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THEME

Rigidity Test Using the Pebble Game Centralized Implementation

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General Introduction

Rigidity theory has a rich history that can be traced back to Leonhard Euler, Baron Augustin Louis Cauchy, James Clerk Maxwell and a host of other mathematicians and engineers. Euler (1766) conjectured that "A closed spatial figure allows no changes, as long as it is not ripped apart" [7]. Maxwell (1864) was exploring whether structures were stable or unstable (deformable), and a host of other engineers worked on different applications, such as determining the structural stability of different truss configurations in bridges and in mechanical linkages.

Later in the half of the nineteenth century, Rigidity theory for plane bar and joint frameworks was already well advanced. Then earlier in twentieth century, previous work was summarized as well as important inductive techniques that are used to construct 2-dimensional bar and joint frameworks starting from the basic building blocks introduced by Henneberg [12]. These inductive techniques with other more recent techniques are still widely used, particularly in proofs and rigidity analysis of frameworks [27]. While a school of work in the rigidity theory continued for decades in the former Soviet Union [8], overall the best discoveries of the nineteenth century in the rigidity theory of bar and joint frameworks was forgotten for at least in the first half of the twentieth century.

An important advancement in rigidity theory begins with the theorem of Dutch mathematician Gerard Laman (1970), this theorem gives a beautiful and complete combinatorial characterization of rigidity and flexibility of bar and joint frameworks in dimension 2. In combinatorial characterization of rigidity, to determine the rigidity and flexibility of a corresponding framework it can simply count the vertices and edges in a graph and its subgraphs. Even though Maxwell and many others, with mechanical engineers, were counting vertices and edges in a graph to analyze rigidity, Laman's theorem gives a first rigorous characterization. Almost in similar time, mathematicians and engineers began to interact and further revive and develop the topic.

Mathematicians and researchers from different fields are continuing to work on problems and applications related to rigidity theory. The number of different applications that rely on this theory is rapidly expending. Some of these include protein and glass network flexibility predictions [14, 24, 25], sensors and communication [1, 4], computer aided design (CAD), decomposition of linkages in mechanical engineering [20, 21] … etc.

In our work we start by outlining some definitions from rigidity theory (Chapter 1). For the sake of simplicity, we will start by looking at the most common and simplest structures known as bar and joint frameworks. Since the fast algorithms for determining the rigidity/flexibility do not rely on the complicated geometry (positions). in other words, the rigidity becomes a graph theoretic property and this gives fast algorithms (pebble game) for determining rigidity/flexibility. We will also briefly present some problems with finding the fast algorithms for determining flexibility of the bar and joint frameworks in 3-dimensional space.

Chapter 2 presents a three different applications related to rigidity theory and we give some detailed examples, we will discuss for the 1- problem of network localization problem, 2-Rigidity and Flexibility of Protein Structures and 3-Application to computer aided design (CAD).

In Chapters 3 we will present some definitions and describes of the pebble game algorithm, also we illustrate example in detailed of the algorithm for enlarging a pebble covering.

Chapter 4 discussing for our implementation of pebble game algorithm and present some describe for the development environment, also our tools uses in implementation, and offer some concluding remarks.

General Conclusion

The rigidity theory gives a beautiful and complete combinatorial characterization of rigidity and flexibility of bar and joint frameworks in 2 dimension. different applications related to rigidity theory have been made in several fields. This theory has facilitated the development of the pebble game algorithms and it is an efficient algorithm for studying flexibility and rigidity and it can be used to explore new and interesting questions which will lead to further research possibilities. We also implement a computer program for the algorithm that Jacobs and Hendrickson has proposed in [10].

In conclusion, for future work, to increase the performance of our computer program we want to

- check if a given graph is rigid, redundantly rigid or globally rigid.
- extract rigid and flexible regions from a given graph.

References

[1] B. Anderson, D. O. P. N. Belhumer, T. Eren, A. S. Morse and W. Whiteley, Operations on rigid formations of autonomous agents, Comunicatios in Informations and Systems, Vol. 3, No. 4, p. 223–258, 2004.

[2] Andrusier N, Mashiach E, Nussinov R, et al. Principles of flexible protein-protein docking. Proteins 2008; 73:271–89.

[3] J. Aspnes and T. Eren and D. Goldenberg, et al., "A theory of network localization,"IEEE Transactions on Mobile Computing, vol. 5, no. 12, 2006.

[4] T. Eren, W. Whiteley, B. D.O. Anderson, A. S. Morse, P. Belhumeur, Information Structures to Secure Control of Rigid Formations with Leader-Follower Structure, In Proc. of the American Control Conference, pp. 2966-2971, Portland, Oregon, 2005.

[5] Fontana A, Filippis V, Laureto PP. Rigidity of thermophilic enzymes. Progress in Biotechnology 1998; 15:277–94

[6] Fox N, Jagodzinski F, Li Y, et al. KINARI-Web: a server for protein rigidity analysis. Nucleic Acids Res 2011; 39:W177–83.

[7] J. Graver, Counting on Frameworks: Mathematics to Aid the Design of Rigid Structures, The Mathematical Association of America, Washington, 2001.

[8] J. Graver, B. Servatius, and H. Servatius, Combinatorial Rigidity, Graduate Studies in Math., AMS, 1993.

[9] H. Gluck, almost all simply connected closed surfaces are rigid, In Geometric Topology, Lecture notes in Mathematics, Springer-Verlag, Berlin, No. 438, p. 225–239, 1975.

[10] T.He, C. Huang, B. Blum, J. Stankovic, and T. Abdelzaher, "Range-free localization schemes in large scale sensor networks," in Proc. Ninth Int'l Conf. Mobile Computing and Networking (MobiCom), pp. 81-95, 2003.

[11] B. Hendrickson and D.J. Jacobs, An algorithm for two dimensional rigidity percolation: The pebble game, J. Comput. Phys., 137:346-365, 1997.

[12] B. Hendrickson, Conditions for unique graph realizations, SIAM J. Comput., 21, pp. 65-84, 1992.

[13] Adnan Sljoka, Counting For Rigidity, Flexibility And Extensions Via The Pebble Game Algorithm, Master Degree Thesis, York University, Toronto, Ontario, September 2006

[14] Jacobs DJ, Rader AJ, Kuhn LA, et al. Protein flexibility predictions using graph theory. Proteins: Struct Funct Genet 2001; 44:150–65.

[15] L.A. Kuhn, D.J. Rader and M.F. Thorpe, Protein flexibility predictions using graph theory, Proteins, 44:150-65, 2001.

[16] G. Laman, "On graphs and rigidity of plane skeletal structures," J. Eng. Math., vol. 4, pp. 331-340, 2002.

[17] A. Mantler and J. Snoeynik, Banana Spiders: A study of connectivity in 3D combinatorial rigidity, CCCG, 44-47, 2004.

[18] D. Moore, J. Leonard, D. Rus, and S.Teller, "Robust distributed network localization with noisy range measurements," in Proc. Second ACM Conf. Embedded Networked Sensor Systems (SenSys), Nov. 2004.

[19] A. Recski, Matroid Theory and its Applications, Springer-Verlag, Berlin, 1989.

[20] A.Savvides, C.-C. Han, and M.B. Strivastave, "Dynamic fine-grained localization in ad-hoc networks of sensors," in Proc.Seventh Int'l Conf. Mobile Computing and Networking (MobiCom), pp. 166-179, 2001.

[21] B. Servatius and W. Whiteley, (1995). Constraining plane configurations in CAD: Combinatorics of directions and lengths, Preprint.

[22] A. Sljoka, O. Shai. and W. Whiteley, Checking Mobility and Decomposition of Linkages via Pebble Game Algorithm, Proceedings for ASME 2011 International Design Engineering Technical Conferences and Computers and Information in Engineering Conference, IDETC/CIE, Aug 28-31, 2011, Washington, USA, 2011.

[23] K. Sugihara, (1985). Detection of structural inconsistency in systems of equations with degrees of freedom, J. Disc Appl. Math. 10, 297–312.

[24] Thomas S,Tang X, Tapia L, et al. Simulating protein motions with rigidity analysis. J Comput Biol 2007; 14:839–55.

[25] M.F. Thorpe, D.J. Jacobs, N.V. Chubynsky and A.J. Rader, Generic rigidity of network glasses. In: Thorpe MF, Duxbury PM, eds. Rigidity theory and applications. New York: Kluwer Academic/ Plenum Publishers, pp 239277, 1999.

[26] M. F. Thorpe, Rigidity percolation Physics of Disordered Materials (Institute for Amorphous Studies Series) (New York: Plenum), 1985.

[27] W. Whiteley, Counting out to the flexibility of molecules, Phys. Biol. 2, S116 S126, 2005.

[28] W. Whiteley, Some Matroids from discrete applied geometry. In J. Bonin, J.Oxley, and B. Servatius, editors, Matroid Theory, volume 197 of Contemp. Math., pages 171-311. Amer. Math. Soc., Providence, 1996.

[29] Merse E. Gáspár and Peter Csermely . Rigidity and flexibility of biological networks. Briefings in Functional Genomics (2012) 11: 443-456.

[30] Sung-Yu Tsai , The Rigidity Property and the Unique Localization Problem of Sensor Networks, Master Degree Thesis, Hsinchu, Taiwan, Republic of China, January 2010.

[31] Brigitte Servatius. The Geometry Of Frameworks: Rigidity, Mechanisms And Cad

Websites

[29] http://kinari.cs.umass.edu, consulted the: 05/02/2016.

[30] https://www.jgraph.com, consulted the: 04/21/2016.

[31] http://linkage.cs.umass.edu/pg/, consulted the: 02/15/2016

Abstract

In this dissertation, we start our study with rigidity theory that can test rigidity and flexibility of bar and joint frameworks in 2 dimension. different applications related to rigidity theory have been made in several fields. This theory has facilitated the development of the pebble game algorithms, and we implement a computer program to test the rigidity and flexibility of a given graph.

Key Words: rigidity theory, rigidity, bar and joint frameworks, 2 dimension, pebble game algorithms, graph.

Résumé

Dans cette thèse, nous commençons notre étude avec la théorie de la rigidité. Cette théorie peut tester la rigidité et souplesse du bar and joint framworks dans 2 dimensions. Des applications différentes reposent sur la théorie de la rigidité ont été faites dans plusieurs domaines. Cette théorie a facilité le développement des algorithmes de pebble game, et nous avons implémenté un programme d'ordinateur pour tester la rigidité et la flexibilité d'un graphe donné.

Mot clés : la théorie de la rigidité, la rigidité, 2 dimension, bar and joint frameworks, les algorithmes de jeu de galets, graphe.

ملخص:

في هاته المذكرة، بدأنا دراستنا بنظرة عامة حول نظرية الصلابة، هذه النظرية التي تمكّن من اختبار قوة ومرونة الأشكال في الأبعاد الثنائية، تعتمد عدة تطبيقات على نظرية الصلابة والتي طُبقت في عدة مجالات، وقد سهلت بتطوير خوارزمية تسمى بلعبة الحصى، حيث قمنا بتطبيق برنامج كمبيوتر يسمح باختبار صلابة ومرونة شكل هندسي معين.

الكلمات المفتاحية: نظرية الصلابة، الصلابة، البعد الثنائي، خوارزمية لعبة الحصى.