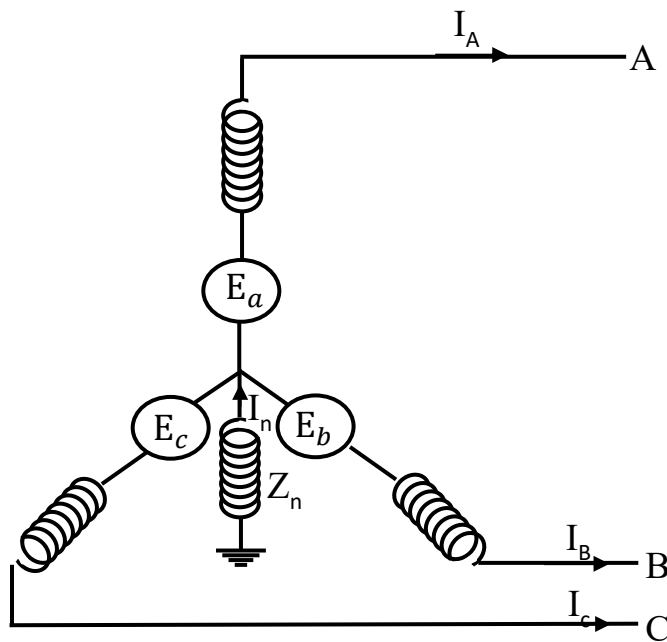
$$V_n = -3I_A^{(0)} Z_n$$

- Since the positive sequence and negative sequence components of currents through the neutral are absent, the drops due to this current are also zero.
- also, for a balanced set of current or voltage, the neutral is at the ground potential; therefore, for positive and negative sequence networks, neutral of the system will be taken as the reference.

IV.6. Sequence network equations:

the main objective when analyzing and balanced faults are:

- 1- Evaluating symmetrical components for currents (positive, negative and zero sequence currents) $I_A^{(0)}, I_A^{(1)}, I_A^{(2)}$
- 2- the evaluating the faults currents I_f



- Then, the phase currents (the original unbalance system parameters) I_A, I_B, I_C can be calculated from The Matrix.

- Also, bus voltage online currents for each sequence can be calculated.

The positive sequence components of voltage and the fourth point is

$$V_A^{(1)} = E_A - I_A^{(1)} Z^{(1)}$$

Similarly, the negative sequence components of voltage of the sport points is:

$$V_A^{(2)} = E_A^{(2)} - I_A^{(2)} Z^{(2)}$$

since the negative sequence voltage generated is zero.

Therefore:

$$E_A^{(2)} = 0$$

$$V_A^{(2)} = -I_A^{(2)} Z^{(2)}$$

Similarly for zero sequence voltages

$$E_A^{(0)} = 0$$

$$V_A^{(0)} = V_n - I_A^{(0)} Z_g^{(0)}$$

$$V_A^{(0)} = -3 I_A^{(0)} Z_n - I_A^{(0)} Z_g^{(0)}$$

$$V_A^{(0)} = -I_A^{(0)} (Z_g^{(0)} + 3 Z_n)$$

where is the zero-sequence impedance of the generator and Z_n is the natural impedance.

- the three sequence Network equations are:

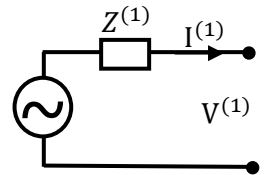
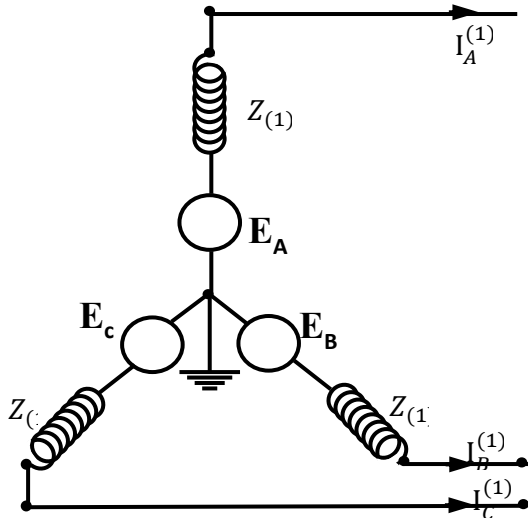
$$V_A^{(1)} = E_A - I_A^{(1)} Z^{(1)}$$

$$V_A^{(2)} = -I_A^{(2)} Z^{(2)}$$

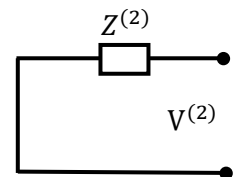
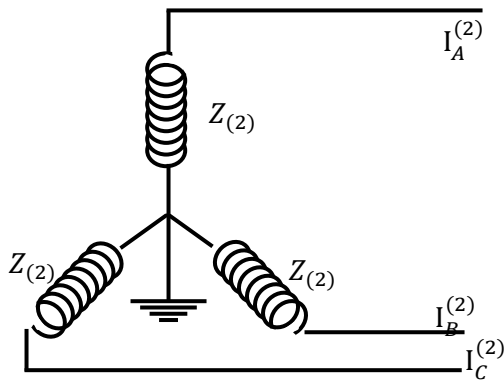
$$V_A^{(0)} = -I_A^{(0)} Z^{(0)}$$

- Where $Z^{(0)} = Z_g^{(0)} + 3Z_n$ and the corresponding sequence Network for the unloaded alternator are shown by:

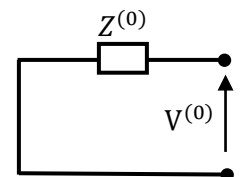
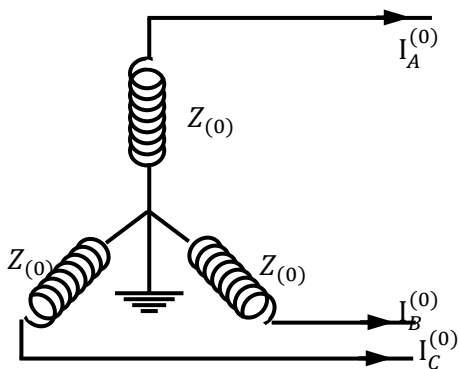
- Positive sequence



- Negative sequence



- Zero sequence



The sequence Network equation with matrix notation will be:

$$\begin{bmatrix} V_A^{(0)} \\ V_A^{(1)} \\ V_A^{(2)} \end{bmatrix} = \begin{bmatrix} 0 \\ E_A \\ 0 \end{bmatrix} \begin{bmatrix} Z^{(0)} & 0 & 0 \\ 0 & Z^{(1)} & 0 \\ 0 & 0 & Z^{(2)} \end{bmatrix} \begin{bmatrix} I_A^{(0)} \\ I_A^{(1)} \\ I_A^{(2)} \end{bmatrix}$$

Chapter V
Fault calculations

V.1. Introduction

Broadly speaking the faults can be classified as:

- 1- shunt faults (short circuits)
 - 2- series faults (open conductor)
- Shunt type of faults involve power conductor or conductors-to-ground or short circuit between conductors.
 - when circuits are controlled by fuses or any device which does not open all three faces, one or two phases of the circuit Maybe opened while the other phases on phase is closed, these are called series type of faults, these faults may also occur with one or two broken conductors.

Shunt faults are characterized buy **increase** in current and **fall** in voltage and frequency, whereas, series Faults are characterized by **increase** in voltage and frequency and **fall** in current in the faulted phases.

V.1.1. Classification of shunt type faults

N°	type of fault	short form	physical representation	symmetrical or unsymmetrical	probability of occurrence
01	three phase line to ground fault	3 LG		symmetrical	2-3%
02	three phase Line to Line faults	3 LL		symmetrical	< 1%
03	single line to ground fault	1 LG		Unsymmetrical	70-80%
04	line to line fault	1 LL		Unsymmetrical	15-20%
05	double lane to ground fault	2 LG		Unsymmetrical	<10%

V.2. Single line to ground fault (LG)

For line to ground fault, consider the following 3 phase system:

From the figure we have:

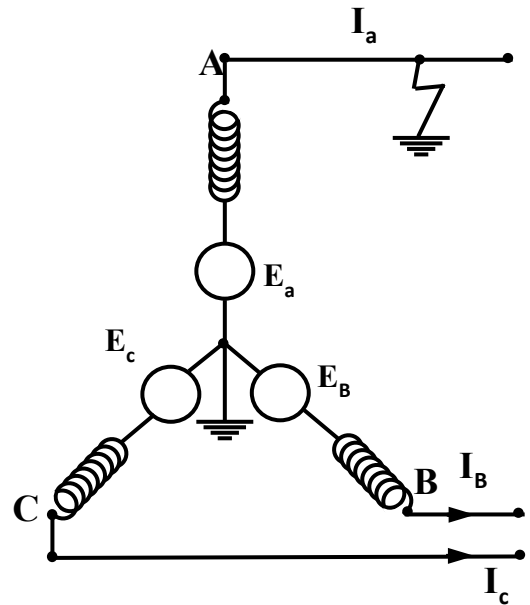
$$V_A = 0, I_B = 0, I_C = 0$$

And the sequence Network equations are:

$$V_A^{(0)} = -I_A^{(0)} Z^{(0)}$$

$$V_A^{(1)} = E_A - I_A^{(1)} Z^{(1)}$$

$$V_A^{(2)} = -I_A^{(2)} Z^{(2)}$$



The solution of these equation will give six unknowns $V_A^{(0)}, V_A^{(1)}, V_A^{(2)}, I_A^{(0)}, I_A^{(1)}, I_A^{(2)}$.

We know

$$\begin{bmatrix} I_A^{(0)} \\ I_A^{(1)} \\ I_A^{(2)} \end{bmatrix} = \frac{1}{3} \begin{bmatrix} 1 & 1 & 1 \\ 1 & a & a^2 \\ 1 & a^2 & a \end{bmatrix} \begin{bmatrix} I_A \\ I_B \\ I_C \end{bmatrix} \quad I_A^{(0)} = I_A^{(1)} = I_A^{(2)} = \frac{1}{3} I_A$$

Substituting the values of I_B and I_C

$$I_A^{(0)} = I_A^{(1)} = I_A^{(2)} = \frac{1}{3} I_A$$

In terms of symmetrical components

$$V_A = 0 = V_A^{(0)} + V_A^{(1)} + V_A^{(2)} = 0$$

Now substituting the values of 1 eq ts in eqs 2:

$$E_A - I_A^{(1)}Z^{(1)} - I_A^{(2)}Z^{(2)} - I_A^{(0)}Z^{(0)} = 0$$

Since

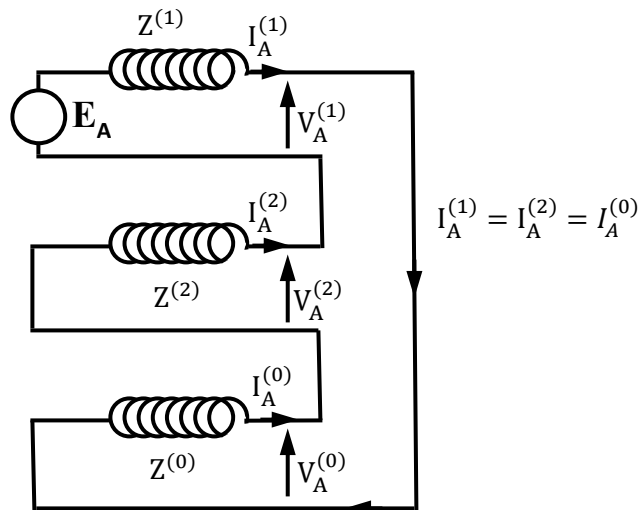
$$I_A^{(0)} = I_A^{(1)} = I_A^{(2)}$$

Equation 3 becomes

$$E_A - I_A^{(0)}Z^{(1)} - I_A^{(0)}Z^{(2)} - I_A^{(0)}Z^{(0)} = 0$$

$$I_A^{(0)} = \frac{E_A}{Z^{(1)} + Z^{(2)} + Z^{(0)}}$$

From this equation, it is clear that to simulate a LG fault for all the three sequence networks are required and since the currents are all equal in magnitude and phase angle.



now in case of the line to ground fault the fault current at the neutral current is:

$$I_n = I_f = I_A^{(0)} + I_A^{(1)} + I_A^{(2)}$$

in same case we have:

$$I_A^{(0)} = I_A^{(1)} = I_A^{(2)}$$

$$I_n = I_A^{(0)} + I_A^{(1)} + I_A^{(2)} = 3I_A^{(0)}$$

In case the neutral is not grounded the zero-sequence impedance $Z^{(0)}$ becomes infinite and, therefore, from equation

$$I_A^{(0)} = \frac{E}{Z^{(1)} + Z^{(2)} + \infty} = 0 = I_A^{(1)} = I_A^{(2)}$$

when the neutral is isolated, there is no return path from the current this means not for the system the fault current

$$I_A = 0$$

now in case of the line to ground faults with Z_f , the fault impedance is Z_f and the neutral impedance is Z_n

The boundary conditions are:

$$V_A = I_A Z_f$$

$$I_B = 0$$

$$I_C = 0$$

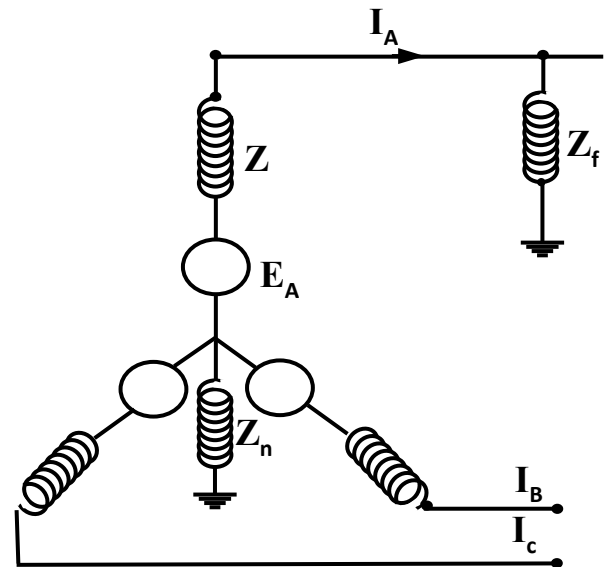
Now:

$$V_A^{(0)} = -I_A(Z_g^{(0)} + 3Z_n)$$

$$V_A^{(1)} = E_A - I_A^{(1)} Z^{(1)}$$

$$V_A^{(2)} = -I_A^{(2)} Z^{(2)}$$

$$I_A^{(0)} = I_A^{(1)} = I_A^{(2)} = \frac{I_A^{(0)}}{3}$$

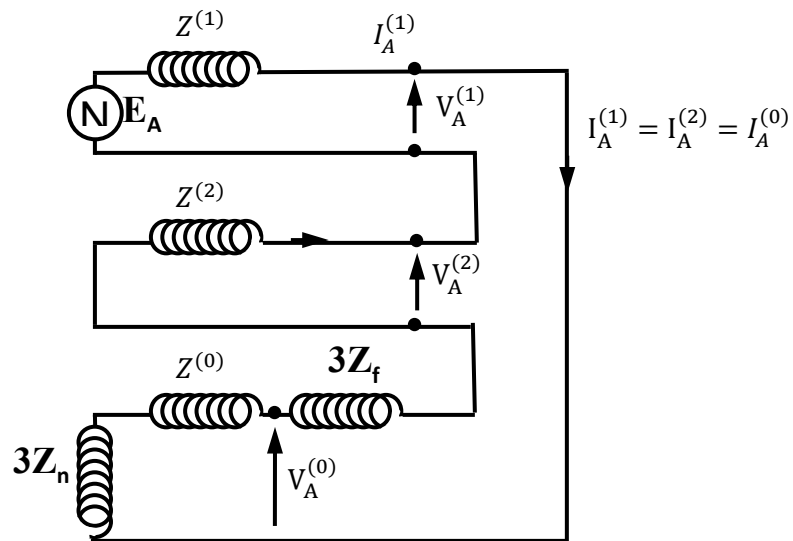


$$V_A^{(1)} + V_A^{(2)} + V_A^{(0)} = V_A = 3I_A^{(1)} Z_f$$

$$E_A - I_A^{(1)} Z^{(1)} - I_A^{(2)} Z^{(2)} - I_A(Z_g^{(0)} + 3Z_n) = 3I_A^{(1)} Z_f$$

$$E_A = I_A^{(1)} [Z^{(1)} + Z^{(2)} + (Z_g^{(0)} + 3Z_n) + 3Z_f]$$

$$I_A^{(0)} = I_A^{(1)} = I_A^{(2)} = \frac{E_A}{[Z^{(1)} + Z^{(2)} + (Z_g^{(0)} + 3Z_n) + 3Z_f]}$$



Exercise:

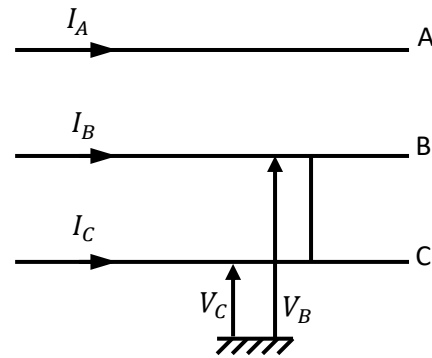
A 11KV ,30MVA generator has $x_1 = x_2 = 0.2pu$ and $x_0 = 0.05pu$ a line to ground fault occurs on the generator terminal.

Find the fault current in Line-to-ground voltages during fault conditions, assume that the generator natural is solidly grounded.

V.3. Line to line Faults

The line to land false takes place on phase b and C
the essential conditions are:

$$\begin{aligned} I_A &= 0 \\ I_B + I_C &= 0, \quad I_B = -I_C \\ V_B &= V_C \end{aligned}$$



Using the relation

$$\begin{bmatrix} I_A^{(0)} \\ I_A^{(1)} \\ I_A^{(2)} \end{bmatrix} = \frac{1}{3} \begin{bmatrix} 1 & 1 & 1 \\ 1 & a & a^2 \\ 1 & a^2 & a \end{bmatrix} \begin{bmatrix} I_A \\ I_B \\ I_C \end{bmatrix}$$

$$\begin{aligned} I_A^{(0)} &= 0 \\ I_A^{(1)} &= \frac{1}{3}(a - a^2)I_B \\ I_A^{(2)} &= \frac{1}{3}(a^2 - a)I_B \\ I_A^{(2)} &= -I_A^{(1)} \end{aligned}$$

Which means for a Line-to-Line fault zero sequence component of current is absent and positive sequence component of current is equal in magnitude but opposite in phase to negative sequence component of current.

Now from:

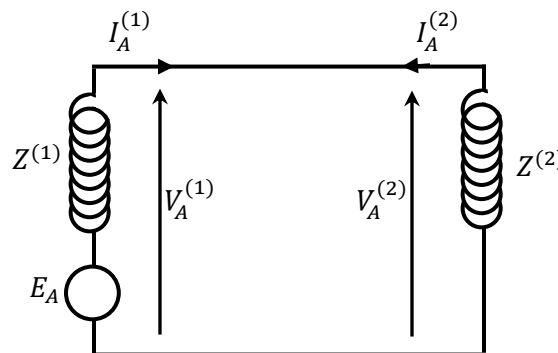
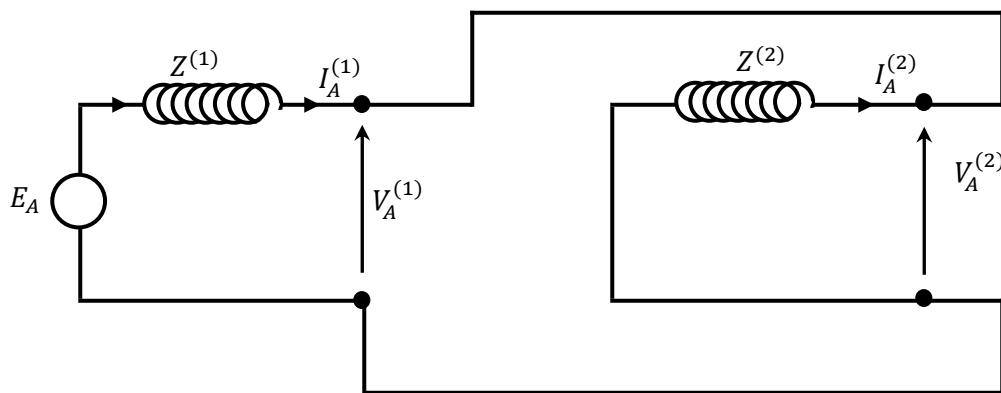
$$\begin{aligned} V_B &= V_C \\ V_A^{(0)} + a^2 V_A^{(1)} + a V_A^{(2)} &= V_A^{(0)} + a V_A^{(1)} + a^2 V_A^{(2)} \\ (a^2 - a)V_A^{(1)} &= (a^2 - a)V_A^{(2)} \\ V_A^{(1)} &= V_A^{(2)} \end{aligned}$$

That is, positive sequence component of voltage equals the negative sequence component of voltage. This also means that the two sequence Networks are connected in opposition, now:

$$E_A - I_A^{(1)} Z^{(1)} = - I_A^{(2)} Z^{(2)} = I_A^{(1)} Z^{(2)}$$

$$I_A^{(1)} = \frac{E_A}{Z^{(1)} + Z^{(2)}}$$

The interconnection of the sequence network for simulation of LL fault is shown in figure below:



V.4. Line to Line fault with Z_f

The fault impedance in Z_f between two lines (B-C)

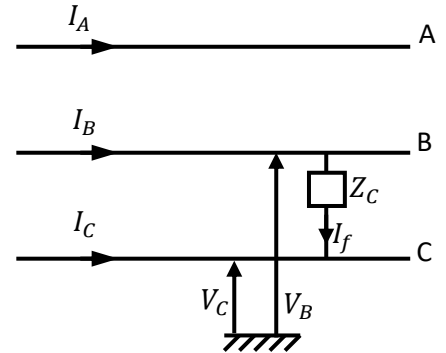
Sine $V_B = V_C + Z_f I_f$

$$I_A = 0$$

$$V_B - V_C = Z_f I_f$$

$$I_B = I_C = I_f$$

$$I_C = -I_B = -I_f$$

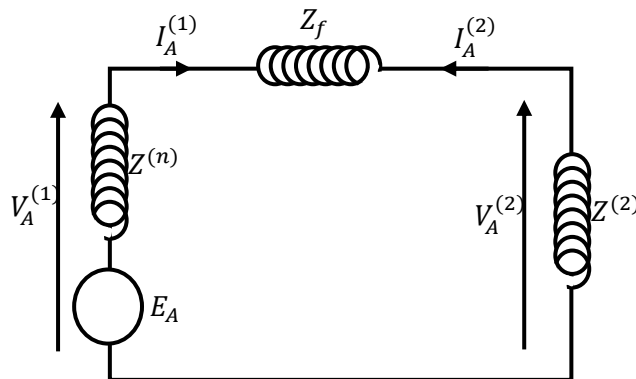


With same work in previous part, we have

$$I_A^{(0)} = 0$$

$$I_A^{(1)} = \frac{E_A}{Z_1 + Z_2 + Z_f}$$

$$I_A^{(2)} = -I_A^{(1)}$$



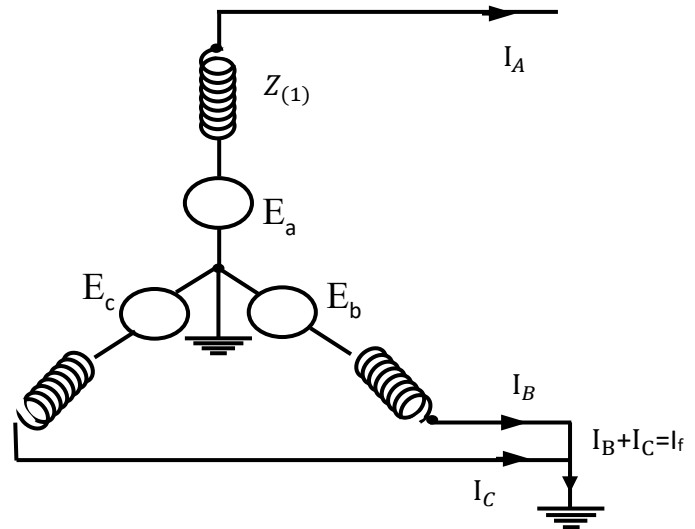
V.5. Double line to ground faults

Double line to ground fault takes place on phases B and C, the boundary conditions are:

$$I_A = 0$$

$$V_B = 0$$

$$V_C = 0$$



$$\begin{bmatrix} V_A^{(0)} \\ V_A^{(1)} \\ V_A^{(2)} \end{bmatrix} = \frac{1}{3} \begin{bmatrix} 1 & 1 & 1 \\ 1 & a & a^2 \\ 1 & a^2 & a \end{bmatrix} \begin{bmatrix} V_A \\ V_B \\ V_C \end{bmatrix}$$

$$V_A^{(0)} = \frac{1}{3}(V_A) = \frac{V_A}{3}$$

$$V_A^{(1)} = \frac{1}{3}(V_A) = \frac{V_A}{3}$$

$$V_A^{(2)} = \frac{1}{3}(V_A) = \frac{V_A}{3}$$

$$V_A^{(0)} = V_A^{(1)} = V_A^{(2)}$$

Using this relation of voltage and substituting in the sequence Network equations.

$$V_A^{(0)} = V_A^{(1)}$$

$$-I_A^{(0)} Z^{(0)} = E_A - I_A^{(1)} Z^{(1)}$$

$$I_A^{(0)} = -\frac{E_A - I_A^{(1)}Z^{(1)}}{Z^{(0)}} \quad (1)$$

Similarly

$$\begin{aligned} V_A^{(2)} &= V_A^{(1)} \\ -I_A^{(2)}Z^{(2)} &= E_A - I_A^{(1)}Z^{(1)} \\ I_A^{(2)} &= -\frac{E_A - I_A^{(1)}Z^{(1)}}{Z^{(2)}} \end{aligned} \quad (2)$$

Now from

$$I_A = I_A^{(1)} + I_A^{(2)} + I_A^{(0)} = 0 \quad (3)$$

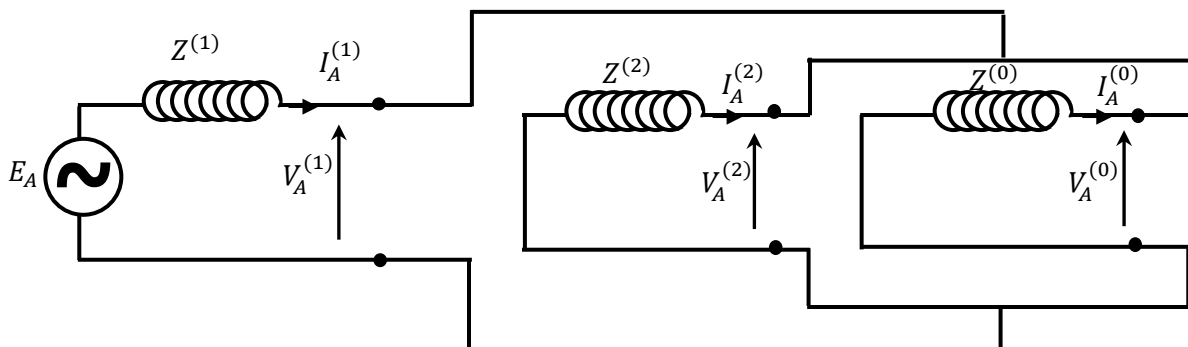
Substituting value of $I_A^{(2)}$ and $I_A^{(0)}$ from equations (1) and (2)

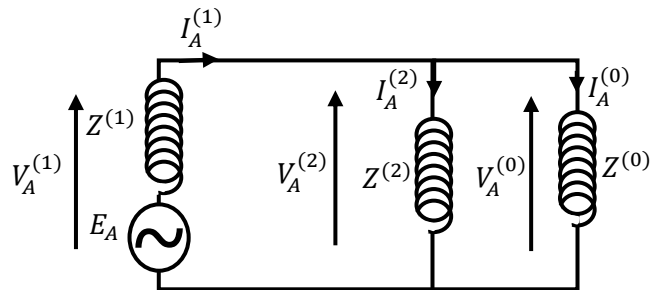
$$I_A^{(1)} - \frac{E_A - I_A^{(1)}Z^{(1)}}{Z^{(2)}} - \frac{E_A - I_A^{(1)}Z^{(1)}}{Z^{(0)}} = 0$$

Gives

$$I_A^{(1)} = \frac{E_A}{Z_1 + \frac{Z^{(0)}Z^{(2)}}{Z^{(0)}+Z^{(2)}}}$$

From this equation, it is clear that all three sequence networks are required to simulate LLG fault and also the zero sequence and the negative sequence Network are connected in parallel





the neutral current

$$I_n = I_f = I_B + I_C$$

$$\begin{bmatrix} I_A^{(0)} \\ I_A^{(1)} \\ I_A^{(2)} \end{bmatrix} = \frac{1}{3} \begin{bmatrix} 1 & 1 & 1 \\ 1 & a & a^2 \\ 1 & a^2 & a \end{bmatrix} \begin{bmatrix} I_A \\ I_B \\ I_C \end{bmatrix}$$

$$I_A^{(0)} = \frac{1}{3}(I_B + I_C) \quad (I_B + I_C) = 3 I_A^{(0)} = I_n$$

$$I_f = 3 I_A^{(0)} = I_n$$

Now:

$$I_A^{(0)} = -\left(\frac{E_A - I_A^{(1)} Z_1}{Z^{(0)}}\right)$$

$$I_A^{(1)} = \frac{E_A}{Z^{(1)} + \frac{Z^{(0)} Z^{(2)}}{Z^{(0)} + Z^{(2)}}}$$

$$I_A^{(2)} = -\left(\frac{E_A - I_A^{(1)} Z_1}{Z^{(2)}}\right)$$

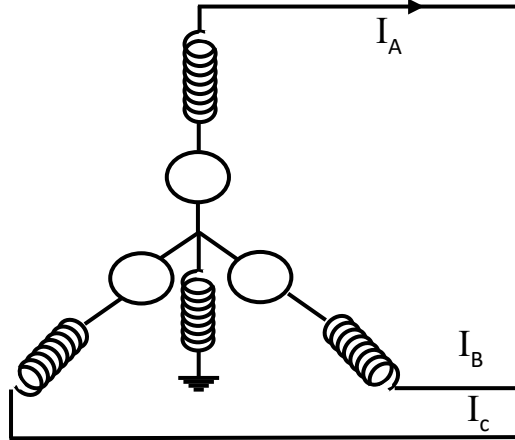
V.6. Three phase faults

As shown in the figure, the boundary conditions are:

$$I_A + I_B + I_C = 0$$

$$V_A = V_B = V_C$$

Since $|I_A| = |I_B| = |I_C|$



And if I_A is the reference $I_B = a^2 I_A$, $I_C = a I_A$

$$\begin{bmatrix} I_A^{(0)} \\ I_A^{(1)} \\ I_A^{(2)} \end{bmatrix} = \frac{1}{3} \begin{bmatrix} 1 & 1 & 1 \\ 1 & a & a^2 \\ 1 & a^2 & a \end{bmatrix} \begin{bmatrix} I_A \\ I_B \\ I_C \end{bmatrix}$$

Now

$$I_A^{(1)} = \frac{1}{3} (I_A + a I_B + a^2 I_C)$$

And substituting the values of I_B and I_C in terms I_A

$$I_A^{(1)} = \frac{1}{3} (I_A + a^3 I_A + a^3 I_A) = I_A \implies I_A^{(1)} = I_A$$

$$I_A^{(2)} = \frac{1}{3} (I_A + a^2 I_B + a I_C)$$

$$I_A^{(2)} = \frac{1}{3} (I_A + a^4 I_A + a^2 I_A) = \frac{1}{3} (1 + a I_A + a^2 I_A) = \frac{1}{3} I_A (1 + a + a^2)$$

$$I_A^{(2)} = 0, \quad I_A^{(0)} = \frac{1}{3} (I_A + I_B + I_C) \quad I_A^{(0)} = 0$$

Which means a three-phase fault zero as well as negative sequence components of current are absent and the positive-sequence component of current is equal to the phase current.

Using the voltage relation we have

$$\begin{bmatrix} V_A^{(0)} \\ V_A^{(1)} \\ V_A^{(2)} \end{bmatrix} = \frac{1}{3} \begin{bmatrix} 1 & 1 & 1 \\ 1 & a & a^2 \\ 1 & a^2 & a \end{bmatrix} \begin{bmatrix} V_A \\ V_B \\ V_C \end{bmatrix}$$

-

$$V_A^{(1)} = \frac{1}{3} V_A (1 + a + a^2) = 0$$

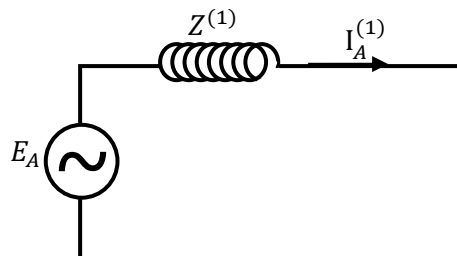
$$V_A^{(2)} = \frac{1}{3} (V_A + a^2 V_B + a V_C) = 0$$

$$V_A^{(0)} = 0$$

$$V_A^{(1)} = 0 = E - I_A^{(1)} Z^{(1)}$$

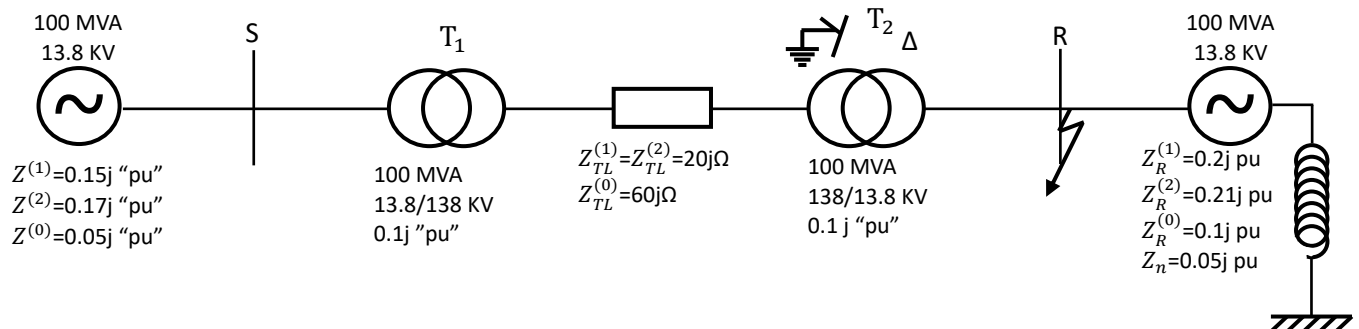
$$I_A^{(1)} = \frac{E_A}{Z^{(1)}}$$

the sequence network is shown in the figure in below



V.7. Applied exercise

Let's consider the below system:



The pre fault voltage at fault location: 1,05 pu and load current is negligible.

Questions:

1. Draw the per unit circuit diagram (positive, negative and zero sequence Networks).
2. Find the short circuit current for the following fault
 - Three phase faults
 - Line to line fault (Bloc)
 - Double line to ground fault
 - Single line to ground fault
3. Find the maximum positive, negative and zero sequence contribution from S and R, (LG fault)
4. Calculate the phase voltage at the fault calculation (LG fault).

Solution:

1)- calculate the current and impedance Bases:

$$A/ \text{ On line Side } S_b = 100MVA, \quad V_b = 138KV$$

$$I_{TLb} = \frac{S_b}{\sqrt{3} V_b} = \frac{100 \cdot 10^6}{\sqrt{3} 138 \cdot 10^3} = 418.37 \text{ A}$$

$$Z_{bTL} = \frac{V_b^2}{S_b} = \frac{138^2 KV}{100 MVA} = 190,44\Omega$$

$$B/ \text{ On bus side } \quad V_b = 13,8KV$$

$$I_b = \frac{S_b}{\sqrt{3} V_b} = \frac{100 \cdot 10^6}{\sqrt{3} 13,8 \cdot 10^3} = 4183,7 \text{ A}$$

$$Z_b = \frac{V_b^2}{S_b} = \frac{(13800)^2}{100 \cdot 10^6} = 1.9 \Omega$$

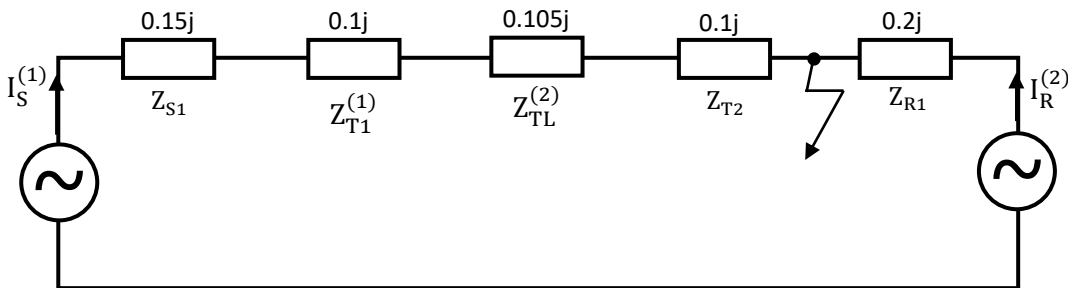
Convert all impedance to “pu” values:

Only transmission line needs to be converted

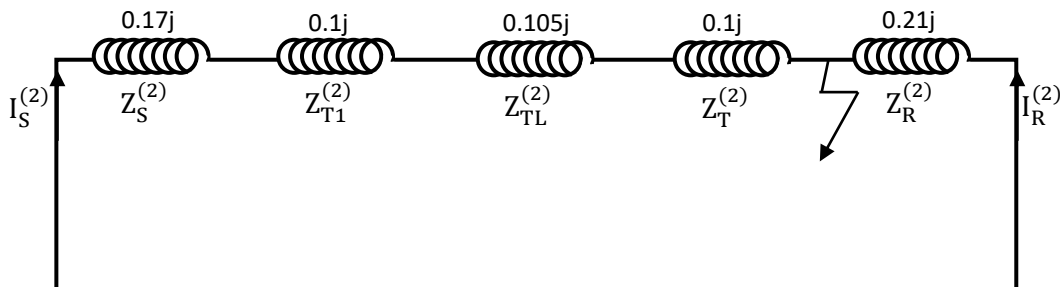
$$Z_{TL}^{(1)} = Z_{TL}^{(2)} = \frac{20j}{190.44} = 0.105j \text{ pu}$$

$$Z_{TL}^{(0)} = \frac{60j}{190,44} = 0.315j \text{ pu}$$

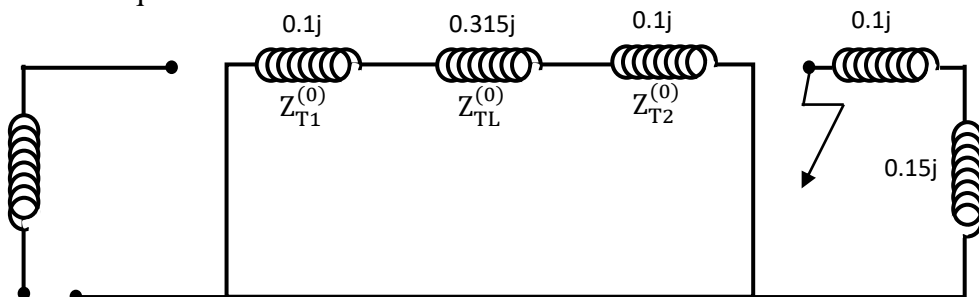
1- Positive sequence network

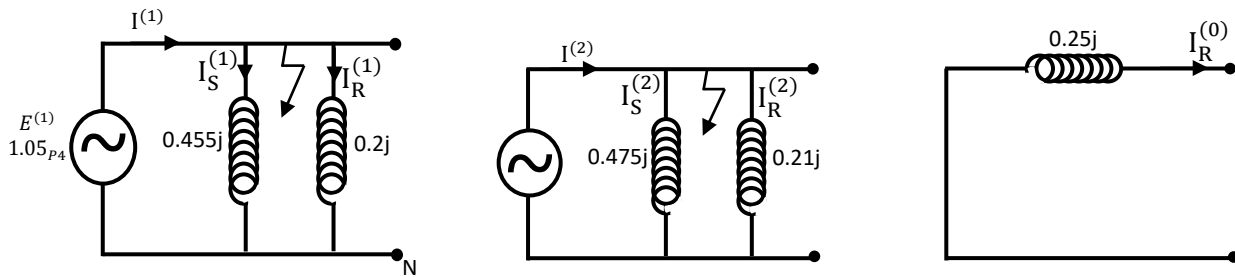


2- Negative sequence network



1- zero sequence network





c- calculate the short circuit for the following faults:

a- Three phase fault:

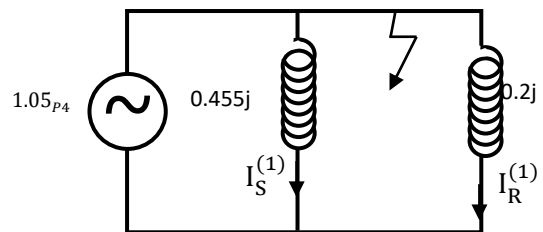
$$I^{(0)} = I^{(2)} = 0$$

$$I^{(1)} = I_S^{(1)} + I_R^{(1)} = \frac{1,05}{0,455j} + \frac{1,05}{0,2j} = 7,55 \angle -90^\circ \text{ pu}$$

$$I_A = I_A^{(0)} + I_A^{(1)} + I_A^{(2)} = 7,55 \angle -90^\circ \text{ pu}$$

$$I_B = I_A^{(0)} + a^2 I_A^{(1)} + a I_A^{(2)} = 7,55 \angle 150^\circ \text{ pu}$$

$$I_C = I_A^{(0)} + a I_A^{(1)} + a^2 I_A^{(2)} = 7,55 \angle 30^\circ \text{ pu}$$



Now convert the current from per unit into Amper (multiply pu * I_{base})

Then

$$I_A = (7,55 \angle -90^\circ)(4183,7) = 31,6 \angle -90^\circ \text{ KA}$$

$$I_B = (7,55 \angle 150^\circ)(4183,7) = 31,6 \angle 150^\circ \text{ KA}$$

$$I_C = (7,55 \angle 30^\circ)(4183,7) = 31,6 \angle 30^\circ \text{ KA}$$

B - LL fault (Bloc)

$$I_A^{(0)} = 0$$

$$I_A^{(1)} = -I_A^{(2)}$$

$$I_A^{(1)} = \frac{1,05 \angle 0^\circ}{0,1389j + 0,1456j}$$

$$I_A^{(1)} = 3,69 \angle -90^\circ \text{ pu}$$

$$I_A^{(2)} = 3,69 \angle 90^\circ \text{ pu}$$

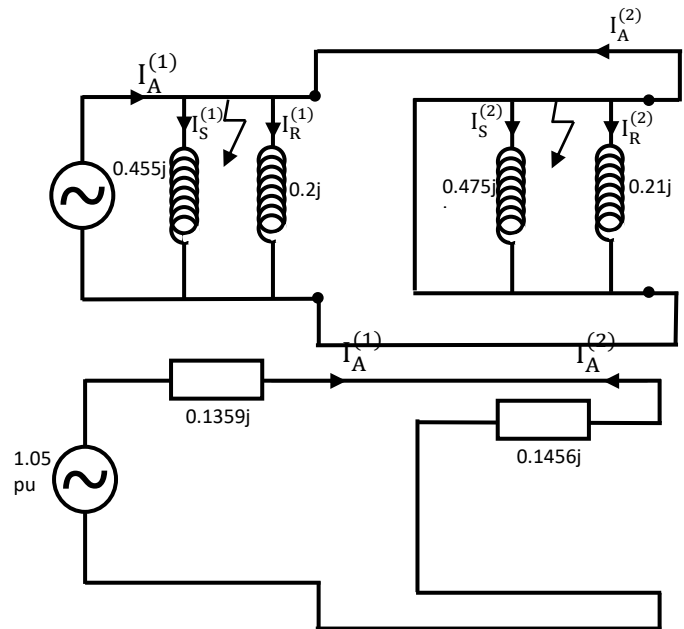
$$I_A = I_A^{(0)} + I_A^{(1)} + I_A^{(2)} = 0$$

$$I_B = I_A^{(0)} + a^2 I_A^{(1)} + a I_A^{(2)}$$

$$I_B = 6,39 \angle 180^\circ \text{ pu}$$

$$I_C = I_A^{(0)} + a I_A^{(1)} + a^2 I_A^{(2)}$$

$$I_C = 6,39 \angle 0^\circ \text{ pu}$$



Now convert the current from per unit into Amper

$$I_A = 0$$

$$I_B = 6,39 \angle 180^\circ * 4183,7 = 26,73 \angle 180^\circ \text{ KA}$$

$$I_C = 6,39 \angle 0^\circ * 4183,7 = 26,73 \angle 0^\circ \text{ KA}$$

C - LLG fault

$$I_A^{(1)} = \frac{1,05}{0,1389j + \frac{0,25j * 0,1456j}{0,25j + 0,1456j}}$$

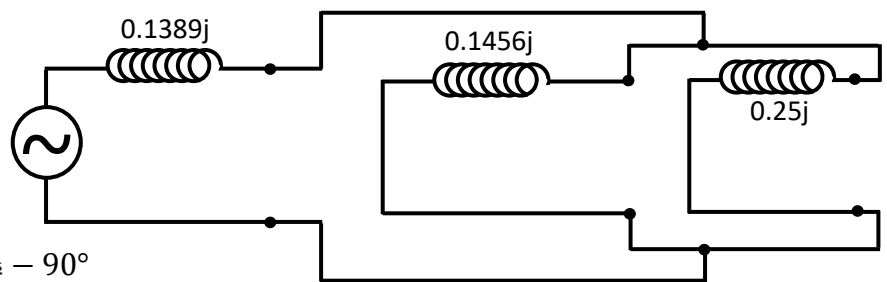
$$I_A^{(1)} = \frac{1,05}{0,1389j + 0,092j} = 4,547 \angle -90^\circ$$

$$I_A^{(2)} = -\left(\frac{V_A - Z^{(1)} I_A^{(1)}}{Z^{(0)}}\right) = -\left(\frac{1,05 - 0,1389j * 4,547 \angle -90^\circ}{0,1456j}\right)$$

$$I_A^{(2)} = 2,87 \angle +90^\circ \text{ pu}$$

$$I_A^{(0)} = -\left(\frac{V_A - Z^{(1)} I_A^{(1)}}{Z^{(0)}}\right) = -\left(\frac{1,05 - 0,1389j * 4,547 \angle -90^\circ}{0,25j}\right)$$

$$I_A^{(0)} = 1,67 \angle -90^\circ \text{ pu}$$



Then $I_A = I_A^{(0)} + I_A^{(1)} + I_A^{(2)} = 0$

$$I_B = I_A^{(0)} + a^2 I_A^{(1)} + a I_A^{(2)} = 6,9 \angle 158,66^\circ pu$$

$$I_C = I_A^{(0)} + a I_A^{(1)} + a^2 I_A^{(2)} = 6,9 \angle 21,33^\circ pu$$

Convert pu to Amperes

$$I_A = 0$$

$$I_B = 6,9 \angle 158,66^\circ * 4183,7 = 28.85 \angle 158,66^\circ KA$$

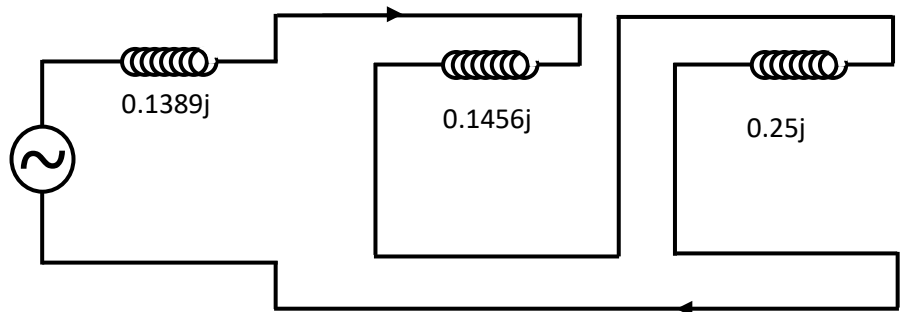
$$I_C = 6,9 \angle 21,33^\circ * 4183,7 = 28.85 \angle 21,33^\circ KA$$

D- LG fault

$$I_A^{(0)} = I_A^{(1)} = I_A^{(2)}$$

$$I_A^{(2)} = \frac{1,05}{0,1389j + 0,1456j + 0,25j}$$

$$= 1,964 \angle -90^\circ pu$$



Then

$$I_A = I_A^{(0)} + I_A^{(1)} + I_A^{(2)} = 5,893 \angle -90^\circ pu$$

$$I_B = I_A^{(0)} + a^2 I_A^{(1)} + a I_A^{(2)} = 0$$

$$I_C = I_A^{(0)} + a I_A^{(1)} + a^2 I_A^{(2)} = 0$$

Convert pu to Amperes

$$I_A = 5,893 \angle -90^\circ * 4183,7 = 24,656 \angle -90^\circ KA$$

$$I_B = 0$$

$$I_C = 0$$

1- find the max positive ,negative and zero sequence current contribution from source S, R (LGF).

a) Source S

$$I_S^{(0)} = 0$$

$$I_S^{(1)} = I^{(1)} * \frac{0,2j}{0,2j+0,455j} = 0,5997 \angle 90^\circ pu \quad (\text{divide of current low})$$

$$I_S^{(2)} = I^{(2)} * \frac{0,21j}{0,21j + 0,475j} = 0,602 \angle -90^\circ pu$$

b) Source R :

$$I_R^{(0)} = I^{(0)} = I^{(1)} = 1,964 \angle -90^\circ pu$$

$$I_R^{(1)} = I^{(1)} * \frac{0,455j}{0,455j + 0,2j} = 1,364 \angle -90^\circ pu$$

$$I_R^{(2)} = I^{(2)} * \frac{0,475j}{0,475j + 0,21j} = 1,362 \angle -90^\circ pu$$

2- calculate the phase voltages at the fault location (Single line to ground)

$$V^{(1)} = V - I^{(1)}Z^{(1)} = 1,05 - (1,964 \angle -90^\circ)(0,1389 \angle 90^\circ)$$

$$V^{(1)} = 0,777 pu$$

$$V^{(2)} = 0 - I^{(2)}Z^{(2)} = 0 - (1,964 \angle -90^\circ)(0,1456 \angle 90^\circ)$$

$$V^{(2)} = -0,286 pu$$

$$V^{(0)} = 0 - I^{(0)}Z^{(0)} = 0 - (1,964 \angle -90^\circ)(0,25 \angle 90^\circ)$$

$$V^{(0)} = -0,419 pu$$

Now let's calculate the phase voltages:

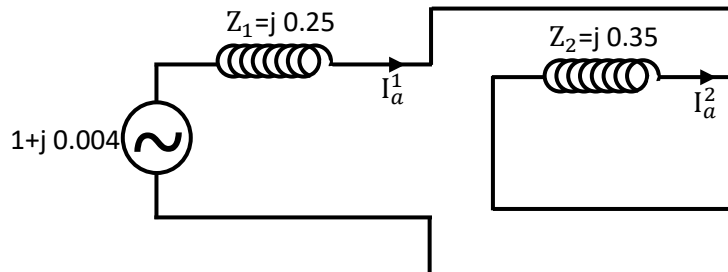
$$V_A = V^{(0)} + V^{(1)} + V^{(2)} = 0$$

$$V_B = V^{(0)} + a^2 V^{(1)} + a V^{(2)} = 1,178 \angle -128,66^\circ pu$$

$$V_C = V^{(0)} + a V^{(1)} + a^2 V^{(2)} = 1,178 \angle 128,66^\circ pu$$

Exercise:

Determine the fault current and line to line voltage at the fault when line to line fault occurs at the terminal of the alternator described in the figure bellow:



Determine the fault current and line to line voltage at the fault when a double line to ground fault occurs at the terminals of this other rotors.

-The base current is 1093A

-Voltage is 13,2KV (LL)

Solution

The sequence network for LL faults shown.

The zero sequence is absent, assuming the pre fault voltage is pu $1 + j0$

$$I_A^1 = \frac{E}{Z_1^{(1)} + Z^{(2)}} = \frac{1 + j0}{j0,25 + j0,35} = \frac{1}{j0,6} = -j1,6$$

Now for LL fault.

$$I_A^{(1)} = -I_A^{(2)} = -j1,667 \quad I_A^{(2)} = j1,667$$

$$I_A^{(0)} = 0$$

And to find out the fault current, $I_B = -I_C$

$$I_B = I_B^{(1)} + I_B^{(2)} + I_B^{(0)} = a^2 I_A^{(1)} + a I_A^{(2)}$$

$$I_B = (-0,5 - j0,866)(-j1,667) + (-0,5 + j0,866)(j1,667)$$

$$I_B = -2,8872 \text{ pu}$$

Fault current: $I_B = 1093 * 2,8872 = 3,155 \text{ KA}$

To find out line-to-line voltage, we found the sequence components of voltages:

$$V_A^{(1)} = E_A - I_A^{(1)} Z^{(1)} = 1 - (-j1,667)(j0,25)$$

$$V_A^{(1)} = 1 - 0,4167 = 0,5833 \text{ pu}$$

Similarly

$$V_A^{(2)} = -I_A^{(2)} Z^{(2)} = (-j 1,667)(j0,35) = 0,5833$$

$$V_A^{(1)} = V_A^{(2)} = 0,5833 \text{ pu}$$

$$V_A = V_A^{(1)} + V_A^{(2)} + V_A^{(0)} = V_A^{(1)} + V_A^{(2)} = 2 * 0,5899 = 1,166 \text{ pu}$$

$$V_B = a^2 V_A^{(1)} + a V_A^{(2)} = (-0,5 - j0,866)(0,5833) + (-0,5 + j0,866)(0,5833)$$

$$V_B = -0,5833 \text{ pu}$$

$$V_B = V_C = -0,5833$$

Line voltage:

$$V_{AB} = V_A - V_B = 1,666 - (-0,5833) = 1,7499 \text{ pu}$$

$$V_{AC} = V_A - V_C = 1,7499 \text{ pu}$$

$$V_{BC} = V_B - V_C = 0,0 \text{ pu}$$

And line to line voltage:

$$V_{AB} = 1,7499 \frac{13,2}{\sqrt{3}} = 13,33 \text{ kv}$$

$$V_{AC} = 13,33 \text{ kv}$$

$$V_{BC} = 0,0 \text{ kv}$$

Recitation works

REC 01

Exercise 01

Write each sum as a single matrix

$$\begin{bmatrix} 2 & 3 & 4 \\ -1 & 6 & 2 \\ 1 & 0 & 3 \end{bmatrix} + \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

$$2 \begin{bmatrix} 6 & 1 \\ 0 & -3 \\ -1 & 2 \end{bmatrix} - 3 \begin{bmatrix} 4 & 2 \\ 0 & 1 \\ -5 & -1 \end{bmatrix}$$

Exercise 02: Write each product as a single matrix

$$\begin{bmatrix} 3 & 1 & -1 \\ 0 & -1 & 2 \end{bmatrix} \begin{bmatrix} 1 & -1 \\ 0 & 2 \\ 1 & 0 \end{bmatrix}$$

$$\begin{bmatrix} 3 & -2 & 2 \end{bmatrix} \begin{bmatrix} 1 \\ 2 \\ -2 \end{bmatrix}$$

$$\begin{bmatrix} 2 & -2 & -1 \\ 1 & 1 & -2 \\ 1 & 0 & -1 \end{bmatrix} \begin{bmatrix} -1 & -2 & 5 \\ -1 & -1 & 3 \\ -1 & -2 & 4 \end{bmatrix}$$

Exercise 03

$$\text{If } A = \begin{bmatrix} 1 & 4 \\ 2 & 1 \end{bmatrix}, \quad B = \begin{bmatrix} -3 & 2 \\ 4 & 0 \end{bmatrix}, \quad C = \begin{bmatrix} 1 & 0 \\ 0 & 2 \end{bmatrix}$$

Q: Find $A^2 + BC$ **Exercise 04:** Find K such that the following matrices are singular

$$\begin{vmatrix} K & 6 \\ 4 & 3 \end{vmatrix}$$

$$\begin{vmatrix} 1 & 1 & -2 \\ 3 & -1 & 1 \\ K & 3 & -6 \end{vmatrix}$$

Exercise 05: Expand the determinants

$$\begin{vmatrix} 1 & 2 & 0 \\ 3 & -1 & 4 \\ -2 & 1 & 3 \end{vmatrix}$$

$$\begin{vmatrix} X & 0 & 0 \\ 0 & X & 0 \\ 0 & 0 & X \end{vmatrix}$$

Exercise 06: What is the cofactor of matrix:

$$\begin{bmatrix} 3 & 1 & -4 \\ 2 & 5 & 6 \\ 1 & 4 & 8 \end{bmatrix} \quad \begin{bmatrix} 2 & -2 & -1 \\ 1 & 1 & -2 \\ 1 & 0 & -1 \end{bmatrix} \quad \begin{bmatrix} -1 & -2 & 5 \\ -1 & -1 & 3 \\ -1 & -2 & 4 \end{bmatrix}$$

Exercise 07: Find the inverse if it exists, of the following matrices

$$\begin{bmatrix} 1 & 3 \\ 2 & -1 \end{bmatrix}$$

$$\begin{bmatrix} 0 & -2 & -3 \\ 1 & 3 & 3 \\ -1 & -2 & -2 \end{bmatrix}$$

$$\begin{bmatrix} 1 & 2 & -1 \\ -3 & 4 & 5 \\ -4 & 2 & 6 \end{bmatrix}$$

REC 02

Exercise 01

Given vectors with

$$A = \langle 6, 5 \rangle, \quad B = \langle 0, 4 \rangle$$

1. Find the magnitude and direction angle for the vector.
2. Use the given vectors to compute $A+B$, $A-B$, $2A-3B$, $A \cdot B$ and $\cos \Theta$

Exercise 02

Find the component form of vector U , given its magnitude and the angle the vector makes with the positive x-axis. Give exact answers when possible.

$$\|\vec{u}\| = 2, \quad \theta = 30^\circ$$

$$\|\vec{u}\| = 6, \quad \theta = 60^\circ$$

$$\|\vec{u}\| = 5, \quad \theta = 90^\circ$$

Exercise 03

Using the operator, $a = 1 \angle 120^\circ$, calculate the following expression:

1. $(a - 1)/(1 + a - a^2)$
2. $(1 + a)(1 + a^2)$

Exercise 04

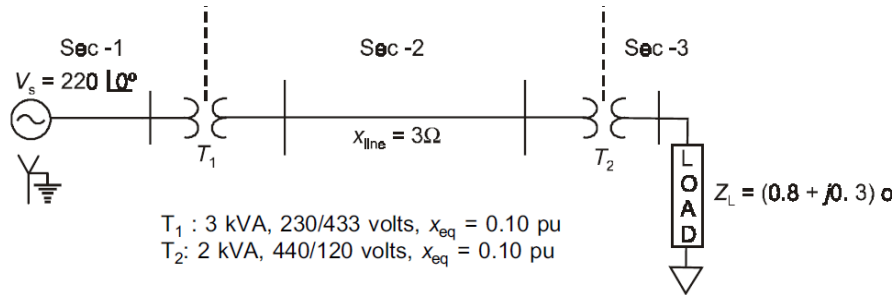
Proved U^{-1} is inverse of U for:

$$U = \begin{bmatrix} 1 & 1 & 1 \\ 1 & a^2 & a \\ 1 & a & a^2 \end{bmatrix}, \quad U^{-1} = \frac{1}{3} \begin{bmatrix} 1 & 1 & 1 \\ 1 & a & a^2 \\ 1 & a^2 & a \end{bmatrix}$$

REC 03

Problem 01

A single line diagram of a single -phase circuit shows Figure below. Using the base values of 3kva and 230 volts? Draw the per-unit circuit diagram and determine the per unit impedance and the per unit source voltage. Also calculate the load current both in per unit and in Amperes.



Problem 02

A single-line diagram of power system shows in the figure below, the rating of the generators and transformers are given below

G1: 25MVA, 6.6kv, $x=20\%$ pu

G2: 15MVA, 6.6kv, $x=15\%$ pu

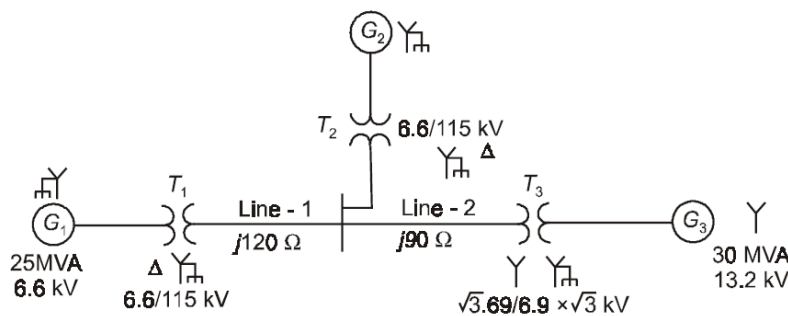
G3: 30MVA, 13.2kv, $x=15\%$ pu

T1: 30MVA, 6.6/115kv, $x=0.10$ pu

T2: 15MVA, 6.6/115kv, $x=0.10$ pu

T3: Single-phase unit each rated 10MVA, 6.9/69kv, $x=0.10$ pu

Draw per unit circuit diagram using base values of 30MVA and 6.6kv in the circuit of G1

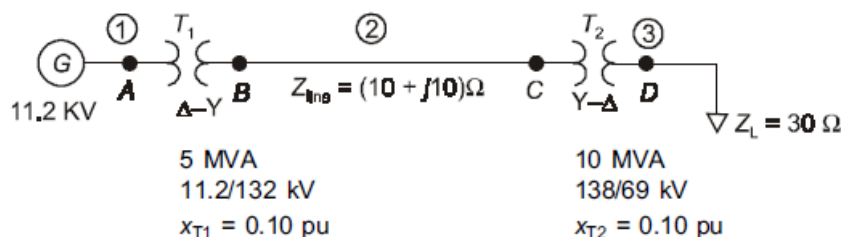


Problem 03

The reactance of a generator designated x'' is given as 0.25 pu based on the generator's nameplate rating of 18KV, 500MVA. The base for calculations is 20KV, 100MVA. Find X'' on the new base.

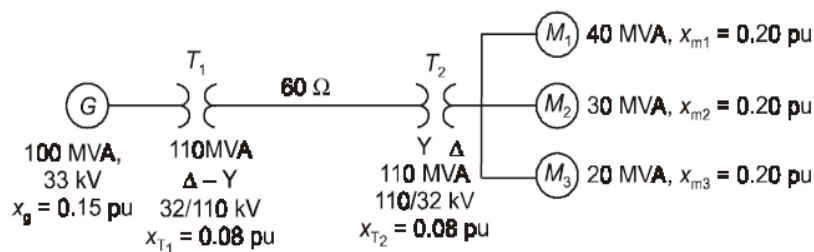
Problem 04

The figure below shows a sample power system network. Find the current supplied by the generator, the transmission line current, the load current, the load voltage and the power consumed by the load



Problem 05

A 100 MVA, 33KV, three phase generator has a reactance of 15%. The generator is connected to the motors through a transmission line and transformers. Motors have rated inputs of 40MVA, 30MVA and 20MVA at 30kv with 20%reactance-each. Draw the per unit circuit diagram.



REC 04

Exercise 03

Given the symmetrical components of unbalanced three phase with:

$$I_A^{(0)} = 0.53 \angle -57.9^\circ$$

$$I_A^{(1)} = 1.27 \angle 72.1^\circ$$

$$I_A^{(2)} = 0.64 \angle 44.1^\circ$$

-Obtain the original unbalanced phasors (I_A , I_B , I_C), and draw the phasor diagram of all parameters.

Exercise 04:

Obtain the symmetrical components for the following set of unbalanced currents

$$I_A = 1.56 \angle 46.8^\circ$$

$$I_B = 1.32 \angle -67^\circ$$

$$I_C = 1.56 \angle -121^\circ$$

Problem 05:

The line-to-ground voltages on the high voltage side of a step-up transformer are 100 kV, 33 kV and 38 kV on phases a, b and c respectively. The voltage of phase a lead that of phase b by 100° and lags that of phase c by 176.5° .

Determine analytically the symmetrical components of voltage.

Problem 06:

The line currents in amperes in phases a, b and c respectively are $500 + j150$, $100 - j600$ and $-300 + j600$ referred to the same reference vector.

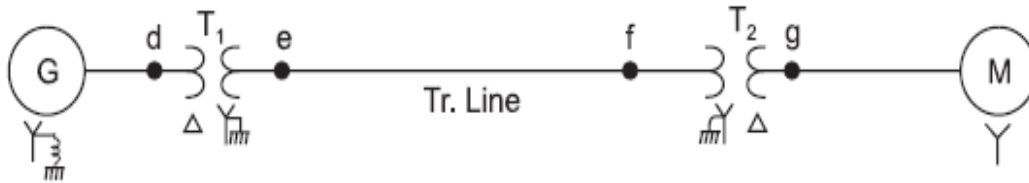
Find the symmetrical component of currents.

Problem 07:

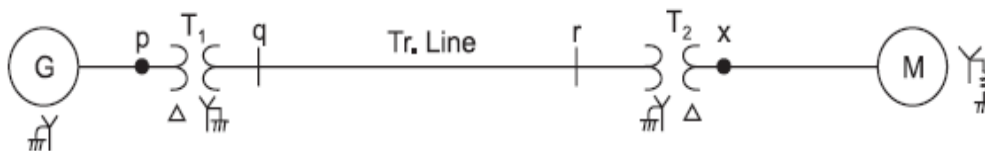
One conductor of a three-phase line is open. The current flowing to the Δ -connected load through line a is 10A. With the current in line a as reference and assuming that line c is open, find the symmetrical components of the line currents

REC 05

Exercise 01: Draw positive, negative and zero sequence network of the power system as shown in figure below



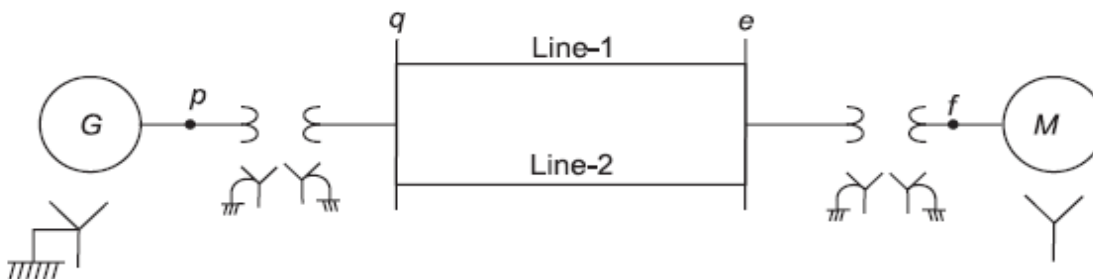
Exercise 02: Draw the zero-sequence network of the system shown in figure below



Exercise 03: Draw the Thevenin equivalent of positive, negative and zero sequence network



Exercise 04: Draw the zero-sequence network of the simple power system shown in the figure; data are given below: G: $X_g^{(0)} = 0.05$ pu; M: $X_M^{(0)} = 0.03$ pu ; T₁: $X_{T1} = 0.12$ pu ; T₂: $X_{T2} = 0.10$ pu; Line 1 : $X_{L1} = 0.70$ pu; Line 1 : $X_{L2} = 0.70$ pu



REC 06

Exercise 01

A 11kv, 30MVA generator has $x_1=x_2=0.2\text{pu}$ and $x_0=0.05\text{pu}$. a line to ground fault occurs on the generator terminal. Find the fault current and line to line voltages during fault conditions. Assume that the generator neutral is solidly grounded.

Exercise 02:

The single line diagram is drawn in figure 1. Before the occurrence of a solid L-G fault at line g the motors were loaded. If the prefault current is neglected, calculate the fault current and sub transient currents in all parts of the system.



Figure 1: Single-line diagram the system

Exercise 03:

Two 11kv, 12 MVA, 3 ph, star connected generators operate in parallel figure 2. The positive, negative and zero sequence reactance of each being $j0.09$, $j0.05$ and $j0.04$ pu respectively. A single line to ground fault occurs at the terminals of one the generators.:

1. Draw the positive, negative and zero sequences
2. Estimate The fault current
3. Estimate Current in grounding resistor
4. Estimate Voltage across grounding resistor

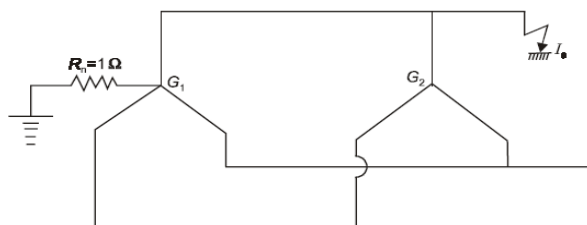


Figure 2: Circuit connection of two generators

Laboratory experiments

Lab N° 01
Operations and Solving Equations Linear Algebra using MATLAB

In this lab you will use Matlab to study and solving the following topics:

- ✓ Solving a system of linear equations.
- ✓ Matrix multiplication and its properties.

Test 01

Compute the following sums of matrices. If the addition is not defined, state how you know

$$\begin{bmatrix} 2 & 3 & 4 \\ -1 & 6 & 2 \\ 1 & 0 & 3 \end{bmatrix} + \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} \qquad 2 \begin{bmatrix} 6 & 1 \\ 0 & -3 \\ -1 & 2 \end{bmatrix} - 3 \begin{bmatrix} 4 & 2 \\ 0 & 1 \\ -5 & -1 \end{bmatrix}$$

Test 02

Compute the following matrix products. If the matrix product is not defined, state how you know.

$$\begin{bmatrix} 3 & 1 & -1 \\ 0 & -1 & 2 \end{bmatrix} \begin{bmatrix} 1 & -1 \\ 0 & 2 \\ 1 & 0 \end{bmatrix} \qquad [3 \quad -2 \quad 2] \begin{bmatrix} 1 \\ 2 \\ -2 \end{bmatrix} \qquad \begin{bmatrix} 2 & -2 & -1 \\ 1 & 1 & -2 \\ 1 & 0 & -1 \end{bmatrix} \begin{bmatrix} -1 & -2 & 5 \\ -1 & -1 & 3 \\ -1 & -2 & 4 \end{bmatrix}$$

Test 03

Given a matrix

$$\text{mat}=[1 \ 2 \ -3;-3 \ -1 \ 1;1 \ -1 \ 1];$$

Calculate the determinant

Get the matrix inverse

Test 04

Given a system of linear equations

$$x+2y-3z=5$$

$$-3x-y+z=-8$$

$$x-y+z=0$$

Construct matrices so the system is described by $Ax=b$

And solve with a single line of code; $ax=b$

Test 05

Solve the following systems of equations:

System 1:

$$x+4y=34$$

$$-3x+y=2$$

System 2:

$$2x-2y=4$$

$$-x+y=3$$

$$3x+4y=2$$

LAB N° 02**Symmetrical Components Analysis Using MATLAB****Objective**

-The obtain symmetrical components of set of unbalanced voltage and current

Lab No:01

Create a program model with Matlab for the obtain a symmetrical component for the following set of unbalanced currents

$$I_A = 1.56 \angle 46.8^\circ$$

$$I_B = 1.32 \angle -67^\circ$$

$$I_C = 1.56 \angle -121^\circ$$

Lab No:02

The line currents in amperes in phases a, b and c respectively are $500 + j150$, $100 - j600$ and $-300 + j600$ referred to the same reference vector.

Find the symmetrical component of currents.

LAB N° 03

Design and implement an Electrical power system

Objective

-Knowing the ETAP Environment and implement SLD of electrical power system

INTRODUCTION:

Electrical Transient and Analysis Program (ETAP) is simulation software that allows the engineer to analyse any electrical system in terms of the load flow analysis, fault analysis, short circuit analysis, relay coordination, transient analysis, etc. Besides these, ETAP also calculates other Electrical parameters efficiently as well. This makes ETAP the most comprehensive environment for the designing, implementation and the study of the Electrical power systems.

In our project we are going to design and implement an Electrical power system, and will do design the Circuit breakers and fuses. But first let's get familiar with the ETAP Environment.

On opening the ETAP, we start by clicking on the "create new a project", after defining the name of project and other general details we finally enter into the ETAP environment. The window opens the editor mode for the SLD's. In the window it can be seen that there are symbols for different components for the SLD's.

Load flow analysis of the SLD can be done. In the upper right corner of the window there are symbols regarding different types of analysis of the SLD. A side bar appears in the window which displays the options regarding the load flow analysis. First option is the display options through which different values can be displays in the windows at each node of the SLD. These values can be of current, voltage and power factor. Then there is the option of the alert view which shows that either any node is behaving abnormally that is under voltage or over voltage.

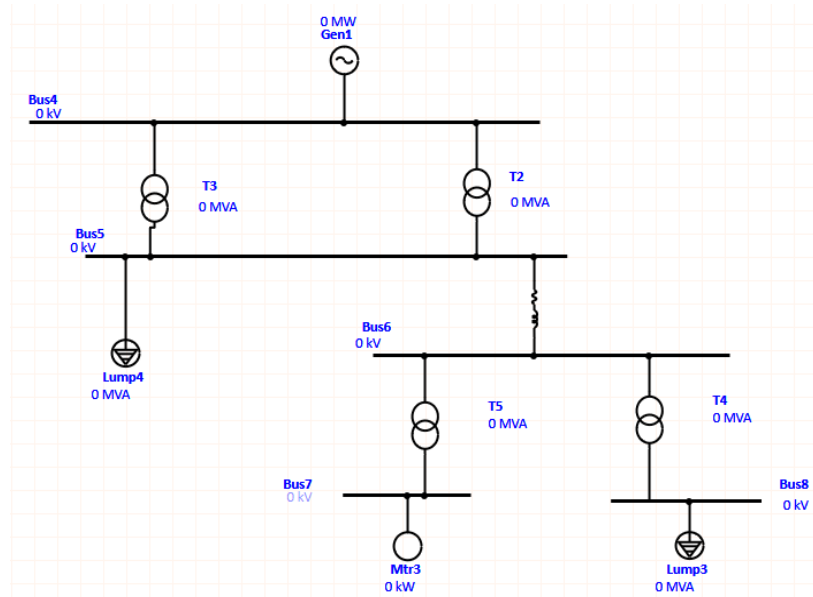
Finally, to generate a load flow report there is an option just below the alert view. It generates a PDF file for all the calculations of the load flow analysis. The report can be made detailed as well as précised depending upon the requirement of the user.

The short circuit analysis of the SLD can also be done. In the upper right corner of the editor window there is an option of short circuit analysis through which short circuit analysis can be done.

A fault is generated on a bus by right clicking it and selecting the fault option and then the required calculations were seen by the display options.

Test 01:

Draw the SLD the circuit below and generate load flow of the system



Generator :250MW, 20Kv

T1,T2 : 100MVA, 20/220 KV, Z :typical, X/R :typical

T3: 40MVA, 220 /60KV, Z :2%, X/R :4

T4 : 20MVA, 220 /30KV, Z :4%, X/R :6

M1 :2MW, Cos ϕ :0.8

Load 1 : 15MW

Load 2 : 25MW

TL :20Km, Z=0.02+j 0.04

LAB N° 04

Implementation of the different types of faults in ETAP

Objective

Require some background knowledge of the different types of faults,

Introduction

For the proper implementation of the design scheme this will require some background knowledge of the different types of faults, their severity and damage to the power system and how such faults can be rectified. For this purpose, we have designed a simple power system to understand the designing, implementation and working of ETAP. A model has been created in the ETAP software which is shown in Figure 1

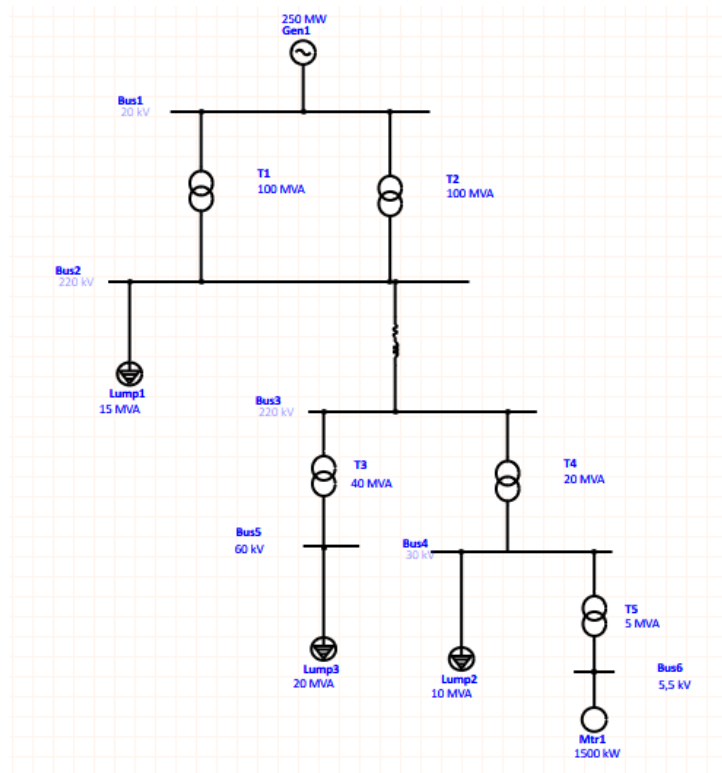


Figure 1: Implementation of Simple Power system. The load flow analysis is done

Faults are created at buses 1 and 4 in the above model in order to study the short circuit analysis of the system.

Faults at Bus 1:

We have made Bus 1 faulty, and then we have applied different fault at Bus 1 like L-G fault, L-L fault, L-L-G fault and 3phase symmetrical fault using Editor as depicts in Figure 2, And the power flow analysis was observed as shown in Figure 3.

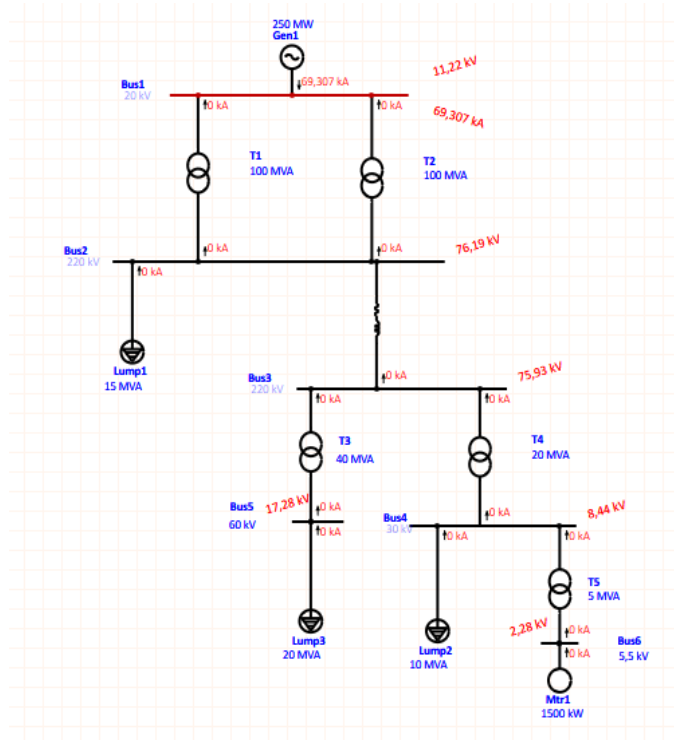


Figure 3: Different Faults on Bus 1

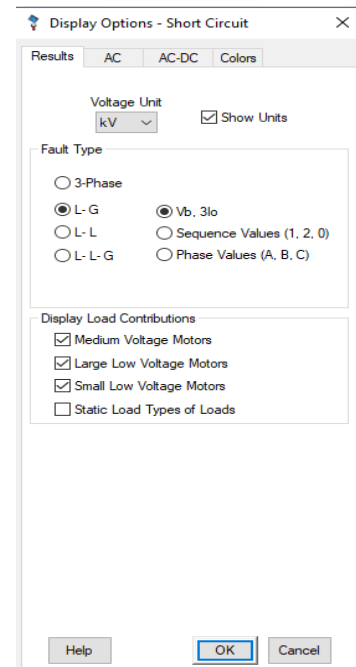


Figure 2: Short circuit editor

Faults at Bus 4:

As previously, we have now made Bus 4 faulty, and then we have applied different fault as Bus 1 like L-G fault, L-L fault, L-L-G fault and 3phase symmetrical fault.

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