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Intitulé

Study of jets in electron-positron collisions

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General introduction

General introduction:

One of the primary objectives of particle physics is to understand the basic structure of matter and the interactions among its constituents. It is known that hadrons, such as protons, neutrons etc., account for almost all the mass of the observable luminous matter of our universe.

A scattering experiment in particle physics consists of directing a beam of particles towards a target. Such experiments yield insight into how the two particles interact and are used to learn about the structure of the particles involved. In 1907, Ernest Rutherford performed his famous gold foil. This was the first evidence that the atom must have a massive, positively charged nucleus. In similar experiments at the Stanford Linear Accelerator Center (SLAC) in 1968, electrons were scattered off by proton targets and the results of this experiment yielded evidence for the existence of quarks. The experiment performed at SLAC was an example of a particular type of scattering experiment: deep inelastic scattering.

Deep inelastic scattering was the first experiment that illustrated our need to a new theory to describe the strong interactions, the Quantum Chromodynamics or shortly QCD.

In theoretical physics, quantum chromodynamics (QCD) is the theory of strong interactions, a fundamental force describing the interactions between quarks and gluons, which make up hadrons. QCD is a type of quantum field theory called a non-abelian gauge theory with symmetry group $SU(3)$. The QCD analog of electric charge is a property called color. Gluons are the force carriers of the theory as photons are for the electromagnetic force in quantum electrodynamics. A large body of experimental evidence for QCD has been gathered over the years.

QCD was built on two peculiar properties:

- Confinement, which means that the force between quarks does not diminish as they are separated. Because of this, when we separate a quark from other quarks, the energy in the gluon field is enough to create another quark pair; they are thus forever bound into hadrons such as the proton and the neutron or the pion .
- Asymptotic freedom, which means that in very high-energy reactions, quarks and gluons interact very weakly creating a quark–gluon plasma. This prediction of QCD was first discovered in the early 1970.

In this work we are aiming to investigate in detail the proton structure and calculate the cross section of 2-jet and 3-jet events also understand the hadronization models, jets and the fragmentation functions.

This work is organized as follows

- *The first chapter:* The Phenomenological study: the theoretical background to understand deep inelastic scattering and its kinematics. It is followed by an introduction to the Bjorken scaling, structure functions ...

Also the theoretical QCD and its properties such as Lagrangian QCD, renormalization ...

- *The second chapter:* We start by an overview on linear accelerator, and the main work on hard process cross section calculation and using the factorization theorem and finally we study hadronization and its models.

- *The last chapter:* we calculate a 3-jet event cross-section. We introduce the jets concept and their shapes, finally a mathematical review on fragmentation functions.

Chapter 1: Introduction to Parton Model

“If, in some cataclysm, all of scientific knowledge were to be destroyed, and only one sentence passed on to the next generation of creatures, what statement would contain the most information in the fewest words? I believe it is the atomic hypothesis (or the atomic fact, or whatever you wish to call it) that all things are made of atoms — little particles that move around in perpetual motion, attracting each other when they are a little distance apart, but repelling upon being squeezed into one another. In that one sentence, you will see, there is an enormous amount of information about the world, if just a little imagination and thinking are applied.”

Richard Feynman, “The Feynman Lectures on Physics” (1964)

1.1-Overview on Deep Inelastic Scattering:

In late 1967, the first of a long series of experiments on highly inelastic electron scattering was started at the two-mile accelerator at the Stanford Linear Accelerator Center (SLAC) using liquid hydrogen and, later, liquid deuterium. Carried out by a collaboration from the Massachusetts Institute of Technology (MIT) and SLAC, the object was to look at large energy loss scattering of electrons from the nucleon a process soon to be called deep inelastic scattering.

Energy beams was up to 21 GeV, the highest electron energies then available, and large electron fluxes, made it possible to study the nucleon to very much smaller distances than had previously been possible. Because quantum electrodynamics provides an explicit and well-understood description of the interaction of electrons with charges and magnetic moments, electron scattering had by 1968, already been shown to be a very powerful probe of the structures of complex nuclei and individual nucleons.

Hofstadter and his collaborators had discovered, by the mid-1960s that as the momentum transfer in the scattering increased, the scattering cross section dropped sharply relative to that from a point charge. The results showed that nucleons were roughly 10^{-23} cm in size, implying a distributed structure. The earliest MIT-SLAC studies, in which California Institute of Technology physicists also collaborated, looked at elastic electron-proton scattering, later ones at electro-production of nucleon resonances with excitation energies up to less than 2 GeV. Starting in 1967, the MIT-SLAC collaboration employed the higher electron energies made available by the newly completed SLAC accelerator to continue such measurements, before beginning the deep inelastic program. Results from the inelastic studies arrived swiftly: the momentum transfer dependence of the deep inelastic cross sections was found to be weak, and the deep inelastic form factors - which embodied the information about the proton structure. These results were inconsistent with the current expectations of most physicists at the time. The belief had been that the nucleon was the extended object found in elastic electron scattering but with the diffuse internal structure seen in pion and proton scattering. The new experimental results suggested point-like constituents but were puzzling because such constituents seemed to contradict well-established beliefs.

Intense interest in these results developed in the theoretical community and, in a program of linked experimental and theoretical advances extending over a number of years, the internal constituents

were ultimately identified as **quarks**, which had previously been devised in 1964 as an underlying, quasi-abstract scheme to justify a highly successful classification of the then-known hadrons. This identification opened the door to development of a comprehensive field theory of hadrons (the strongly interacting particles), called Quantum Chromodynamics (QCD) that replaced entirely the earlier picture of the nucleons and mesons. QCD in conjunction with electroweak theory, which describes the interactions of leptons and quarks under the influence of the combined weak and electromagnetic fields, constitutes the Standard Model, all of whose predictions are in satisfactory agreement with experiment. The contributions of the MIT-SLAC inelastic scattering program were recognized by the award of the 1990 Nobel Prize in Physics. [2] [3]

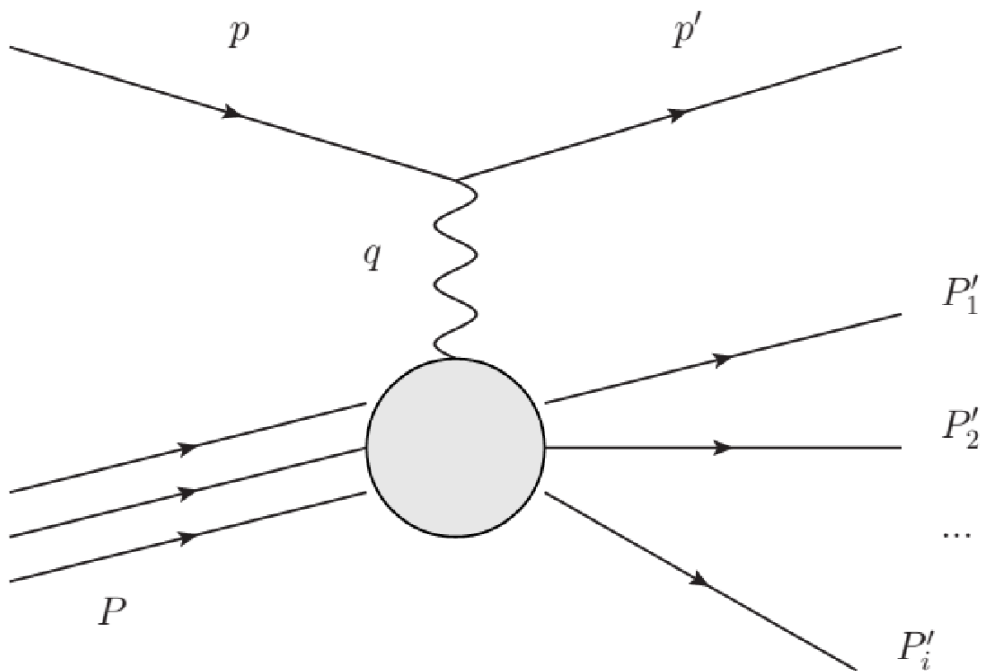


Figure (1.1) DIS inelastic proton-electron scattering

1.2-Phenomenological study

1.2.1-Kinematics

The kinematics of the process is described by two independent Lorentz invariant quantities. The four-momentum transfer squared Q^2 is given by:

$$Q^2 = -q^2 - (k - k')^2 \quad (1.1)$$

Where k (k') is the four-momentum of the incoming (outgoing) electron (see Figure 1.2). Another Lorentz invariant quantity is the inelasticity (y)

$$y = \frac{p(k - k')}{pk} \quad (1.2)$$

Where p is the momentum of incoming proton. The inelasticity y is dimensionless and corresponds to the fraction of momentum lost by the electron in the proton rest frame.

A further frequently used variable, called Bjorken x_B , is defined as the ratio of the four-momentum transfer squared and the energy transfer in the proton rest frame,

$$x_B = \frac{Q^2}{2p(k - k')} \quad (1.3)$$

As in the case of the inelasticity y , the Bjorken x_B variable is dimensionless and limited to the range $[0,1]$

Two of the three mentioned Lorentz invariant variables are independent since the following approximations relation holds

$$Q^2 = x_B y S \quad (1.4)$$

Where s is the square of the center of mass energy defined as: $s = (k + p)^2$. Neglecting the masses of the interacting particles the center of mass energy can be evaluated as $s = 4E_e E_p$

Where E_e and E_p are the energies of the electron and the proton beam respectively.

In addition, it is convenient to define the following Lorentz invariant variable

$$W^2 = (q + p)^2 \tag{1.5}$$

Which is the total mass squared of the hadronic final state (or invariant mass of the virtual boson-proton system) squared and the following relation holds

$$W^2 = \frac{1-x}{x} Q^2 + M_p^2 \tag{1.6}$$

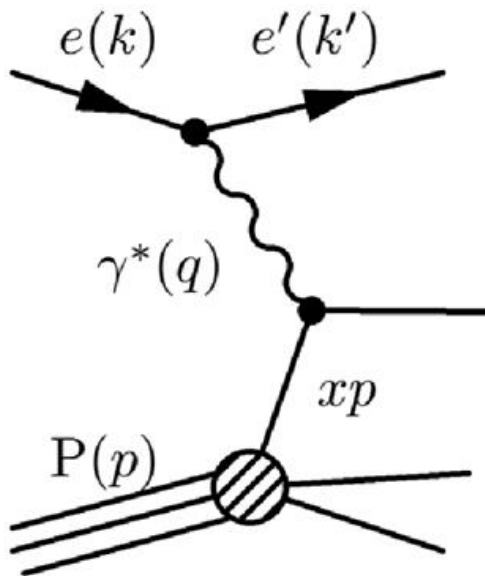


Figure (1.2): The deep inelastic scattering process

1.2.2-The Bjorken scaling

The early deep-inelastic scattering experiments performed at SLAC showed that the structure functions F_1 and F_2 are approximately Q^2 -independent in the large momentum transfer Region:

$$F_{1,2}(x, Q^2) \approx F_{1,2}(x)(Q^2 \gg M^2)$$

This phenomenon, predicted by the Quark Parton Model became known as Bjorken scaling or scale invariance, the Bjorken limit

$$\lim_{\text{Bj}} = \begin{cases} Q^2 \rightarrow \infty \\ \nu \rightarrow \infty \\ x \text{ fixed} \end{cases} \quad (1.7)$$

$$x_B = \frac{Q^2}{2m_p \nu} \quad (1.8)$$

where is

$$\nu = \frac{p(k - k')}{m_p} \quad (1.9)$$

This implies an inner structure of the proton. The observed scaling behavior could be successfully accounted by considering scattering from point-like constituents within the proton, rather than from the proton as a whole. [7]

1.2.3-Structure Functions

We modified a typical fermion current, $J^\mu \sim \bar{\psi}\gamma^\mu\psi$ to a hadron current, $j^\mu \sim \bar{\psi}\Gamma^\mu\psi$ by changing $\gamma^\mu \rightarrow \Gamma^\mu$ and constructing the most general Γ^μ with the form factors.

The expression for differential cross section

$$\frac{d\sigma}{d^3p'} = \frac{2\alpha_{em}^2}{sQ^4p'^0} L_{\mu\nu}^e W^{\mu\nu} \quad (1.10)$$

Here $L_{\mu\nu}^e$ is the normal Lepton tensor , and we have the Hadron tensor , $W^{\mu\nu}$

$$\begin{aligned}
 d\sigma &= \frac{1}{2s} \frac{d^3p'}{(2\pi)^3 2p'^0} \frac{e^4}{2Q^2} Tr \left[\not{k}' \gamma^\mu \not{k} \gamma^\nu \right] \left(\sum_X \int \langle P, s | J_\mu(0) | X \rangle \times \langle X | J_\nu(0) | P, s \rangle \right) \\
 &\quad \times (2\pi)^4 \delta^4(P + q - \sum_i P_i) \\
 &= \frac{d^3p'}{2s p'^0 Q^4} \left(\frac{e^4}{(8\pi)^2} Tr \left[\not{k}' \gamma^\mu \not{k} \gamma^\nu \right] \right) \left(\frac{1}{4\pi} \right) \left(\sum_X \int \langle P, s | J_\mu(0) | X \rangle \times \right. \\
 &\quad \left. \langle X | J_\nu(0) | P, s \rangle \right) (2\pi)^4 \delta^4(P + q - \sum_i P_i)
 \end{aligned} \tag{1.11}$$

Where we group the lepton and hadron tensors in the following way

$$W_{\mu\nu} = \left(\frac{1}{4\pi} \right) \left(\sum_X \int \langle P, s | J_\mu(0) | X \rangle \langle X | J_\nu(0) | P, s \rangle \right) (2\pi)^4 \delta^4(P + q - \sum_i P_i) \tag{1.12}$$

$$L^{\mu\nu} = \frac{1}{2} Tr \left[\not{k}' \gamma^\mu \not{k} \gamma^\nu \right] \tag{1.13}$$

We start by constructing the most general form of $W_{\mu\nu}$, similar to what we did to Γ^μ the most general form is constructed by the introduction of the metric $g^{\mu\nu}$ and the independence momenta p and q , [12]

We write

$$W_{\mu\nu} = -W_1 g^{\mu\nu} + \frac{W_2}{M^2} P^\mu P^\nu + \frac{W_3}{M^2} q^\mu q^\nu + \frac{W_4}{M^2} (P^\mu q^\nu + q^\mu P^\nu) \tag{1.14}$$

From current conservation, $\partial_\mu J^\mu = 0$, it can be shown that

$$q_\mu W^{\mu\nu} = q_\nu W^{\mu\nu} = 0 \quad (1.15)$$

By applying this condition to $W^{\mu\nu}$ we get:

$$q_\mu W^{\mu\nu} = -W_1 q^\nu + \frac{W_2}{M^2} (q \cdot P) P^\nu + \frac{W_3}{M^2} q^2 q^\nu + \frac{W_4}{M^2} ((P \cdot q) q^\nu + q^2 P^\nu) = 0 \quad (1.16)$$

There are only two independent structure functions here, let us look at terms individually of just P^ν and q^ν , First

$$\frac{W_2}{M^2} (q \cdot P) P^\nu + \frac{W_4}{M^2} (q^2 P^\nu) = 0 \quad (1.17)$$

This imply

$$W_4 = -\frac{(q \cdot P)}{q^2} W_2 \quad (1.18)$$

And second ,

$$-W_1 q^\nu + \frac{W_3}{M^2} q^2 q^\nu + \frac{W_4}{M^2} (P \cdot q) q^\nu = 0 \quad (1.19)$$

Then

$$W_3 = W_1 \frac{M^2}{q^2} - W_4 \frac{(P \cdot q)}{q^2} \quad (1.20)$$

W_3 Simplifies to

$$W_3 = \left(\frac{P \cdot q}{q^2}\right)^2 W_2 + \frac{M^2}{q^2} W_1 \quad (1.21)$$

Now we see that $W^{\mu\nu}$ can be expressed in terms of just two structure functions and combination of momenta, $W^{\mu\nu}$ reduce to

$$W^{\mu\nu} = W_1 \left(-g^{\mu\nu} + \frac{q^\mu q^\nu}{q^2} \right) + W_2 \frac{1}{M^2} \left(P^\mu - \frac{P \cdot q}{q^2} q^\mu \right) \left(P^\nu - \frac{P \cdot q}{q^2} q^\nu \right) \quad (1.22)$$

This is the expression of the hadronic tensor in which we express the structure functions W_1 and W_2

The x_B is called Bjorken x_B . It is also standard to redefine the two remaining structure functions in the following way:

$$F_1 = W_1 \quad (1.23)$$

$$F_2 = \frac{P \cdot q}{M^2} W_2 \quad (1.24)$$

First, we will contract $-g_{\mu\nu}$ with $W^{\mu\nu}$ from equation (1.22), thus we get

$$\begin{aligned} -g_{\mu\nu} W^{\mu\nu} &= W_1 \left(4 - \frac{q^2}{q^2} \right) W_2 \left[p^2 - \frac{2(P \cdot q)^2}{q^2} + \frac{(P \cdot q)^2}{q^2} \right] \\ &= 3W_1 - W_2 \left[P^2 - \frac{(P \cdot q)}{x_B} \right] \end{aligned} \quad (1.25)$$

Working in the massless regime and converting to the F form factors, we see that

$$-g_{\mu\nu} W^{\mu\nu} = 3 \left[F_1 - \frac{F_2}{2x_B} \right] + \frac{F_2}{x_B} \quad (1.26)$$

The contraction of (1.22) with $P_\mu P_\nu$ with $W^{\mu\nu}$ is

$$\begin{aligned} P_\mu P_\nu W^{\mu\nu} &= W_1 P_\mu P_\nu \left(\frac{q^\mu q^\nu}{q^2} - g^{\mu\nu} \right) + W_2 P_\mu P_\nu \left(P^\mu - \frac{P \cdot q}{q^2} q^\mu \right) \left(P^\nu - \frac{P \cdot q}{q^2} q^\nu \right) \\ &= \frac{Q^2}{4x_B^2} \left(\frac{F_2}{2x_B} - F_1 \right) \end{aligned} \quad (1.27)$$

We can rearrange these two equations in the following convenient form [12]

$$F_1 = \left(2 \frac{x_B^2}{Q^2} P_\mu P_\nu - \frac{g_{\mu\nu}}{2} \right) W^{\mu\nu} \quad (1.28)$$

$$F_2 = x_B \left(\frac{12x_B^2}{Q^2} P_\mu P_\nu - g_{\mu\nu} \right) W^{\mu\nu} \quad (1.29)$$

1.2.4-Parton Distribution Functions (PDF)

The parton model was proposed by Richard Feynman in 1969 as a way to analyze high-energy hadron collisions. Any hadron (for example, a proton), can be considered a composition of a number of point-like constituents, named "partons". [9]

We define the structure functions relative to the parton model in a way that accomplished this :

$$F_1 = \sum_i \int_{x_B}^1 \frac{dx}{x} \hat{F}_1(\hat{x}_B) f_i(x) \quad (1.30)$$

$$F_2 = \sum_i \int_{x_B}^1 dx f_i(x) \hat{F}_2(\hat{x}_b) \quad (1.31)$$

The index i runs over all types of partons (Quarks of different flavor as well as gluons). This function represents the probability of finding a parton of momentum fraction x in the hadron. This statement is saying that the sub-process multiplied by the probability of finding a certain Quark, with particular momentum fraction x is the structure function of the parent process.

All the momentum fractions have to add up to 1 which constrains $f_i(x)$

$$\sum_i \int dx. x. f_i(x) = 1 \quad (1.32)$$

This is another way to say that a parton with a momentum fraction exist

The bound of the integrand is the allowed region of x , x is bound from $x_B < x < 1$

This function $f(x)$ is what used to contain all the information of the structure of the proton , in the parton model we defined $W^{\mu\nu}$ as :

$$W^{\mu\nu} = \sum_i \int_{x_B}^1 \frac{dx}{x} f_i(x) \hat{W}^{\mu\nu} \tag{1.33}$$

If we rewrite this expression $\hat{W}^{\mu\nu}$ explicitly , after rearranging the delta function we have:

$$W^{\mu\nu} = \sum_i \frac{e_i^2}{4Q^2} \int_{x_B}^1 \frac{dx}{x} \text{tr}[\gamma^\mu \not{k} \gamma^\nu \not{k}] \delta(1 - \hat{x}_B) f_i(x) \tag{1.34}$$

Applying the F_2 projection

$$\begin{aligned} F_2 &= -x_B g_{\mu\nu} W^{\mu\nu} \\ &= -x_B g_{\mu\nu} \sum_i \int_{x_B}^1 \frac{dx}{x} f_i(x) \hat{W}^{\mu\nu} \end{aligned} \tag{1.35}$$

By using $-g_{\mu\nu} \hat{W}^{\mu\nu} = \sum_i e_i^2 \delta(1 - \hat{x}_B)$ so the expression reduces to

$$F_2 = \sum_i \int \frac{dx}{x} f_i(x) e_i^2 \delta(1 - \hat{x}_B) x_B \tag{1.36}$$

$$F_2 = \sum_i e_i^2 x_B f_i(x_B) \tag{1.37}$$

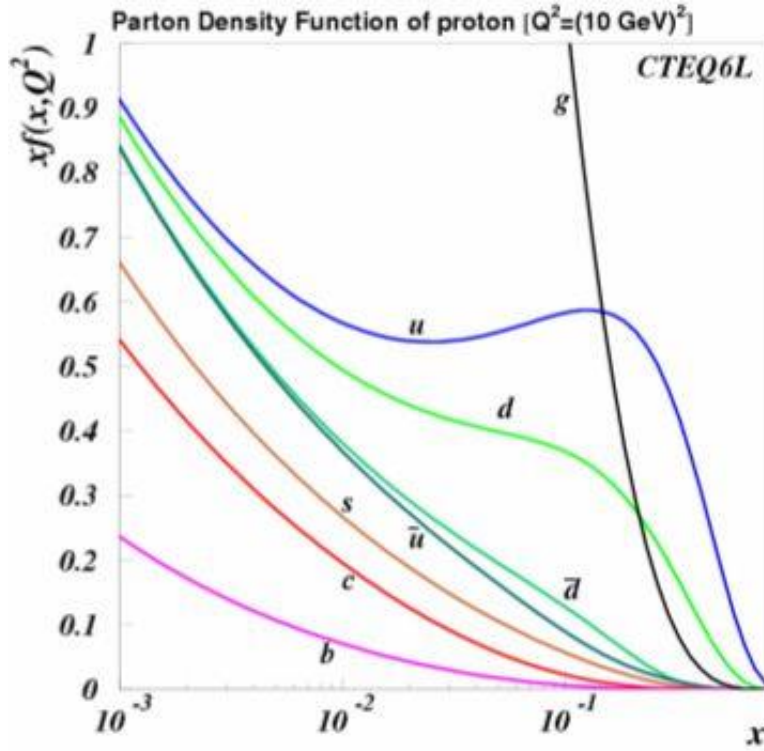


Figure (1.3) :(PDF) Parton distribution function of a Proton

1.3-Theoretical QCD

1.3.1-The QCD Lagrangian

In order to obtain the lagrangian of QCD we start with the dirac lagrangian for N_f Quarks [4]

$$\mathcal{L}_{Dirac} = \sum_f^{u,d,s,c,b,t} \bar{\psi}_f (i\gamma^\mu \partial_\mu - m) \psi_f \quad (1.38)$$

Where the six Quarks ψ_f are triplets in $SU(3)$ color space

$$\psi_f = \begin{pmatrix} \psi_{f,r} \\ \psi_{f,g} \\ \psi_{f,b} \end{pmatrix}, \quad f = u, d, s, c, b, t \quad (1.39)$$

And behave under local $SU(3)_c$ transformation as

$$\psi_f \rightarrow \exp \left\{ -i \sum_{a=1}^{N_c^2-1} \theta_a(x) T^a \right\} \psi_f \equiv U_c(x) \psi_f \quad (1.40)$$

With T^a being the eight generators of $SU(3)$, which are equal to half the Gell-Mann matrices $T^a = \lambda^a/2$.

The next step is to claim a local $SU(3)$ symmetry. As a consequence we have to replace the partial derivative in \mathcal{L}_{Dirac} by the covariant derivative

$$D_\mu = \partial_\mu + igA_\mu \quad (1.41)$$

Containing the coupling constant g of the strong interaction and the gauge field

$$A_\mu = \sum_{a=1}^{N_c^2-1} A_\mu^a T_a \quad (1.42)$$

Which transforms under local $SU(3)$ transformation as :

$$A_\mu \rightarrow A'_\mu = U_c(x) \left(A_\mu - \frac{i}{g} \partial_\mu \right) U_c^\dagger(x) \quad (1.43)$$

These eight fields describe the exchange bosons, named gluons, of the strong interaction, Hence they need a kinematic term, respectively a self-interaction term, in the lagrangian :

$$\mathcal{L}_{Self-interaction} = -\frac{1}{4} G_{\mu\nu}^a G_a^{\mu\nu} \quad (1.44)$$

With the field strength tensor

$$G_{\mu\nu}^a = \partial_\mu A_\nu^a - \partial_\nu A_\mu^a + gf^{abc} A_{b,\mu} A_{c,\nu} \quad (1.45)$$

where f^{abc} are the structure constants of $SU(3)$. The field strength tensor transforms in the following way under local $SU(3)$ transformations :

$$(G_{\mu\nu}^a T_a) \rightarrow (G_{\mu\nu}^a T_a)' = U_c(x)(G_{\mu\nu}^a T_a)U_c^\dagger(x) \tag{1.46}$$

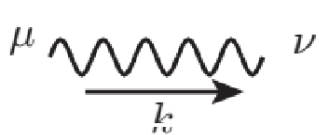
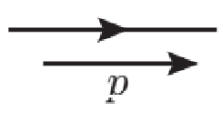
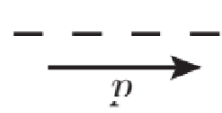
Finally , the gauge invariant QCD Lagrangian is given by :

$$\mathcal{L}_{QCD} = \sum_{f=1}^{N_f} \bar{\psi}_f (i\gamma^\mu D_\mu - m)\psi_f - \frac{1}{4} G_{\mu\nu}^a G_a^{\mu\nu} \tag{1.47}$$

Where the first term describes the coupling of gluon with quarks and the last term contains the kinetic term and self-interaction of gluons

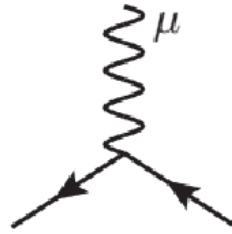
1.3.2-Feynman Rules

Propagators :

gauge boson		$\frac{-i}{k^2 + i\epsilon} g^{\mu\nu}$
fermion		$\frac{i(\not{p} + m)}{p^2 - m^2 + i\epsilon}$
Higgs (or other scalar particle)		$\frac{i}{p^2 - M_H^2 + i\epsilon}$

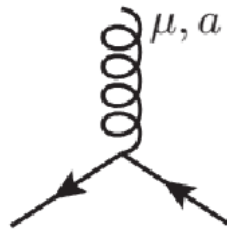
Vertices :

QED



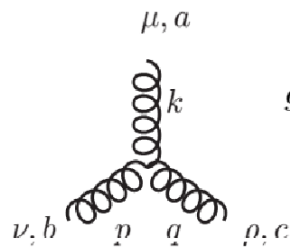
$$iQe\gamma^\mu$$

QCD



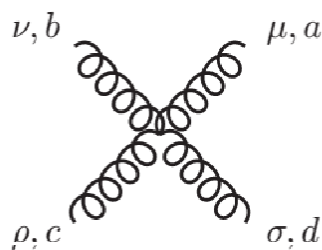
$$ig\gamma^\mu T^a$$

3 gluon



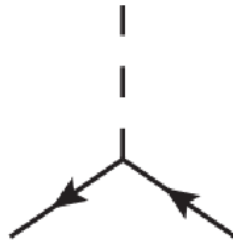
$$gf^{abc}[g^{\mu\nu}(k-p)^\rho + g^{\nu\rho}(p-q)^\mu + g^{\rho\mu}(q-k)^\nu]$$

4 gluon



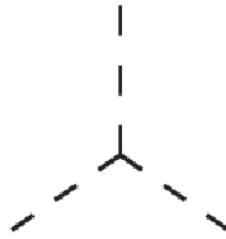
$$-ig^2[f^{abe} f^{cde}(g^{\mu\rho}g^{\nu\sigma} - g^{\mu\sigma}g^{\nu\rho}) + f^{ace} f^{bde}(g^{\mu\nu}g^{\rho\sigma} - g^{\mu\sigma}g^{\nu\rho}) + f^{ade} f^{bce}(g^{\mu\nu}g^{\rho\sigma} - g^{\mu\rho}g^{\nu\sigma})]$$

Yukawa coupling



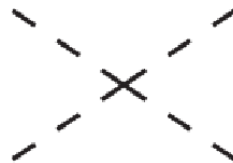
$$-i \frac{m_i}{v}$$

3 Higgs coupling



$$-i \frac{3M_H^2}{v}$$

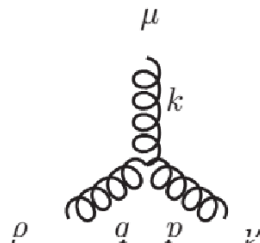
4 Higgs coupling



$$-i \frac{3M_H^2}{v^2}$$

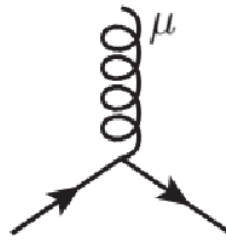
Color Ordered feynman Rules :

3 gluon coupling



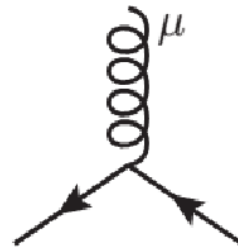
$$\frac{i}{\sqrt{2}} (g^{\mu\nu}(k-p)^\rho + g^{\rho\mu}(q-k)^\nu + g^{\nu\rho}(p-q)^\mu)$$

QCD



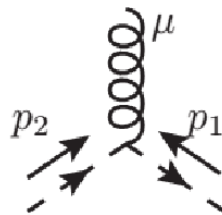
$$-\frac{i}{\sqrt{2}}\gamma_{\mu}$$

QCD



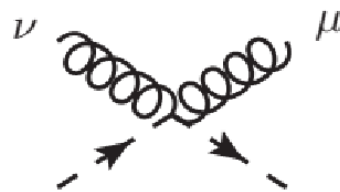
$$\frac{i}{\sqrt{2}}\gamma_{\mu}$$

scalar theory



$$\frac{i}{\sqrt{2}}(p_1 - p_2)^{\mu}$$

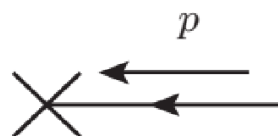
scalar theory



$$\frac{i}{2}g^{\mu\nu}$$

External Fermions :

incoming fermion



$$u^s(p)$$

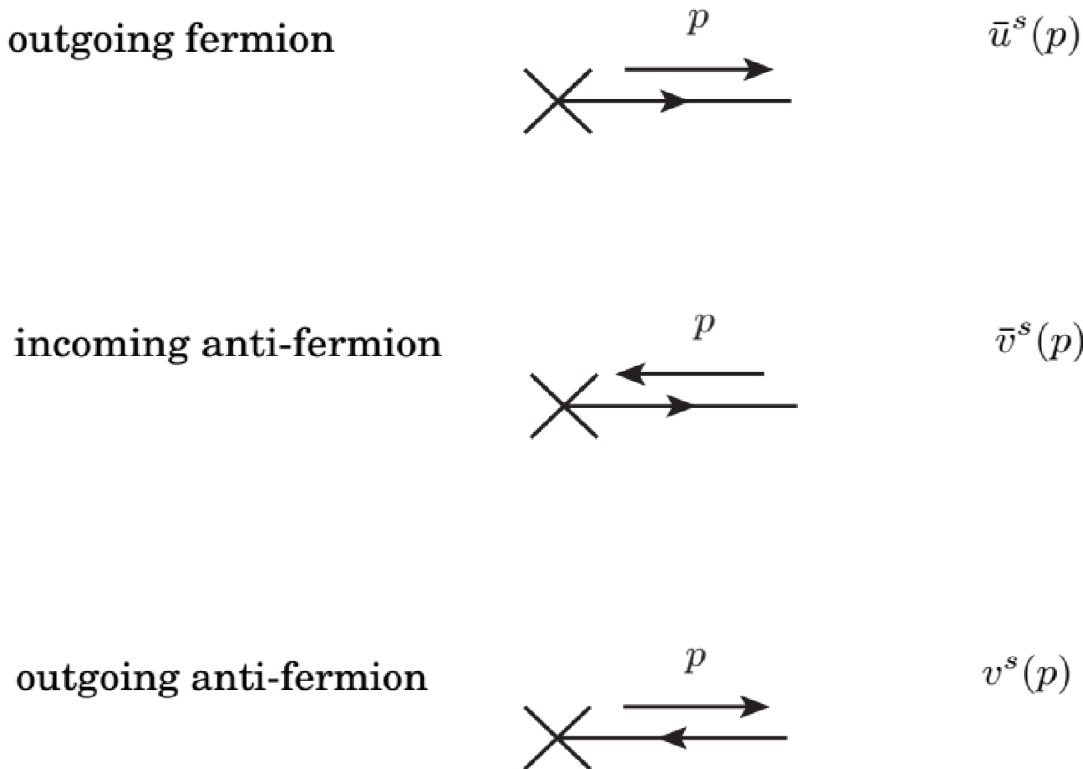


Figure (1.4): feynman rules for QCD[1] [2] [7]

$u^s(p)$ and $v^s(p)$ are spinors satisfying the Dirac equation.

1.3.3-Renormalization

Any process can be calculated perturbatively using the rules of Feynman derivatives which are obtained from the replacement of the covariant derivatives by momentum calculation usually requires Four dimensions plus intermediate arising from the gluons Quantum fluctuations, which suffer from ultraviolet divergences.

The procedure for renormalization is necessary to eliminate these fluctuations, which essentially means that the Lagrangian is rewritten so that the extra masses and the forces are eliminated in favor of their physically measurable value, giving rise to a Renormalized Lagrangian. [1]

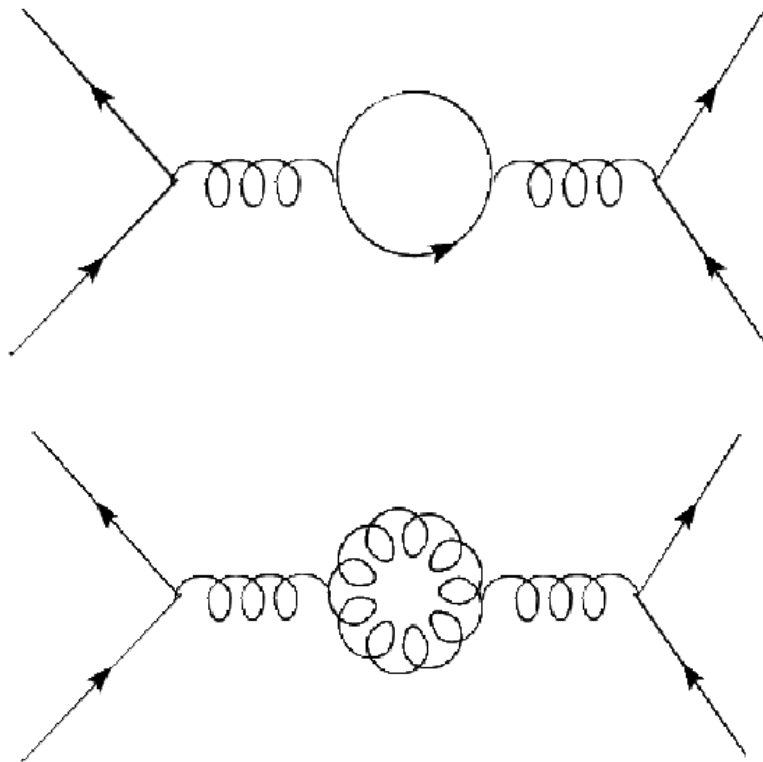


Figure (1.5): vacuum fluctuations

1.3.4-Asymptotic Freedom

One of the properties of QCD is the asymptotic freedom, which state that the interaction strength $\alpha_s = g^2/4\pi$ between quarks becomes smaller as the distance between them gets shorter.

To explain asymptotic freedom, let us first recall that in electrodynamics the force between two charges q_1 and q_2 in vacuum is describe by Coulomb's law.

$$F = \frac{1}{4\pi} \frac{q_1 q_2}{r^2} \tag{1.48}$$

In the other hand, the two charges are submerged in a medium with dielectric constante $\epsilon > 1$ (charge screening) the force becomes

$$F = \frac{1}{4\pi\epsilon} \frac{q_1 q_2}{r^2} \tag{1.49}$$

In the quantum field theory, the vacuum is not empty because it is the lowest energy state of a field system and is filled with electrons of negative energies from one point of view. When a photon passes through the vacuum, it can induce transition of an electron from negative to positive energy states, virtually creating a pair of electron and positron, known as vacuum fluctuation (see figure 1.5)

Because of this, the interaction between two electrons in the vacuum

$$F = \frac{e_{eff}^2}{4\pi r^2} = \frac{\alpha_{em}(r)}{r^2} \quad (1.50)$$

The dependence of the fine constant structure on the distance or momentum scale can be determined in a differential equation in QED

$$\mu \frac{d\alpha(\mu)}{d\mu} = \beta(\alpha(\mu)) \quad (1.51)$$

Where μ is a momentum scale roughly corresponding to $1/r$ the above is also an example of renormalization group equations the (beta) function may be calculated in perturbation theory because α_{em} is small and at one loop order $\beta = 2\alpha_{em}^2/3\pi > 0$ the solution is thus

$$\alpha_{em}(\mu) = \frac{\alpha_{em}(\mu_0)}{1 - \frac{\alpha_{em}(\mu_0)}{3\pi} \ln \frac{\mu^2}{\mu_0^2}} \quad (1.52)$$

In QCD, the same differential equation for the strong coupling constant holds however the β function now is different

$$\beta(\alpha) = -\frac{\beta_0}{2\pi} \alpha^2 + \dots \quad (1.53)$$

The coupling constant of QCD can be shown to have the following scale-dependence

$$\alpha_s(\mu) = \frac{2\pi}{\beta_0 \ln(\mu/\Lambda_{QCD})} \quad (1.54)$$

The running coupling introduces a dimensional parametre Λ_{QCD}

$$\Lambda_{QCD} \sim 250 \text{ MeV}$$

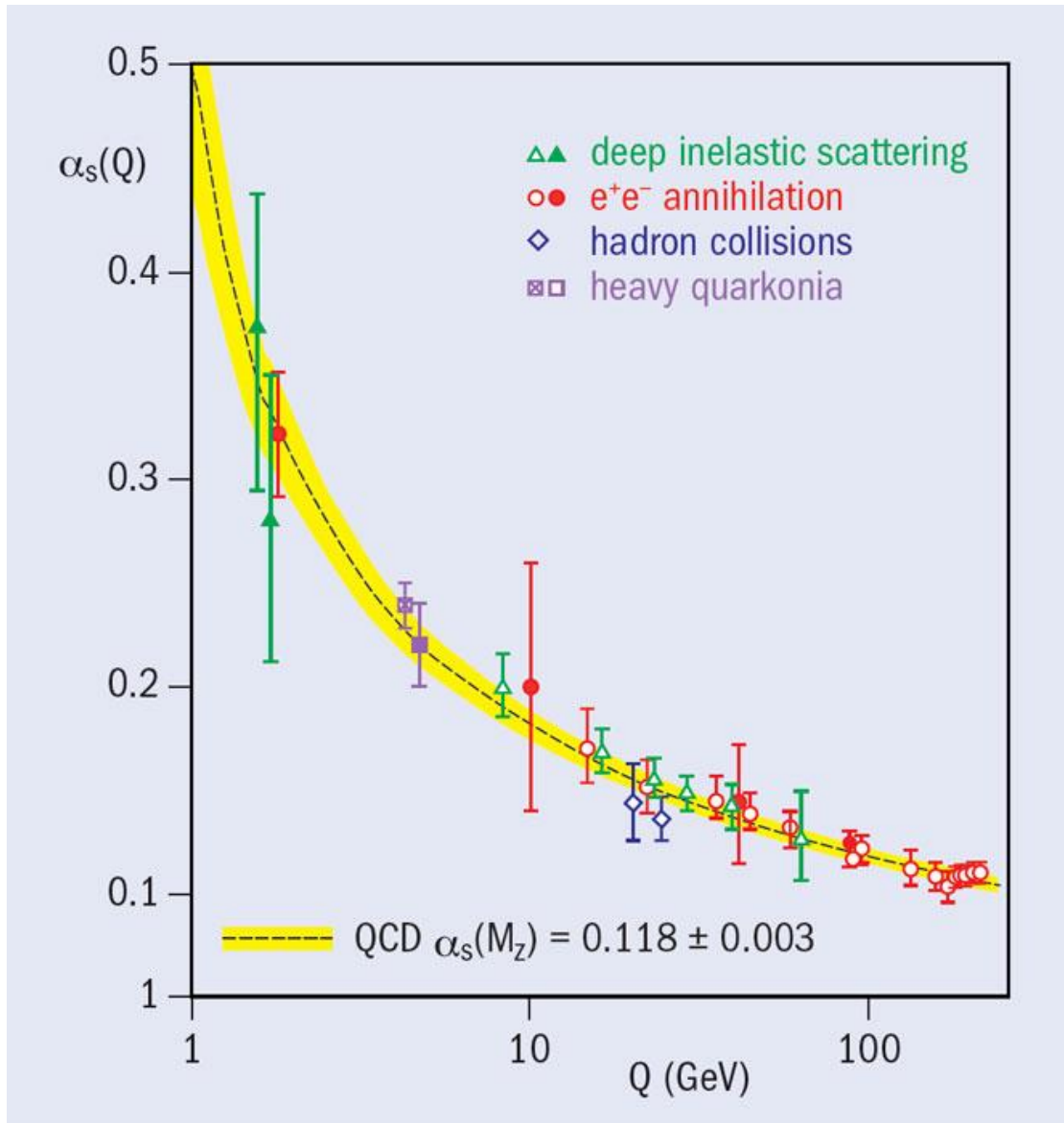


Figure (1.6): Running coupling constant diagram $\alpha_s(Q)$

After this general overview on parton model, let us have a further look on another important process in QCD e^-e^+ collision

Chapter 2: Study of e^-e^+ collision

*Der erste Schluck aus dem Glas Naturwissenschaften wird dich zu einem
Atheisten machen, aber am Grunde des Glases wartet Gott auf dich.*

- Werner Heisenberg

*“The first gulp from the glass of natural sciences will turn you into an atheist,
but at the bottom of the glass God is waiting for you.”*

- Werner Heisenberg

Introduction:

One of the important things in high energy physics is the e^-e^+ collision in linear accelerators this collision at high energy allows to understand the jet physics because the process is clean in the initial state we don't need parton distribution functions in the initial state which make it easy when we calculate the total cross section we do need only fragmentation functions in the final state those function organize and give us a good predictions about jets.

2.1- Overview on linear accelerators

The design of an accelerator depends on the type of particle that is being accelerated: electrons, protons or ions. A linear particle accelerator is a type of particle accelerator that greatly increases the kinetic energy of charged subatomic particles or ions by subjecting the charged particles to a series of oscillating electric potentials along a linear beamline. one of the most important linear accelerators is Linacs in CERN (often shortened to linac), Linacs has many applications: serves as particle injectors for higher-energy accelerators, and is used directly to achieve the highest kinetic energy for light particles (electrons and positrons) for particle physics.

Linear accelerators range in size from a cathode ray tube (which is a type of linac) to the 3.2-kilometer-long.

High power linear accelerators are also being developed for production of electrons at relativistic speeds, required since fast electrons traveling in an arc will lose energy through synchrotron radiation ,this limits the maximum power that can be imparted to electrons in a synchrotron of given size. Linacs are also capable of producing a nearly continuous stream of particles, whereas a synchrotron will only periodically raise the particles to sufficient energy. The device is also practical for the production of antimatter particles, which are generally difficult to obtain, being only a small fraction of a target's collision products. These may then be stored and further used to study matter-antimatter annihilation. [9] [10] [11]

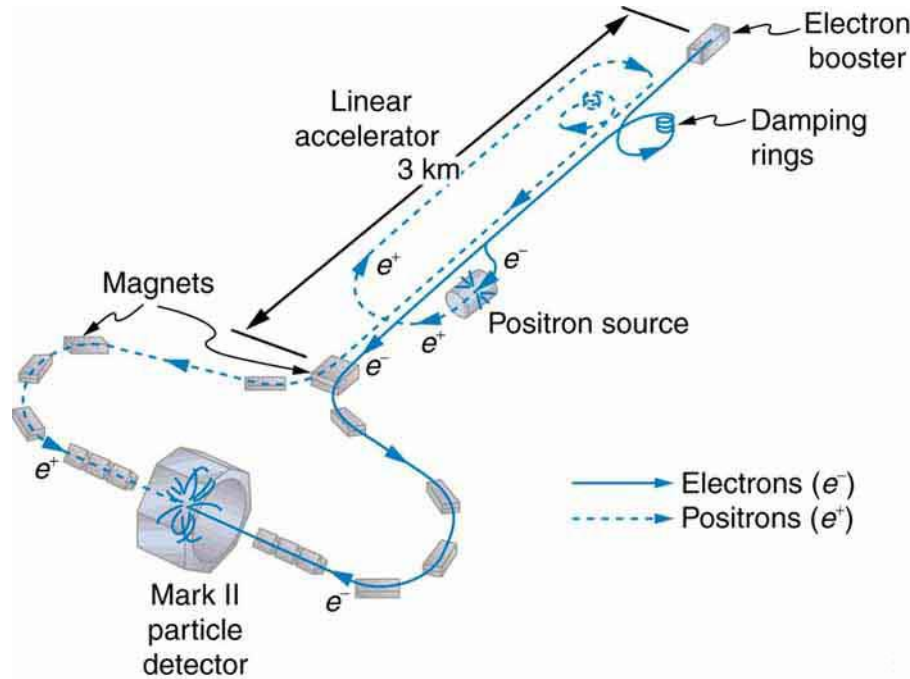


Figure (2.1) : SLAC Linear accelerators

2.2-Hard process cross section

In our current understanding of the strong interaction, given by Quantum Chromodynamics (QCD), all hadrons are composed of Dirac fermions called quarks. Quarks appear in a variety of types, called flavors, each with its own mass and electric charge. A quark also carried an additional quantum number, color, which takes one of three values. Color serves as the charge of QCD.

According to QCD, the simplest e^-e^+ process that ends in hadrons is

$$e^-e^+ \rightarrow q\bar{q}$$

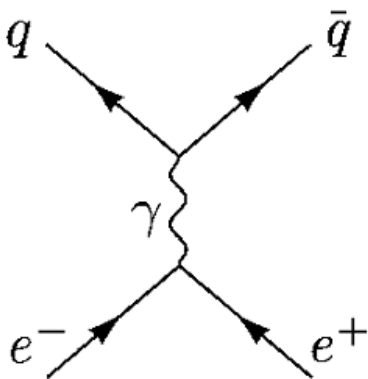


Figure (2.2): Feynman diagram for the annihilation of an electron and a positron, through a vertical photon, into a quark-antiquark pair

2.2.1-The scattering matrix:

From the Feynman diagram, we have the matrix element

$$i\mathcal{M}_{fi} = \bar{v}(P_2)(ie\gamma^\alpha)u(P_1)\frac{-i}{q^2}(g_{\alpha\beta} - (1 - \xi)\frac{q_\alpha q_\beta}{q^2})\bar{u}(P_3)(-ie\gamma^\beta)v(P_4) \quad (2.1)$$

Where p, p' are the incoming electron and positron, respectively k, k' are the outgoing quark and anti-quark respectively: $q = p_1 + p_2 = p_3 + p_4$, ξ allows choice of gauge

The gauge term drops out immediately, since

$$\begin{aligned} (1 - \xi)\frac{q_\alpha q_\beta}{q^2}\bar{u}(p_3)(-ie\gamma^\beta)v(p_4) &= (1 - \xi)\frac{q_\alpha}{q^2}\bar{u}(p_3)(-ieq_\beta\gamma^\beta)v(p_4) \\ &= -e(1 - \xi)\frac{q_\alpha}{q^2}\bar{u}(p_3)(i\gamma^\beta p_{3\beta} + i\gamma^\beta p_{4\beta})v(p_4) \\ &= -e(1 - \xi)\frac{q_\alpha}{q^2}(\bar{u}(p_3)i\gamma^\beta p_{3\beta})v(p_4) \\ &\quad + \bar{u}(p_3)(i\gamma^\beta p_{4\beta})v(p_4) \\ &= -e(1 - \xi)\frac{q_\alpha}{q^2}(-m\bar{u}(p_3)v(p_4) + \bar{u}(p_3)mv(p_4)) \\ &= 0 \end{aligned} \quad (2.2)$$

Now set $q^2 = S$

$$\begin{aligned} |\mathcal{M}_{fi}|^2 &= \frac{e^4}{S^2}(\bar{v}(p_2)\gamma^\alpha u(p_1)g_{\alpha\beta}\bar{u}(p_3)\gamma^\beta v(p_4)) \\ &\quad \times (\bar{v}(p_4)\gamma^\mu u(p_3)g_{\mu\nu}\bar{u}(p_1)\gamma^\nu v(p_2)) \end{aligned} \quad (2.3)$$

The complex conjugate can be as follow:

$$(\bar{v}\gamma^\mu u)^* = u^\dagger(\gamma^\mu)^\dagger(\gamma^0)^\dagger v = u^\dagger(\gamma^\mu)^\dagger\gamma^0 v = u^\dagger\gamma^0\gamma^\mu v = \bar{u}\gamma^\mu v \quad (2.4)$$

The expression for $|\mathcal{M}_{fi}|^2$ simplifies considerably when we throw away the spin information, we want to compute

$$\frac{1}{4} \sum_{\text{quarks}} \sum_{\text{initial spins}} \sum_{\text{final spins}} e_i^2 N |\mathcal{M}_{fi}|^2 = \frac{1}{4} \sum_{\text{all spins}} \sum_{\text{quarks}} e_i^2 N |\mathcal{M}_{fi}|^2 \quad (2.5)$$

The N factor is the color factor, with e_i are the quark charge

$$e_i = 2/3 \text{ if } i = u, c, t, \quad e_i = -1/3 \text{ if } i = d, s, b$$

For this, let us use the outer product identities:

$$\sum_{s=1}^2 u(p_1) \bar{u}(p_1) = \not{p}_1 + m_e, \quad \sum_{s=1}^2 u(p_3) \bar{u}(p_3) = \not{p}_3 + m_q \quad (2.6)$$

$$\sum_{s=1}^2 v(p_1) \bar{v}(p_1) = \not{p}_1 - m_e, \quad \sum_{s=1}^2 v(p_3) \bar{v}(p_3) = \not{p}_3 - m_q \quad (2.7)$$

Thus, we have:

$$\begin{aligned} & \frac{1}{4} \sum_{\text{all spins}} \sum_{\text{quarks}} e_i^2 N |\mathcal{M}_{fi}|^2 \\ &= \frac{e^4}{S^2} \frac{1}{4} N \sum_{\text{all spins}} \sum_{\text{quarks}} e_i^2 (\bar{v}_a(p_2) [\gamma^\alpha]_{ab} u_b(p_1) g_{\alpha\beta} \bar{u}_c(p_3) [\gamma^\beta]_{cd} v_d(p_4)) \\ & \times (\bar{v}_e(p_4) [\gamma^\mu]_{ef} u_f(p_3) g_{\mu\nu} \bar{u}_g(p_1) [\gamma^\nu]_{gh} v_h(p_2)) \\ &= \frac{e^4}{S^2} \frac{1}{4} N \sum_{\text{all spins}} \sum_{\text{quarks}} e_i^2 [\gamma^\alpha]_{ab} u_b(p_1) \bar{u}_g(p_1) g_{\alpha\beta} [\gamma^\beta]_{cd} v_d(p_4) \bar{v}_e(p_4) [\gamma^\mu]_{ef} \\ & \times u_f(p_3) \bar{u}_c(p_3) g_{\mu\nu} [\gamma^\nu]_{gh} v_h(p_2) \bar{v}_a(p_2) \\ &= \frac{e^4}{S^2} \frac{1}{4} N \sum_{\text{all spins}} \sum_{\text{quarks}} e_i^2 v_h(p_2) \bar{v}_a(p_2) [\gamma^\alpha]_{ab} u_b(p_1) \bar{u}_g(p_1) g_{\alpha\beta} u_f(p_3) \bar{u}_c(p_3) \\ & \times [\gamma^\beta]_{cd} v_d(p_4) [\gamma^\mu]_{ef} g_{\mu\nu} [\gamma^\nu]_{gh} \end{aligned} \quad (2.8)$$

By using the outer product identities (2.6) and (2.7) we will obtain

$$\begin{aligned}
 & \frac{1}{4} \sum_{\text{all spins}} \sum_{\text{quarks}} e_i^2 N |\mathcal{M}_{fi}|^2 \\
 &= \frac{e^4}{S^2} \frac{1}{4} N \sum_{\text{quarks}} e_i^2 (\not{p}_2 - m_e)_{ha} [\gamma^\alpha]_{ab} (\not{p}_1 + m_e)_{bg} g_{\alpha\beta} (\not{p}_3 + m_q)_{fe} \\
 & \quad \times [\gamma^\beta]_{cd} (\not{p}_4 - m_q)_{de} [\gamma^\mu]_{ef} g_{\mu\nu} [\gamma^\nu]_{gh} \\
 &= \frac{e^4}{S^2} \frac{1}{4} N \sum_{\text{quarks}} e_i^2 g_{\alpha\beta} g_{\mu\nu} ((\not{p}_2 - m_e)_{ha} [\gamma^\alpha]_{ab} (\not{p}_1 + m_e)_{bg} [\gamma^\nu]_{gh}) \\
 & \quad \times ((\not{p}_3 + m_q)_{fe} [\gamma^\beta]_{cd} (\not{p}_4 - m_q)_{de} [\gamma^\mu]_{ef}) \\
 &= \frac{e^4}{S^2} \frac{1}{4} N \sum_{\text{quarks}} e_i^2 g_{\alpha\beta} g_{\mu\nu} \text{tr}((\not{p}_2 - m_e) \gamma^\alpha (\not{p}_1 + m_e) \gamma^\nu) \\
 & \quad \times \text{tr}((\not{p}_3 + m_q) \gamma^\beta (\not{p}_4 - m_q) \gamma^\mu)
 \end{aligned} \tag{2.9}$$

2.2.2-Traces of Gamma matrices:

Now we compute the traces using the fundamental relation $\{\gamma^\alpha, \gamma^\beta\} = 2g^{\alpha\beta}$, and the cycle property of the trace, $\text{tr}(A \dots BC) = \text{tr}(CA \dots B)$. First we can show the traces of the product of any odd number of γ - matrices vanishes by using $\gamma_5^2 = 1, \{\gamma_5, \gamma^\alpha\} = 0$

$$\begin{aligned}
 \text{tr}(\gamma^\alpha \gamma^\beta \gamma^\mu \gamma^\nu) &= \text{tr}((-\gamma^\beta \gamma^\alpha + 2g^{\alpha\beta} 1) \gamma^\mu \gamma^\nu) \\
 &= -\text{tr}(\gamma^\beta \gamma^\alpha \gamma^\mu \gamma^\nu) + 2g^{\alpha\beta} \text{tr}(\gamma^\mu \gamma^\nu) \\
 &= -\text{tr}(\gamma^\beta (-\gamma^\mu \gamma^\alpha + 2g^{\alpha\beta}) \gamma^\nu) + 2g^{\alpha\beta} \text{tr}(\gamma^\mu \gamma^\nu) \\
 &= \text{tr}(\gamma^\beta \gamma^\mu \gamma^\alpha \gamma^\nu) - 2g^{\mu\alpha} \text{tr}(\gamma^\beta \gamma^\nu) + 2g^{\alpha\beta} \text{tr}(\gamma^\mu \gamma^\nu) \\
 &= \text{tr}((\gamma^\beta \gamma^\mu \gamma^\nu \gamma^\alpha) + 2g^{\nu\alpha}) - 2g^{\mu\alpha} \text{tr}(\gamma^\beta \gamma^\nu) + 2g^{\alpha\beta} \times \text{tr}(\gamma^\mu \gamma^\nu) \\
 &= -\text{tr}(\gamma^\beta \gamma^\mu \gamma^\nu \gamma^\alpha) + 2g^{\nu\alpha} \text{tr}(\gamma^\beta \gamma^\mu) - 2g^{\mu\alpha} \text{tr}(\gamma^\beta \gamma^\nu) \\
 & \quad + 2g^{\alpha\beta} \text{tr}(\gamma^\mu \gamma^\nu) \\
 2\text{tr}(\gamma^\alpha \gamma^\beta \gamma^\mu \gamma^\nu) &= 2g^{\nu\alpha} \text{tr}(\gamma^\beta \gamma^\mu) - 2g^{\mu\alpha} \text{tr}(\gamma^\beta \gamma^\nu) + 2g^{\alpha\beta} \text{tr}(\gamma^\mu \gamma^\nu)
 \end{aligned}$$

$$\text{tr}(\gamma^\alpha \gamma^\beta \gamma^\mu \gamma^\nu) = 4g^{\nu\alpha} g^{\beta\mu} - 4g^{\mu\alpha} g^{\beta\nu} + 4g^{\alpha\beta} g^{\mu\nu} \quad (2.10)$$

We use this to evaluate the traces:

$$\begin{aligned} \text{tr}((\not{p}_2 - m_e)\gamma^\alpha(\not{p}_1 + m_e)\gamma^\nu) &= \text{tr}(\not{p}_2\gamma^\alpha\not{p}_1\gamma^\nu + \not{p}_2\gamma^\alpha m_e\gamma^\nu - m_e\gamma^\alpha\not{p}_1\gamma^\nu - m_e^2\gamma^\alpha\gamma^\nu) \\ &= \text{tr}(\not{p}_2\gamma^\alpha\not{p}_1\gamma^\nu) + \text{tr}(\not{p}_2\gamma^\alpha m_e\gamma^\nu) - m_e\text{tr}(\gamma^\alpha\not{p}_1\gamma^\nu) - m_e^2\text{tr}(\gamma^\alpha\gamma^\nu) \\ &= P_{2\rho}P_{1\sigma}\text{tr}(\gamma^\rho\gamma^\alpha\gamma^\sigma\gamma^\nu) + m_e P_{2\rho}\text{tr}(\gamma^\rho\gamma^\alpha\gamma^\nu) - m_e P_{1\rho}\text{tr}(\gamma^\alpha\gamma^\rho\gamma^\mu) \\ &\quad - m_e^2\text{tr}(\gamma^\alpha\gamma^\nu) \\ &= P_{2\rho}P_{1\sigma}(4g^{\rho\alpha}g^{\sigma\nu} - 4g^{\rho\sigma}g^{\alpha\nu} + 4g^{\rho\nu}g^{\sigma\alpha}) - 4m_e^2g^{\alpha\nu} \\ &= (4P_2^\alpha P_1^\nu - 4(P_2P_1)g^{\alpha\nu} + 4P_2^\nu P_1^\alpha) - 4m_e^2g^{\alpha\beta} \end{aligned} \quad (2.11)$$

With the same method, we get:

$$\text{tr}((\not{p}_3 - m_q)\gamma^\beta(\not{p}_4 + m_q)\gamma^\mu) = (4P_4^\beta P_3^\mu - 4(P_4P_3)g^{\beta\mu} + 4P_4^\mu P_3^\beta) - 4m_q^2g^{\beta\mu} \quad (2.12)$$

With these traces, the matrix element becomes:

$$\begin{aligned} \frac{1}{4} \sum_{\text{all spins}} \sum_{\text{quarks}} e_i^2 N |\mathcal{M}_{fi}|^2 &= \frac{e^4}{S^2} \frac{1}{4} N \sum_{\text{quarks}} e_i^2 g_{\alpha\beta} g_{\mu\nu} (4P_2^\alpha P_1^\nu - 4(P_2P_1)g^{\alpha\nu} \\ &\quad + 4P_2^\nu P_1^\alpha - 4m_e^2g^{\alpha\nu})(4P_4^\beta P_3^\mu - 4(P_4P_3)g^{\beta\mu} + 4P_4^\mu P_3^\beta - 4m_q^2g^{\beta\mu}) \\ &= \frac{4e^4}{S^2} N \sum_{\text{quarks}} e_i^2 g_{\alpha\beta} g_{\mu\nu} (P_2^\alpha P_1^\nu - (P_2P_1)g^{\alpha\nu} + P_2^\nu P_1^\alpha \\ &\quad - m_e^2g^{\alpha\nu})(P_4^\beta P_3^\mu - (P_4P_3)g^{\beta\mu} + P_4^\mu P_3^\beta - m_q^2g^{\beta\mu}) \end{aligned}$$

$$\begin{aligned}
 &= N \sum_{quarks} e_i^2 \left(\frac{4e^4}{S^2} g_{\alpha\beta} g_{\mu\nu} ((P_1 P_3)(P_2 P_3) - (P_4 P_3)(P_2 P_1) + (P_2 P_3) \right. \\
 &\quad \left. - m_q^2 (P_2 P_1)) - \frac{4e^4}{S^2} (P_2 P_1)(P_4 P_3) - 4(P_4 P_3) + (P_4 P_3) - 4m_q^2 \right) \\
 &\quad + \frac{4e^2}{S^2} ((P_1 P_4)(P_2 P_3) - (P_4 P_3)(P_2 P_1) \\
 &\quad + (P_1 P_3)(P_2 P_4) - m_q^2 (P_2 P_1) \\
 &\quad \left. - \frac{4e^4}{S^2} m_e^2 ((P_4 P_3) - 4(P_4 P_3) - 4m_q^2) \right)
 \end{aligned} \tag{2.13}$$

Collecting terms:

$$\begin{aligned}
 \frac{1}{4} \sum_{all\ spins} \sum_{quarks} e_i^2 N |\mathcal{M}_{fi}|^2 &= \sum_{quarks} e_i^2 N \left(\frac{8e^4}{S^2} ((P_1 P_3)(P_2 P_4) - (P_4 P_3)(P_2 P_1) \right. \\
 &\quad \left. + (P_2 P_3)(P_1 P_4) - m_q^2 (P_2 P_1)) + \frac{4e^4}{S^2} ((P_2 P_1 + m_e^2)(2(P_4 P_3) + 4m_q^2)) \right)
 \end{aligned} \tag{2.14}$$

2.2.3-Relativistic kinematics:

We have

$$\begin{cases} P_1 = (E, P_1) \\ P_2 = (E, -P_1) \\ P_3 = (E, P_3) \\ P_4 = (E, -P_3) \end{cases} \tag{2.15}$$

Where s is related to the energy

$$\begin{aligned}
 s &= (P_1 + P_2)^2 \\
 &= ((E, P_1) + (E, -P_1))^2 \\
 &= (2E, 0)^2 \\
 &= 4E^2
 \end{aligned} \tag{2.16}$$

$$\begin{cases} P_1^2 = E^2 - m_e^2 = \frac{S}{4} - m_e^2 \\ P_3^2 = \frac{S}{4} - m_q^2 \end{cases} \quad (2.17)$$

We will need the inner product and using (2.17)

$$\begin{aligned} P_1 P_2 &= E^2 + P_1^2 \\ &= \frac{S}{2} - m_e^2 \end{aligned} \quad (2.18)$$

$$\begin{aligned} P_1 P_3 &= P_2 P_4 = E^2 - P_1 P_3 \\ &= E^2 - \sqrt{\frac{S}{4} - m_e^2} \sqrt{\frac{S}{4} - m_q^2} \cos \theta \\ &= \frac{S}{4} \left(1 - \sqrt{1 - \frac{4m_e^2}{S}} \sqrt{1 - \frac{4m_q^2}{S}} \cos \theta \right) \end{aligned} \quad (2.19)$$

$$P_1 P_4 = P_2 P_3 = \frac{S}{4} \left(1 + \sqrt{1 - \frac{4m_e^2}{S}} \sqrt{1 - \frac{4m_q^2}{S}} \cos \theta \right) \quad (2.20)$$

$$\begin{aligned} P_4 P_3 &= E^2 + P_3^2 \\ &= \frac{S}{2} - m_q^2 \end{aligned} \quad (2.21)$$

Now we substitute these expressions into the matrix element

$$\begin{aligned}
 & \frac{1}{4} \sum_{\text{all spins}} \sum_{\text{quarks}} e_i^2 N |\mathcal{M}_{fi}|^2 \\
 &= \frac{8e^4}{s^2} \sum_{\text{quarks}} e_i^2 N \left[\frac{s^2}{16} \left(1 + \sqrt{1 - \frac{4m_e^2}{s}} \sqrt{1 - \frac{4m_q^2}{s}} \cos \theta \right)^2 \right. \\
 & \quad \left. - \left(\frac{s}{2} - m_q^2 \right) \left(\frac{s}{2} - m_e^2 \right) + \frac{s^2}{16} \left(1 + \sqrt{1 - \frac{4m_e^2}{s}} \sqrt{1 - \frac{4m_q^2}{s}} \cos \theta \right) \right] \\
 & \quad + \frac{4e^4}{s^2} \sum_{\text{quarks}} e_i^2 N \left(\left(\left(\frac{s}{2} - m_e^2 \right) + m_e^2 \right) \left(2 \left(\frac{s}{2} - m_q^2 \right) \right) + m_q^2 \right)
 \end{aligned}$$

Further simplifications

$$\begin{aligned}
 &= \frac{8e^4}{s^2} \sum_{\text{quarks}} e_i^2 N \left(\frac{s^2}{8} \left(1 + \left(1 - \frac{4m_e^2}{s} \right) \left(1 - \frac{4m_q^2}{s} \right) \cos^2 \theta \right) - \frac{s}{2} \frac{s}{2} \right. \\
 & \quad \left. + \frac{s}{2} m_e^2 \right) + \frac{8e^4}{s^2} \sum_{\text{quarks}} e_i^2 N \left(\frac{s}{2} s - m_e^2 s - 2m_q^2 \frac{s}{2} + 2m_q^2 m_e^2 + 2 \frac{s}{2} m_e^2 \right. \\
 & \quad \left. - 2m_e^2 m_q^2 + 4m_q^2 \frac{s}{2} - 4m_q^2 m_e^2 + 4m_q^2 m_e^2 \right) \\
 &= e^4 \sum_{\text{quarks}} e_i^2 N \left(\left(1 - \frac{4m_e^2}{s} \right) \left(1 - \frac{4m_q^2}{s} \right) \cos^2 \theta - 1 + \frac{4m_e^2}{s} \right) \\
 & \quad + 4e^4 \left(1 + \frac{4m_q^2}{s} \right) \\
 &= e^4 \sum_{\text{quarks}} e_i^2 N \left(\left(1 - \frac{4m_e^2}{s} \right) \left(1 - \frac{4m_q^2}{s} \right) \cos^2 \theta + 1 + \frac{4}{s} (m_e^2 + m_q^2) \right)
 \end{aligned} \tag{2.22}$$

2.2.4-Differential cross section

Now we can find the differential cross section. In the center of mass form, the 2-particle-to-2-particle differential cross section is: [7]

$$d\sigma = \frac{hc}{(8\pi)^2} \frac{|\mathcal{M}_{fi}|^2}{s|P_1|} \frac{d^3P_3 d^3P_4}{E_3 E_4} \delta^4(P_1 + P_2 - P_3 - P_4) \quad (2.23)$$

The δ term define as:

$$\delta^4(P_1 + P_2 - P_3 - P_4) = \delta\left(\frac{E_1 + E_2 - E_3 - E_4}{c}\right) \delta^3(-P_3 - P_4) \quad (2.24)$$

We have $E = c\sqrt{m^2c^2 + p^2}$ and $P_4 \equiv P_3$ we obtain:

$$d\sigma = \frac{hc}{(8\pi)^2} \frac{|\mathcal{M}_{fi}|^2}{s|P_1|} \frac{\delta((E_1 E_2 / c) - \sqrt{m^2_3 c^2 + P^2_3} - \sqrt{m^2_4 c^2 + P^2_3})}{\sqrt{m^2_3 c^2 + P^2_3} \sqrt{m^2_4 c^2 + P^2_3}} d^3P_3 \quad (2.25)$$

We work with derivative relation and solid angle Ω :

$$d^3P_3 = P_3^2 dP_3 d\Omega \quad (2.26)$$

$$d\Omega = \sin\theta d\theta d\varphi \quad (2.27)$$

By applying these modifications in, (2.25) we get

$$\begin{aligned} d\sigma &= \frac{hc}{(8\pi)^2} \frac{|\mathcal{M}_{fi}|^2}{s|P_1|} \int |\mathcal{M}_{fi}|^2 \frac{\delta((E_1 E_2 / c) - \sqrt{m^2_3 c^2 + P^2_3} - \sqrt{m^2_4 c^2 + P^2_3})}{\sqrt{m^2_3 c^2 + P^2_3} \sqrt{m^2_4 c^2 + P^2_3}} P_3^2 dP_3 d\Omega \end{aligned} \quad (2.28)$$

Finally, we obtain

$$d\sigma = \frac{1}{64\pi^2 s} \frac{|P_3|}{|P_1|} |\mathcal{M}_{fi}|^2 d\Omega \quad (2.29)$$

This immediately give us the result:

$$d\sigma = \frac{e^4}{64\pi^2 s} \frac{\sqrt{1 - \frac{4m_q^2}{s}}}{\sqrt{1 - \frac{4m_e^2}{s}}} \left(1 + \frac{4}{\delta} (m_e^2 + m_q^2 + \left(1 - \frac{4m_e^2}{s}\right) \left(1 - \frac{4m_q^2}{s}\right) \cos^2 \theta) \right) d\Omega \quad (2.30)$$

If the energy is much larger than either mass $m_e < m_q \ll \sqrt{s} < 90 \text{ Gev}$ than this approximately

$$d\sigma = \frac{e^4}{64\pi^2 s} (1 + \cos^2 \theta) d\Omega \quad (2.31)$$

Writing e^2 in terms of the fine structure constant

$$e^2 = \frac{e^2}{hc} = 4\pi\alpha \quad (2.32)$$

This is

$$d\sigma = \frac{e^2}{4s} (1 + \cos^2 \theta) d\Omega \quad (2.33)$$

Then the total cross section is

$$\begin{aligned} \sigma &= \int d\sigma \\ &= \frac{\alpha^2}{4s} \int (1 + \cos^2 \theta) d \cos\theta d\varphi \end{aligned}$$

$$\begin{aligned}
 &= \frac{2\pi\alpha^2}{4s} \int_{-1}^1 (1+x^2) dx \\
 &= \frac{2\pi\alpha^2}{4s} \left(x + \frac{1}{3}x^3 \right) \Big|_{-1}^1 \\
 &= \frac{4\pi\alpha^2}{3s}
 \end{aligned}$$

(2.34)

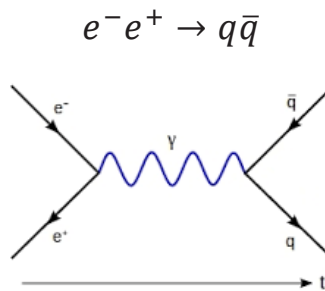
Where is $\cos \theta \rightarrow x$

$$\sigma(e^-e^+ \rightarrow q\bar{q}) = \sum_{i=u,d,s,c,t,b} e_i^2 N \frac{4\pi\alpha^2}{3s}$$

(2.35)

2.3-Factorization theorem

Perturbative QCD can be used to calculate cross-sections at the parton level, provided that the energy/momentum scale of the process Q is large enough so that $\alpha_s(Q)$ is sufficiently small. For example, we can calculate the cross-section for the process [13]



We have the relation:

$$\frac{d\sigma(\hat{s})}{P_T}$$

With the momentum P_T transverse to the direction of the incoming partons, where $\sqrt{\hat{s}}$ is the center-of-mass energy of the incoming partons

In order to obtain the differential cross-section for electron-positron scattering into two jets of final state hadrons with transverse momentum, P_T we can invoke the factorization theorem.

If we pull a hadron of type 1 from one of the incoming partons, with a fraction x_1 of the momentum of the parent parton, and a hadron of type 2 from the second parton, with a fraction x_2 of its momentum, then (in the case of relativistically moving particles whose energy E and momentum p are related by $\approx |P|c$) the center-of-mass energy of the two partons is given by

$$\hat{s} = x_1 x_2 s \tag{2.36}$$

Factorization tells us that if $D_q^h(x_1)$ and $D_{\bar{q}}^h(x_2)$ are the fragmentation functions for hadrons 1 and 2, then the contribution to the electron-positron differential cross-section is:

$$\int_0^1 dx_1 \int_0^1 dx_2 D_q^h(x_1) D_{\bar{q}}^h(x_2) \frac{d\hat{\sigma}(x_1 x_2 s)}{dP_T}$$

Thus, we finally obtain an expression for the electron-positron differential cross-section

$$\frac{d\sigma_{e^-e^+}(s)}{dP_T} = \sum_{i,j} \int_0^1 dx_1 \int_0^1 dx_2 D_q^h(x_1) D_{\bar{q}}^h(x_2) \frac{d\hat{\sigma}(x_1 x_2 s)}{dP_T} \tag{2.37}$$

Where the sum over i, j means sum over all possible partons.

QCD calculations based on this factorization theorem agree well with experiment.

2.4-Hadronization

Hadronic jet production at high-energy colliders has proved to be one of the most valuable test for quantum chromodynamics (QCD). At high energies, precisely at large momentum transfers, the QCD coupling becomes small and perturbation theory becomes more reliable. Perturbative predictions to next-to-leading order, and to higher order in a few cases, give a good account of jet production processes in hadron-hadron, and lepton-hadron collisions. [13] [10]

A barrier to further progress in jet physics is our lack of understanding of the process of hadronization, in which the quarks and gluons of perturbative QCD are converted into the hadrons that are seen in the detectors.

At present the only detailed descriptions of the hadronization process are provided by models.

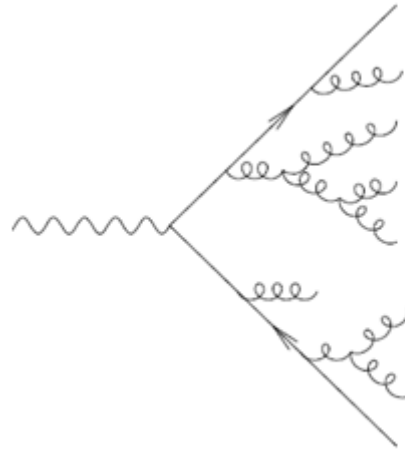
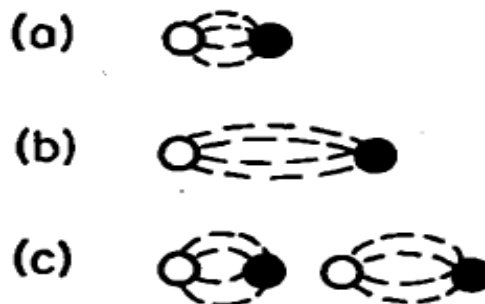


Figure (2.3) : parton cascade in electron-positron annihilation

2.5- Hadronization models

2.5.1-Naive string model:



Typical effects of breaking a hadron:

(a) Original quark antiquark pair held by color lines of force

(b) Lines of force are stretched

(c) Lines of force break with the creation of a new quark antiquark pair [8]

2.5.2-String model:

The string model of hadronization is most easily described for annihilation. The produced quark and anti-quark move out in opposite directions, losing energy to the color field. The string has a

uniform energy per unit length, corresponding to a linear quark confining potential, the string breaks up into hadron-sized pieces through spontaneous pair production.

The string may be broken up starting at either the quark or the antiquark end, or both simultaneously.

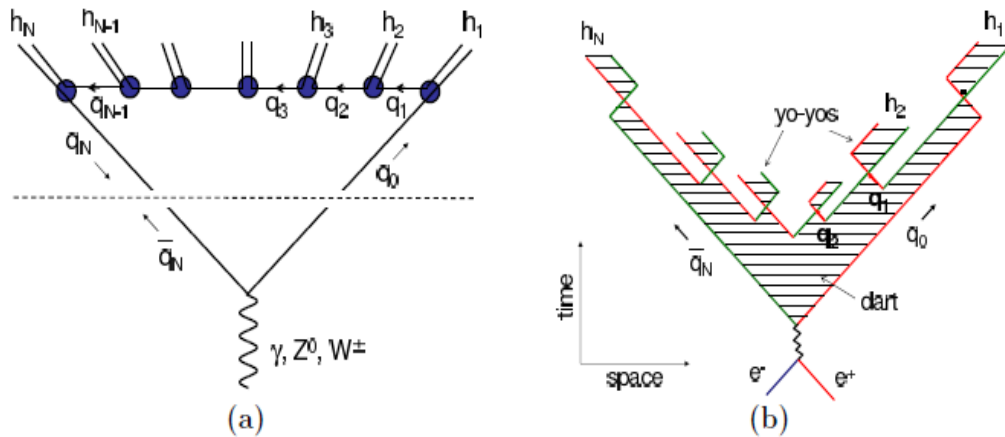
The $q\bar{q}$ pairs are created according to the probability of a tunneling process $\exp(-\pi m_{q,\perp}^2/k)$, which depends on the transverse mass squared $m_{q,\perp}^2 \equiv m_q^2 + P_{q,\perp}^2$ and the string tension $k \approx 1\text{Gev}/fm$. The transverse momentum $P_{q,\perp}$ is locally compensated between quark and antiquark. Due to the dependence on the parton mass m_q and/or hadron mass, m_h , the production of strange and, in particular, heavy-quark hadrons is suppressed. The light-cone momentum fraction .
[5] [6]

$$z \approx \frac{(E + p_{\parallel})_h}{(E + p)_q} \tag{2.38}$$

Where p_{\parallel} is the momentum of the formed hadron h along the direction of the quark q , is given by the string-fragmentation function

$$f(z) \sim \frac{1}{z} (1 - z)^a \exp\left(-\frac{bm_{h,\perp}^z}{z}\right) \tag{2.39}$$

Where a and b are free parameters. These parameters need to be adjusted to bring the fragmentation into accordance with measured data, *e.g.*, $a = 0.11$ and $b = 0.52 \text{ Gev}^{-2}$



Figure(2.4): the string model creation

2.5.3-Yoyo mode (string model):

The yoyo mode is used to describe e^-e^+ annihilation events and as a simple model for stable hadrons. In this mode the two charges at the endpoint of the string move like a point-particles

As the quarks fly apart they are decelerated by the string tension, accelerated back together and then fly apart once more, executing periodic oscillations (known as yo-yo modes), see Fig(2.5) .[8]

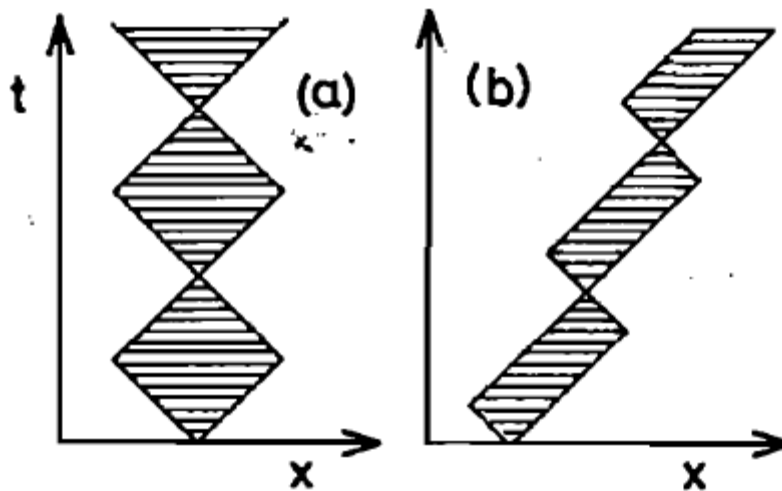


Figure (2.5): shows the Yoyo mode in string hadronization

2.5.4-Cluster model:

Assuming a local compensation of color based on the *pre-confinement* property of perturbative QCD, the remaining gluons at the end of the parton shower evolution are split non-perturbatively

into quark-antiquark pairs. Color singlet clusters of typical mass of a couple of GeV are then formed from quark and antiquark of color-connected splittings. These clusters decay directly into two hadrons unless they are either too heavy, when they decay into two clusters, or too light, in which case a cluster decays into a single hadron, requiring a small rearrangement of energy and momentum with neighboring clusters.

The decay of a cluster into two hadrons is assumed to be isotropic in the rest frame of the cluster except if a perturbative-formed quark is involved. A decay channel is chosen based on the phase-space probability, the density of states, and the spin degeneracy of the hadrons.

Cluster fragmentation has a compact description with few parameters, due to the phase-space dominance in the hadron formation. [5] [6]

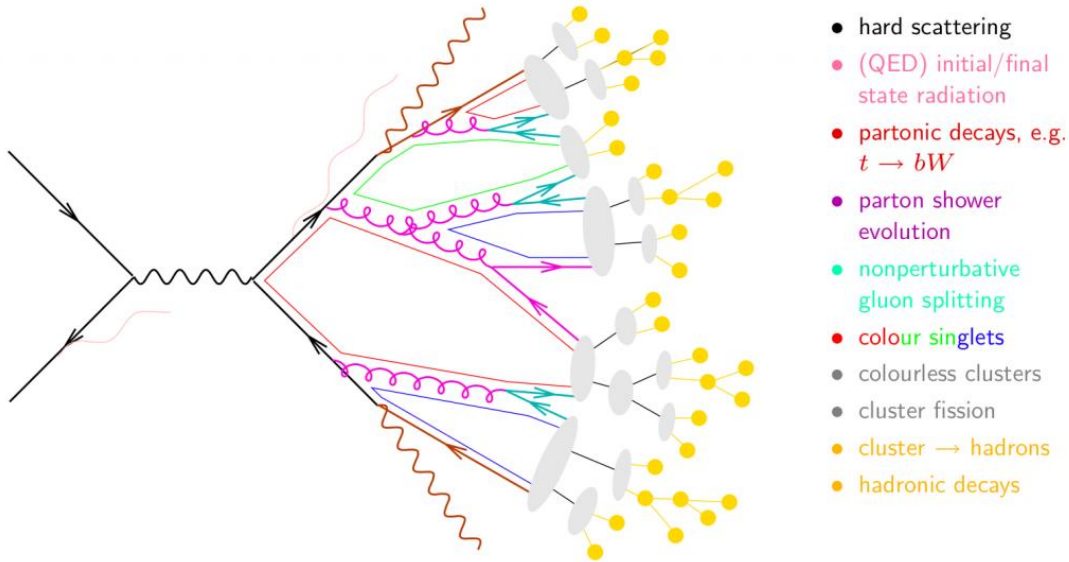


Figure (2.6): shows the cluster model creation

*Chapter 3: study of
jets in e^-e^+
collisions*

*Gleichungen sind wichtiger für mich, weil die Politik für die Gegenwart ist,
aber eine Gleichung etwas für die Ewigkeit*

-Albert Einstein

*Equations are more important to me because politics is for the present,
but an equation is something for eternity*

-Albert Einstein

-Introduction

We studied the process e^-e^+ collision in QCD at leading order this could be detected as an event of two jets in the detector. Higher order emissions have the possibility to trigger an event of three or more jets. The jet event is studied with the help of fragmentation functions.

3.1- e^-e^+ collisions cross-section at next to leading order

Now let us calculate the Cross section of the following interaction

$e^-e^+ \rightarrow q\bar{q}g:$

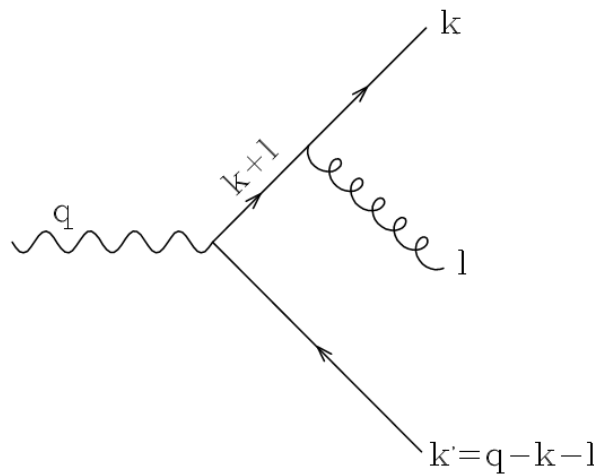


Figure (3.1): Feynman diagram for three jet events

We will use energy-momentum conservation principle to obtain:

$$\vec{q} + \vec{k}' = \vec{k} + \vec{l} \tag{3.1}$$

From Feynman diagram

$$\mathcal{M}_{q\bar{q}g} = \mathcal{M}_1 + \mathcal{M}_2 \tag{3.2}$$

Where \mathcal{M}_1 is the amplitude from the outgoing fermion

Where \mathcal{M}_2 is the amplitude from the outgoing anti-fermion and gluon

We have

$$\mathcal{M} = \bar{u}(k)\epsilon^\mu\gamma_\mu v(k') \quad (3.3)$$

We can write this amplitude in other form

$$\mathcal{M} = \bar{u}(k)\mathcal{N} \quad (3.4)$$

$$\mathcal{N} = \epsilon^\mu\gamma_\mu v(k') \quad (3.5)$$

Where ϵ is the virtual photon polarization

Where k momentum of the fermion

Where k' momentum of the anti-fermion

Where q momentum of the incoming photon

From the diagram the amplitude is given by

$$\mathcal{M}_1 = \bar{u}(k)(-i)\gamma_\alpha i \frac{\not{k} + \not{l}}{(k+l)^2} \mathcal{N} \quad (3.6)$$

From relation (3.1) we get:

$$\begin{aligned} \vec{k}' &= \vec{q} - \vec{k} - \vec{l} \\ \vec{q} + \vec{k}' &= \vec{k} + \vec{l} \\ \vec{q} + \vec{k}' - \vec{k} - \vec{l} &= 0 \end{aligned} \quad (3.7)$$

In the last relation if we can chose $\vec{q} = \vec{l}$ then $\vec{k}' = \vec{k}$ and verse versa, our choice is:

$$\vec{q} = \vec{k} = \vec{l} = \vec{k}' \quad (3.8)$$

In addition, we have: ($l^2 = k^2 = 0$)

Now for the amplitude \mathcal{M}_1 we find:

$$\mathcal{M}_1 = \bar{u}(k)(-i)\gamma_\alpha i \frac{k + l}{(k + l)^2} \mathcal{N}$$

Using equation (2.18) to find:

$$\begin{aligned} \mathcal{M}_1 &= \bar{u}(k)\gamma_\alpha \frac{k + k}{2k.l} \mathcal{N} \\ &= \bar{u}(k)\gamma_\alpha \frac{2k}{2k.l} \mathcal{N} \\ &= \bar{u}(k)\gamma_\alpha \frac{k}{k.l} \mathcal{N} \end{aligned}$$

By using this notation: $\gamma_\alpha k = k_\alpha$

$$\begin{aligned} \mathcal{M}_1 &= \bar{u}(k) \frac{k_\alpha}{k.l} \mathcal{N} \\ &= \frac{k_\alpha}{k.l} \mathcal{M} \end{aligned}$$

(3.9)

From the second amplitude $\mathcal{M}_2(k')$, we use the same method of $\mathcal{M}_1(k)$ to obtain:

$$\begin{aligned} \mathcal{M}_2 &= \bar{u}(k) \frac{k'_\alpha}{k'.l} \mathcal{N} \\ &= \frac{k'_\alpha}{k'.l} \mathcal{M} \end{aligned}$$

(3.10)

Thus, the total amplitude is

$$\mathcal{M}_{q\bar{q}g} = \mathcal{M}_1 + \mathcal{M}_2 = \left(\frac{k_\alpha}{k.l} - \frac{k'_\alpha}{k'.l} \right) \mathcal{M}$$

(3.11)

For calculating the cross section, we calculate **first** \mathcal{M}^2 :

$$\begin{aligned}
 \mathcal{M}_{q\bar{q}g}^2 &= \left| \left(\frac{k_\alpha}{k \cdot l} - \frac{k'_\alpha}{k' \cdot l} \right) \mathcal{M} \right|^2 \\
 &= \left| \left(\frac{k_\alpha^2}{k^2 \cdot l^2} + \frac{k'_\alpha{}^2}{k'^2 \cdot l^2} \right) - 2 \left(\frac{k_\alpha}{k \cdot l} \frac{k'_\alpha}{k' \cdot l} \right) \right| \mathcal{M}^2 \\
 &= 2 \left(\frac{k_\alpha}{k \cdot l} \frac{k'_\alpha}{k' \cdot l} \right) \mathcal{M}^2 \\
 &= 2 \left(\frac{k}{k \cdot l} \frac{k'}{k' \cdot l} \right) \mathcal{M}^2
 \end{aligned}
 \tag{3.12}$$

Second:

We use polarization of the gluon

$$\sum_{pol} \varepsilon^\sigma(k) \varepsilon^{\sigma'}(k) = -g^{\sigma\sigma'}
 \tag{3.13}$$

And the cross section law (Fermi golden rule) for \mathcal{M}^2 to find Born cross section (we already discussed that in chapter 2)

From all this we can find the total cross section

$$\sigma_{q\bar{q}g} = C_F g_s^2 \sigma_{q\bar{q}}^{Born} \int \frac{d^3l}{2l^0 (2\pi)^3} 2 \frac{k \cdot k'}{(k \cdot l)(k' \cdot l)}
 \tag{3.14}$$

Where

$$C_F = \frac{N_c^2 - 1}{2N_c} = \frac{4}{3} \text{ for } N_c = 3
 \tag{3.15}$$

Let us call the angle that the Gluon makes with the fermion direction

$$2 \frac{k \cdot k'}{(k \cdot l)(k' \cdot l)} = \frac{4}{l_0^2(1 - \cos\theta)(1 + \cos\theta)} \quad (3.16)$$

Using $\alpha_s = g_s^2/4\pi$

$$\sigma_{q\bar{q}g} = C_F \frac{\alpha_s}{2\pi} \sigma_{q\bar{q}}^{Born} \int d\cos\theta \frac{dl^0}{l^0} \frac{4}{(1 - \cos\theta)(1 + \cos\theta)} \quad (3.17)$$

Our task is to introduce kinematical variables to describe such events. The momentum vectors of the q , and \bar{q} and g which are produced by a virtual photon (γ^*) at rest, are displayed in Fig (3.2) we work with the energies, and with the longitudinal and transverse momenta of the partons.

We introduce

$$x_q \equiv \frac{2E_q}{Q}, \quad x_{\bar{q}} \equiv \frac{2E_{\bar{q}}}{Q}, \quad x_g \equiv \frac{2E_g}{Q} \quad (3.18)$$

And

$$x_T \equiv \frac{2E_T}{Q} \quad (3.19)$$

The four-momentum fractions in fig (3.2) are

$$\begin{aligned} (x_q, 0, 0, -x_q) & \quad \text{for } q \\ (x_{\bar{q}}, x_T, 0, x_1) & \quad \text{for } \bar{q} \\ (x_g, -x_T, 0, x_q, x_L) & \quad \text{for } g \end{aligned} \quad (3.20)$$

Longitudinal and transverse momentum conservation are embodied in (3.20), but energy conservation introduces the additional requirement that

$$x_q + x_{\bar{q}} + x_g = 2 \quad (3.21)$$

The zero mass of the q and g leads to the further constraints

$$\begin{aligned} x_{\bar{q}}^2 - x_T^2 - x_L^2 &= 0 \\ x_g^2 - x_T^2 - (x_L - x_q)^2 &= 0 \end{aligned} \quad (3.22)$$

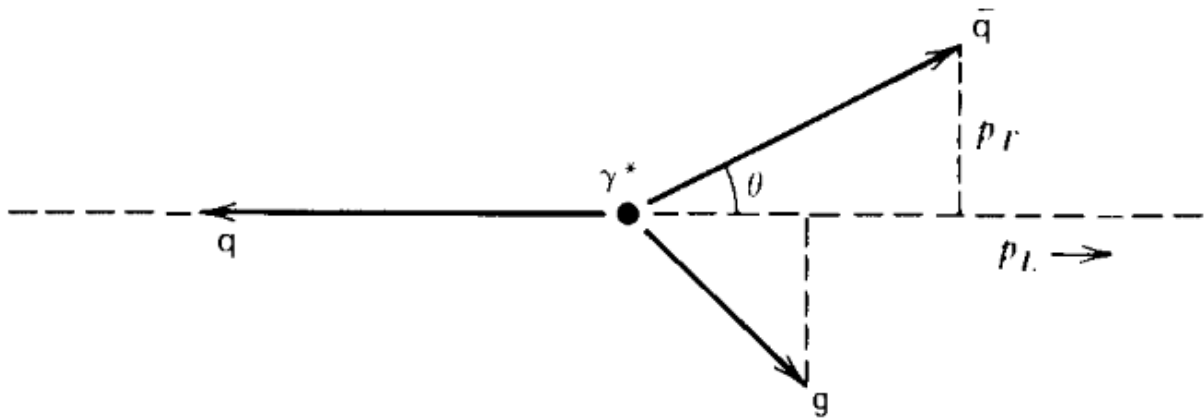


Figure (3.2): the process $e^- e^+ \rightarrow \gamma^* \rightarrow q \bar{q} g$ in the center of mass frame [1]

From (3.21) and (3.22), it follows that

$$x_T^2 = \frac{4}{x_q^2} (1 - x_q)(1 - x_{\bar{q}})(1 - x_g) \quad (3.23)$$

3.2- Jets in Quantum Chromo Dynamics

Jets of partons arise in QCD due to the fact that final state particles cluster into bunches. If such a bunch carries large transverse momentum, P_T it is referred to as a jet and its transverse momentum is associated with that of the original parton that participated in the hard scattering. Because of the

latter, jets are the signatures of large momentum transfer through local interactions and they form direct evidence of processes taking places at distances $\sim 1/P_T$.

In order to relate the jets of hadrons, which are registered by detectors, to the jets of partons, which can be computed within perturbative QCD, one needs a precise and robust *jet definition*. Only then, one is able to meaningfully compare the experimental data with theoretical predictions and fully exploit the information about the hard-interaction carried by jets.

We define the quantity $\langle f_n \rangle$, which has the perturbative expansion as follows:

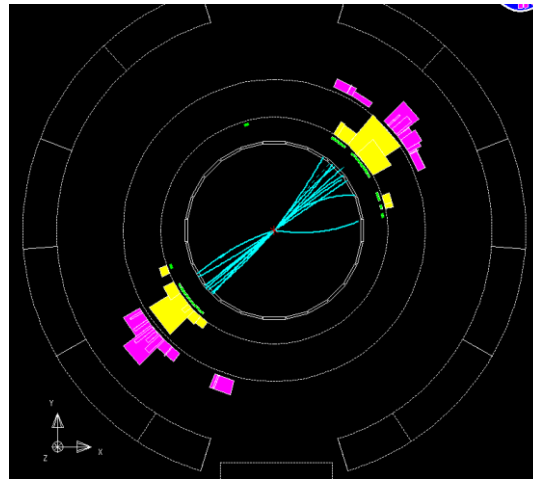
$$\langle f_n \rangle = C_1(f_2)\alpha_s + C_2(f_3)\alpha_s^2 + C_3(f_4)\alpha_s^3 + \dots \quad (3.24)$$

Where f_n is the total fraction of the jets and α_s is the running coupling this definition says that a final state is classified as a 2-jet event if at least a fraction $1 - \epsilon$ of the total available energy is contained in a pair of cones of half-angle δ .

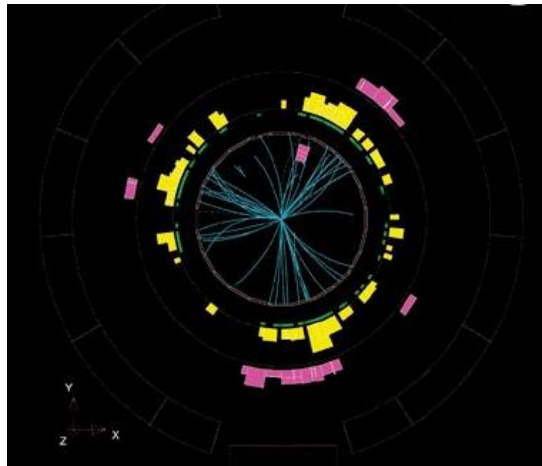
This definition says that a final state is classified as a 2-jet event if at least a fraction of the total available energy is contained in a pair of cones of half-angle. This simple definition can be used to compute fractions of 2- and 3-jet events. At leading order we have $e^-e^+ \rightarrow q\bar{q}$ and all events fall into the 2-jet class. At next to leading order, if the gluon emissions is sufficiently large-angled and carries more than the fraction ϵ of the total energy, the event corresponds to a 3-jet configuration. [1]
[13]

The exact 3-jet fraction at NLO is given by

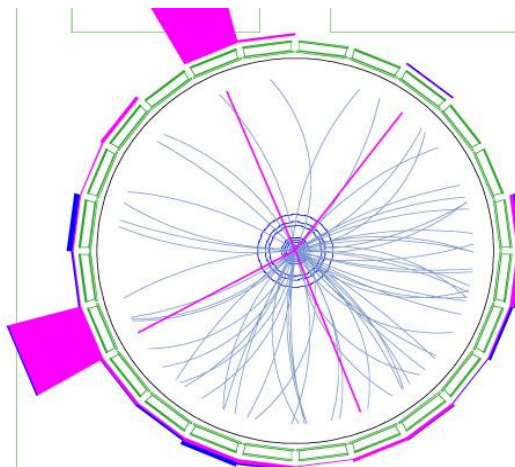
$$f_3 = \frac{\alpha_s^2}{3\pi^2} \left(3\ln\delta + 4\ln\delta \ln 2\epsilon + \frac{\pi^2}{3} - \frac{7}{4} \right) \quad (3.25)$$



2-jet events



3-jet events



Multi-jet events

Figure (3.3): presentation of jet events at linear accelerators [13]

3.3-Fragmentation Function

Fragmentation functions are dimensionless functions that describe the final-state single particle energy distributions in hard scattering processes. The total e^-e^+ fragmentation function for hadrons of type h , energy \sqrt{s} , via an intermediate vector boson $V = \gamma/Z_0$, is defined as [13]

$$F^h(\hat{x}, s) = \frac{1}{\sigma_{tot}} \frac{d\sigma}{d\hat{x}}(e^-e^+ \rightarrow V \rightarrow hX) \quad (3.26)$$

Where $x = 2E_h/\sqrt{s} \leq 1$ is the scale hadron energy.

Its integral with respect to x gives the average multiplicity of those hadrons:

$$\langle n_h(s) \rangle = \int_0^1 dx F^h(\hat{x}, s) \quad (3.27)$$

The fragmentation function can be represented as the sum of contribution from the different parton type $i = u, \bar{u}, d, \bar{d}, \dots, g$

$$F^h(\hat{x}, s) = \sum_i \int_x^1 \frac{dx}{x} C_i(s; x, \alpha_s) D_i^h(\hat{x}/x, s) \quad (3.28)$$

Where D_i^h are the parton fragmentation functions. At lowest order in α_s the coefficient function

C_g for gluon is zero, while for quarks $C_i = g_i(s)\delta(1-x)$, where $g_i(s)$ is the appropriate electroweak coupling

We can compute the function D_i^j that would describe the fragmentation of parton of type i into parton of type j , they would be finite since the probability of emitting a gluon or light quark is divergent, all such divergence is factorizable.

We can write:

$$D_i^j(\hat{x}, t) = \sum_k \int_{\hat{x}}^1 dx K_i^k(x, t, t_0) D_k^i(\hat{x}/x, t_0) \quad (3.29)$$

Where the kernel function K_i^k is

$$K_i^k(x, t, t_0) = \delta_{ik} \delta(1 - x) \quad (3.30)$$

Conclusion

Conclusion:

From what has been seen in this work, it can be said that the field of perturbative QCD is a vast field of research in the physics of elementary particles. We tried to describe in a simple way jet physics in QCD.

We started with a detailed study of the proton structure, which led to the calculation of the cross section of the hard process electron-positron annihilation. In order to understand the final state of the e^-e^+ collision that manifests in two jet or three jets event in the detectors, we talked about the hadronization models, and jet definition.

By studying and measuring jets in particle detectors, we can determine the properties of the original quark. The 2-jet events and 3-jet events process is very important in QCD and elementary particle in general, through this process we have discovered several key particles in the fundamental structure of matter and their interactions. The importance of this process, theoretically and experimentally takes place in the final state of the interaction, of which we need the fragmentation function to describe the hadronization process.

In my perspective, the importance of this work is further investigation about jet physics by comparing the detector data with the analytical calculation, with the help of a numerical method like Monte Carlo recursive jets.

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Abstract

In this work we studied the perturbative QCD, We started with the parton model, then we calculated the cross section of electron-positron annihilation (hard process) at leading and high order, we explained some hadronization models, finally we included a brief definition of a jet event and fragmentation function.

Key Words: perturbative QCD, parton model, jet, fragmentation function

Résumé

Dans ce travail, nous avons étudié la QCD perturbative, nous avons commencé avec le modèle des Partons, alors nous avons calculé la section efficace d'annihilation électron-positron (processus dur) à ordre élevé, nous présentons ici quelques modèles hadronization, enfin, nous avons inclus une brève définition un événement à réaction et la fonction de fragmentation.

Mots clés : QCD perturbative, modèle des partons, jet, la fonction de fragmentation

ملخص

في هذا العمل قمنا بدراسة الكروموديناميك الكمومي الاضطرابي، بدأنا مع نموذج البارتون، مما أدى إلى حساب المقطع الفعال لانحلال الإلكترون-البوزيترون. من أجل فهم الحالة النهائية للانحلال استخدمنا نماذج الهدرنة، وفي نهاية الاطروحة عرفنا المقذوفات ودالة التجزئة

الكلمات المفتاحية. الكروموديناميك الكمومي الاضطرابي، نموذج البارتون، المقذوفات، دالة التجزئة