PEOPLE'S DEMOCRATIC REPUBLIC OF ALGERIA
MINISTRY OF HIGHER EDUCATION AND SCIENTIFIC RESERACH

Mohamed Boudiaf University of M'sila
Faculty of Mathematics and Computer Sciences
Departement of Mathematics

Master of Mathematics

Domain : Mathematics and Computer Science
Specialty: Mathematics
Option : Partial Differential Equations and Applications

Theme

A Free Surface Flow under Gravity

Presented by :
Saadi Manel

Publicly presented on : June 5,2024.

Before the jury :

N.Benhamidouche	P.r,	University of M'sila	**Presedent**.
N.Bounab	M.C.B,	University of M'sila	**Supervisor**.
W.Delloum	M.A.A,	University of M'sila	**Examiner**.

Academic Year 2023/2024

CONTENTS

LIST OF FIGURES

Acknowledgments

First and foremost, I would like to thank "**Allah** " who bless me to finish this work.
The prophet Muhammad, may Allah bless him and grant him peace, said, "Allah does not thank the person who does not thank people".
I am very grateful to my supervisor, **Dr .BOUNAB Noura**, for his constant support, guidance, helped and encouraged me, and thanks to my teacher **Dr.ABDELKEBIR Saad** for his helpful, kind, and taking care of us and make every thing simple.
My since thanks to the presedent of the jury, **Dr.N.Benhamidouche** to accept this task and to give interest to my work. Also, my thank to **Dr.W.Delloum** to accept being the examiner of this thesis.
It is important for to thanks my familly : my parents, my sisters, and my brothers who have always been an inexhaustible source of encouraguse.
A big thanks to my freinds specail , my colleagues and all teachers of the mathematics departement for their dedecation and their generosity.

Dedication

I dedicate this thesis to my dear parents my Mother and my Father**Salem** the first reason for my success and the most beautiful blessings of God upon me . Thank you for always being with me because they were proud of me and who have always been by my side and have always supported me throughout these long years of study .As a sign of recognition , that they find here , the expression of my deep gratitude for all the efforts and means they have made to see me succeed in my studies

To my sisters **Samra**, **Hadjer** , **Rouwaida** , **Sirine** and my brothers **Chihab** and **Mohamed**source of joy and of happiness.

Special thanks to my friends **Achouak ,Chahra, Djohina ,Souad** who accompanied me throughout my academic journey in all circumstances and supported me and my supportive **T.Mohamed** , thank you for being the pillar of strength and encouragement,as I navigated the challenges of my master's thesis. your unwavering faith and endless support

To all my family And to all my friends, to my supervisor **Dr.BOUNAB NOURA** and to my teachers To all my distinguished teachers and to all who love good work and don't back down from life's obstacles. To you dear reader.

NOTATION

ρ	Density
$\overrightarrow{u}$	The speed vector
m	The mass
$\overrightarrow{a}$	Acceleration
P	Pressure
S	The surface
V	The Volume
f	the complex function
g	Gravity acceleration
F_r	Froude number
z	The complex variable
(u, v)	Components of the speed vector
ψ	stream function
ϕ	Potential Function
$M_0(x_0, y_0, z_0)$	Initial position
$\vec{a} = (M_0, t)$	The position vector
ds	surface element of normal n

INTRODUCTION

Fluid mechanics is the science of the laws of fluid flow. It is the basis for sizing fluid pipes and fluid transfer mechanisms. It is a branch of physics which studies the flows of fluids, that is to say liquids and gazez when they under go forces or constraints, It has a long history dating back to ancient civilization. The ancient Greeks were the first to study this science. Archimedes laid down the basic principles of mechanics, then it witnessed development in the 17th and 18th centuries, as it was conducted by Isaac Newton and Pascal, which led to a deeper understanding of the forces affecting fluids and their movement, fluid mechanics includes two major branches:

- Fluid statics or hydrostatics which studies fluids , It is historically the beginning of fluid mechanics, with Archimedes' thrust and the study of pressure.
- Fluid dynamics which studies fluids in motion , We also distinguish other disciplines linked to fluid mechanics: hydraulics, hydrodynamics, aerodynamics and other numerous applications in various fields such as naval engineering, aeronautics such as meteorology, climatology and even oceanography etc.

A new approach has emerged over the past few decades: digital fluid mechanics (CFD or Computational Fluid Dynamics in English), which simulates the flow of fluids by solving the equations that govern them using very powerful computers: supercomputers.

In fluid mechanics the problems of free surface flows fluids are studied thanks to their importance of application in several fields ,analytical and numerical methods using various technique such as method based on the conformal mapping transformation like Schwartz-Christoffel transformation ,the series truncation technique and the boundary integral method which consist to determine the forme of free surface for many problems of potential flow over different giving shapes of obstacle . It were used by several auther ,such as Vanden-Broeck [12],F.Dais [6],a flow by emerging a two-dimensional spout under a angle.were studied by Broeck ,H.Mekias and B.Bouderah N.Bounab [3], A.Gasmi [2] and H.Mekias,M.B. Abd-el-Malek and S.Z. Masoud

studies Linearized solution of a flow over a ramp, Appl. Math. Modelling,[14],and others...
In this work we have treated numerically a two-dimensional potential flow problem of an incompressible and inviscid fluid in channel with obstacle in the bottom inclined and forms un angle α.where the forces of gravity are considered but the tension of the superficielle surface is neglected

Our work contains three chapters and appendix, organized as follows:

The first chapter is remainder of definitions and preliminary notions concerning the theory of potential flows and the general equations of fluid motion.

The second chapter is concerned to numerically study a problem of a potential flow f an incompressible and inviscid fluid in channel of an inclined obstacle formed with the axis $(x'ox)$ an angle $\alpha = \frac{\pi}{3}$ with height H, the flow is assumed to be uniform at speed U,where we taked in account the force of gravity.

The third chapter contains a numerical procedure of our problem described in the second chapter using the series truncation technique and some discussion of our results .
Finally we finished this work with a general conclusion and an appendix.

CHAPTER 1

PRELIMINARY

Certain assymptions about the behavior and physical properties of the fluid are made to simplify the equations of motion which are complex, analytical solutions do not can only be found in certain simple situations, the simplest flow equations are obtained by considering a perfect fluid i.e. the effect of viscosity and thermodynamic effects are neglected.
In this chapter we present some definitions and properties of fluids **It contains** :

1.1 Fluids .
1.2 Description of a fluid in motion
1.3 Use of complex variable theory
1.4 Flows Fluid
1.5 Some fluid mechanics equations
1.6 Free streamline theory
1.7 Schwartz-Christoffel transformation
1.8 Froude number

1.1 Fluids

A fluid [17] is a continuous material medium, perfectly deformable and non-rigid, which takes the form of its container , fluids can be divided into liquids and gases. Gases represent compressible fluids, while liquids are practically incompressible.

Incompressible Fluids

A fluid is said to be incompressible when the volume occupied by a given mass does not vary, not depending on external pressure. Liquids can be considered fluids incompressible (water, oil, etc.)

Compressible Fluids

A fluid is said to be compressible when the volume occupied by a given mass varies in function of external pressure, Gases are compressible fluids For example air, hydrogen and methane in the gaseous state are considered compressible fluids.

1.2 Description of a fluid in motion

There are two ways to describe the movement of particles of a fluid [22]

1.2.1 Description of Lagrange

The Lagrangian description requires observing the position and velocity of each parcel of fluid individually as a function of time, that is to say each particle of fluid.
the identity of a particle is given by its initial position $M_0(x_0, y_0, z_0)$
The description of the movement is therefore to determine the position vector $\overrightarrow{r} = \overrightarrow{r}(M_0, t)$asset instant for all particles of the fluid.

$$\overrightarrow{r} = \overrightarrow{r}(M_0, t) or \overrightarrow{r} = \overrightarrow{r}(x_0, y_0, z_0, t)$$

That is to say:

$$x_i = x_i(x_0, y_0, z_0)$$

and

$$\overrightarrow{u} = \overrightarrow{u}(M_0, t) = \frac{\partial \overrightarrow{r}}{\partial t}, \overrightarrow{a} = \overrightarrow{a}(M_0, t) = \frac{\partial \overrightarrow{u}}{\partial t}$$

1.2.2 Description of Euler

In the Eulerian description of the flow of a fluid, we define a finite element called a flow domain or volume element through which the flow enters and leaves. We do not need to know the position and velocity of the mass of the fluid particles. Instead, we define, inside the volume element, a field variable, functions of space and time. For example, the field pressure is a scalar field variable for fluid flows not stationary in Cartesian coordinates.
pressure field: $p = (x, y, z, t)$ We define the velocity field as a vector field variable of the same way.
velocity field: $\vec{u} = \vec{u}(u, v, w)$
Likewise, the acceleration field is also a vector field variable Acceleration field $\vec{a} = \vec{a}(x, y, z, t)$
All of these field variables define the flow field. the velocity field from the equation can be expanded in Cartesian coordinates $(x, y, z), (i, j, k)$

$$\vec{u} = (u, v, w) = u(x, y, z, t)\vec{i} + v(x, y, z, t)\vec{j} + w(x, y, z, t)\vec{k}$$

1.3 Use of complex variable theory

Let φ andϕ a potential function and the current function respectively of a flow two-dimensional potential. We relate the flow plane to the complex plane by writing $z = x + iy$ then we define the complex function $f(z)$ [4]

$$f(z) = \phi + i\psi \tag{1.1}$$

Such as $i^2 = -1$is called the complex potential of the flow. Since the real part and the imaginary part of $f(z)$ verify the Laplace equation, moreover
we have :

$$\begin{cases} u = -\frac{\partial\phi}{\partial x} = -\frac{\partial\psi}{\partial y} \\ v = -\frac{\partial\phi}{\partial y} = -\frac{\partial\psi}{\partial x} \end{cases}$$

Then the Cauchy–Riemann relations

$$\begin{cases} \frac{\partial\phi}{\partial x} = \frac{\partial\psi}{\partial y} \\ -\frac{\partial\phi}{\partial y} = \frac{\partial\psi}{\partial x} \end{cases} \tag{1.2}$$

The theory of complex variables offers a very powerful method for obtaining solutions some flow. If the plan (x, y) is considered a plan of $z = x + iy$ function$f(z)$ will be analytical in the field of flow. Additionally, the complex speed is defined by :

$$\frac{\partial f}{\partial z} = \frac{\partial\phi}{\partial x} + i\frac{\partial\psi}{\partial y} = u - iv \tag{1.3}$$

The flow plan will also be analytical. This very important property will configure us to subsequently use the theory of complex analytic functions to solve our problem considered.

1.4 Flows Fluid

1.4.1 stationary flows

$$\partial_t u = \partial_t \rho = \partial_t T = \partial_t p = 0$$

Such a flow is possible when the domain, the applied mass forces, the sources heat and edge conditions are also independent of time. For example the stationary Navier–Stokes equations are written

$$\begin{cases} -v\Delta u + \rho(u\nabla)u + \nabla p \\ divu = 0 \end{cases} \tag{1.4}$$

1.4.2 Incompressible flow

A flow is said to be incompressible when the volume occupied by a given mass does not vary depending on the external pressure its density is constant.

$$\rho = cte$$

1.4.3 Potential flow

We say that the flow is potential if its velocity vector is derived from a potential, that is to say

$$\overrightarrow{u} = \nabla\phi$$

$$u = \frac{d\phi}{dx}, v = \frac{d\phi}{dy}$$

the function $\phi(x, y)$ is the speed potential.

1.4.4 Irrotational flow

A flow is called irrotational flow if:

$$rot\overrightarrow{u} = 0 \tag{1.5}$$

Naturally, a flow that is not irrotational is said to be rotational. A flow potential is an irrotational flow. In fact, we have:

$$u = \nabla\phi = (\frac{\partial\phi}{\partial x}, \frac{\partial\phi}{\partial y}, \frac{\partial\phi}{\partial z})$$

$rotu = (\frac{\partial\phi}{\partial y}(\frac{\partial\phi}{\partial z}) - \frac{\partial\phi}{\partial z}(\frac{\partial\phi}{\partial y}), \frac{\partial\phi}{\partial z}(\frac{\partial\phi}{\partial x}) - \frac{\partial\phi}{\partial x}(\frac{\partial\phi}{\partial z}), \frac{\partial\phi}{\partial y}(\frac{\partial\phi}{\partial x}) - \frac{\partial\phi}{\partial x}(\frac{\partial\phi}{\partial y}))$

1.4.5 Uniform flow

a flow is said to be uniform if the velocities of all the particles are the same at every point of the fluid [5]

1.5 Some fluid mechanics equations

we have this equations [13]

1.5.1 Continuity equation

Consider a part of a fluid of density delimited by a closed surface S (of volume V).
Let dS be an elementary vector of this surface, oriented outwards to the closed surface. The fluid part has mass

$$m = \int\int_v\int \rho dv$$

The mass flow leaving the surface S is equal to R $\int\int_s \rho u dS$
The conservation of mass is written as:

$$\frac{dm_s}{dS} - \int\int_s \rho\overrightarrow{u}dS = \int\int_v\int \frac{\partial\rho}{\partial t}dV$$

or $\frac{dm_s}{dS}$represents the mass flow rate of internal fluid at the volume considered, counted positively if it is a source and negatively if it is a sink. Considering the theorem of Ostrogradsky to transform the surface integral into a volume integral.

$$\int\int_s \overrightarrow{u}dS = \int\int_v\int div(\rho\overrightarrow{u})dV$$

The conservation of mass equation written:

$$\frac{dm_s}{dS} = \int\int_v\int (div(\rho\overrightarrow{u} + \frac{\partial\rho}{\partial t})$$

The equality written above is valid whatever the volume V considered and the integral is zero, which leads to the local expression of the conservation of mass:

$$div(\rho\overrightarrow{u}) + \frac{\partial\rho}{\partial t} = 0 \tag{1.6}$$

1. If the fluid is incompressible, the density does not change over time and the mass conservation equation reduces to:

   $$div\overrightarrow{u} = 0 \tag{1.7}$$

   For stationary or non-stationary flow. This flow is called isovolume. The equation (1.7)expresses the conservation of the volume of an element of fluid during of its deformation by the flow.

2. The case of a stationary flow $\frac{\partial\rho}{\partial t} = 0$ SO :

   $$div(\rho u) = 0 = \rho div(u\nabla)\rho$$

Apart from case 1, there is the possibility of isovolume flows such that $(u\nabla)\rho = 0$ that is to say the variations in density are orthogonal, at all points, to the velocity vector.

1.5.2 Conservation of fluid energy

We will evaluate the temporal evolution of the kinetic energy of a fluid element of unit volume and mass, limiting ourselves to the flows of incompressible fluids:

$$\frac{\partial}{\partial t}\left(\frac{\rho u^2}{2}\right) = \rho u_i \frac{\partial u_i}{\partial t} \tag{1.8}$$

Using the equation of motion to express the Eulerian derivative of velocity (1.8)
becomes :

$$\frac{\partial}{\partial t}\left(\frac{\rho u^2}{2}\right) = \rho u_j \frac{\partial u_i}{\partial x_j} + u_i \frac{\partial \sigma_{i,j}}{\partial x_j} + u_i f_i$$

Or, by decomposing the stress tensor as previously into an isotropic part$-p\delta_{i,j}$,and in a deviator$d_{i,j}$

$$\frac{\partial}{\partial t}\left(\frac{\rho u^2}{2}\right) = u_j \frac{\partial}{\partial x_j}\left(\frac{\rho u^2}{2} - p\right) + \frac{\partial u_i d_{i,j}}{\partial x_j} - d_{i,j}\frac{\partial u_i}{\partial x_j} + u_i f_i \tag{1.9}$$

Or, in vector notation:

$$\frac{\partial}{\partial t}\left(\frac{\rho u^2}{2}\right) = u.\nabla\left(\frac{\rho u^2}{2} - p\right) + \nabla.(u.d) - d.\nabla u + u.f \tag{1.10}$$

Finally, taking into account the condition of incompressibility $(\nabla.u = 0)$,we can put the first term on the right hand side of (1.10)in the form of a divergence, i.e.:

$$\frac{\partial e_c}{\partial t} = \nabla.\left[u\nabla\left(\frac{\rho u^2}{2} - p\right) + u.d)\right] - d.\nabla u + u.f \tag{1.11}$$

Let us rewrite this equation for the evolution of kinetic energy in integral form, integrating each of the terms on a fixed volume V and using the divergence theorem:

$$\frac{\partial}{\partial t}\left(\int_v e_c dV\right) = \int_s \frac{\rho u^2}{2} u.ndS + \int_s (\sigma.u).ndS + \int_v u.fdV - \int_v \sigma.\nabla udV \tag{1.12}$$

What is the physical meaning of the different terms of (1.12)

1. the first term of the second member is the flow of kinetic energy ”convected” by the flow through the surface S.
2. the second term is the work, per unit of time, of the stresses exerted on the surface S.
3. the third term is the work, per unit of time, of the forces in volume
4. the third term is the work, per unit of time, of the forces in volume
5. finally, the fourth term is associated with the deformation of the volume V. It represents the energy dissipated by viscosity during this deformation.

1.5.3 Line and stream Function

power lines

we call the streamline the curve which, at each of its points, is tangent to the vector speed . Its differential equation is written:

$$\frac{dx}{u(x,y,z)} = \frac{dy}{v(x,y,z)}$$

stream function

if we consider the flow is incompressible $\frac{D\rho}{Dt} = 0$ then the continuity equation will be given:

$$div\overrightarrow{u} = 0$$

Or

$$\frac{\partial u}{\partial x} = \frac{\partial v}{\partial t} \tag{1.13}$$

We present a new function ψ of x and y which we call current function, checking:

$$u = \frac{\partial \psi}{\partial y}, v = -\frac{\partial \psi}{\partial x} \tag{1.14}$$

The surfaces defined by $\psi = cte$are current lines, in fact, the exact differential of given :

$$d\psi = \frac{\partial \psi}{\partial x}dx + \frac{\partial \psi}{\partial y}dy = -vdx + udy \tag{1.15}$$

Since $\psi = cte$ so $\partial\psi = 0$we find the equation of the current line according to (1.14) Let it be,C a fine curve which goes from one streamline to another characterized by $\psi = \psi_1$ and $\psi = \psi_2$ respectively.

Let it be $\overrightarrow{n}$ a unit vector normal to C and oriented in the direction of the flow, the flow at through C given by:

$$Q = \int_c \overrightarrow{u}.\overrightarrow{n} = \int_c \left(-u\frac{\partial y}{\partial t} + v\frac{\partial x}{\partial t}\right) = \int_c (vdx - udy)dt$$

or

$$Q = \int_c \left(\frac{\partial \psi}{\partial x}dx + \frac{\partial \psi}{\partial y}dy\right) = \int_c d\psi$$

Therefore

$$Q = \psi_1 + \psi_2 \tag{1.16}$$

1.5.4 Equation of fluid motion

By the fundamental relation of dynamics, the temporal variation of the quantity of movement of an element of volume V is equal to the sum of the forces exerted on this volume element, i.e.:

$$\frac{d}{dt}\left(\int_v \rho u dx\right) = \int_v \rho\frac{du}{dt}dx$$

The integral of the surface forces can be written using Ostrogradsky's theorem under the shape $\int_v div\sigma dx$ By making the volume V tend towards zero the equation of motion becomes:

$$\rho(\partial_t u + (u.\nabla)u) = f + div\sigma \tag{1.17}$$

1.5.5 Differential equations of functions ψ and ϕ

Consider a two-dimensional, irrotational and stationary flow of an incompressible fluid non–viscous. Since [4] :

$$\overrightarrow{u} = grad\overrightarrow{\phi}$$

and

$$div\overrightarrow{u} = 0$$

It turns out that:

$$div\left(grad\overrightarrow{\phi}\right) = 0$$

That's to say :

$$\Delta\phi = 0 \tag{1.18}$$

Likewise, according to:

$$\overrightarrow{u} = (u, v) = \left(-\frac{\partial\psi}{\partial x}, \frac{\partial\psi}{\partial y}\right)$$

and

$$rot\overrightarrow{u} = 0$$

We find :

$$\frac{\partial u}{\partial y} = \frac{\partial v}{\partial x}$$

or

$$\frac{\partial^2\psi}{\partial x^2} = \frac{\partial^2\psi}{\partial y^2}$$

that is to say

$$\Delta\psi = 0$$

Hence, the potential function ϕ and the power line function ψ and verify the equation of The place.
A two-dimensional, irrotational and stationary flow of an incompressible fluid, not viscous is potential flow.

1.5.6 Stokes equations

The Navier–Stokes equation [18]:

$$\begin{cases} -v\nabla u + \rho(u.\nabla)u + \nabla p = f \\ divu = 0 \end{cases} \tag{1.19}$$

By neglecting the proportional terms in the stationary incompressible Navier–Stokes equation to the density of the fluid $(u.\nabla)u$, we obtain the Stokes equation

$$\begin{cases} -v\nabla u+\nabla p=f \\ divu=0 \end{cases} \tag{1.20}$$

The smaller the flow speed compared to the dimensions of the and the viscosity value, plus the Stokes model is a valid approximation of the Navier–Stokes equations. The fundamental difference between the two equations is that the nonlinear term in velocity has gone, the Stokes equation is a linear partial differential equation.

1.5.7 Bernoulli's theorem

Bernoulli's theorem is an application of the conservation of energy to the case of fluids in motion.

Bernoulli's first theorem

In a stationary flow, along a trajectory we have conservation of charge

$$H=\frac{\hat{p}}{\rho g}+\frac{u^2}{2g}=z+\frac{p}{\rho g}+\frac{u^2}{2g}=const$$

$\hat{p}=pgz$

Bernoulli's second theorem

In a potential flow the Euler equation is written:

$$\rho\left[\nabla\left(\frac{\partial\phi}{\partial t}\right)+\nabla\frac{u^2}{2}\right]=-\nabla\hat{p}$$

$$\Leftrightarrow\frac{\partial\phi}{\partial t}+\frac{u^2}{2}+\frac{\hat{p}}{\rho}=\frac{\partial\phi}{\partial t}+\frac{u^2}{2}+\frac{p}{\rho}+gz=\frac{\partial\phi}{\partial t}+gH$$

1.5.8 Energy conservation equation

The energy conservation equation is obtained from the first law of thermodynamics. This principle connects the different forms of energy,
either :

$$\rho C_p\frac{dT}{dt}=div\left[\lambda.\overrightarrow{gradT}\right]+T.\beta\frac{dP}{dt}+\varphi+P_s \tag{1.21}$$

1.6 Free streamline theory

The theory of free current lines consists of studying potential flow problems, limited by rigid rectilinear walls and free current lines of unknown shapes, on which the pressure is assumed to be constant.

If free streamlines are not present and gravity effects are neglected, the flow region in the physical plane is a polygon.

Also the free current lines present and the effects of gravity as well as the effects of surface tension are neglected, the flow region can be transformed by a transformation conforms to a polygonal region
This region is a perfect of the defined hodograph plane

$$\Omega = log\left(U/\frac{df}{dz}\right) = log\left(\frac{u}{u-iv}\right) = log\left(\frac{u}{q}\right) + i\theta \tag{1.22}$$

or $f = \varphi + i\psi$, $\frac{df}{dz} = u - iv$, $q = \sqrt{u^2+v^2}$, $(u,v)$are the components of the following velocity vector of the axis x and the y axis respectively θ is the angle that the velocity vector makes with the horizontal and U the reference speed

- real part of Ω is constant on the free current line, i.e.$log\left(\frac{u}{q}\right)$
- The imaginary part of is constant on each rectilinear wall i.e $\theta = cte$

Therefore, the flow is represented by a plane figure with straight sides (polygon) note Ω
Using Schwarz-Christoffel transformation, the domain Ωpolygonal is transformed into a upper half plane of the auxiliary variable λ So, in plan $\lambda$the flow is uniform represented by the potential function $F(\lambda) = c\lambda$ To illustrate the above, we give some properties of the Schwarz-Christoffel conformal transformation

1.7 Schwartz-Christoffel transformation

We consider a polygon,in the plan Ωhaving as vertices $A_1, A_2, A_3, ..., A_n$ the points corresponding respectively to $\lambda_1, \lambda_2, ..., \lambda_n$ of the real axis of the plane of the λSchwarz–Christoffel transformation, transforms the interior of a polygon
in the upper (or lower) half-plane of another plane. The transformation is given by [18]:

$$\frac{d\Omega}{d\lambda} = \alpha(\lambda-\lambda_1)^{\frac{\alpha_1}{\pi}-1}(\lambda-\lambda_2)^{\frac{\alpha_2}{\pi}-1}...(\lambda-\lambda_n)^{\frac{\alpha_n}{\pi}-1} \tag{1.23}$$

Or

$$\Omega = \alpha\int(\lambda-\lambda_1)^{\frac{\alpha_1}{\pi}-1}(\lambda-\lambda_2)^{\frac{\alpha_2}{\pi}-1}...(\lambda-\lambda_n)^{\frac{\alpha_n}{\pi}-1} + \beta \tag{1.24}$$

or αandβ are complex constants. It will be noted that:

1. Among the points $\lambda_1, \lambda_2, ..., \lambda_n$we can choose three arbitrarily.
2. Constants α and β determine the orientation and position of the polygon.
3. It is convenient to choose a point, for example $\lambda_n$to infinity, case in which factor of (1.24)does not exist.
4. Infinite non-closed polygons can be considered as limited cases of polygons.

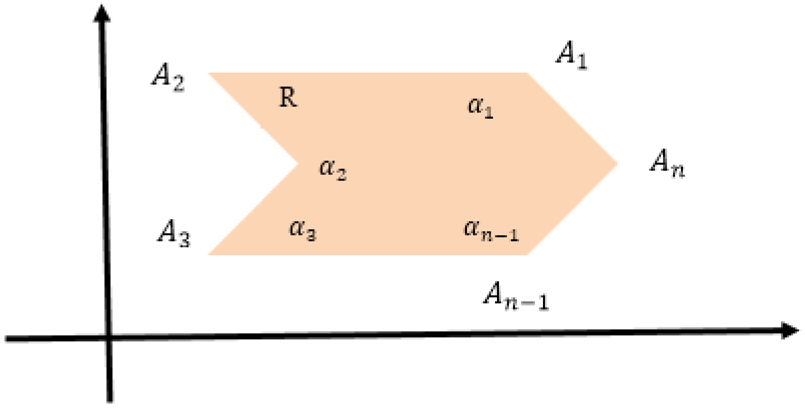

Figure 1.1: plan of Ω.

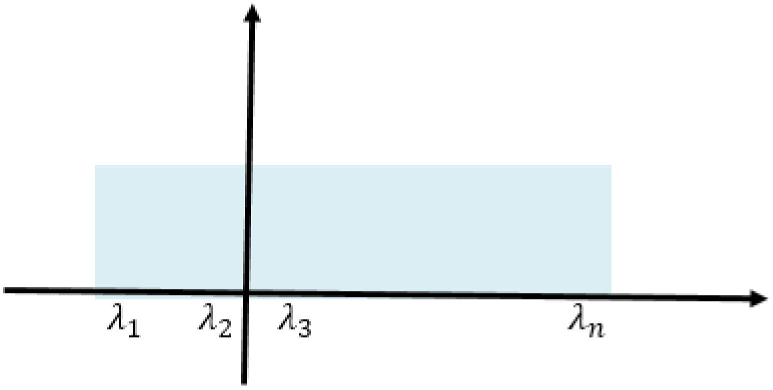

Figure 1.2: variable planeλ.

Froude number

The Froude number, denoted as (Fr), is a dimensionless number in fluid mechanics that is used to indicate the influence of gravity on the motion of fluids. It is defined as the ratio of the flow inertia to the external field, which in many applications is due to gravity. The Froude number is expressed as:

$$Fr = \frac{U}{\sqrt{gH}}$$

Here, (U) represents the velocity of the flow, (g) is the acceleration due to gravity, and (L) is a characteristic length, such as the depth of the flow or the length of a wave.

CHAPTER 2

PROBLEM POSITION

In this chapter we presented and treated numerically nonlinear super-critical flow of ideal fluid over apolygonal obstacle by specifying uniform flow upstream of the obstcale, finally we have formulate problem using ,It contains :

2.1 Problem's position
2.2 Formulation of the problem
2.3 Numerical procedure
2.4 Asymtotic behaviour the neighborhood of w

2.1 Problem's position

Let's consider a two-dimensional potential flow of an incompressible and inviscid fluid in front of an inclined obstacle formed with the axis $(x'ox)$ an angle $\alpha = \frac{\pi}{3}$ and height H, the flow is assumed to be uniform at speed U. As shown in the following figure:

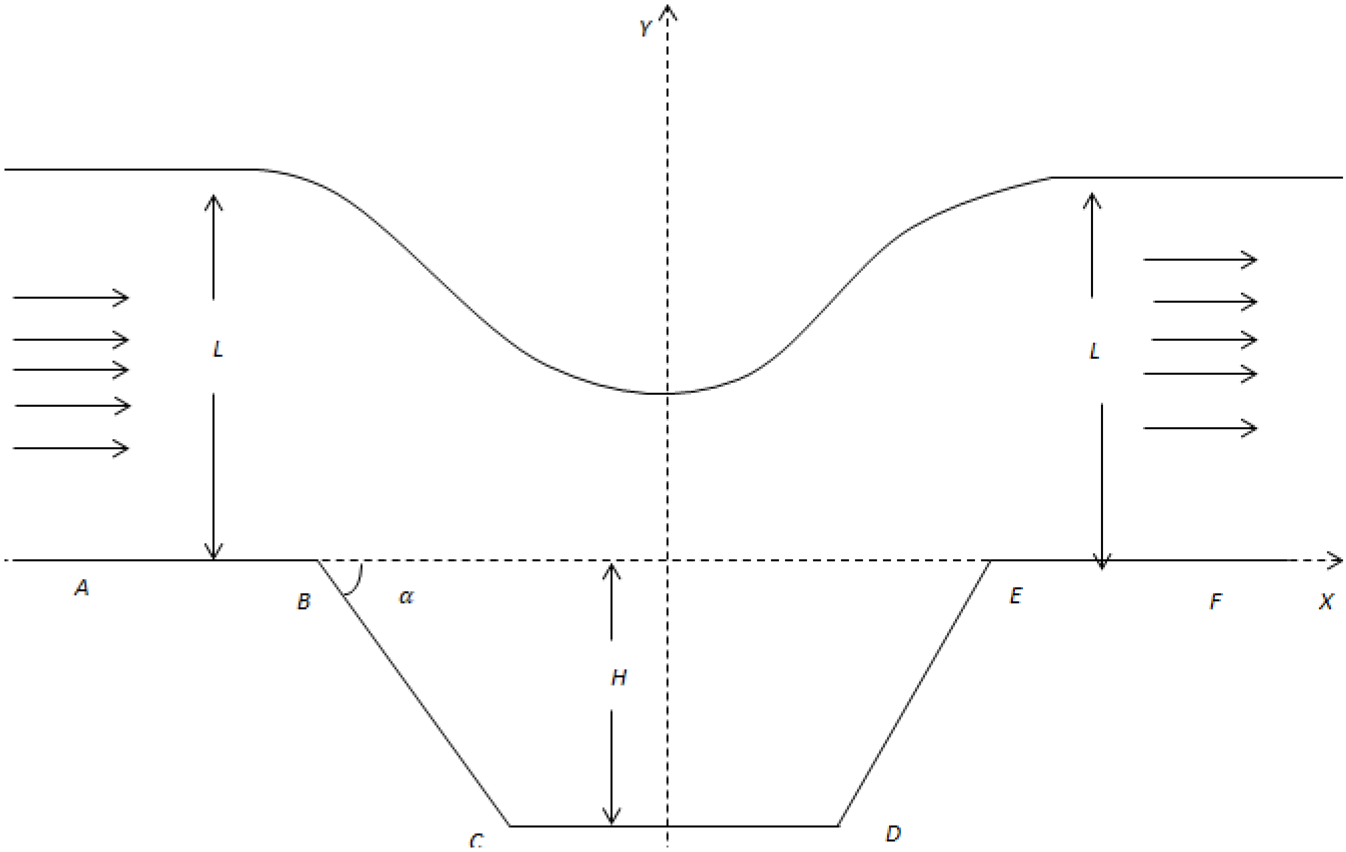

Figure 2.1: Physical plane of the flow and of the coordinates and the general symmetric obstacle, z-plane.

2.2 Formulation of the problem

Let's consider a two-dimensional potential flow of an incompressible and inviscid fluid in front of an inclined obstacle formed with the axis $(x'ox)$ an angle $\alpha = \frac{\pi}{3}$ and height H, the flow is assumed to be uniform at speed U.As the free stream line and the free surface are indicated by $(A'B'C')$ on note $\xi = u - iv$ such as u,v are the components of the velocity vector and by

$$f = \phi + i\psi$$

The complex potential function where ϕ, ψ are respectively the potential function and the stream function.The conditions on the plane f ,$\phi = 0$on point $(0,0)$,$\psi = 0$ sur ABC
$\psi = HU = 1$ on $(A'B'C')$.

The Bernoulli equation on the free surface $(A'B'C')$ is given by:

$$\frac{1}{2}q^2 + \frac{1}{F^2}k = \frac{1}{2} \tag{2.1}$$

F represent the Froude number,q the flow rate ,K is the free surface curvature.and since
$\xi = u - iv$ is analytical ,We define the function $\tau - i\theta$ the relation is

$$\xi = u - iv = exp(\tau - i\theta) \tag{2.2}$$

θ is the angle between the velocity vector and the horizon, the removal of curvature K

given :

$$\vec{u} = exp(\tau)(cos(\theta)\vec{i} + sin(\theta)\vec{j})$$

and in curvilinear coordinates

$$\vec{u} = |\vec{u}|.\vec{u_T}$$

$\vec{u_T} = cos(\theta)\vec{i} + sin(\theta)\vec{j}$ is the unit tangent vector

$$\begin{aligned}
\vec{u_n} &= R.\frac{d\vec{u_T}}{ds} = R.\frac{d\vec{u_T}}{dt}.\frac{dt}{ds} \\
\vec{u_n} &= Rexp(\tau)\left(\frac{\partial\theta}{\partial\phi}.\frac{\partial\phi}{\partial x}.\frac{\partial x}{\partial t} + \frac{\partial\theta}{\partial\phi}.\frac{\partial\phi}{\partial y}.\frac{\partial y}{\partial t}\right)\frac{d\vec{u_T}}{d\theta} \\
\vec{u_n} &= R.exp(\tau)(\frac{\partial\theta}{\partial\phi})(cos\theta\vec{i} - sin\theta\vec{j})
\end{aligned}$$

$\vec{u_n}$: is the unit normal vector and, ds represents the length element on the free surface, R the radius of curvature of the free surface.
Finalement ,on obtient

$$k = \frac{1}{R} = exp(\tau)\left|\frac{\partial\theta}{\partial\phi}\right|$$

We replace k by its value in the equation(2.1) we find

$$\frac{1}{2}q^2 + \frac{1}{F^2}exp(\tau)\left|\frac{\partial\theta}{\partial\phi}\right| = \frac{1}{2}$$

On the other hand, we know that$\theta(\phi)$ is an increasing function $(-\infty < \phi < +\infty)$ on the free surface $(A'B'C')$, therefore the Bernoulli equation in the plane f is written:

$$exp(2\tau) + \frac{2}{F^2}exp(\tau)\left|\frac{\partial\theta}{\partial\phi}\right| = 1 \qquad (2.3)$$

With the conditions $\Delta\phi = 0$ and $\psi = 1 = UH$ The mathematical problem is to determine the function $\tau - i\theta$ which is analytical in the band $0 < \psi < H = 1$ and who to check the conditions equations (2.3)

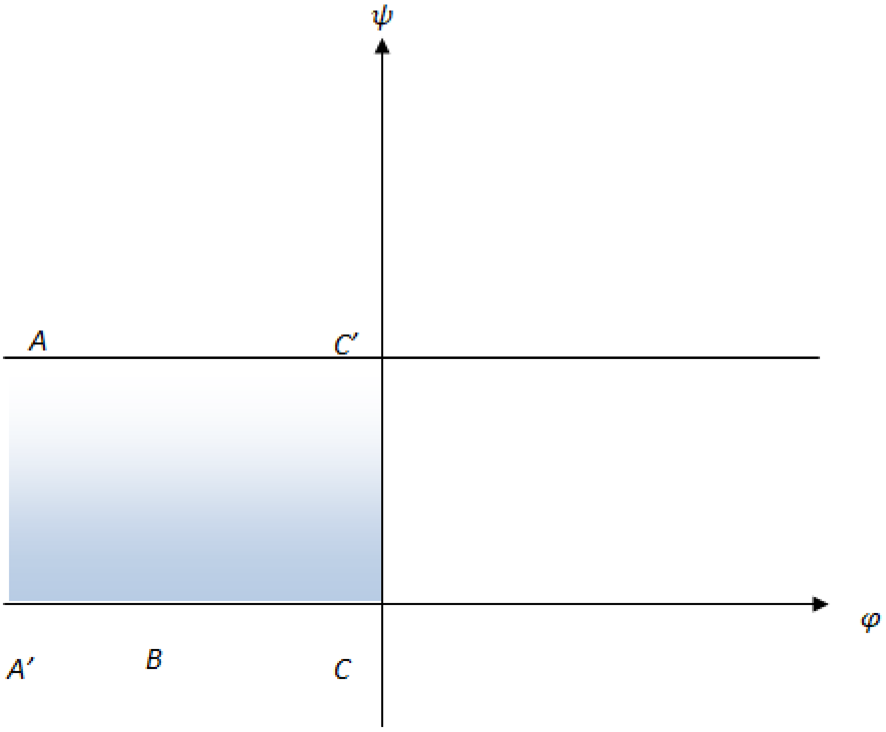

Figure 2.2: f plane with symetrie

2.3 Numerical procedure

To numerically solve this problem, we apply the truncation technique of the series. We transform the domain occupied by the fluid in the plane$f$.In a half disk unity in the variable planeω figure
By transformation:

$$f = \frac{2}{\pi} ln \frac{1-\omega}{1+\omega} \tag{2.4}$$

The points A' B' C' and A B C in the plane f are transformed respectively to the points

A=A'	(1,0)
B	(0.65,0)
B'	(-1,0)
C	(0,0)
C'	(0,-1)

the free surface ($A'B'C'$) is transformed into a half disk figure [2.4]

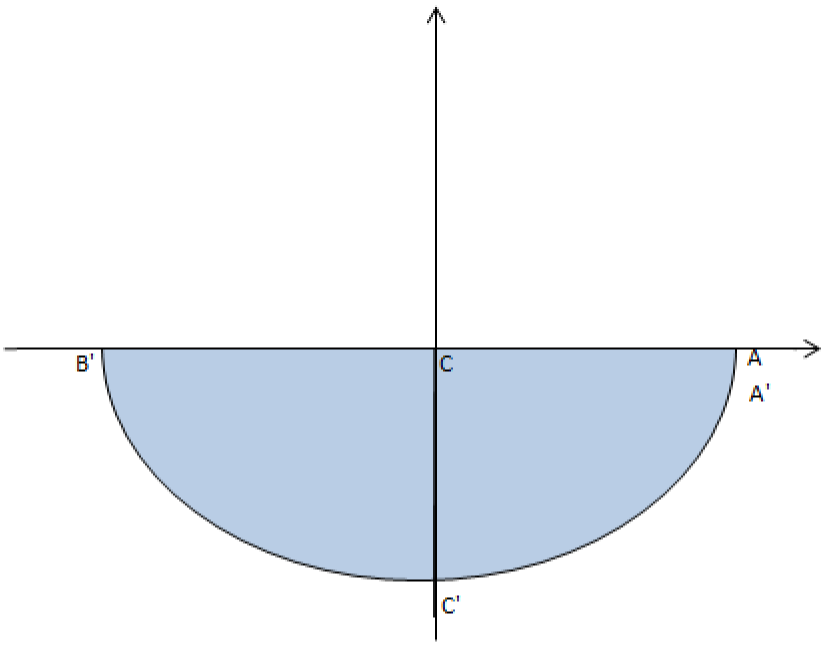

Figure 2.3: The upper half circle-plane, w-plane

The points of the free surface in the plane w are given by the relation

$$\omega = exp(i\sigma) \quad such\ that \quad \pi < \sigma < -\pi \tag{2.5}$$

And in the plane f by the relation

$$f = \phi, \qquad -\infty < \phi < +\infty \tag{2.6}$$

To solve the problem, we must write the equation

$$\frac{1}{2}exp(2\tau) + \frac{1}{F^2}exp(\tau)\left|\frac{\partial\theta}{\partial\phi}\right| = \frac{1}{2}$$

By substituting(2.5) in (2.4) we obtain

$$f = \phi = \frac{2}{\pi}log(-itan(\frac{\sigma}{2}))$$

Which implies

$$df = d\phi = \frac{2}{\pi sin(\sigma)}d\sigma$$

From where

$$\frac{\partial\phi}{\partial\sigma} = \frac{2}{\pi sin(\sigma)}$$

Then, the Bernoulli equation on the free surface $(A'B'C')$

$$exp(2\tau) + \frac{\pi}{F^2}sin(\sigma)exp(\tau)\frac{\partial\theta}{\partial\sigma} = 1 \tag{2.7}$$

We now have determined the local behaviour of the flow near the singularities

2.4 Asymtotic behaviour the neighborhood of $\omega = \omega_B$

The flow, in the neighborhood of ω_B is a flow in angle $\alpha = \frac{4\pi}{3}$, in a plane z at the neighborhood $z = z_\omega$ in the complex function is given by:

$$f(z) \sim \frac{a}{n}(z - z_B)^n, n = \frac{\pi}{\alpha} = \frac{3}{4} \;\; when \;\; z \to z_B \tag{2.8}$$

this implies

$$f(z) \sim \frac{4a}{3}(z - z_B)^{\frac{3}{4}} \;\; when \;\; z \to z_B \tag{2.9}$$

since $\xi = \frac{df}{dz}$ we find

$$\xi \sim \frac{df}{dz} \sim a(z - z_B)^{\frac{-1}{4}} \;\; when \;\; z \to z_B \tag{2.10}$$

we have

$$f = \phi = \frac{2}{\pi} log\left(\frac{1-w}{1+w}\right) \quad on\ the\ free\ surface \tag{2.11}$$

of the equation (2.8) and (2.11) we find

$$\frac{2}{\pi} log\left(\frac{1-w}{1+w}\right) = \frac{4a}{3}(z - z_B)^{\frac{3}{4}} \tag{2.12}$$

so we find

$$\frac{6}{4\pi a} log\left(\frac{1-w}{1+w}\right) = (z - z_B)^{\frac{3}{4}}$$

so

$$\left(\frac{3}{2\pi a} log\left(\frac{1-w}{1+w}\right)\right)^{\frac{4}{3}} = (z - z_B) \;\; when\ w_B \to -b$$

so we find

$$z \sim \left(\frac{3}{2\pi a} log\left(\frac{1-w}{1+w}\right)\right)^{\frac{4}{3}} - b \tag{2.13}$$

by substituting (2.10) and (2.13) we find

$$\xi = \frac{df}{dz} \sim a\left[\left(\frac{3}{2\pi a} log\left(\frac{1-w}{1+w}\right)\right)^{\frac{4}{3}} - b + b\right]^{\frac{-1}{4}} \tag{2.14}$$

so

$$\xi = \frac{df}{dz} \sim a\left(\frac{3}{2\pi a} log\left(\frac{1-w}{1+w}\right)\right)^{\frac{-1}{3}}$$

we have

$$log\left(\frac{1-w}{1+w}\right) \sim -2w \ utilization\ taylor's\ theorem$$

so we find

$$\xi \sim \frac{df}{dz} \sim a\left(\frac{3}{a}\right)^{\frac{-1}{3}}\left(\frac{-w}{\pi}\right)^{\frac{-1}{3}}$$

$$\xi \sim \frac{df}{dz} \sim c_1 w^{\frac{-1}{3}}$$

such as

$$c_1 \sim a\left(\frac{3}{a}\right)^{\frac{-1}{3}}\left(\frac{-1}{\pi}\right)^{\frac{-1}{3}}$$

2.5 Asymtotic behaviour the neighborhood of $\omega = \omega_C$

the flow, in the neighborhood of ω_C is a flow in angle $\alpha = \frac{2\pi}{3}$, in a plane z at the neighborhood $z = z_C$ in the complex function is given by:

$$f(z) \sim \frac{a}{n}(z - z_B)^n, n = \frac{\pi}{\alpha} = \frac{3}{2} \;\; when \;\; z \to z_C \tag{2.15}$$

this implies

$$f(z) \sim \frac{2a}{3}(z - z_C)^{\frac{3}{2}} \;\; when \;\; z \to z_C \tag{2.16}$$

since $\xi = \frac{df}{dz}$ we find

$$\xi \sim \frac{df}{dz} \sim a(z - z_C)^{\frac{1}{2}} \;\; when \;\; z \to z_C \tag{2.17}$$

we have

$$f = \phi = \frac{2}{\pi} log\left(\frac{1-w}{1+w}\right) \quad on\ the\ free\ surface \tag{2.18}$$

of the equation (2.17) and (2.18) we find

$$\frac{2}{\pi} log\left(\frac{1-w}{1+w}\right) = \frac{2a}{3}(z - z_C)^{\frac{3}{2}} \tag{2.19}$$

so we find

$$\frac{6}{2a\pi} log\left(\frac{1-w}{1+w}\right) \sim (z - z_C)^{\frac{3}{2}}$$

so

$$\left(\frac{6}{2\pi a} log\left(\frac{1-w}{1+w}\right)\right)^{\frac{2}{3}} = (z + c) \;\; when\ z \to -c$$

so we find

$$z = \left(\frac{3}{\pi a} log\left(\frac{1-w}{1+w}\right)\right)^{\frac{2}{3}} - c \tag{2.20}$$

by substituting (2.20) and (2.17) we find

$$\xi = \frac{df}{dz} \sim a\left[\left(\frac{3}{\pi a} log\left(\frac{1-w}{1+w}\right)\right)^{\frac{2}{3}} - c + c\right]^{\frac{1}{2}} \tag{2.21}$$

so

$$\xi = \frac{df}{dz} \sim a\left(\frac{3}{\pi a} log\left(\frac{1-w}{1+w}\right)\right)^{\frac{1}{3}}$$

we have

$$log\left(\frac{1-w}{1+w}\right) \sim -2w\ utilization\ taylor's\ theorem$$

so we find

$$\xi \sim \frac{df}{dz} \sim a\left(\frac{3}{a}\right)^{\frac{1}{3}} \left(\frac{-2w}{\pi}\right)^{\frac{1}{3}}$$

$$\xi \sim \frac{df}{dz} \sim c_2 w^{\frac{1}{3}}$$

such as

$$c_2 \sim a\left(\frac{3}{a}\right)^{\frac{1}{3}} \left(\frac{-2}{\pi}\right)^{\frac{1}{3}}$$

After this step ,we will try in the next chapter to write the function ψ in the form of an analytical series where we will use the series truncation technique

CHAPTER 3

RESULTS AND DISCUSSION

In this chapter we have solved numerically our non linear problem discribed in this chapter2,using series truncation technique .where obtained the form of the free surface basid on the froud number,Its contains:

3.1 Formulation of the series
3.2 The form of the free surface
3.3 Numerical result and discussion

3.1 Formulation of the series

We define the function $\xi(w)$ therefore

$$\xi(w) = g(w)\Omega(w)$$

where $g(w)$ contains singularities and zeros, the function $\Omega(w)$ is analytical, it is develops in series, that is to say:

$$\xi(w) = g(w)exp\left(\sum_{n=1}^{+\infty} a_n w^n\right)$$

where $g(w)$ contains the stagnation and zeros of at point B and point C therefore:

$$\xi = u - iv = \left((w+b)^{\frac{-1}{3}}(w+c)^{\frac{-1}{3}}\right) exp\left(\sum_{k=1}^{+\infty} a_{2k} w^{2k}\right) \tag{3.1}$$

where thea_k are real constants to be determined. The equation (3.1)checked all conditions to the limits

$$exp(2\tau) + \frac{\pi}{F^2} sin(\sigma) exp(\tau) \frac{\partial\theta}{\partial\sigma} = 1 \quad on\ the\ free\ surface\ A'B'C'$$

We substitute the equation $w = exp(i\sigma) = 1$ in the equation (3.1) we obtain

$$exp(\tau - i\theta) = (1+b)^{\frac{-1}{3}}(1+c)^{\frac{1}{3}} exp\left(\sum_{k=1}^{+\infty} a_{2k} e^{2i\sigma k}\right)$$

so we find

$$\tau - i\theta = log\left((1+b)^{\frac{-1}{3}}(1+c)^{\frac{1}{3}}\right) + \sum_{k=1}^{+\infty} a_{2k} cos(2k\sigma) + i\sum_{k=1}^{+\infty} a_{2k} sin(2k\sigma)$$

so

$$\tau - i\theta = \frac{-1}{3} log(1+b) + \frac{1}{3} log(1+c) + \sum_{k=1}^{+\infty} a_{2k} cos(2k\sigma) + i\sum_{k=1}^{+\infty} a_{2k} sin(2k\sigma)$$

By separating the real part and the imaginary part, we find:
The real part:

$$\tau(\sigma) = \frac{-1}{3} log(1+b) + \frac{1}{3} log(1+c) + \sum_{k=1}^{+\infty} a_{2k} cos(2k\sigma) \tag{3.2}$$

The imaginary part:

$$\theta(\sigma) = \sum_{k=1}^{+\infty} a_{2k} sin(2k\sigma) \tag{3.3}$$

Substituting and (3.2) into (3.3) on the free surface we find:

$$exp\left(\frac{-2}{3} log(1+b) + \frac{2}{3} log(1+c) + 2\sum_{k=1}^{+\infty} a_{2k} cos(2k\sigma)\right) + \frac{\pi}{F^2}$$

$$sin\sigma exp\left(\frac{-1}{3} log(1+b) + \frac{1}{3} log(1+b) + \sum_{k=1}^{+\infty} a_{2k} cos(2k\sigma)\right) \sum_{k=1}^{+\infty} 2k a_{2k} cos(2k\sigma) = 1 \tag{3.4}$$

We will determinate the cofficient a_n
here $\tau(\sigma)$ and $\theta(\sigma)$ denote the values of θ and τ on the free surfaces $A'B'C'$ and ABC we solve the problem numerically by truncating the infinity series in (2.7) after N terms .we find the $N-1$ coefficients a_n and the angle α by collocation .thus we introduce the N mesh points

$$\sigma_1 = \frac{\pi}{N}(I - \frac{1}{2}), I = 1, ..., N \tag{3.5}$$

we use (3.4) and (3.5) we obtain a system of N +1 equation
We use Newton's method to solve this system.

3.2 The form of the free surface

After finding the coefficients a_k, the form of the free surface is determined as following : Of the relations

$$\frac{1}{u-iv} = exp(-\tau+i\theta) = \frac{\partial x}{\partial\phi} + i\frac{\partial y}{\partial\phi} \tag{3.6}$$

with

$$dz = dx + idy = exp(-\tau+i\theta) = (d\phi + id\psi) \tag{3.7}$$

$\psi = 0$ on the free surface

we have

$$\begin{cases} \frac{\partial x}{\partial\phi} = exp(-\tau)cos\theta \\ \frac{\partial y}{\partial\phi} = exp(-\tau)sin\theta \end{cases} \tag{3.8}$$

and

$$\begin{cases} \frac{\partial x}{\partial\sigma} = \frac{\partial x}{\partial\phi}\frac{\partial\phi}{\partial\sigma} \\ \frac{\partial y}{\partial\sigma} = \frac{\partial y}{\partial\phi}\frac{\partial\phi}{\partial\sigma} \end{cases}$$

we find

$$\begin{cases} \frac{\partial x}{\partial\sigma} = exp(-\tau)cos\theta\frac{\partial\phi}{\partial\sigma} \\ \frac{\partial y}{\partial\sigma} = exp(-\tau)sin\theta\frac{\partial\phi}{\partial\sigma} \end{cases}$$

Substituting (3.2) and (3.3) and $\frac{\partial\phi}{\partial\sigma}$=$\frac{2}{\pi sin\sigma}$ we find $\frac{\partial x}{\partial\sigma}, \frac{\partial y}{\partial\sigma}$ At every point $\sigma_I, I = 1, ..., N+1$ so find

$$\begin{cases} \frac{\partial x}{\partial\sigma} = \frac{2}{\pi sin\sigma}exp(\frac{1}{3}log(1+b) - \frac{1}{3}log(1+c) - \sum_{k=1}^{+\infty} a_{2k}cos(2k\sigma))cos(\sum_{k=1}^{+\infty} a_{2k}sin(2k\sigma)) \\ \frac{\partial y}{\partial\sigma} = \frac{2}{\pi sin\sigma}exp(\frac{1}{3}log(1+b) - \frac{1}{3}log(1+c) - \sum_{k=1}^{+\infty} a_{2k}cos(2k\sigma))sin(\sum_{k=1}^{+\infty} a_{2k}sin(2k\sigma)) \end{cases} \tag{3.9}$$

3.3 Numerical result and discussion

In our work ,focused on the study of the effect of Froude number on the from of free surface .the numerical method based on series truncation which presented above used to solve our problem described in chapter 2 for given values of the froude number F_r as shown in figure 3.3 ,In order to find the coefficients a_k we have solved the system (3.9) which has k nonlinear equation with k unknowns by the Newton's method .

without loss a generality , and when we have taken $L = H = 1$, we found the shape of the free surface for $F_r = 1$ such as in the figure 3.2 .

figure 3.3,shows that the depth of the free surface varied between $F_r = 0.1, 0.2, 0.5, 1$.It is clear that the minimum of the depth of the free surface profile o, corresponding to $F_r = 0.1$ it is about 0.1 and maximum depth is about 0.65 corresponding to $F_r = 1$.

then we can say that the depth of shape of the free surface in crease by the increase of the froude number .

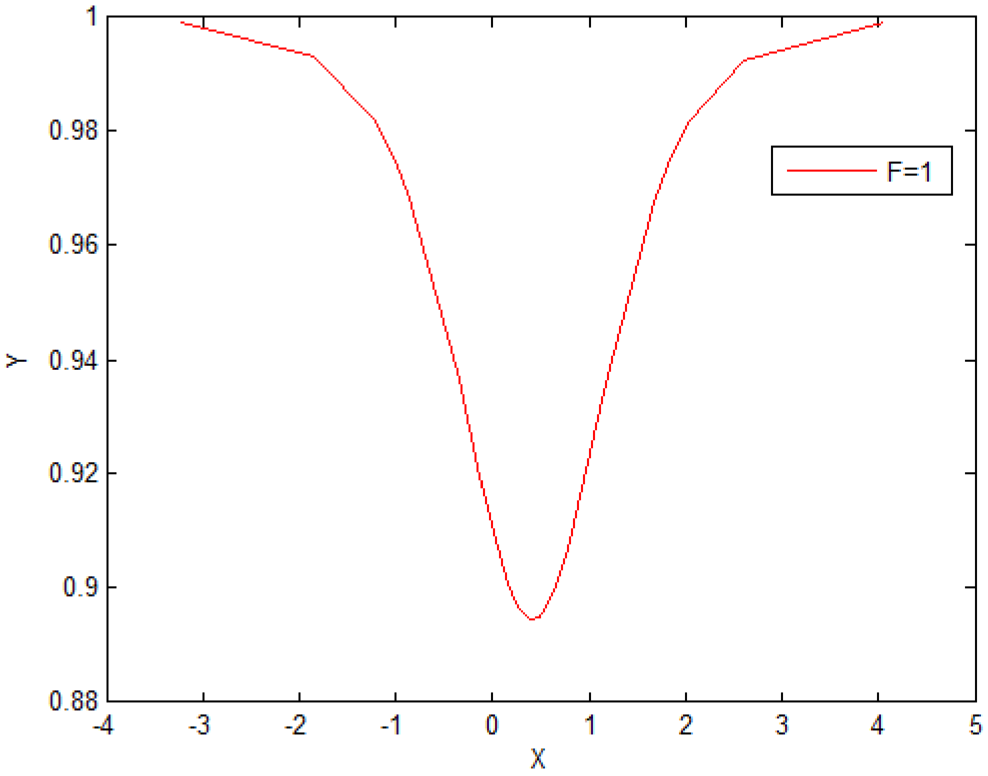

Figure 3.1: The form of free surface for the froude number equals 1

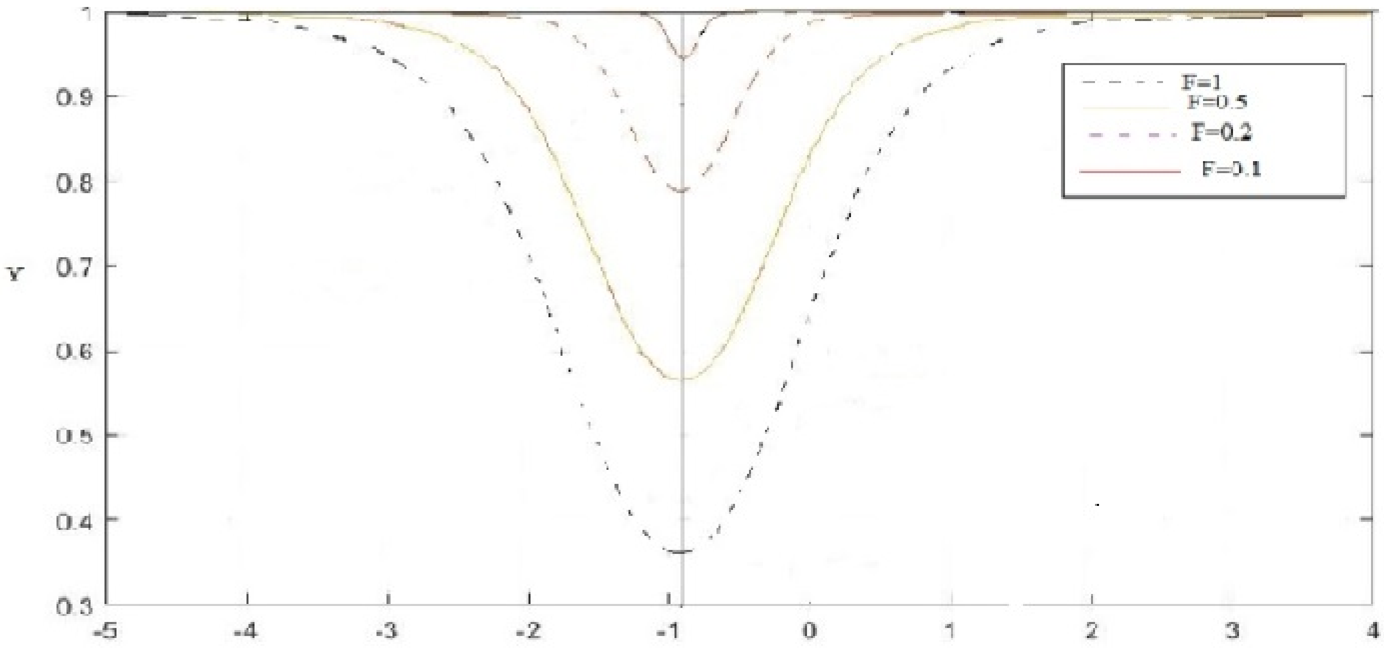

Figure 3.2: The form of free surface for different the froude number

CONCLUSION

Nonlinear problems of two-dimensional free surface flows appear in many fileds like: industry, urbain planning for example: jet pumps, dams, sources or wells. The numerical solution of these problems is becoming very important and difficult, esspitially when the force of gravity and the surface tension are considered, due to the apperance of the nonlinear term in the Bernoulli equation.

In our study, we have studied numerically a two dimensional flow of a potential and incompressible fluid in channel with a symmetric trapezoidal obstacle in the bottom which forms an angle $\alpha = \frac{\pi}{3}$ with the horizontal axis $(x'ox)$), The effects of the force of gravity is taken in account but the tension surface is neclegted. We have used a conformal mapping especially Schwartz-Christoffel transformation and the series truncation technique to obtain the shape of free surface for various value of Froude number . A numerical solution is succefully found depended to the Froude number, where we have obtained it for each number of Froude vafied between: $F_r = 0.1, F_r = 0.2, F_r = 0.5, F_r = 1$

In a forthcoming work, we will presente and study other problems in the domain of potential flows over different geometry of the obstacle, with a new boundary conditions by adopting other numerical methods like finit difference and finits volumes to find the approch solution and the shape of the free surface

APPENDIX A

1. Newton's method
2. Newton's algorithm for solving nonlinear systems $f(x) = 0$
3. Jordan's algorithm with implicit total pivotation

In our problem, we use Newton's method for the resolution of a nonlinear system with Jordan's algorithm with implicit tital rotation. In this appendix, we give a brief description of the method used.

A.1 Newton's method

In this section, we consider the problem of finding a solution to the equation system:

$$f_1(x_1, x_2, ...x_n) = 0$$
$$\begin{cases} f_2(x_1, x_2, ...x_n) = 0 \\ \dots\dots\dots\dots\dots\dots\dots\dots\dots\dots \end{cases}$$
$$f_n(x_1, x_2, ...x_n) = 0$$

that we can write $f(x) = 0$,or f and x are two vectors with n components we suppose that f is of class C^1 on a certain convex open D.
the matrix $E(x) = \left(\frac{\partial f_i(x)}{\partial x_j}\right)_{i,j=1....n}$ is called Jacobian of f at the point x. the Taylor expansion in the neighborhood of an approximation x^* of x is written

$$f(x) = f(x^*) + E(x^*)(x - x^*) + R(x^*, x - x^*)$$

or

$$R(x^*, x - x^*) = (x - x^*).\int_0^1 (E(x^* + t(x - x^*)) - E(x^*))dt$$

Also,voisinage of x the linear expression

$$f_*(x) = f(x^*) + E(x^*)(x - x^*)$$

is a good approximation of $f(x)$ It is clear that a better approximation of the solution x than x^* can be obtained by solving the equation

$$f_*(x) = 0$$

that's to say

$$f_*(x) + E(x^*)(x - x^*) = 0$$

Following the preceding procedure, Newton's general method consists of taking an initial approximation x to the solution x, then attempting to improve it iteratively as follows:

$$x_{k+1} = x_k - S_k^{-1}.f_k \qquad k = 1, 2, ...n$$

by taking $f_k = f(x_k)$and with the definition of the Jacobian matrix

$$S_k = E(x_k)$$

we continue until $|f_k(x_k)| < \epsilon$

A.2 Newton's algorithm for solving nonlinear systems $f(x) = 0$

non-linear $f(x) = 0$
since:

1. calculate:

   $$E_{ij}^{(k)} = \frac{\partial f_i(x)}{\partial x} \qquad x_k, j = 1, 2, ...neti = 1, 2...n$$

   $$f_i^k = -f_i(x_k)$$

2. Solve the linear system:

   $$\sum_{j=1}^{n} E_{ij}^{(k)} \Delta X_i^{(k)} \qquad i = 1, 2, ..n$$

3. calculate

   $$x_i^{(k+1)} = x_i^{(k)} + \Delta X_j^{(k)} \qquad i = 1, 2, ...n$$

4. if

$$|f_k(x_{k+1})| < \epsilon \qquad i = 1, 2, ...n$$

is verified, stop

A.3 Jordant algorithm with implicit total rotation

Choice of pivot

$P_k = a_{l_k \ c_k}$ or $a_{lk \ c_k} = max_{ij}|a_{ij}| \qquad i = 1, 2, ...n \quad i =\neq l_1, l_2, ..., l_{k-1}$
$j = 1, 2, ...n \quad j \neq c_1, c_2, ..., c_{k-1}$
Normalisation

$k = 1, 2, ...n$
$a_{lkj} = \frac{a_{lkj}}{P_K} \qquad j = 1, 2, ...n + 1$
Reduction

$$\left.\begin{array}{c} w = a_{ick} \\ a_{ij} = a_{ij} - w.a_{lkj} \end{array}\right\} j = 1, n + 1 \left.\vphantom{\begin{array}{c} w \\ a \end{array}}\right\} i = 1.neti \neq l_k$$

Putting things in order

$x_{ck} = a_{lk,n+1}$

BIBLIOGRAPHY

[1] A. Gasmi. free surface of a jet flow in u-shaped channel witout gravity effect. 118(3):p.393-400,(2018).

[2] A. Gasmi and H. Mekias. A jet from a container and flow past avertical flat plate, Appl. math, sci. 2687-2698,(2007).

[3] B. Bouderah N. Bounab. Computing solution of the flow due to a nozzle. I. J of computer application, 67 (13) (2013), 14-18.

[4] C. Ancey. Analyse différentielle outils mathématiques pour la dynamique des fluides. école polytechnique fédérale de lausanne,écublens.

[5] C. Grossetete. Mécanique des fluides. Ellipses Marketing, (1991)

[6] F.Dias and J.M. Vanden-Broeck. fows emerging from a nozzle and falling under gravit. 213, 465(1990).

[7] F. and P. Christodoulides. Ideal jets falling under gravity. physics of fluids, vol. 3, no. 7, american institute of physics. July 1991, pp.1711–17.

[8] F. Toison and J. Hureau. Potential flow issuing from a two-dimensional curved nozzle, Phys. fluids. 13(3) (2001), 793-798.

[9] G. Birkhoff and Jets E.H. Zarantonello. Wakes and cavities (academic press, new york)(1957).

[10] H. Lamb. Hydrodynamics. 6th, (1945).

[11] H. Mekias F. Guechi and A. Merzougui. Free surface flow with surface tension and gravity, applied mathematical sciences. Vol. 1, 2007, no. 49, 2399 - 2408.

[12] J. M. Vanden-Broeck. The influence of surface tension on cavitating flow past a curved obstacle, j. fluid mech. J. Fluid Mech. 133 (1983), 255-264.

[13] J. Roussel-E. N. S. C. R. Mécanique des fuides. (2010).

[14] M.B. Abd el Malek and S.Z.Masoud. Linearized solution of a flow over a ramp . appl .math modelling. 406-410(1988).

[15] M.B. Abd-el-Malek S.N. Hanna, M.N. Abdel-Malek. Super-critical free-surface flow over a trapezoidal obstacle. revised 31 May 1995.

[16] M. Damou. Mécanique des fluides, (o. p. u). (1993).

[17] M. Seddak. Mécanique des fluides 1(cours). (2022).

[18] N.Bounab. Simulation numérique de quelques écoulements potentiels en 2d ”curved nozzle” thése de doctorat, universite mohamed boudiaf m’sila ,(2015).

[19] P. Budden and J. Norury. Uniquness of free-surface flow under gravity,arch. rat. math. 78, 361-380(1982).

[20] S. Douadi. Mécanique des fluides. universite de bordj bou arréridj(cours). (2015).

[21] T.Blizak and A.Gasmi. Nonlinear free surface flow past a wedge in channel. INCAS Bulletin 15(1),(2023).

[22] Y. A .Cengel and J. M .Cimbala. Mécanique des fluides. De Boeck supérieur, (2017).

الملخص:

في هذه المذكرة قمنا بالدراسة العددية لمسألة تدفق كموني ثنائي الابعاد داخل قناة ذات عائق شبه منحرف يصنع زاوية $\propto=\frac{\pi}{3}$ مع المحور $(x'ox)$ مع أخد بعين الاعتبار تأثير الجاذبية واهمال تأثيرات السطح، وللبحث عن الحل الموجود قمنا باستعمال تحويلات المتطابقة و طريقة تقطيع السلاسل

الكلمات المفتاحية: سطح الحر ، التدفق الكموني ، تحويل شوار تز كريستو فيل ، طريقة تقطيع السلسلة ، عدد فرود

Abstract

In this thesis , we have studied numerically a problem of two-dimensional potential flow in a channel with trapezoidal obstacle in the bottom , witch makes an angle $\propto=\frac{\pi}{3}$ *with the axis* $(x'ox)$ *taking into account the effect of gravity but surface tension efforts are neglected. We have used some conformal transformations and series truncation technique for obtain the numerical solution.*

Key- words: *Free surface, Potential flow , Schwartz –Christoffel, Transformation, Truncation of the series, Froude number*

Résume:

Dans ce memoire ,on a etudié numériquement un problème d'un écoulement bidimensionnel de tension a l'interieur d'un canal avec obstacle trapezondal formant un angle $\propto=\frac{\pi}{3}$ *avec l'axe* $(x'ox)$*,*en prenant en compte l-effet de la gravite et negligent l'effet tension des surface ,pour rechercher la solution , nous avons utilise des transformatio conformes puis troncation des series

Mots-clès: *surface free, l'ecoulement potentail,transformation de schwartz-christoffel,troncation de la serie,nombre de froud*