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Theme

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p-Factorable non linear operators

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Abstract

This memory deals with the concepts of p-factorable linear operators and multilinear p-factorable operators.

Résumé

Ce mémoire traite les concepts linéires et multilinéaires d'opérateurs p-factorable

Mots-clés: Application multilinéaire, Opérateur p-factorable.

Notation

$\mathbb{K}$	The field of real or complex numbers.
p^*	The conjugate of the number p $(1 \leq p \leq \infty)$, that is $\frac{1}{p} + \frac{1}{p^*} = 1$
X^*	The topological dual of X.
B_X	The closed unit ball of X
$L(X;Y)$	The set of all linear operators.
$\mathcal{L}(X;Y)$	The set of all continuous linear operators.
w	The weak topology.
w^*	The weak * topology.
T^*	The adjoint operator of T.
$\mathcal{K}$	The set of all compact linear operators.
$\mathcal{L}_f$	The set of all finite rank linear operators.

Introduction

The theory of linear operator ideals between normed (or Banach) spaces have been developed by Albert Pietsch [11], and it is nowadays well established. A linear operator ideal is a subclass $\mathcal{A}$ of the class of all continuous linear operators, such that for every Banach spaces E and F, the set $\mathcal{A}(E, F)$ is a vector subspace of $\mathcal{L}(E, F)$ that is invariant by the composition of linear operators on the right or and on the left and which contains the linear operators of finite rank. The reader can find a lot of information about in the cxcellcnt book [6]. The ideal of p-factorable is due to Kwapien [7]. Recently, Cerna extended the notion of p-factorable operators to multilinear operators [3]. In the last decades this linear theory has spreaded to non-linear contexts that include multilinear mappings, polynomials, holomorphic functions or Lipschitz mappings among others. Transferring summability properties to nonlinear mappings is not an obvious task as shows the variety of different generalizations of several classes of summing operators to the multilinear case, and to hit the multilinear class that is closest, in some sense, to the original linear class is not trivial (see e.g. [1, 4, 2, 12]). In this memory we will deals with the ideal of p-factorable linear operators and multilinear p-factorable operators.

Our memory is organized as follows,

In the first Chapter is an overview of notions and basic concepts and results needed in the following chapters. These include multilinear mappigs, also operator ideals. In the second Chapter We describe the ideal of p-factorable operators.

In the last Chapter we study multilinear version of p-factorable operators, of the theory given in [11, 6] about linear p-factorable operators, we have come to generalize some concepts and theorems.

Chapter 1

Continuous multilinear operators

In this chapter we will recall some fundamental definitions, properties in multilinear operators, we end this chapter by given various examples of ideals of linear summing operators.

1.1 Notation and background

Throughout this chapter X, Y will denoted vector spaces over a field $\mathbb{K}$ which may be either the real or complex numbers. A Banach space is a complete normed vector space. The closed unit ball of X is denote by B_X.

A linear map $T : X \to Y$ is continuous if and only if

$$\|T\| = \sup\{\|T(x)\| : \|x)\| \leq 1, x \in X\} < \infty,$$

this value is a norm on the vector space $L(X,\ Y)$ of all linear operators from X into Y. We denoted by $\mathcal{L}(X,\ Y)$ the space of continuous linear operators, and by X^* the dual space of X, the norm of $x^* \in X^*$ is given by

$$\|x^*\| = \sup\{|\langle x^*, x\rangle| : x \in B_X\}.$$

We will denoted by (Ω, Σ, μ) a measure space, if $\mu(\Omega) = 1$ then is (Ω, Σ, μ) called probability space. For any measure space (Ω, Σ, μ) we define the space $L_p(\mu) = L_p(\Omega, \Sigma, \mu)$, $1 \leq p \leq \infty$, to be the space of Σ-measurable functions such that $\int |f(w)|^p d\mu(w) < \infty$, we mean $\sup ess\,|f(w)| < \infty$. We will use the notation

$$\|f\|_p = \left(\int |f(w)|^p d\mu(w)\right)^{\frac{1}{p}} \text{ for } p \geq 1, \text{ and } \|f\|_\infty = \sup ess\,|f(w)|.$$

We know that:

1. A linear operator $T : X \to Y$ is invertible if there exist a linear operator noted $T^{-1} : Y \to X$ such that $T^{-1} \circ T = id_X$, and $T \circ T^{-1} = id_Y$, where id_X (or id_Y) is identity operator on X (or on Y).

2. A linear operator $T : X \to Y$ between two normed spaces X and Y is isomorphism if T is a continuous bijection whose inverse T^{-1} is also continuous. In this case, the spaces X and Y are isomorphism in addition if $\|T(x)\| = \|x\|$, for all $x \in X$. Then T be isometric isomorphism.

3. Let $T : X \to Y$ be continuous linear operator. Then the continuous linear operator $T^* : Y^* \to X^*$ defined by

$$T^*(y^*)(x) = y^*T(x)$$

for every $y^* \in Y^*$ and $x \in X$ is called the adjoint of T and he have the property

$$\|T\| = \|T^*\|.$$

Also we have $(T \circ u)^* = u^* \circ T^*$ for all continuous linear operator u.

Let X be a Banach space and $1 \leq p \leq \infty$, p^* is the conjugate of p, i.e., $\frac{1}{p} + \frac{1}{p^*} = 1$. We denote by $\ell_p(X)$, the space of all *absolutely p-summable sequences* in X; that is, sequences $(x_i)_i$ in X such that

$$\|(x_i)_i\|_p := (\sum_i \|x_i\|^p)^{1/p} < \infty,$$

if $1 \leq p < \infty$ or,

$$\|(x_i)_i\|_\infty := \sup_i \|x_i\|,$$

if $p = \infty$.

$\ell_p^w(X)$, the space of all *weakly p-summable sequences* in X; that is, sequences $(x_i)_i$ in X such that

$$\|(x_i)_i\|_{p,w} := \sup_{x^* \in X^*, \|x^*\| \leq 1} (\sum_i |x^*(x_i)|^p)^{1/p} < \infty,$$

if $1 \leq p < \infty$ or,

$$\|(x_i)_i\|_{\infty,w} := \sup_i \sup_{x^* \in X^*, \|x^*\| \leq 1} |x^*(x_i)| = \sup_i \|x_i\|,$$

if $p = \infty$, where X^* denotes the topological dual of X. The closed unit ball of X will be denoted B_X.

Note that $\ell_p(X)$ is a linear subspace of $\ell_p^w(X)$ and

$$\|(x_i)_i\|_{p,w} \leq \|(x_i)_i\|_p \text{ for all } (x_i)_i \in \ell_p(X).$$

Then $\ell_p(X) = \ell_p^w(X)$ for some $1 \leq p < \infty$ if, and only if, $\dim(X) < \infty$. If $p = \infty$, we have $\ell_\infty(X) = \ell_\infty^w(X)$, also if we take $X = \mathbb{K}$ (or $n = 1$), then the spaces $\ell_p(\mathbb{K})$ and $\ell_p^w(\mathbb{K})$ coincide

A sequences $(x_i)_{i=1}^{\infty}$in X is said to be *unconditionally p-summable* if

$$\lim_{n\to\infty} \|(x_i)_{i=n}^{\infty}\|_{p,\omega} = 0.$$

We denote by $\ell_p\langle X\rangle$, the space of all *strongly p-summable sequences* in X; that is, sequences $(x_i)_i$ in X such that

$$\|(x_i)_i\|_{\ell_p\langle X\rangle} := \sup_{\|(x_i^*)_i\|_{p^*,\omega}\leq 1} \left|\sum_i x_i^*(x_i)\right| < \infty.$$

It well know that for $1 \leq p < \infty$ and $(\varphi_i)_{i=1}^n \in \ell_{p^*,\omega}^n(Y^*)$ we have

$$\|(\varphi_i)_i\|_{p^*,\omega} = \sup_{\psi\in B_{Y^{**}}} \left(\sum_{i=1}^n |\psi(\varphi_i)|^{p^*}\right)^{\frac{1}{p^*}} = \sup_{y\in B_Y} \|(\varphi_i(y))_i\|_{p^*}.$$

If u is a linear bounded operator from X into Banach Y for $\varphi_i \in Y^*$, $i = 1, ..., n$ we have

$$\|(\varphi_i \circ u)_i\|_{p,\omega} \leq \|u\| \, \|(\varphi_i)_i\|_{p,\omega}. \tag{1.1}$$

1.2 Multilinear mappings

Let $m \in \mathbb{N}$ and $X_j\,(1 \leq j \leq m)$ be a Banach spaces over $\mathbb{K}$ ($\mathbb{K} = \mathbb{R}$ ou $\mathbb{C}$). We consider the Cartesian product

$$X_1 \times ... \times X_m = \left\{\left(x^1, ..., x^m\right) : x^j \in X_j,\ \forall 1 \leq j \leq m\right\},$$

which is a normed space equipped with the norm

$$\left\|\left(x^1,...,x^m\right)\right\| := \max\left\{\left\|x^j\right\|;\ x^j \in X_j, \forall 1 \leq j \leq m\right\}. \tag{1.2}$$

Definition 1.1. *An application $T : X_1 \times ... \times X_m \to Y$ is called multilinear (or m-linear) if the mappings*

$$\begin{array}{rccl} T_j: & X_j & \to & Y \\ & x^j & \mapsto & T\left(x^1,...,x^j,...,x^m\right), \end{array}$$

are linear for each $x^k \in X_k$, $k \neq j$, in other words

$$T\left(x^1,...,\lambda x^j + y^j,...,x^m\right) = \lambda T\left(x^1,...,x^j,...,x^m\right) + T\left(x^1,...,y^j,...,x^m\right);$$

for all $\lambda \in \mathbb{K}$ and x^j, $y^j \in X_j$ $(1 \leq j \leq m)$, we denoted by $L\left(X_1,...,X_m;Y\right)$ the space of all m-linear applications T from $X_1 \times ... \times X_m$ into Y. The set $\mathcal{S}$ of all vectors in Y of the form $T\left(x^1,...,x^m\right)$, $x^j \in X_j$ $(1 \leq j \leq m)$ is not in general vector subspace of Y (see [5, Section 1.1]).

Now we define the following linear operations

$$\begin{aligned} (S+T)((x^1,...,x^m) &= S\left(x^1,...,x^m\right) + T\left(x^1,...,x^m\right) \\ (\lambda T)\left(x^1,...,x^m\right) &= \lambda T\left(x^1,...,x^m\right), \lambda \in \mathbb{K} \end{aligned}$$

which gives to $L\left(X_1,...,X_m;Y\right)$ a structure of a vector space. If $Y = \mathbb{K}$, we write $L\left(X_1,...,X_m\right)$

Definition 1.2. *The multilinear application $T : X_1 \times ... \times X_m \to Y$ is continuous if it is continuous as a function between two normed spaces.*

As a consequence of this definition, and the following equality

$$\begin{aligned} T\left(x^1,...,x^m\right) - T\left(y^1,...,y^m\right) &= T\left(x^1-y^1,...,x^m\right) + T\left(x^1,x^2-y^2,...,x^m\right) \\ &\quad + \cdots + T\left(x^1,...,x^m-y^m\right), \end{aligned}$$

we have a result that gives a characterization of continuous m-linear mapping.

Proposition 1.3. *Let $X_1,...,X_m$, Y be normed spaces. For all $T \in L\left(X_1,...,X_m;Y\right)$, the following statements are equivalent*

(1) T *is continuous.*

(2) *T is continuous in $(0,...,0)$.*

(3) *There exists a constant $C > 0$ such that*

$$\left\| T\left(x^1,...,x^m\right)\right\| \leq C \left\| x^1\right\| ... \left\| x^m\right\|, \tag{1.3}$$

for all $x^j \in X_j$, $(\forall 1 \leq j \leq m)$.

If X_j, $(\forall 1 \leq j \leq m)$ and Y are a normed spaces, then we provide the space $\prod X_j$ of topology of product vector spaces, and we denote by $\mathcal{L}(X_1,...,X_m)$ the vector space of all the continuous m-linear applications of $\prod X_j$ into Y.

Proposition 1.4. *Let Y be a Banach space, the vector space $\mathcal{L}(X_1,...,X_m;Y)$ is a Banach space endowed with the norm $\|T\|$ define by:*

$$\begin{aligned} \|T\| &= \sup_{\|x^j\|\leq 1,\ j=1,...m} \left\| T\left(x^1,...,x^m\right)\right\| \\ &= \sup_{x^j\neq 0,\ j=1,...m} \frac{\left\| T\left(x^1,...,x^m\right)\right\|}{\|x^1\|...\|x^m\|} \\ &= \inf\left\{C : \left\| T\left(x^1,...,x^m\right)\right\| \leq C\left\|x^1\right\|...\left\|x^m\right\|\right\}. \end{aligned}$$

If $T \in \mathcal{L}(X_1,...,X_m;Y)$ we define the adjoint operator of T by

$$T^* : Y^* \to \mathcal{L}(X_1,...,X_m),\ y^* \mapsto T^*(y^*) : X_1 \times ... \times X_m \to \mathbb{K}$$

with

$$T^*(y^*)\left(x^1,...,x^m\right) = y^*\left(T\left(x^1,...,x^m\right)\right).$$

Now we define the multilinear mapping

$$K : X_1 \times ... \times X_m \to \mathcal{L}(X_1,...,X_m)^*$$

with

$$K\left(x^1,...,x^m\right)(\phi) = \phi\left(x^1,...,x^m\right)$$

for all $x^j \in X_j$ and $\phi \in \mathcal{L}(X_1,...,X_m)$. Then K is continuous and $\|K\| = 1$.

Proposition 1.5. *Let $T \in \mathcal{L}(X_1,...,X_m)$. Then*

1. *T^* is a linear operatars.*

2. *$(u \circ T)^* = T^* \circ u^*$ with $T \in \mathcal{L}(X_1,...,X_n;Y)$ and $u \in \mathcal{L}(Y,W)$*

3. $\|T^*\| = \|T\|$

A good result in multilinear operator theory is the isometric isomorphic identification described in the following proposition

Proposition 1.6. *[9, Theorem 1.10] Let $X_1, ..., X_m$ and Y be Banach spaces. Then we have the isometric isomorphism identification.*

$$\mathcal{L}(X_1, ..., X_m, Y) = \mathcal{L}(X_1, ..., X_m; Y^*). \tag{1.4}$$

1.3 Normed operator ideals

In this section, we recall some basic facts and properties about operator ideals. We also recall some of the classical examples.

Let X and Y be Banach spaces. It is well known that an operator $T \in \mathcal{L}(X, Y)$ is of finite rank if and only if there exist functionals $x_1^*, ...x_n^* \in X^*$, and vectors $y_1, ...y_n \in Y$ such that

$$T(x) = \sum_{k=1}^{n} x_k^*(x) y_k, x \in X.$$

The class of all finite rank operator is denoted by $\mathcal{F}$.

Following the standard tensor-product notation, for $x^* \in X^*$ and $y \in Y$ the operator $x \longmapsto x^*(x)y$, $x \in X$, is denoted by $x^* \otimes y$. It is clear that $x^* \otimes y$ is a rank one operator if and only if $x^* \neq 0$ and $y \neq 0$. Therefore, $T \in \mathcal{F}(X, Y)$ if and only if T can be represented as a finite sum of rank one operators

$$T = \sum_{k=1}^{n} x_k^* \otimes y_k.$$

Definition 1.7. *An operator ideal $\mathcal{A}$ is a subclass of $\mathcal{L}$ such that the components*

$$\mathcal{A}(X, Y) := \mathcal{A} \cap \mathcal{L}(X, Y),$$

satisfy the following conditions:

(i) $\mathcal{A}(X, Y)$ is a linear subspaces of $\mathcal{L}(X, Y)$ for all Banach spaces X and Y.

(ii) The subclass $\mathcal{A}$ contains all Finite rank operator.

(iii) The ideal property: if X, Y, Z, W are Banach spaces and $R \in \mathcal{L}(X,Y)$, $T \in \mathcal{A}(Y,Z)$, $S \in \mathcal{L}(Z,W)$, then $STR \in \mathcal{A}(X,W)$.

If $\|.\|_{\mathcal{A}} : \mathcal{A} \to \mathbb{R}_+$ satisfies

(i') $(\mathcal{A}(X,Y), \|.\|_{\mathcal{A}})$ is a normed (Banach) spaces for all Banach spaces X and Y.

(ii') $\|Id_{\mathbb{K}}\|_{\mathcal{A}} = 1$.

(iii') If $R \in \mathcal{L}(X,Y)$, $T \in \mathcal{A}(Y,Z)$, $S \in \mathcal{L}(Z,W)$, then

$$\|STR\|_{\mathcal{A}} \leq \|S\| \, \|T\| \, \|R\| \, ,$$

then $(\mathcal{A}, \| \, . \, \|_{\mathcal{A}})$ is called normed (Banach) operators ideal.

Examples

1. **Approximable operators.** An operator $T \in \mathcal{L}(X,Y)$ is called Approximable operators if there are $T_n \in \mathcal{F}(X,Y)$, with

   $$\lim_n \|T - T_n\| = 0.$$

   We denote by $\overline{\mathcal{F}}(X,Y)$ the ideal space of all approximable operators from X to Y.

2. **Compact linear operators.** A linear operator $T \in \mathcal{L}(X,Y)$ is compact if the image of the unit ball B_X of X into a relatively compact subset of Y.

   In other words, T is compact if and only if for every norm bounded sequence $\{x_n\}$ of X, the sequence $\{T(x_n)\}$ has a norm convergent subsequence in Y. We denote by $\mathcal{K}(X,Y)$ the ideal space of compact operators from X to Y.

3. **The ideal of p-summing linear operators.** Let $1 \leq p < \infty$. A linear operator $u : X \longrightarrow Y$ between Banach spaces is said to be *absolutely p-summing* or just *p-summing* if it takes weakly p-summable sequences $(x_i)_{i=1}^{\infty}$ of X to absolutely p-summable sequences $(u(x_i))_{i=1}^{\infty}$ of Y.

This means that $\widehat{u} : (x_i)_{i=1}^{\infty} \longmapsto (u(x_i))_{i=1}^{\infty}$ defines a linear mapping from $\ell_p^{\omega}(X)$ into $\ell_p(Y)$ that is bounded in view of the closed graph theorem (see[10],[6]). Hence there exists a constant $C \geq 0$ such that

$$\left(\sum_{i=1}^{n} \|u(x_i)\|^p\right)^{\frac{1}{p}} \leq C \sup_{\|\xi\|_{X^*} \leq 1} \left(\sum_{i=1}^{n} |\xi(x_i)|^p\right)^{\frac{1}{p}}, \tag{1.5}$$

for every finite family $(x_i)_{i=1}^{n} \subset X$. This inequality characterizes p-summing operators. The set of all p-summing operators, is denoted by $\Pi_p(X,Y)$, which constitute a Banach ideal under the ideal norm

$$\pi_p(u) := \inf \{C, \text{ for all } C \text{ verifying the inequality (1.5)}\},$$

moreover we have

$$\pi_p(u) = \|\widehat{u}\|.$$

The nowadays classical Pietsch's domination theorem characterizes the p-summability of an operator by means of a norm domination uniform inequality. Concretely, it says that the mapping $u \in \mathcal{L}(X,Y)$ is p-summing if and only if there exist a constant C and a regular Borel probability measure μ on B_{X^*} (with the weak star topology) such that

$$\|u(x)\| \leq C \left(\int_{B_{X^*}} |\langle x, x^* \rangle|^p \, d\mu(x^*)\right)^{\frac{1}{p}}, \quad x \in X. \tag{1.6}$$

In this case, $\pi_p(u)$ is the least of all the constants C so that (1.6) holds. This inequality also provides a factorization of u through the natural mapping $C(B_{X^*}) \to L^p(\mu)$.

1.4 Ideal multilinear operators

Let $m \in \mathbb{N}$ and $X_1, ..., X_m, Y$ be Banach spaces over $\mathbb{K}$ (real or complex scalars field), and let $\mathcal{L}(X_1, ..., X_m; Y)$ the Banach space of all continuous m-linear mappings from $X_1 \times ... \times X_m$ to Y. We denote by $\mathcal{L}_f(X_1, ..., X_m; Y)$, the space of all m-linear mappings of finite type, which is generated by the mappings of the special form

$$T_{y \otimes_{j=1}^{m} x_j^*} = x_1^* \otimes ... \otimes x_m^* \otimes y : \left(x^1, ..., x^m\right) \to x_1^* \left(x^1\right) ... x_m^* \left(x^m\right) .y,$$

for some non-zero $x_j^* \in X_j^*$ $(1 \leq j \leq m)$ and $y \in Y$.

According to Pietsch in [12]. An ideal of multilinear mappings (or multi-ideal) is a subclass $\mathcal{M}$ of all continuous multilinear mappings between Banach spaces such that for all $m \in \mathbb{N}$, Banach spaces $X_1, ..., X_m$ and Y, the components

$$\mathcal{M}(X_1, ..., X_m; Y) := \mathcal{L}(X_1, ..., X_m; Y) \cap \mathcal{M}$$

satisfy:

(i) $\mathcal{M}(X_1, ..., X_m; Y)$ is a linear subspace of $\mathcal{L}(X_1, ..., X_m; Y)$ which contains the m-linear mappings of finite type.

(ii) The ideal property: If $T \in \mathcal{M}(G_1, ..., G_m; F)$, $u_j \in \mathcal{L}(X_j, G_j)$ for $j = 1, ..., m$ and $v \in \mathcal{L}(F, Y)$, then $v \circ T \circ (u_1, ..., u_m)$ is in $\mathcal{M}(X_1, ..., X_m; Y)$.

If $\|.\|_{\mathcal{M}} : \mathcal{M} \to \mathbb{R}^+$ satisfies

(i') $(\mathcal{M}(X_1, ..., X_m; Y), \|.\|_{\mathcal{M}})$ is a normed (Banach) space for all Banach spaces $X_1, ..., X_m$ and Y and all m,

(ii") $\|T : \mathbb{K}^m \to \mathbb{K} : T(x^1, ..., x^m) = x^1 \cdots x^m\|_{\mathcal{M}} = 1$ for all m,

(iii'") If $T \in \mathcal{M}(G_1, ..., G_m; F)$, $u_j \in \mathcal{L}(X_j, G_j)$ for $j = 1, ..., m$ and $v \in \mathcal{L}(F, Y)$, then $\|v \circ T \circ (u_1, ..., u_m)\|_{\mathcal{M}} \leq \|v\| \, \|T\|_{\mathcal{M}} \, \|u_1\| ... \|u_m\|$, then $(\mathcal{M}, \|.\|_{\mathcal{M}})$ is called a normed (Banach) multi-ideal.

We begin by presenting different classes of ideals of multilinear mappings related to the concept of absolutely summing operators:

• **Absolutely $(p; q_1, ..., q_m; r)$-summing multilinear operators**

The notion of Absolutely $(p; q_1, ..., q_m; r)$-summing multilinear operators was introduced by Achour in [1]. A bounded multilinear operator T is absolutely $(p; q_1, ..., q_m; r)$-summing for all $0 < p, q_1, ..., q_m, r < \infty$ with $\frac{1}{p} \leq \frac{1}{q_1} + ... + \frac{1}{q_m} + \frac{1}{r}$ if there is a constant $C \geq 0$ such that

$$\left(\sum_{i=1}^{n} \left| \varphi_i \left(T \left(x_i^1, ..., x_i^m \right) \right) \right|^p \right)^{\frac{1}{p}} \leq C \prod_{j=1}^{m} \left\| \left(x_i^j \right)_{1 \leq i \leq n} \right\|_{q_j, \omega} \left\| (\varphi_i)_{1 \leq i \leq n} \right\|_{r, \omega} \tag{1.7}$$

with $\varphi_i \in Y^*$and $x_i^j \in X_j$, for all $n \in \mathbb{N}$, $i = 1, ..., n$ and $j = 1, ..., m$.

For $p \geq 1$, the space $\left(\mathcal{L}_{as(p; q_1, ..., q_m; r)}(X_1, ..., X_m; Y), \pi^m_{(p; q_1, ..., q_m; r)}(.) \right)$ is a Banach multi-ideal, where $\pi^m_{(p; q_1, ..., q_m; r)}(.)$ is the smallest constant $C > 0$ such that the inequality 1.7 holds.

Proposition 1.8. *Let $\mathcal{M}$ be an ideal, then*

1. $\|T\| \leq \|T\|_{\mathcal{M}}$ *for all* $T \in \mathcal{M}(X_1, ..., X_m, Y)$.

2. $\|x_1^* \otimes ... \otimes x_m^* \otimes y\|_{\mathcal{M}} = \prod_{j=1}^{m} \|x_j^*\| \|y\|$, $x_j^* \in X_j^*, j = 1, ..., m$ *and* $y \in Y$.

Chapter 2

p-factorable linear operators

2.1 Definition and properties

Definition 2.1. *Let $1 \leq p < \infty$ and $X; Y$ be Banach spaces. The operator $u : X \longrightarrow Y$ is said p-factorable if there exist a measure space $(\Omega; \Sigma; \mu), v \in \mathcal{L}(L_p(\mu); Y^{**})$ and $w \in \mathcal{L}(X; L_p(\mu))$ such that:*

$$\begin{array}{ccccc} X & \xrightarrow{u} & Y & \xrightarrow{K_Y} & Y^{**} \\ & \searrow^{w} & & \nearrow_{v} & \\ & & L_p(\mu) & & \end{array}$$

In other words

$$K_Y u = v \circ w.$$

where K_Y is the isometric embedding of Y into Y^{**} We denote by $\Gamma_{p-fat}(X; Y)$ the space of all p-factorable linear operators, which is a Banach space under the norm

$$\gamma_p(u) = \inf \|v\| \|w\|,$$

where the infimum is taken over all factorizations above.

Definition 2.2. *Let $p = \infty$ and $X; Y$ be Banach spaces.An operators u belongs to $\Gamma_\infty(X, Y)$ if and only if $j_Y u$ factors through the space $C(K)$, for some compact Hausdorff space K :*

$$\begin{array}{ccccc} X & \xrightarrow{u} & Y & \xrightarrow{j_Y} & Y^{**} \\ & \searrow^{w_1} & & \nearrow_{w_2} & \\ & & C(K) & & \end{array} \tag{2.1}$$

Then $\gamma_\infty(u) = \inf \|w_2\| \|w_1\|$, where the infimum is taken over all factorizations (2.1).

Proposition 2.3. *For each* $1 \le p \le \infty, [\mathit{\Gamma}_p, \gamma_p]$ *is a Banach ideal.*

Proof. We shall indicate the proof of the triangle inequality for γ_p.
Let X and Y be Banach space . Let u and $w \in \Gamma_p(X,Y)$.The argument is based on a suitable normalization of factorizations of u and w . Assume that $j_Y u = u_2 u_1$ with $u_1 \in L(X, L_p(\Omega_1, \mu_1))$ and $u_2 \in L(L_p(\Omega_1, \mu_1), Y^{**})$ and that $j_Y w = w_2 w_1$ with $w_1 \in L(X, L_p(\Omega_2, \mu_2))$ and $w_2 \in L(L_p(\Omega_2, \mu_2), Y^{**})$. Set
$a = \|u_1\| \|u_2\|$, $b = \|w_1\| \|w_2\|$.
Without loss of generality assume that

$$\|u_1\| = a^{1/p}, \|u_2\| = a^{1/p^*}, \|w_1\| = b^{1/p}, \|w_2\| = b^{1/p^*}.$$

Set $Z = (L_p(\Omega_1, \mu_1) + L_p(\Omega_2, \mu_2))_p$. Define operators $v_1 \in L(X, Z)$ and $v_2 \in L(X, Y^{**})$ by

$$v_1 x = (u_1 x, w_1 x) \in Z \text{ for } x \in X,$$

$$v_2(z_1, z_2) = u_2 z_1 + w_2 z_2 \in Y^{**} \text{for } (z_1, z_2) \in Z.$$

It is easy to check that $j_Y(u + w) = v_2 v_1$. Since Z can be identified with some $L_p(\Omega, \nu)$, then $\gamma_p(u + w) \le \|v_1\| \|v_2\| \le a + b$ Since the estimate holds for all factorizations of $j_Y u$ and $j_Y w$, it follows that $\gamma_p(u + w) \le \gamma_p(u) + \gamma_p(w)$,completing the proof . □

Proposition 2.4. *Let* p^* *be the index conjugate to* $1 \le p \le \infty$ *and* $u \in \mathcal{L}(X, Y)$.*The following statements are equivalent.*

(i) $u \in \mathit{\Gamma}_p(X, Y)$,

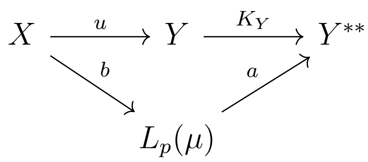
(ii) $u^* \in \mathit{\Gamma}_{p^*}(Y^*, X^*)$,

(iii) $u^{**} \in \mathit{\Gamma}_p(X^{**}, Y^{**})$,

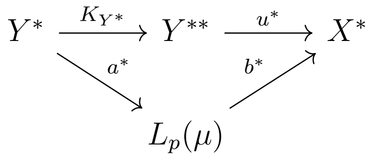
(iv) $K_Y u \in \mathit{\Gamma}_p(X, Y^{**})$.

In this case, $\gamma_p(u) = \gamma_{p^*}(u^*) = \gamma_p(u^{**}) = \gamma_p(K_Y u)$.

Proof. $(i) \Longrightarrow (ii)$: Let



De a typical factorization of $u \in \Gamma_p(X,Y)$. We can use the identity $\kappa_Y^* \kappa_{Y^*} = id_{Y^*}$ to obtain the factorization ,



Since $L_p(\mu)^*$ is an $L_{p^*}(\nu)$-space, u^* is a p^*-factorable operator, with $\gamma_{p^*}(u^*) \leq \|b^*\| \|a^* \kappa_{Y^*}\| = \|a\| \|b\|$,The inequality $\gamma_{p^*}(u^*) \leq \gamma_p(u)$ is immediate.

$(ii) \Longrightarrow (iii)$: A repetition of the argument above, starting with $u^* \in \Gamma_{p^*}(Y^*, X^*)$, gives $u^{**} \in \Gamma_p(X^{**}, Y^{**})$ and $\gamma_p(u^{**}) \leq \gamma_{p^*}(u^*)$

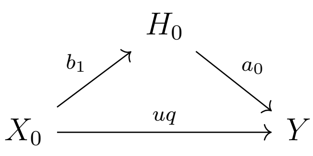
$(iii) \Longrightarrow (iv)$: Observe that $\kappa_Y u = u^{**} \kappa_X$. If $u^{**} \in \Gamma_p(X^{**}, y^{**})$, the ideal property makes it clear that $\kappa_Y u \in \Gamma_p(X, Y^{**})$, and $\gamma_p(\kappa_Y u) \leq \gamma_p(u^{**})$.

$\square$

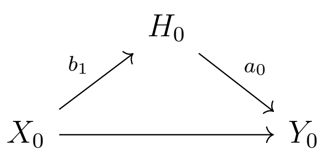
2.2 Characterization of 2-factorable operators

Proposition 2.5. *Let X_0, X, Y, Y_0 be Banach spaces,let $q \in \mathcal{L}(X_0, X)$ be a quotient map, and let $j \in \mathcal{L}(Y, Y_0)$ be an isometric embedding. An operator $u : X \longrightarrow Y$ is 2-factorable if and only if $juq : X_0 \longrightarrow Y_0$ is and in this case $\gamma_2(u) = \gamma_2(juq)$.*

Proof. One half of the assertion and the inequality $\gamma_2(juq) \leq \gamma_2(u)$ follow from the ideal property of $[\Gamma_2, \gamma_2]$. For the other part, start with a factorization $X_0 \underset{\rightarrow}{b} H \underset{\rightarrow}{a} Y_0$ of juq through a Hilbert space H. Consider the closed subspace $H_0 := \ker(a)^\perp \bigcap \{h \in H : ah \in j(Y)\}$ and Let $a_0 : H_0 \longrightarrow Y$ be the operator such that jao is a's restriction to H_0. Notice that a_0 is injective with $\|a_0\| \leq \|a\|$. Let p be the orthogonal projection of H onto H_0 and set $b_1 := pb$. Then $\|b_1\| \leq \|b\|$ and uq has the factorization



The injectivity of ao implies the existence of a $b_0 \in \mathcal{L}(X, H_0)$ such that $b_0 q = b_1$ since q is a quotient map, we have $\|b_0\| = \|b_1\|$ Now we aredone: since



we see that $u \in \Gamma_2(X,Y)$ and $\gamma_2(w) \leq \|a_0\|\|b_0\| \leq \|a\|\|b\|$. Since we started with an arbitrary Γ_2-factorization of juq, $\gamma_2(u) \leq \gamma_2(juq)$ follows. $\square$

Theorem 2.6. *[6] The following statements about an operator $u : X \longrightarrow Y$ are equivalent:*

(i) *$u \in \Gamma_2(X,Y)$;*

(ii) *There is a $C \geq 0$ such that*

$$\left|\sum_{i,j=1}^{n} a_{ij} < y_i^*, ux_j >\right| \leq C \sup\{\left|\sum_{i,j=1}^{n} a_{ij} s_i t_j\right| : (s_i), (t_j) \in B_{\ell_\infty^n}\}$$

regardless of the choice of $n \in \mathbb{N}$, the $n \times n$ scalar matrix (a_{ij}), and the vectors $x_1, ..., x_n \in B_X$ and $y_1^*, ..., y_1^* \in B_{Y^*}$. In case (i) and (ii) hold, we may take $C = \kappa_G.\gamma_2(u)$ where κ_G is Grothendieck's constant.

Theorem 2.7. *Let X and Y be Banach spaces. The operator $u : X \longrightarrow Y$ belongs to $\Gamma_2(X,Y)$ with $\gamma_2 \leq C$ if and only if, for any positive integer n, any $n \times n$ matriz $a = (a_{ij})$ of scalars and any vectors $x_1, ..., x_n \in X, y_1^*, ..., y_n^* \in F^*$ we have*

$$\left|\sum_{i,j=1}^{n} a_{ij} < y_i^*, ux_j >\right| \leq C\|a\|(\sum_{j=1}^{n} \|x_j\|^2)^{\frac{1}{2}}.(\sum_{i=1}^{n} \|y_i^*\|^2)^{\frac{1}{2}} \tag{2.2}$$

Proof. We have already observed that each $u \in \Gamma_2(X,F)$ satisfies condition (1), with $C = \gamma_2(u)$.

For the converse, rephrase inequality (1) as

$$\sum_{i=1}^{n} \| \sum_{j=1}^{n} a_{ij} ux_j\|^2 \leq C^2 \|a\|^2 \sum_{j=1}^{n} \|x_j\|^2; \tag{2.3}$$

we leave the verification of the equivalence to the reader.

Our aim is to use the lemma in the special case $Z = X$. Accordingly,suppose we are given sequences $(x_j)_{j=1}^n$ and $(z_i)_{i=1}^m$ in X with

$$\sum_{i=1}^{m} |< x^*, z_i >|^2 \leq \sum_{j=1}^{n} |< x^*, x_j >|^2; \forall x^* \in X^* \tag{2.4}$$

By adding zeros if necessary we may as well assume that $m = n$. Let s and $t$be the operators in $\mathcal{L}(\ell_2^n, X)$ which are defined via $e_i \longmapsto z_i$ and $e_i \longmapsto x_i$ respectively $(1 \leq i \leq n)$. Note that (3) can be restated as

$$\|s^* x^*\| \leq \|t^* x^*\|; \forall x^* \in X^* \tag{2.5}$$

Since t^*'s range is a Hilbert space, this signifies that there exists an operator $a \in \mathcal{L}(\ell_2^n)$ norm sit most one, such that $s^* = a^* t^*$, that is, $s = ta$ if (a_{ij}) is the matriz of a then, with suitably chosen $y_1^*, ..., y_n^* \in B_{Y^*}$ get

$$\sum_{i=1}^{n} \|uz_i\|^2 = \sum_{i=1}^{n} |< y_i^*, use_i >|^2$$

$$= \sum_{i=1}^{n} |< y_i^*, utae_i >|^2 = \sum_{i=1}^{n} |< y_i^*, \sum_{j=1}^{n} a_{ij} ux_j >|^2$$

$$\leq \sum_{i=1}^{n} \| \sum_{j=1}^{n} a_{ij} ux_j \|^2$$

$$\leq C^2 \sum_{j=1}^{n} \|x_j\|^2$$

by (2) and since $\|a\| \leq 1$. □

Chapter 3

p-factorable multilinear mappings

3.1 Definition and properties

Definition 3.1. *Let $1 \leq p \leq \infty, T \in \mathcal{L}(X_1, ..., X_m; Y)$ is said to be p-factorable,if there is a measure space (Ω, Σ, μ) operators $a \in \mathcal{L}(L_p(\mu), Y^{**})$ and $T \in \mathcal{L}(X_1, ..., X_n; L_p(\mu))$ such that the following diagram commute:*

$$\begin{array}{ccccc} X_1 \times ... \times X_m & \xrightarrow{T} & Y & \xrightarrow{K_Y} & Y^{**} \\ & \searrow^{T} & & \nearrow_{a} & \\ & & L_p(\mu) & & \end{array}$$

The collection of the p-factorable multi-linear operators of $X_1, ..., X_m$ to Y will be denoted by $\mathcal{L}_{p-fact}(X_1, ..., X_m, Y)$ and $\hat{\gamma}_p = \inf \|a\| \|T\|$, where the infimum is taken over all factorization above.

Theorem 3.2. *[3] For $1 \leq p \leq \infty$,$[\mathcal{L}_{p-fat}(X_1, ..., X_m; Y), \hat{\gamma}_p]$ is a Banach ideal.*

Proposition 3.3. *Let $1 \leq p \leq \infty$ and $T \in \mathcal{L}(X_1, ..., X_m; Y)$. The following statements are equivalent.*

(i) $T \in \mathcal{L}_{p-fact}(X_1, ..., X_m; Y)$

(ii) $T^* \in \mathcal{L}_{p^*-fact}(Y^*; \mathcal{L}(X_1, ..., X_m))$

(iii) $T^{**} \in \mathcal{L}_{p-fact}(\mathcal{L}(X_1, ..., X_m)^*, Y^{**})$

(iv) $K_Y T \in \mathcal{L}_{p-fact}(X_1, ..., X_m; Y^{**})$*.in this case*

$$\gamma_{p^*}(T^*) = \hat{\gamma}(T) = \gamma_p(T^{**}) = \hat{\gamma}_p(K_Y T)$$

proof. $(i) \Rightarrow (ii)$ Let $T \in \mathcal{L}_{p-fat}(X_1, ..., X_m; Y)$ so existe the space of measure (Ω, Σ, μ) operators $a \in \mathcal{L}(L_p(\mu); Y^{**})$ and $T \in \mathcal{L}(X_1, ..., X_n; L_p(\mu))$ such that $K_Y \circ T = a \circ T$ if:

$$T^* \circ a^* = T^* \circ K_Y^*$$

we have

$$K_Y^* \circ K_{Y^*} = id_{Y^*}$$

this tow last relations we have :

$$T^* = T^* \circ a^* \circ K_{Y^*}$$

Like this

$$K_{\mathcal{L}(X_1,...,X_m;\Bbbk)} \circ T^* = K_{\mathcal{L}(X_1,...,X_m;\Bbbk)} \circ T^* \circ a^* \circ K_{Y^*}$$

taken $b = a^* \circ K_{Y^*} \in \mathcal{L}(Y^*; L_{p^*(\mu)})$ and $c = K_{\mathcal{L}(X_1,...,X_m;\Bbbk)} \circ T^* \in \mathcal{L}(L_p(\mu), \mathcal{L}(X_1, ..., X_m; \Bbbk)^{**})$,we have to $T^* \in \mathcal{L}_{p^*-fat}(Y^*, \mathcal{L}(X_1, ..., X_m; \Bbbk))$ is other that:

$$\begin{aligned} \gamma_{p^*}(T^*) &\leq \|a^* \circ K_{Y^*}\| \|K_{(X_1,...,X_m;\Bbbk)} \circ T^*\| \\ &\leq \|a\| \|T^*\| \\ &= \|a\| \|T\| \end{aligned}$$

like this has if:

$$\gamma_{p^*}(T^*) \leq \hat{\gamma}_p(T) \tag{3.1}$$

$(ii) \Rightarrow (iii)$ let $T^* \in \mathcal{L}_{p^*-fat}(Y^*, \mathcal{L}(X_1, ..., X_m; \Bbbk))$ so existe a space of measure (Ω, Σ, μ), operators $a \in \mathcal{L}(Y^*, L_{p^*(\mu)})$ and $b \in \mathcal{L}(L_{p^*}(\mu), \mathcal{L}(X_1, ..., X_m; \Bbbk)^{**})$ such that

$$K_{\mathcal{L}(X_1,...,X_m;\Bbbk)} \circ T^* = b \circ a \tag{3.2}$$

this relation like :

$$T^{**} \circ K^*_{\mathcal{L}(X_1,...,X_m;\Bbbk)} = a^* \circ b^*$$

Doing $\circ$ Even in $(i) \Rightarrow (ii)$,Like this :$T^{**} \in \mathcal{L}_{p-fat}(\mathcal{L}(X_1, ..., X_m; \Bbbk)^*, Y^{**})$, and other that :

$$\gamma_p(T^{**}) \leq \gamma_{p^*}(T^*) \tag{3.3}$$

$(iii) \Rightarrow (iv)$: Let $T^{**} \in \mathcal{L}_{p-fat}(\mathcal{L}(X_1, ..., X_m; \Bbbk)^*, Y^{**})$ so existe a space of measure (Ω, Σ, μ) and operators $b \in (\mathcal{L}(X_1, ..., X_m; \Bbbk)^*, L_p(\mu)), a \in \mathcal{L}(L_p(\mu); Y^{****})$ such that :

$$a \circ b = K_{Y^{**}} \circ T^{**} \tag{3.4}$$

Let

$$K : X_1 \times ... \times X_n \longrightarrow \mathcal{L}(X_1, ..., X_n; \Bbbk)^*,$$

$$(x_1, ..., x_m) \longmapsto K_{(x_1,...,x_m)}$$

with $K_{x_1,...,x_n}$ given away for :

$$K_{(x_1,...,x_n)} : (X_1 \times ... \times X_n; \Bbbk) \longrightarrow \Bbbk$$

$$T \longmapsto T(x_1, ..., x_m)$$

claramente K ,$\|K\| = 1$ .to we have

$$\begin{aligned}(K_Y \circ T)(x_1, ..., x_n)(y^*) &= y^*(T(x_1, ..., x_m)) \\ &= (y^* \circ T)(x_1, ..., x_m) \\ &= T^*(y^*)(x_1, ..., x_m).\end{aligned} \tag{3.5}$$

Besides that;

$$\begin{aligned}(T^{**} \circ K)(x_1, ..., x_m)(y^*) &= (K_{(x_1,...,x_m)} \circ T^*)(y^*) \\ &= (K_{(x_1,...,x_m)}(T^*(y)) \\ &= T(y^*)(x_1, ..., x_m)\end{aligned} \tag{3.6}$$

of (3.5) and (3.6) Has if

$$K_Y \circ T = T^{**} \circ K \tag{3.7}$$

like this, of (3.6) and (3.7) we have to :

$$K_{Y^{**}} \circ K_Y \circ T = K_{Y^{**}} \circ T^{**} \circ K = a \circ b \circ K$$

this last reluation is clear that $K_Y \circ T \in \mathcal{L}_{p-fat}(X_1, ..., X_m; Y^{**})$ form complete proof let us also see that $\|K\| = 1$.

$$\begin{aligned}\|K\| &= \sup_{\|x_i\|=1; i=1,...,n} \|K(x_1, ..., x_m)\| \\ &= \sup_{\|x_i\|=1; i=1,...,m} \|K_{(x_1,...,x_n)}\| \\ &= \sup_{\|x_i\|=1; \|T\|=1; i=1,...,m} \{ \sup_{\|T\|=1} | T(x_1, ..., x_m) | \}.\end{aligned}$$

such that $\|K\| \leq 1$.on the other hand

$$\begin{aligned}\|K\| &\geq \|K(x_1, ..., x_m)\| \\ &= \|K_{(x_1,...,x_m)}\| \\ &\geq | T(x_1, ..., x_m) |,\end{aligned}$$

for all $x_i \in X_i$ with $\|x_i\| = 1, i = 1, ..., m$ and $\|T\| = 1$.

$T \in \mathcal{L}(X_1, ..., X_m; \Bbbk)$ with T is overjector we have to :$\|K\| \geq 1$. Lake :

$$\hat{\gamma}(K_Y T) \leq \|a\|\|b\|\|K\|$$

Dai segue-if that :

$$\hat{\gamma}(K_Y T) \leq \gamma_p(T^{**}) \tag{3.8}$$

$(iv) \Rightarrow (i)$: Let $K_Y \circ T \in \mathcal{L}_{p-fat}(X_1, ..., X_n; Y^{**})$, so existe in space of measure (Ω, Σ, μ) and operators $a \in \mathcal{L}(L_p(\mu), Y^{****}), T \in \mathcal{L}(X_1, ..., X_m; L_p(\mu))$ such that :

$$a \circ T = K_{Y^{**}} \circ K_Y \circ T \tag{3.9}$$

with the :

$$K^*_{Y^*} \circ K_{Y^{**}} = id_{Y^{**}} \tag{3.10}$$

das relation (3.9) and (3.10) we have

$$K^*_{Y^*} \circ a \circ T = K_Y \circ T$$

like this we have $\circ$ seguinte:

$$\hat{\gamma}_p(T) \leq \|K^*_{Y^*} a\|\|T\| \leq \|a\|\|T\|$$

Lake,in relation $(3.3), (3.5), (3.8)$ and we have :

$$\hat{\gamma}_p(T) \leq \hat{\gamma}_p(K_Y \circ T) \leq \gamma_p(T^{**}) \leq \gamma_{p^*}(T^*) \leq \hat{\gamma}_p(T)$$

if $1 < p < \infty$.

Form $p = 1$, ou $p = \infty$ is usar $\circ$ fact that $L_1^{**}(\mu)$ is ismorphisme isometricamente a $L_1(\nu)$ form some measure apropriate ,form iste vide . □

3.2 Characterization of 2-factorable multilinear operators

Proposition 3.4. *A mulitilinear operator $T \in \mathcal{L}(X_1, ..., X_m; Y)$ belongs to $\mathcal{L}_{2-fact}(X_1, ..., X_m; Y)$ if and only if*

$$X_1 \times ... \times X_m \xrightarrow{T} H \xrightarrow{b} Y$$

where H is a Hilbert space in this case :

$$\hat{\gamma}_2(T) = \inf \|T\| \|b\|$$

Proposition 3.5. *Let $E_i, X_i, Y, Y_0, i = 1, ..., n$ are Banach spaces and $q = (q_1, ..., q_m) \in \mathcal{L}(E_1 \times ... \times E_m, X_1 \times ... \times X_m)$ where $q_i, i = 1, ..., m$ are application quotientes with $q_i \in \mathcal{L}(E_i, X_m)$ and $j \in \mathcal{L}(Y, Y_0)$ an isometric. An operators $T \in \mathcal{L}(X_1, ..., X_m; Y)$ is 2-factorable if and only if $j \circ T \circ q : E_1 \times ... \times E_m \longrightarrow Y_0$ is,in this case $\hat{\gamma}_2(T) = \hat{\gamma}_2(j \circ T \circ q)$.*

Theorem 3.6. *Let $m \in \mathbb{N}$. The following are equivalent for $T \in \mathcal{L}(X_1, ..., X_m; Y)$.*

(i) $T \in \mathcal{L}_{2-fact}(X_1, ..., X_m; Y)$.

(ii) *Existe an constant $C \geq 0$ such that*

$$\left| \sum_{i,j=1}^{n} a_{ij} < T_i^*, y_i^* \circ T > \right| \leq C \sup\{ \left| \sum_{i,j=1}^{n} a_{ij} s_i t_j \right| : (s_i), (t_j \in \mathbf{B}_{\ell_\infty^n})\}$$

regardless of the choice of $n \in \mathbb{N}$, the $n \times n$ scalar matrix (a_{ij}), and the vectors $T_1^*, ..., T_n^* \in \mathbf{B}_{\mathcal{L}(X_1,...,X_m;\mathbb{K})^*}$ and $y_1^*, ..., y_n^* \in B_{Y^*}$. In case (i) and (ii) hold, we may take $C = \kappa_G \hat{\gamma}_2(T)$ where κ_G is Grothendieck's constant.

Proof. $T \in \mathcal{L}_{2-fat}(X_1, ..., X_m; Y)$ if and only if $T^* \in \mathcal{L}_{2-fat}(Y^*, \mathcal{L}(X_1, ..., X_m; \mathbb{K}))$
$(i) \implies (ii)$ If $T \in \mathcal{L}_{2-fat}(X_1, ..., X_m; Y)$ we have to T^* is 2-factorable.There existe an constante $C = \kappa_G \gamma_2(T^*) \geq 0$ such that:

$$| \sum_{i,j=1}^{n} a_{ij} < T_i^*, y_i^* \circ T > | \leq \kappa_G \gamma_2(T^*) \sup\{ | \sum_{i,j=1}^{n} a_{ij} s_i t_j | : (s_i), (t_j \in \mathbf{B}_{\ell_\infty^n})\}$$

for any $y_1^*, ..., y_n^* \in B_{Y^*}$ and $T_1^*, ..., T_n^* \in B_{\mathcal{L}(X_1,...,X_m)^*}$ with the $T^*(y_i^*) = y_i^* \circ T$ and $\hat{\gamma}_2(T) = \gamma_2(T^*)$ we have
$| \sum_{i,j=1}^{n} a_{ij} < T_i^*, y_i^* \circ T > | \leq K_G \hat{\gamma}_2(T) \sup\{ | \sum_{i,j=1}^{n} a_{ij} s_i t_j | : (s_i), (t_j \in \mathbf{B}_{\ell_\infty^n})\}$
$(ii) \implies (i)$ Using a relation ship $T^*(y_i^*) = y_i^* \circ T$, we have that T^* is p-factorable , we have that $T \in \mathcal{L}_{2-fat}(X_1, ..., X_m; Y)$ □

Theorem 3.7. *Let $m \in \mathbb{N}$ and $X_1, ..., X_m; Y$ of Banach spaces, operators $T : X_1, ..., X_m \longrightarrow Y$ pertaence a $\mathcal{L}_{2-fact}(X_1, ..., X_m; Y)$ with $\hat{\gamma}_2(T) \leq c$,if and only if ,form any all n,and any matrix $a = (a_{ij})_{n \times n}$ and any vectores $T_1^*, ..., T_n^* \in \mathcal{L}(X_1, ..., X_m; \mathbb{K}), y_1^*, ..., y_n^* \in Y^*$ we have:*

$$\left| \sum_{i,j=1}^{n} a_{ij} < T_i^*, y_i^* \circ T > \right| \leq c \| \, |a| \, \| (\sum_{j=1}^{n} \|T_j^*\|^2)^{\frac{1}{2}} (\sum_{i=1}^{n} \|y_i^*\|^2)^{\frac{1}{2}}$$

$\| \, |a| \, \| = \sup | \sum_{i,j} a_{ij}(x_i \mid y_j) |$ *with suprem taken on all the of Hilbert spaces and* $\|x_i\|_H = \|y_i\|_H = 1, i = 1, ..., n$.

Corollary 3.8. *Let* $1 < p < \infty$ *and* $n \in \mathbb{N}$,*if* $X_1, ..., X_n; Y$ *are Banach spaces so* $\mathcal{L}_{2-fact}(X_1, ..., X_n; Y)$ *this count in* $\mathcal{L}_{p-fat}(X_1, ..., X_n; Y)$.

Proposition 3.9. *Let* $2 < p < \infty$ *and* $n \in \mathbb{N}$ *if* $X_1, ..., X_n; Y$ *are Banach spaces with* Y *Cotype 2, then*

$$\mathcal{L}_{p-fact}(X_1, ..., X_n; Y) = \mathcal{L}_{2-fact}(X_1, ..., X_n; Y)$$

Corollary 3.10. *Let* $1 < p < \infty$ *and* $n \in \mathbb{N}$ *if* $X_1, ..., X_n$ *are Banach spaces and* H *Hilbert spaces, then*

$$\mathcal{L}_{p-fact}(X_1, ..., X_n; H) = \mathcal{L}_{2-fact}(X_1, ..., X_n; H)$$

Moreover:

$$\hat{\gamma}_2(T) \leq \hat{\gamma}_p(T)$$

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