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كلية التكنولوجيا
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مستخلص اللجنة العلمية لقسم الهندسة الميكانيكية
بخصوص تقييم مطبوعة جامعية

بناءا على التقارير الإيجابية الواردة من السادة أعضاء لجنة دراسة وتقييم مطبوعة جامعية , والآتية أسماؤهم :

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• الأستاذ : سي عامر محمد	أستاذ	جامعة باتنة

صادق أعضاء اللجنة العلمية على قبول المطبوعة المنجزة باللغة الإنجليزية من طرف

الأستاذ ة : بختي فاطمة الزهراء

أستاذة محاضرة - أ - قسم الهندسة الميكانيكية , كلية التكنولوجيا , جامعة محمد بوضياف بالمسيلة.

- تحت عنوان **Heat Transfer**

وفق البرنامج المقترح لطلبة السنة الثانية جذع مشترك للهندسة الميكانيكية .

رئيس اللجنة العلمية للقسم

جامعة محمد بوضياف بالمسيلة
رئيس اللجنة العلمية لقسم الهندسة الميكانيكية
كلية التكنولوجيا

الأستاذ : نزاوي موسي

Abstract

This Course Handout serves as a comprehensive introduction to heat transfer, tailored for undergraduate mechanical engineering students specializing in mechanical construction. It adheres to the official academic curriculum and emphasizes the fundamental principles and mechanisms of heat transfer: conduction, convection, and radiation. Structured into three chapters, the course begins with a general overview of three heat transfer modes, accompanied by practical examples to simplify understanding. Subsequent chapters delve into steady-state one-dimensional conduction with detailed equations for various geometries, followed by an exploration of free and forced convection, including methods for calculating fluxes and key correlations. The final chapter addresses heat transfer by radiation, introducing black body concepts, radiative surface properties, and radiation in semi-transparent media. The course is enriched with solved exercises and appendices containing essential thermophysical data, providing students with practical tools to master theoretical concepts and apply them effectively in engineering contexts.

Keywords: Heat transfer, conduction, convection, radiation, temperature, heat flux.

Algerian People's Democratic Republic
Ministry of Higher Education and Scientific Research
Mohamed Boudiaf University – M'sila
Faculty of Technology
Department of Mechanical Engineering

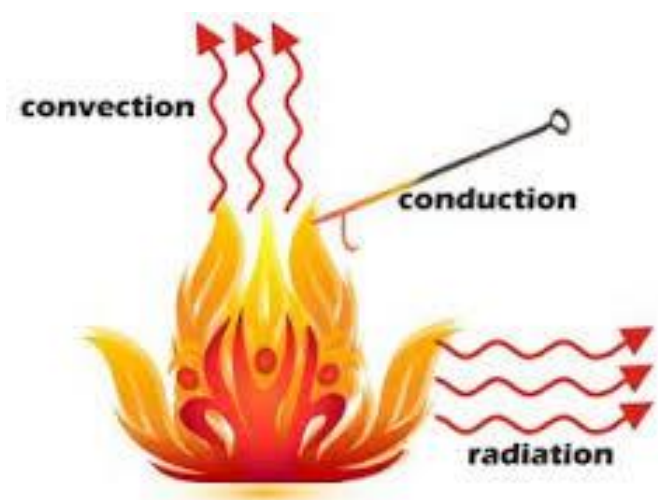
Course Handout

Intended for students of the 3rd year Bachelor's Degree in Mechanical Engineering, Mechanical Construction option

Heat Transfer

Prepared by

Dr. BAKHTI Fatima zohra



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Preface

The multiple processes used in the industry very often involve heat transfers, either because it is the desired goal (furnaces, casting, heat exchangers, thermoforming, induction, quenching, cooling), or because they occur in an unavoidable manner. (Thermal shocks, heat losses, radiation). Basic knowledge in this field is therefore necessary for the production or development engineer to understand the physical phenomena they observe and to master the processes and thus the quality of the products.

This handout for the heat transfer course is intended for students in the Bachelor's program in Mechanical Engineering with a specialization in Mechanical Construction. It is in accordance with the official program issued by the Ministry of Higher Education and Scientific Research.

It is designed to enable students to acquire the basic mechanisms and concepts related to the three modes of heat transfer: conduction, convection, radiation. According to the chapters (particularly convection), certain prerequisites in thermodynamics, continuum mechanics, fluid mechanics, differential calculus, and/or numerical analysis are necessary.

The handout is limited to three chapters. In the first chapter, we will present the three modes of heat transfer in general terms with examples related to each mode, with the aim of introducing the modes of thermal transfer in a simple and easy manner before moving on to more difficult cases. This facilitates the understanding of theoretical considerations. Also, the goal is to gradually familiarize the student with these phenomena that coexist naturally and practically in our daily lives.

Next, we will present one-dimensional conduction in steady state where the general differential equation of conduction has been developed in detail for the different configurations encountered in practice (prismatic, cylindrical, and spherical). Also, the problem of one-dimensional conduction with loss through the lateral surfaces (the typical conduction problem in a solid, cooling by fins) is presented.

The second chapter provides a brief overview of free and forced convection. The methods for quantifying the fluxes under this regime are presented for both types of convection. The commonly used formulas in free and forced convection and the different dimensionless numbers are also presented.

This section also includes the main correlations that allow the calculation of the convection coefficient for different regimes and various geometries.

The last chapter concerns heat transfer by radiation. The concepts of black body and the properties of radiative surfaces between surfaces are introduced, as well as some essential notions on radiation in semi-transparent media, which are presented at the end of this chapter.

At the end of each chapter, we offer carefully selected solved exercises to better understand the principles of the three modes of heat transfer and to reinforce the acquired knowledge.

This course is finally complemented by a series of appendices gathering the main thermophysical data necessary for solving a thermal problem.

Chapiter I
Heat Transfer by Conduction

1.1. Introduction

From the study of thermodynamics, we have learned that energy can be transferred by interactions of a system with its surroundings. These interactions are called work and heat. However, thermodynamics deals with the end states of the process during which an interaction occurs and provides no information concerning the nature of the interaction or the time rate at which it occurs. The objective of this course is to extend thermodynamic analysis through the study of the modes of heat transfer and through the development of relations to calculate heat transfer rates.

Heat transfer within a system only occurs if there are temperature gradients between different parts of the system, which implies that it is not in thermodynamic equilibrium (the temperature is not uniform throughout the system). During the transformation of the system towards a final equilibrium state, the temperature will evolve both in time and space. The purpose of heat transfer analysis is to identify the modes of transfer involved during the transformation and to quantitatively determine how the temperature varies at each point in the system over time.

I.2.The three modes of heat transfer

Heat can be transferred in three different modes: conduction, convection, and radiation. All modes of heat transfer require the existence of a temperature difference and all modes are from the high temperature medium to lower temperature one.

There are three modes of heat transfer: conduction, convection, and radiation. Any energy exchange between bodies occurs through one of these modes or a combination of them. Conduction is the transfer of heat through solids or stationery fluids. Convection uses the movement of fluids to transfer heat. Radiation does not require a medium for transferring heat; this mode uses the electromagnetic radiation emitted by an object for exchanging heat.

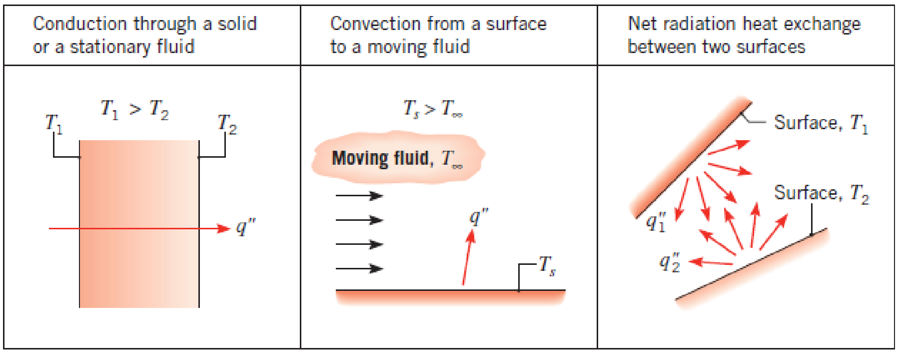


Figure I.1. Conduction, convection, and radiation heat transfer modes.

I.2.1. Conduction

It is the transfer of heat within an opaque medium, without the movement of matter, under the influence of a temperature difference. The propagation of heat by conduction within a body is due to microscopic physical phenomena (agitation of atoms or molecules, flow of free electrons...). It can be seen as a transfer of energy from the most energetic particles (the hot particles with high vibrational energy) to the less energetic particles (the cold particles with lower vibrational energy), due to collisions between particles. In solids, energy transfer can also occur due to the movement of free electrons within the crystal lattice (for example, in metals). Thus, good conductors of electricity are generally also good conductors of heat.

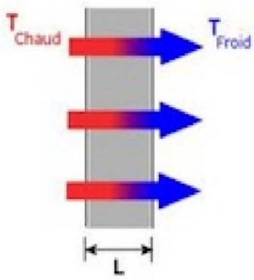


Figure I.2. Phenomenon of conduction in a solid.

I.2.2. Heat transfer by convection

Thermal convection is the transfer of energy between two media, at least one of which is a fluid, through molecular movement. It involves the combined effects of conduction and fluid movement. In the absence of fluid movement, heat transfer is ensured exclusively by pure conduction.

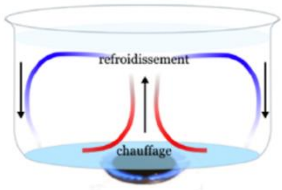


Figure I.3. Phenomenon of convection in a liquid.

We distinguish between two types of convection:

- *Natural convection*: the movements are due to variations in density within a fluid subjected to the gravitational field. Density variations can be generated by temperature gradients (warm air is lighter than cold air) and/or by composition gradients.

- *Forced convection*: the movement of the fluid is caused by external mechanical actions (pump, fan...).

- We will talk about *mixed convection* when both types of convection coexist in a system.

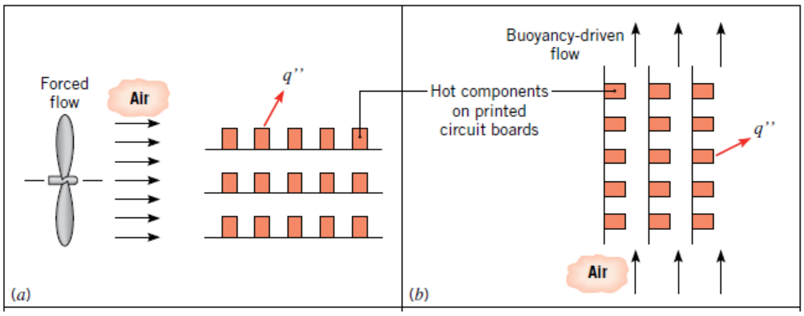


Figure 1.4. Convection heat transfer processes.(*a*) Forced convection. (*b*) Natural convection.

I.2.3. Heat transfer by radiation

Every material body emits and absorbs energy in the form of electromagnetic radiation.

The transfer of heat by radiation between two bodies separated by a vacuum or a semi-transparent medium occurs through electromagnetic waves, thus without a material medium.

The phenomenon of a body's emission corresponds to the conversion of material energy (the agitation of the electrons constituting the matter, the intensity of which depends on the temperature) into radiative energy. The phenomenon of absorption is the reverse conversion.

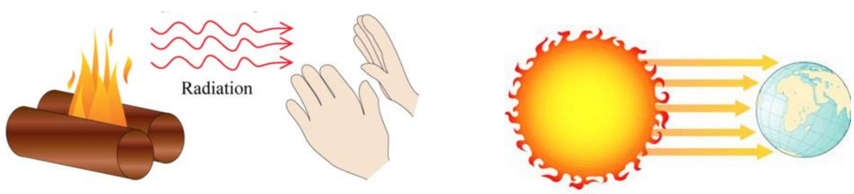


Figure I.5. Examples of thermal radiation.

I.3. Definitions

I.3.1. Temperature Field

Energy transfers are determined based on the evolution of temperature in space and time: T= f(x, y, z, t). The instantaneous value of the temperature at every point in space is a scalar called the temperature field. We distinguish two cases:

-*Temperature field independent of time:* the regime is said to be ***permanent or stationary.***

- *Evolution of the temperature field over time:* the regime is said to be ***variable or unsteady.***

I.3.2. Temperature Gradient

If we gather all the points in space that have the same temperature, we obtain a surface called an isothermal surface. The variation in temperature per unit length is maximum along the normal to the isothermal surface. This variation is characterised by the temperature gradient:

$$\overrightarrow{\mathrm{grad}}(T) = \vec{n}.\frac{\partial T}{\partial n}\ , \quad \vec{\nabla} T = \overrightarrow{\mathrm{grad}} T = \left| \begin{array}{l} \frac{\partial T}{\partial x} \\ \frac{\partial T}{\partial y} \\ \frac{\partial T}{\partial z} \end{array} \right. \qquad (I.1)$$

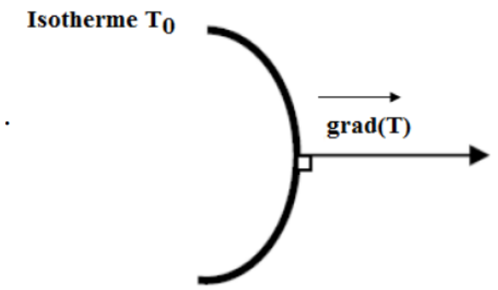


Figure I.6. Isothermal surface and thermal gradient.

I.3.3. Heat Flux

A heat flux is an amount of energy transferred in the form of heat per unit of time. It is therefore a power, expressed in Watts (J/s):

$$\varphi = \frac{dQ}{dt} \quad [W] \qquad (I.2)$$

I.3.4. Heat flux density

In general, the flux exchanged through a surface is not uniform across the entire surface. We then define a heat flux density ϕ, which corresponds to a heat flux per unit area.

$$\phi = \frac{1}{S}\frac{dQ}{dt} \quad \left[\frac{W}{m^2}\right] \qquad (I.3)$$

I.4. Basic laws of heat transfer

For each mode of heat transfer, there is a law that provides the expression for the heat flux.

I.4.1. Heat flux exchanged by conduction – Fourier's law

This transfer mechanism is governed by a phenomenological law established by Joseph Fourier in 1822, stipulating that the flux exchanged by conduction is proportional to the temperature gradient. This law, called Fourier's law, is written as:

In general

$$\vec{\varphi} = -\lambda . S . \overrightarrow{grad}(T) \qquad (I.5)$$

$$\varphi = -\lambda . S . \left(\frac{\partial T}{\partial X}\right) \qquad (I.6)$$

With:

φ: Heat flux transmitted by conduction (W)

λ: Thermal conductivity of the medium ($W.\ m^{-1}.°C^{-1}$)

x: Spatial variable in the direction of the flow (m)

S: Area of the heat flow passage section (m^2)

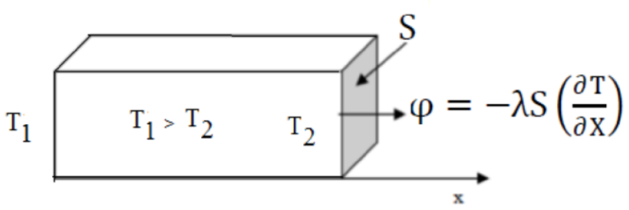


Figure I.7. Diagram of conductive heat transfer.

The sign (-) in Fourier's equation reflects the second law of thermodynamics: the heat flow goes in the opposite direction of the temperature gradient, meaning that the flow moves from the highest temperature to the lowest. So, this sign indicates that a negative temperature gradient (decrease in temperature in the positive x direction) multiplied by the sign (–)gives a positive heat flux in the positive direction of x.

The exchange surface S is the surface that is crossed by the heat flow. Figure 1.7 shows how to calculate this area in different cases.

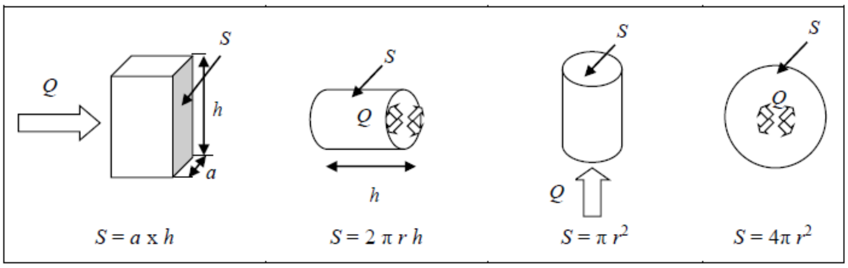


Figure I.8. Calculation of the exchange surface.

Thermal conductivity λ depends on the nature of the material in question and generally depends on the temperature. It translates the ability of a material to conduct heat. Thus, for a given temperature gradient, the heat flux will be greater the higher the conductivity. For heat-conducting materials, λ will be high, and conversely, it will be low for insulators.

In Table I.1, you will find the thermal conductivity values λ of some of the most common materials.

Table I.1. Thermal conductivity of certain materials.

Material	Thermal conductivity (W/m K)
Copper (pure)	399
Gold (pure)	317
Aluminum (pure)	237
Iron (pure)	80.2
Carbon steel (1 %)	43
Stainless Steel (18/8)	15.1
Glass	0.81
Plastics	0.2 – 0.3
Wood (shredded/cemented)	0.087
Cork	0.039
Water (liquid)	0.6
Ethylene glycol (liquid)	0.26
Hydrogen (gas)	0.18
Benzene (liquid)	0.159
Air	0.026

I.4.2. Heat flux exchanged by convection – Newton's law

This transfer mechanism is governed by Newton's law, which states that the heat flux exchanged between a solid wall and a flowing fluid is proportional to the temperature difference that gave rise to it.

The heat flux entering the fluid or leaving the solid if Tp > T∞ is expressed by Newton's law:

$$\vec{\varphi} = h.S.(T_P - T_\infty).\vec{n} \qquad (I.7)$$

$$\varphi = h.S.(T_P - T_\infty) \qquad (I.8)$$

With:

φ: Heat flux transmitted by convection (W)

h: Coefficient of heat transfer by convection (W/m² °C)

T_p: Surface temperature of the solid (°C)

T∞: Temperature of the fluid far from the solid surface (°C)

S: Area of the solid/liquid contact surface (m^2)

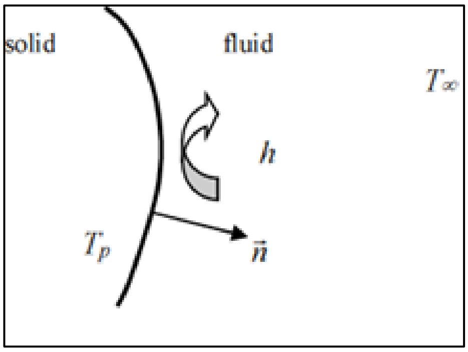


Figure I.9. Diagram of convective heat transfer

h is a positive quantity called the convective heat transfer coefficient, in ($W.m^{-2}.K^{-1}$). This coefficient depends on numerous parameters (fluid, type of flow, surface condition...) and is therefore extremely difficult to quantify precisely.

Table I.2. Typical values of the convection heat transfer coefficient.

Process	h ($W/m^2 \cdot K$)
Free convection	
Gases	2–25
Liquids	50–1000
Forced convection	
Gases	25–250
Liquids	100–20,000
Convection with phase change	
Boiling or condensation	2500–100,000

I.4.3. Heat flux exchanged by radiation – Stefan-Boltzmann law

All surfaces emit energy in the form of electromagnetic waves. In the absence of an intermediate medium, the net and total heat transfer occurs between two surfaces at different temperatures. The maximum value of the heat flux emitted by a surface is given by the Stefan-Boltzmann law:

$$\varphi = \sigma . S . T_P^4 \qquad (I.9)$$

Such a body is called an ideal body or a black body. In this expression, σ is the Stefan-Boltzmann constant:

$\sigma = 5.67 \times 10^{-8}$ (W m^{-2} K^{-4});

T p: Surface temperature (K).

The heat flux emitted by a real surface is less than that of a blackbody at the same temperature and is given by

$$\varphi = \sigma . \varepsilon . S . T_P^4 \qquad (I.10)$$

Where ε is a radiative property of the surface termed the ***emissivity***. With values in the range $0 \leq \varepsilon \leq 1$, this property provides a measure of how efficiently a surface emits energy relative to a blackbody. It depends strongly on the surface material and finish.

The emissivities of some surfaces are given in Table I.3.

Table I.3. Emissivities of some materials at 300 K.

Material	Emissivity
Aluminum foil	0.07
Anodized aluminum	0.82
Polished copper	0.03
Polished gold	0.03
Polished silver	0.02
Polished stainless steel	0.17
Black paint	0.98
White paint	0.90
White paper	0.92–0.97
Asphalt pavement	0.85–0.93
Red brick	0.93–0.96
Human skin	0.95
Wood	0.82–0.92
Soil	0.93–0.96
Water	0.96
Vegetation	0.92–0.96

Another important radiation property of a surface is its **absorptivity** a, which is the fraction of the radiation energy incident on a surface that is absorbed by the surface. Like emissivity, its value is in the range $0 \leq \alpha \leq 1$. A blackbody absorbs the entire radiation incident on it. That is, a blackbody is a perfect absorber ($\alpha = 1$) as it is a perfect emitter.

In general, both ε and α of a surface depend on the temperature and the wave length of the radiation. Kirchhoff's law of radiation states that the emissivity and the absorptivity of a surface at a given temperature and wave length are equal. In many practical applications, the surface temperature and the temperature of the source of incident radiation are of the same order of magnitude, and the average absorptivity of a surface is taken to be equal to its average emissivity.

$$\varphi_{\text{absorbed}} = \alpha.\ \varphi_{\text{incident}} \quad (\mathrm{W}) \qquad (\mathrm{I}.11)$$

Where $\varphi_{\text{incident}}$ is the rate at which radiation is incident on the surface and a is the absorptivity of the surface. For opaque (nontransparent) surfaces, the portion of incident radiation not absorbed by the surface is reflected back.

When a surface of emissivity ε and surface area S at a thermodynamic temperature T_p is completely enclosed by a much larger (or black) surface at thermodynamic temperature T_∞ separated by a gas (such as air) that does not intervene with radiation, the net rate of radiation heat transfer between these two surfaces is given by

$$\varphi = \sigma.\varepsilon.S.(T_P^4 - T_\infty^4) \qquad (\mathrm{I}.12)$$

In this special case, the emissivity and the surface area of the surrounding surface do not have any effect on the net radiation heat transfer.

I.5.General equation of conduction

In its one-dimensional form, it describes the unidirectional heat transfer through a flat wall

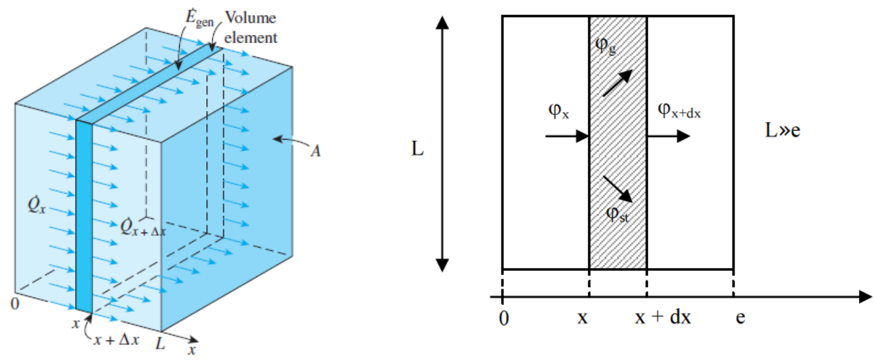


Figure I.10. One-dimensional heat conduction through a volume element in a large plane wall.

Consider a thin element of thickness Δx in a large plane wall, as shown in figure I.10. Assume the density of the wall is ρ, the specific heat is c_p, and the area of the wall normal to the direction of heat transfer is S. An energy balance on this thin element during a small time interval dt can be expressed as

$$\begin{pmatrix}\text{Rate of heat}\\ \text{conduction}\\ \text{at } x\end{pmatrix} - \begin{pmatrix}\text{Rate of heat}\\ \text{conduction}\\ \text{at } x+\Delta x\end{pmatrix} + \begin{pmatrix}\text{Rate of heat}\\ \text{generation}\\ \text{inside the}\\ \text{element}\end{pmatrix} = \begin{pmatrix}\text{Rate of change}\\ \text{of the energy}\\ \text{content of the}\\ \text{element}\end{pmatrix}$$

$$\varphi_x - \varphi_{x+dx} + \varphi_g = \varphi_{st} \qquad (I.13)$$

With

$$\varphi_x = -\left(\lambda.S.\frac{\partial T}{\partial x}\right)_x \qquad (I.14)$$

and $$\varphi_{x+dx} = -\left(\lambda.S.\frac{\partial T}{\partial x}\right)_{x+dx} \qquad (I.15)$$

$$\varphi_g = \dot{q}_s.S.dx \qquad (I.16)$$

$$\varphi_{st} = \rho.c_p.S.dx.\frac{\partial T}{\partial t} \qquad (I.17)$$

By substituting into the energy balance and dividing by dx, we obtain:

$$\frac{\left(\lambda.S.\frac{\partial T}{\partial x}\right)_{x+dx}-\left(\lambda.S.\frac{\partial T}{\partial x}\right)_{x}}{dx}+\dot{q}_s.S=\rho.c_p.S.\frac{\partial T}{\partial t}\Rightarrow\frac{\partial}{\partial x}\left(\lambda.S.\frac{\partial T}{\partial x}\right)+\dot{q}_s.S=\rho.c_p.S.\frac{\partial T}{\partial t} \qquad (I.18)$$

And in the three-dimensional case, we obtain the heat equation in the most general case:

$$\frac{\partial}{\partial x}\left(\lambda_x\frac{\partial T}{\partial x}\right)+\frac{\partial}{\partial y}\left(\lambda_y\frac{\partial T}{\partial y}\right)+\frac{\partial}{\partial z}\left(\lambda_z\frac{\partial T}{\partial z}\right)+\dot{q}_s=\rho.c_p.\frac{\partial T}{\partial t} \qquad (I.19)$$

$$\rho.c_p.\frac{\partial T}{\partial t}=\mathrm{div}\left[\lambda.\overrightarrow{\mathrm{grad}}T\right]+\dot{q}_s \qquad (I.20)$$

The thermal conductivity λ of a material, in general, depends on the temperature T (and therefore x), and thus it cannot be taken out of the derivative. However, the thermal conductivity in most practical applications can be assumed to remain constant at some average value. The equation above in that case reduces to

$$\nabla^2T+\frac{\dot{q}_s}{\lambda}=\frac{1}{a}.\frac{\partial T}{\partial t} \qquad (I.21)$$

The ratio $a=\lambda/(\rho.c_p)$ is called thermal diffusivity ($m^2.s^{-1}$), which characterises the speed of heat flow propagation through a material.

-In steady state, we obtain the Laplace equation

$$\nabla^2T=0 \qquad (I.22)$$

This equation can be written in cylindrical or spherical coordinates as follows,

-Heat equation in cylindrical coordinates:

$$\frac{\partial^2T}{\partial r^2}+\frac{1}{r}\frac{\partial T}{\partial r}+\frac{1}{r^2}\frac{\partial^2T}{\partial\theta^2}+\frac{\partial^2T}{\partial z^2}+\frac{\dot{q}_s}{\lambda}=\frac{1}{a}\frac{\partial T}{\partial t} \qquad (I.23)$$

In the case of a problem with cylindrical symmetry where the temperature depends only on r and t, equation (I.23) can be written in a simplified form:

$$\frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial T}{\partial r}\right)+\frac{\dot{q}_s}{\lambda}=\frac{1}{a}\frac{\partial T}{\partial t} \qquad (I.24)$$

-Heat equation in spherical coordinates:

$$\frac{1}{r}\frac{\partial^2(rT)}{\partial r^2}+\frac{1}{r^2\sin\theta}\cdot\frac{\partial}{\partial\theta}\left(\sin\theta.\frac{\partial T}{\partial\theta}\right)+\frac{1}{r^2.\sin^2\theta}\frac{\partial^2 T}{\partial\varphi^2}+\frac{\dot{q}_s}{\lambda}=\frac{1}{a}\frac{\partial T}{\partial t} \qquad (I.25)$$

I.6. Spatiotemporal boundary conditions for the solution of the heat equation

The general heat equation expresses a relationship between the temperature function T and the variables x, y, z, and t. The mathematical solution to this second-order linear partial differential equation theoretically admits an infinite number of solutions. Also, its resolution requires knowledge, on the one hand, of the initial condition, that is to say, the initial distribution of temperatures at every point in the medium T(x, y, z, 0), and on the other hand, the law of variation over time of the temperature or its derivative on the surface S. These are the spatiotemporal boundary conditions.

- **Initial condition**

It is the temperature distribution at the moment t=0, that is T_0=f(x, y, z, 0). Generally, this condition is known.

- **Boundary conditions**

At the boundaries of a material, different types of boundary conditions can appear in problems commonly encountered in heat transfer.

a) Dirichlet conditions (1er type)

The temperature distribution Tp at the considered boundary surface is given as a function of time and for all points on the surface. T=f(x,y,z). The most common case is when Tp depends neither on t nor on space (uniform in space).

Tp = constant

b) Neumann boundary conditions (2nd type)

We impose the flux density on the surface, for all points on the surface as a function of time: $\varphi_s(x,y,z,t)$

$$\phi_s=-\lambda\left(\frac{dT}{dn}\right)_S=f(M_S,t) \qquad (I.26)$$

Where $\left(\frac{dT}{dn}\right)_S$ is the normal derivative at the surface.

Particular case of the adiabatic boundary: in this case, the heat flux crossing the boundary is zero.

$$\phi_s = -\lambda \left(\frac{dT}{dn}\right)_S = 0 \qquad (I.27)$$

c) Robin boundary conditions (3rd type)

-Convective exchanges on a wall

When a wall exchanges heat by convection with the outside, the density of the flux it exchanges is proportional to the temperature difference between the wall and the external fluid medium, multiplied by an exchange coefficient h in W/m².K that takes into account the different physical and kinematic properties of the fluid in contact with the wall. We impose a temperature on the ambient fluid, which we will denote as T_∞, that will be known.

$$\phi_s = -\lambda \left(\frac{dT}{dn}\right)_S = h(T_P - T_\infty) \qquad (I.28)$$

-Radiative exchanges on a wall

The radiation from a wall can be a mode of exchange to consider, especially if the temperature is high (above approximately 100 degrees). The flux density of a wall at temperature T_p exchanges by radiation with the external medium at temperature T_∞ is:

$$\phi = \sigma.\varepsilon.\left(T_p^4 - T_\infty^4\right) \qquad (I.29)$$

In such a case, the boundary condition at the border will therefore become:

$$-\lambda\left(\frac{\partial T}{\partial n}\right)_{wall} = \sigma.\varepsilon.\left(T_p^4 - T_\infty^4\right) \qquad (I.30)$$

d) The solid under study is in contact with another material

-Perfect contact

At the interface S of the two media with different conductivities λ_1 and λ_2, the conservation of flux is written as:

$$\lambda_1.\overrightarrow{grad}T_1 = \lambda_2.\overrightarrow{grad}T_2 \qquad (I.31)$$

A second condition is obtained in the case of perfect contact. It concerns the temperatures on S:

$T_1=T_2$

-Contact imperfect

In reality, this condition is not met: the two surfaces are not strictly in contact due to the air present between the mediums. The temperatures are no longer equal. We consider a contact thermal resistance Rc [$m^2.K.W^{-1}$].

The condition obtained on the interface is then written as:

$$\lambda_1.\overrightarrow{\mathrm{grad}}T_1 = \lambda_2.\overrightarrow{\mathrm{grad}}T_2 \qquad \text{(I.32)}$$

$$\Phi = \frac{1}{R_C}(T_1 - T_2) \qquad \text{(I.33)}$$

I.7. Steady-state conduction without internal heat dissipation

I.7.1. A simple wall without a source

We consider a homogeneous and undeformable solid (or a fluid at rest) of thickness e and large transverse dimensions, whose extreme faces are at temperatures T_1 and T_2. We assume that the thermal conductivity of the material is constant. Let's revisit the heat equation established previously:

$$\rho.c_p.\frac{\partial T}{\partial t} = \lambda\nabla^2 T + \dot{q}_s \qquad \text{(I.34)}$$

In steady-state (permanent) conditions and without internal heat dissipation

$$\dot{q}_s = 0 \quad \text{et} \quad \frac{\partial T}{\partial t} = 0$$

Where ;

$$\Delta T = 0$$

Boundary conditions of the problem: The two faces of the wall are maintained at fixed temperatures over time (Dirichlet conditions)

We will take x=0 for one of the faces and x=e for the other.

$T=T_1$ if $x=0$ and $T=T_2$ if $x=e$

The problem addressed is a one-dimensional problem. The temperature is solely a function of the variable x.

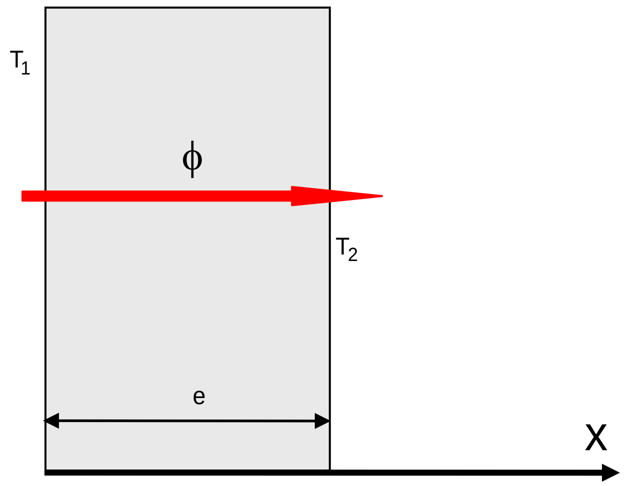


Figure I.11. One-dimensional conduction in a simple wall.

$\Delta T = 0 \Rightarrow \frac{\partial^2 T}{\partial x^2} = 0$

$\frac{dT}{dx} = A$

$\Rightarrow T(x) = A.x + B$

If $x=0$, $T=T_1 \Rightarrow T(0)= A.(0)+B=T_1$

$\Rightarrow B=T_1$

If $x=e$, $T=T_2 \Rightarrow T(e)=A.e+T_1= T_2$

Where $A = \frac{T_2-T_1}{e}$

$$T(x) = \frac{T_2-T_1}{e} x + T_1 \qquad (I.35)$$

$\varphi = -\lambda.S.\frac{dT}{dx}$

Where ; $\varphi = -\lambda.S.\frac{T_2-T_1}{e}$

$$\varphi = \lambda.S.\frac{T_1 - T_2}{e} \qquad (I.36)$$

The relation (I.36) can also be expressed as: $\varphi = \frac{(T_1-T_2)}{\frac{e}{\lambda s}}$, this relation is analogous to Ohm's law in electricity, which defines the current intensity as the ratio of the difference in electric potential to the electrical resistance ($V_1-V_2=R.I \Rightarrow I = \frac{V_1-V_2}{R}$). The temperature thus appears as a thermal potential and the term $\frac{e}{\lambda S}$ appears as the thermal resistance of a flat wall of thickness e, thermal conductivity λ, and lateral surface S. We therefore refer to the equivalent diagram shown in Figure I.12.

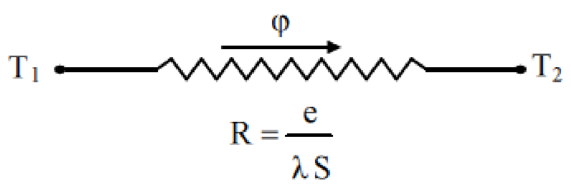


Figure I.12. Equivalent electrical circuit of a simple wall.

I.7.2. Simple wall with conductivity that varies with temperature

If the range of temperatures encountered in a conduction problem is such that the values of λ are different from one end to the other of this range, we can no longer assume λ is constant.

In this case, we can make the approximation that the thermal conductivity varies linearly with temperature, i.e., $\lambda=\lambda_0(1+bT)$ with λ_0 being the conductivity at T=0, and b depending on the material.

For a wall, a one-dimensional problem, we must then return to the general equation of conduction in the case of non-uniform thermal conductivity:

$\frac{d}{dx}\left(\lambda\frac{dT}{dx}\right)=0$ (Without heat source $\dot{q}$=0, in steady state $\frac{dT}{dt}=0$)

$$\frac{d}{dx}\left(\lambda_0(1+bT)\frac{dT}{dx}\right)=0$$

$$\lambda_0(1+bT)\frac{dT}{dx}=E$$

$$\lambda_0 T+\frac{\lambda_0 bT^2}{2}=EX+D \qquad (I.37)$$

x=0 $\quad T=T_1$ $\qquad T_1>T_2$

x=e $\quad T=T_2$

The distribution of temperatures is therefore parabolic within the wall.

We can solve the problem by considering two Dirichlet conditions:

Which lead to

$$D=\lambda_0\left(T_1+\frac{bT_1^2}{2}\right)$$

And

$$E=\frac{\lambda_0}{e}\left[\frac{b}{2}(T_2^2-T_1^2)+(T_2-T_1)\right]$$

By substituting into (I.37) and expressing T(x), we derive:

$$T(x)=-\frac{1}{b}+\sqrt{\left(\frac{1}{b}+T_1\right)^2+\frac{2.E.x}{b.\lambda_0}} \qquad (I.38)$$

Three cases need to be considered. :

b>0, b=0 et b<0

b=0 :$\lambda=\lambda_0$: linear case between T_1 and T_2

b>0 : concavity upwards
b<0 : concavity downwards.
(see Figure I.13)

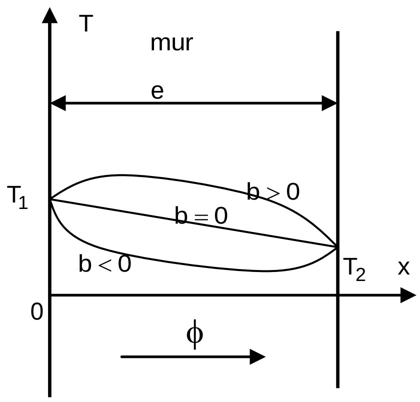


Figure I.13. Simple wall with conductivity varying with temperature (case already handled)

I.7.3. Multi-layer wall

This is the case for real walls (schematized in Figure I.14) composed of several layers of different materials, where only the temperatures T_{f1} and T_{f2} of the fluids in contact with the two faces of the wall with lateral surface S are known.

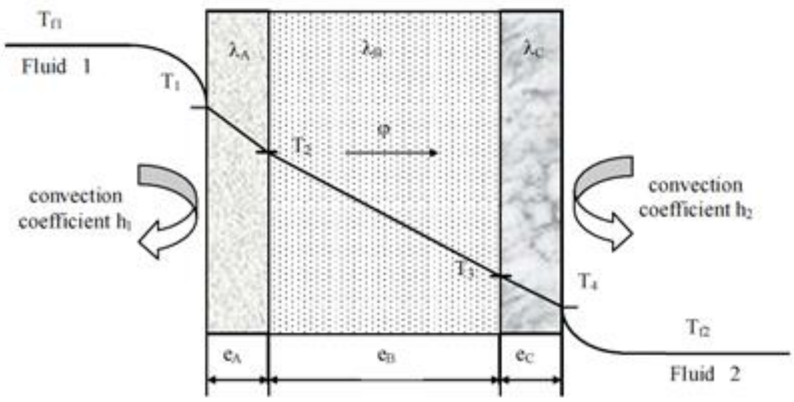


Figure I.14. Schematic representation of the flows and temperatures in a multilayer wall.

In steady state, the heat flux is conserved as it crosses the wall and is expressed as:

$$\varphi = h_1.S.(T_{f1} - T_1) = \frac{\lambda_A S(T_1 - T_2)}{e_A} = \frac{\lambda_B S(T_2 - T_3)}{e_B}$$
$$= \frac{\lambda_C S(T_3 - T_4)}{e_C} = h_2.S.(T_4 - T_{f2}) \qquad (I.39)$$

Where ;

$$\varphi = \frac{T_{f1} - T_{f2}}{\frac{1}{h_1 S} + \frac{e_A}{\lambda_A S} + \frac{e_B}{\lambda_B S} + \frac{e_C}{\lambda_C S} + \frac{1}{h_2 S}} \qquad (I.40)$$

It was assumed that the contacts between layers of different natures were perfect and that there was no temperature discontinuity at the interfaces. In reality, given the roughness of the surfaces, a micro-layer of air exists between the hollows of the opposing surfaces, which contributes to the creation of a thermal resistance (air is an insulator) called contact thermal resistance. The previous formula is then written as:

$$\varphi = \frac{T_{f1} - T_{f2}}{\frac{1}{h_1 S} + \frac{e_A}{\lambda_A S} + R_{AB} + \frac{e_B}{\lambda_B S} + R_{BC} + \frac{e_C}{\lambda_C S} + \frac{1}{h_2 S}} \qquad (I.41)$$

The equivalent electrical diagram is shown in Figure I.15.

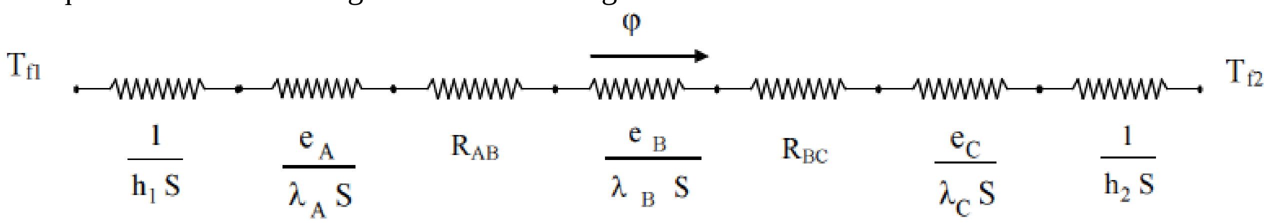


Figure I.15. Equivalent electrical diagram of a multilayer wall.

I.7.4. Composite wall

This is the most commonly encountered case in reality where the walls are not homogeneous. Let's consider, for example, a wall of width L made of hollow blocks. (Figure I.16).

Assuming unidirectional transfer and taking into account the axes of symmetry, we can reduce the calculation of the flux through the isolated element on the right side of the figure and calculate the equivalent thermal resistance R of a portion of the wall with a width L and a height $l = l_1 + l_2 + l_3$ using the laws of association of resistances in series and parallel by the relation:

$$R = R_1 + R_2 + \frac{1}{\frac{1}{R_3} + \frac{1}{R_4} + \frac{1}{R_5}} + R_6 + R_7 \qquad (I.42)$$

Where ;

$$R_1 = \frac{1}{h_1.l.L};\ R_2 = \frac{e_1}{\lambda_1.l.L};\ R_3 = \frac{e_2}{\lambda_2.l_1.L};\ R_4 = \frac{e_2}{\lambda_1.l_2.L};\ R_5 = \frac{e_2}{\lambda_2.l_3.L};\ R_6 = \frac{e_3}{\lambda_1.l.L};\ R_7 = \frac{1}{h_2.l.L}$$

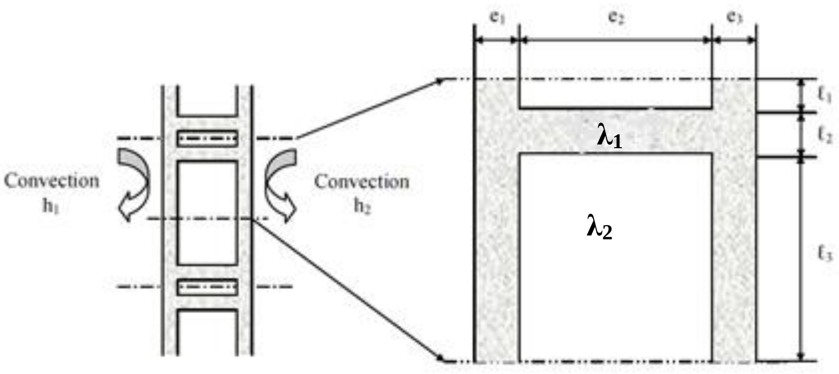


Figure I.16. Schematic of a composite wall.

This can be represented by the equivalent electrical diagram shown in Figure I.17.

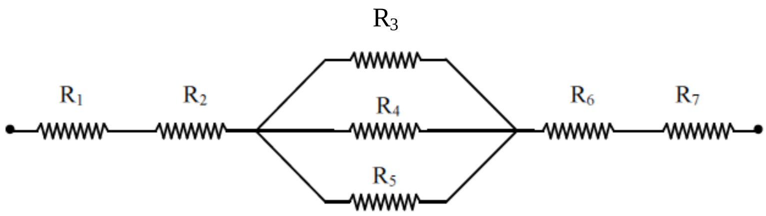


Figure I.17. Equivalent electrical diagram of the composite wall.

I.7.5. Long hollow cylinder

We consider a hollow cylinder with thermal conductivity λ, an inner radius r_1, an outer radius r_2, and a length L, with the temperatures of the inner and outer surfaces being T_1 and T_2, respectively. (cf. figure I.18). It is assumed that the longitudinal temperature gradient is negligible compared to the radial gradient.

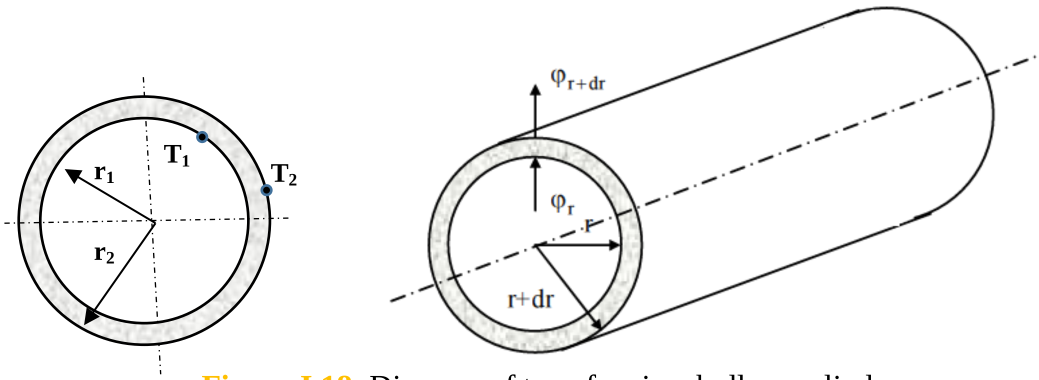


Figure I.18. Diagram of transfers in a hollow cylinder.

The conduction equation is:

$$\lambda.\Delta T + \dot{q}_s = \rho.c_p.\frac{\partial T}{\partial t} \qquad (I.43)$$

In the case of conduction without an internal source ($\dot{q}_s$=0 in steady state $\left(\frac{\partial T}{\partial t} = 0\right)$):

$$\Delta T = 0$$

In the case of the cylinder, the problem is radial and depends only on r. We recall that the Laplacian in cylindrical coordinates (independent of θ and z) is written as:

$$\Delta T = \frac{d^2T}{dr^2} + \frac{1}{r}\frac{dT}{dr}$$

Let: $r\frac{d^2T}{dr^2} + \frac{dT}{dr} = 0$ or again $\frac{d}{dr}\left(r\frac{dT}{dr}\right) = 0$

After an initial integration:

$$r\frac{dT}{dr} = A$$

$$\frac{dT}{dr} = \frac{A}{r}$$

Where from :

$$T(r) = A.\ln r + B \qquad (I.44)$$

Let's consider Dirichlet boundary conditions:

$T=T_1$ if $r=r_1$
$T=T_2$ if $r=r_2$

We obtain the system: $\begin{cases} T_1 = A.\ln r_1 + B \\ T_2 = A.\ln r_2 + B \end{cases}$

Let

$$A = \frac{T_1 - T_2}{\ln\left(\frac{r_1}{r_2}\right)} \quad \text{et } B = \frac{(T_2 \ln r_1 - T_1 \ln r_2)}{\ln\frac{r_1}{r_2}}$$

Where from

$$T(r) = \frac{T_1 - T_2}{\ln\left(\frac{r_1}{r_2}\right)} \ln r + \frac{(T_2 \ln r_1 - T_1 \ln r_2)}{\ln\frac{r_1}{r_2}} \qquad (I.45)$$

And by applying Fourier's law: $\varphi = -\lambda.S.\frac{dT}{dr}$

Let:

$\varphi = -\lambda.(2.\pi.L.r).\frac{dT}{dr}$ if we consider a length L of the cylinder.

$$\frac{dT}{dr} = \left[\frac{T_1 - T_2}{\ln\left(\frac{r_1}{r_2}\right)}\right]\frac{1}{r}$$

Where from :

$$\varphi = -\frac{\lambda.2.\pi.L}{\ln\left(\frac{r_1}{r_2}\right)}(T_1 - T_2)$$

$$\varphi = \frac{\lambda.2.\pi.L}{\ln\left(\frac{r_2}{r_1}\right)}(T_1 - T_2) \qquad (I.46)$$

Thermal resistance is defined as:

$R_{12} = \frac{T_1 - T_2}{\varphi}$

Let $\qquad R_{12} = \frac{\ln\left(\frac{r_2}{r_1}\right)}{\lambda.2.\pi.L} \qquad (I.47)$

We have thus defined a new thermal resistance for a cylindrical pipe.

This relationship is represented by the equivalent electrical diagram:

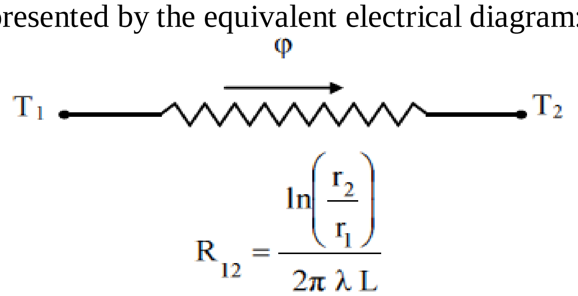


Figure I.19. Equivalent electrical circuit of a hollow cylinder.

I.7.6. Multi-layer hollow cylinder

This is the practical case of a tube covered with one or more layers of different materials, where only the temperatures T_{f1} and T_{f2} of the fluids in contact with the inner and outer faces of the cylinder are known; h_1 and h_2 are the heat transfer coefficients by convection between the fluids and the inner and outer faces. (see figure I.20)

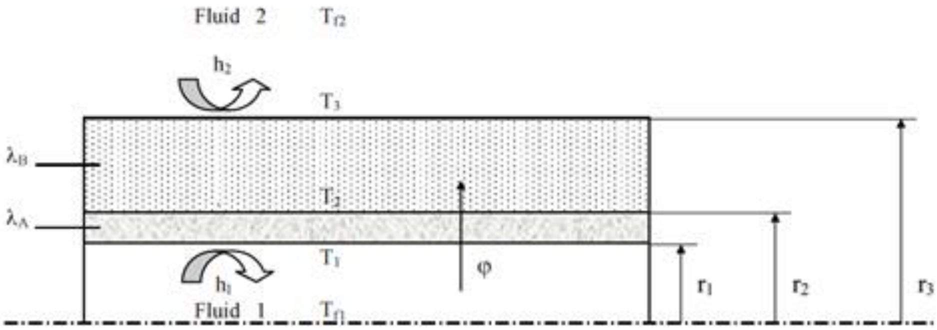


Figure I.20. Diagram of transfers in a multilayer hollow cylinder.

In steady state, the heat flux φ is conserved as it passes through the different layers and is expressed as:

$$\varphi = h_1 . 2\pi . r_1 . L(T_{f1} - T_1) = \frac{2\pi\lambda_A L(T_1 - T_2)}{\ln\left(\frac{r_2}{r_1}\right)} = \frac{2\pi\lambda_B L(T_2 - T_3)}{\ln\left(\frac{r_3}{r_2}\right)} = h_2 . 2\pi r_3 L(T_3 - T_{f2})$$

Where from :

$$\varphi = \frac{T_{f1} - T_{f2}}{\frac{1}{h_1 2\pi r_1 L} + \frac{\ln\left(\frac{r_2}{r_1}\right)}{2\pi\lambda_A L} + \frac{\ln\left(\frac{r_3}{r_2}\right)}{2\pi\lambda_B L} + \frac{1}{h_2 2\pi r_3 L}} \qquad (I.48)$$

Which can be represented by the equivalent electrical diagram in Figure I.21.

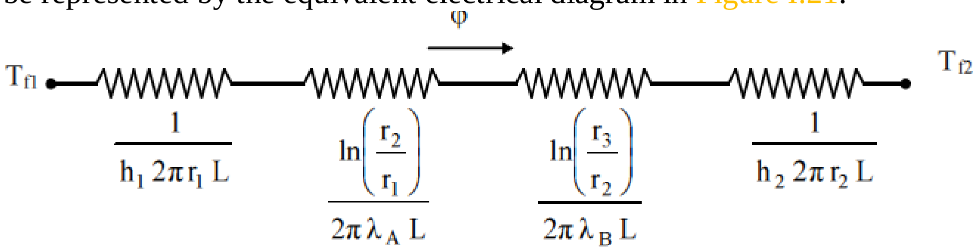


Figure I.21. Equivalent electrical diagram of a multilayer hollow cylinder.

In the case of a composite circular tube, composed for example of n superimposed materials limited by cylinders r_0, r_1… r_n with respective conductivities λ_1, λ_2, ……λ_n.

The thermal resistance of each cylinder is:

$$R_i = \frac{\ln\left(\frac{r_i}{r_{i-1}}\right)}{2\pi\lambda_i L}$$

The total thermal resistance of the tube is: $R_T = \sum_{i=1}^{n} R_i$ (series resistors) .

If the extreme temperatures T_1 and T_2 are imposed, the flux can be calculated using the relation:

$$\varphi = \frac{T_1 - T_2}{R_T}$$

Where R_T is the total thermal resistance.

I.7.7. Concentric Spheres

In the case of conduction without an internal source $\dot{q}_s=0$ in steady state $\left(\frac{\partial T}{\partial t} = 0\right)$:

$\Delta T=0$

Let's consider a hollow sphere with an outer radius r_e and an inner radius r_i. The problem is radial (r). In spherical coordinates, we have:

$$\frac{d^2T}{dr^2} + \frac{2}{r}\frac{dT}{dr} = 0$$

Either

$$\frac{d}{dr}\left(r^2 \frac{dT}{dr}\right) = 0$$

Which leads to

$$T(r) = \frac{-A}{r} + B \qquad \text{(I.50)}$$

Considering Dirichlet boundary conditions:

$T=T_e$ if $r=r_e$

$T=T_i$ if $r=r_i$

We shoot

$$\begin{cases} T_i = \dfrac{-A}{r_i} + B \\ T_e = \dfrac{-A}{r_e} + B \end{cases}$$

$$\Rightarrow \begin{cases} A = \dfrac{(T_e - T_i)}{\dfrac{1}{r_i} - \dfrac{1}{r_e}} \\ B = T_i + \dfrac{1}{r_i} \cdot \dfrac{(T_e - T_i)}{\dfrac{1}{r_i} - \dfrac{1}{r_e}} \end{cases}$$

$$\mathrm{T(r) = T_i + (T_i - T_e)} \frac{\dfrac{1}{\mathrm{r}} - \dfrac{1}{\mathrm{r_i}}}{\dfrac{1}{\mathrm{r_i}} - \dfrac{1}{\mathrm{r_e}}} \qquad (\mathrm{I.51})$$

And by applying Fourier's law:

$$\varphi = \frac{4\pi\lambda}{\dfrac{1}{\mathrm{r_i}} - \dfrac{1}{\mathrm{r_e}}} (\mathrm{T_i - T_e}) \qquad (\mathrm{I.52})$$

This relationship can also be expressed as:

$$\varphi = \frac{\mathrm{T_i - T_e}}{\mathrm{R_{12}}}$$

With:

$$\mathrm{R_{12}} = \frac{\dfrac{1}{\mathrm{r_i}} - \dfrac{1}{\mathrm{r_e}}}{4\pi\lambda} \qquad (\mathrm{I.53})$$

I.8. The fins

I.8.1.Definition and interest in using fins

The fins are good conductors of heat, with one dimension being large compared to the others. They are used to improve the heat dissipation of a confined solid system in which the heat flux densities are high.

The applications of fins are now very numerous and highly developed:

- Fins placed on hot water steam pipes to ensure heating (radiator)
-Engine coolings
- Heat exchangers (thermal power plants)
- Electricity: "radiators" for cooling electrical components, like in transformers
- Microelectronics and micro informatics

As an example, below are some photos of fins sold commercially for microelectronics or electricity, which are among the most important fields of application.

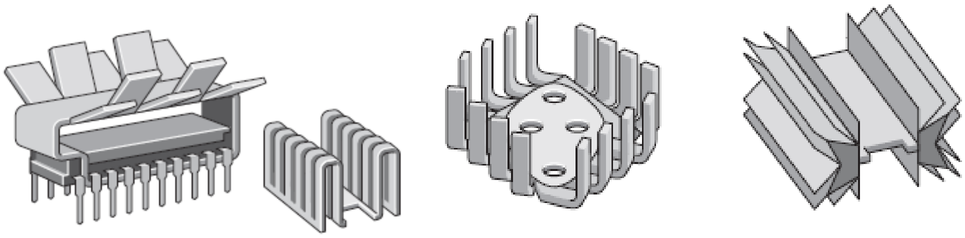


Figure I.22. Some innovative fin designs.

In the previous paragraphs, heat transfer by conduction within the solid and heat transfer by convection from its boundaries occurred in the same direction.

In systems with fins, the direction of the convective heat flow is perpendicular to the main direction of the heat flow in the solid.

Let's consider the plane wall in Figure I.23(a). The heat flux evacuated from the wall by convection is expressed by Newton's law:

$$\varphi = h.S_{éch}.(T_P - T_\infty) \qquad (I.54)$$

If Tp is fixed, there are two possibilities to increase the heat flow removed: Increase the convective heat transfer coefficient, h, by increasing the flow velocity and/or decreasing the fluid temperature T_∞. In most applications, increasing h to the maximum is not enough to dissipate the desired heat flow, and often the cost is too high (installation of powerful and bulky pumps or fans). Reducing T_∞ is often unfeasible in the installation. The second solution is much simpler to implement: it involves increasing the heat exchange surface by using fins extending from the solid into the surrounding medium (see Figure I.23 (b)). The thermal conductivity of the material constituting the fin must be high in order to minimize the temperature gradients between the base and the tip of the fin.

The increase in heat flux will be maximum if the fin is at a uniform temperature at T_p (infinite conductivity).

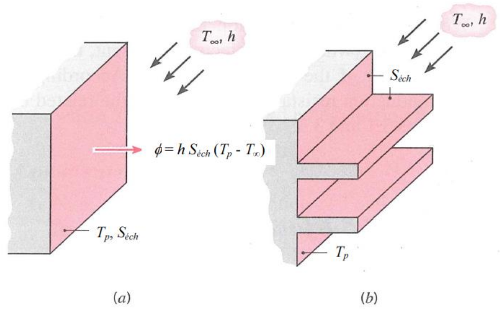


Figure I.23. Use of fins to increase the heat flow removed from the wall: (a) flat wall, (b) wall with fins.

There are several configurations of fins (see figure below), the choice of which, in practice, is conditioned by numerous criteria: the available space in the system, the weight, the ease of manufacturing, the costs...It is also necessary to take into account the disruption of the flow caused by the presence of the fins (pressure losses).

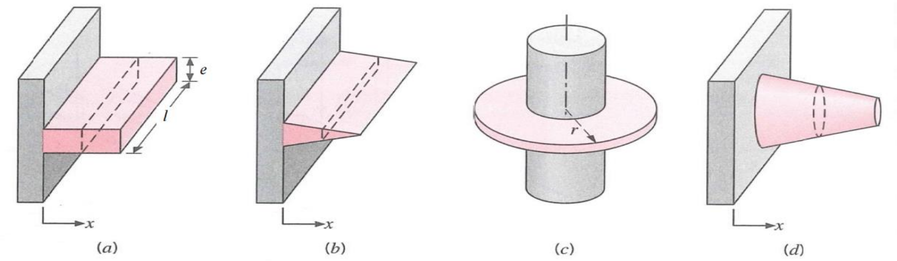


Figure I.24. Different types of fins: (a) straight fin with constant section, (b) straight fin with variable section, (c) annular fin, (d) needle-shaped fin with variable section.

I.8.2.Heat transfer in rectangular fins

We seek to determine to what extent the presence of fins can improve the heat transfer from a solid surface to the surrounding fluid. Let's consider the fin with a constant cross-section schematized in Figure I.25, immersed in a fluid in motion at the temperature T_∞. To quantify the heat transfer associated with this fin, we must first determine the temperature distribution along the fin based on an energy balance that we will establish by making the following assumptions:

- The regime is steady and there is no internal heat dissipation.
- The thermal conductivity of the fin λ is constant.
- The convective heat transfer coefficient h is uniform over the entire surface of the fin.
- We neglect heat transfer by radiation.

- The problem is one-dimensional, meaning that the heat flow only propagates in one direction (the x direction). It is thus considered that the temperature is uniform in a given section of the fin in x, which is generally ensured by the use of thin fins.

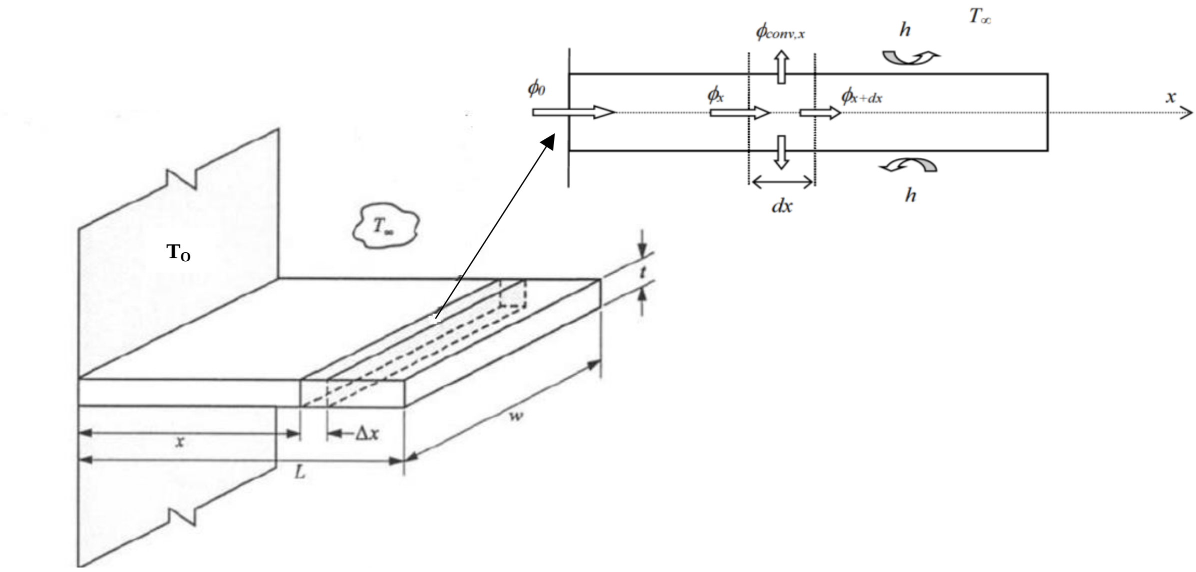


Figure I.25. Energy balance on a fin slice of thickness dx.

Let's perform an energy balance on the system consisting of a slice of the fin between x and x + dx:

$$\varphi_x = \varphi_{x+dx} + \varphi_{conv}$$

- φ_x heat flux transmitted by conduction in x :

  $$\varphi_x = -\lambda . S . \left(\frac{dT}{dx}\right)_x$$

- φ_{x+dx} heat flux transmitted by conduction at x + dx:

  $$\varphi_{x+dx} = -\lambda . S . \left(\frac{dT}{dx}\right)_{x+dx}$$

- $\Phi_{conv,x}$ flux evacuated by convection at the boundary between x and x + dx:

  $$\varphi_{conv,x} = h . p . dx(T(x) - T_\infty)$$

Where :

S is the area of the cross-section through which the conduction flow passes.

p is the perimeter of the fin (perimeter of convective flow exchange).

$$\lambda.S.\left(\frac{dT}{dx}\right)_{x+dx} - \lambda.S.\left(\frac{dT}{dx}\right)_{x} = h.p.dx(T(x) - T_{\infty})$$

$$\Leftrightarrow \left(\frac{dT}{dx}\right)_{x+dx} - \left(\frac{dT}{dx}\right)_{x} = \frac{h.p}{\lambda S}dx(T(x) - T_{\infty})$$

$$\left(\frac{dT}{dx}\right)_{x+dx} - \left(\frac{dT}{dx}\right)_{x} = \left[\left(\frac{dT}{dx}\right)_{x} + d\left(\frac{dT}{dx}\right)\right] - \left(\frac{dT}{dx}\right)_{x} = \frac{d^2T}{dx^2}dx$$

$$\Rightarrow \frac{d^2T}{dx^2}dx = \frac{h.p}{\lambda S}dx(T(x) - T_{\infty})$$

$$\Rightarrow \frac{d^2T}{dx^2} = \frac{h.p}{\lambda S}(T(x) - T_{\infty}) \qquad (I.55)$$

The temperature field in the fin T(x) is thus determined by solving this equation (sometimes called the beam equation) associated with two boundary conditions, written at the base and at the tip of the fin.

Let's ask

$$\theta(x) = (T(x) - T_{\infty}) \quad \text{et} \quad m^2 = \frac{h.p}{\lambda.S}$$

Equation (I.55) becomes:

$$\frac{d^2\theta}{dx^2} - m^2\theta = 0 \qquad (I.56)$$

• h convective exchange coefficient ($W.m^{-2}.K^{-1}$).
• λ thermal conductivity of the material constituting the fin ($W.m^{-1}.K^{-1}$).
• S fin section (m^2).
• p fin perimeter (m)
For a rectangular fin of thickness e and width l: S = t.w and p = 2.(t+w)
For a cylindrical fin of radius R: $S=\pi.R^2$ and $p=2\pi.R$

The differential equation (I.56), of order 2, linear and homogeneous with constant coefficients, admits a general solution of the form:

$$\theta = Ae^{mx} + Be^{-mx} \qquad (I.57)$$

Where $\theta = A_1 ch(mx) + B_1 sh(mx)$ (I.58)

The constants A, B, A_1, and B_1 are determined from the boundary conditions at the base and the tip of the fin.

I.8.3. Infinitely long rectangular fin of constant cross-section

For a fin of "infinite" length, the temperature at the tip will be equal to the temperature of the surrounding medium T_∞.

Thus, $\theta_L \to 0$ if $L \to \infty$. The boundary condition in x = L is then written:

In $x = 0: \ \theta(0) = T_0 - T_\infty$ (a)

In $x = L: \ \theta(L) = 0$ (b)

(b) $\Rightarrow A = 0$

(a) $\Rightarrow B = T_0 - T_\infty$

Where;

$$\frac{T(x) - T_\infty}{T_0 - T_\infty} = e^{-mx} \qquad (I.59)$$

The flux dissipated over the entire surface of the fin can be calculated by integrating the local convection flux:

$$\varphi_p = \int_0^L h.p\,[T(x) - T_\infty]dx$$

Or more easily by noticing that in the case of the steady state, it is the same as that transmitted by conduction at the base of the fin, namely: $\varphi_p = \varphi_{c(x=0)}$

$$\varphi_c = -\lambda.S.\left(\frac{dT}{dx}\right)_{x=0} = -\lambda.S.(T_0 - T_\infty)(-m)e^{(-mx)} \quad \text{avec} \quad m = \sqrt{\frac{h.p}{\lambda.S}}$$

Where from :

$$\varphi_p = \sqrt{h.p.\lambda.S}(T_0 - T_\infty) \qquad (I.60)$$

Note:

In practice, the assumption $L \to \infty$ will be considered valid if

$$\frac{\theta_L}{\theta_0} < 0.01 (= 1\%)(\text{ ou } T_L = 0.99 T_\infty),$$

$$mL \geq \ln(10)$$

$$\text{Where } L \geq \frac{4.6}{m} \quad \text{witg} \quad m = \sqrt{\frac{h.p}{\lambda.S}}$$

I.8.4. Rectangular fin with constant cross-section isolated at the tip

The solution of the second-order differential equation is of the type:

$$\theta(x) = A_1 ch(mx) + B_1 sh(mx) \qquad (I.61)$$

We determine A_1 and B_1 from the boundary conditions:

$$\begin{cases} T(x=0) = T_0 \Rightarrow \theta(0) = T_0 - T_\infty = \theta_0 \\ -\lambda.S.\left(\frac{dT}{dx}\right)_{x=L} = 0 \text{ (conservation of heat flux at } x = L) \Rightarrow \left(\frac{d\theta}{dx}\right)_{x=L} = 0 \end{cases}$$

$$\theta_0 = A_1 ch\big(m.(0)\big) + B_1 sh\big(m.(0)\big) \Rightarrow A_1 = \theta_0$$

$$\left(\frac{d\theta}{dx}\right)_{x=L} = A_1.sh(mL) + B_1 ch(mL) = 0 \Rightarrow B_1 = \frac{-\theta_0.sh(mL)}{chmL}$$

The temperature distribution is then written as:

$$\theta(x) = \theta_0 ch(mx) - \frac{\theta_0 sh(mL)}{chmL} sh(mx)$$

$$\Rightarrow \theta(x) = \theta_0 \left[\frac{ch(mx).chmL - sh(mL).sh(mx)}{chmL}\right]$$

$$\Rightarrow \theta(x) = \theta_0 \left[\frac{ch(m(L-x))}{chmL}\right]$$

The solution can be written as:

$$\frac{(T(x) - T_\infty)}{T_0 - T_\infty} = \frac{ch(m(L-x))}{chmL} \qquad (I.62)$$

And the total flow dissipated by the fin is expressed as:

$$\varphi_p = m.\lambda.S.\tanh(m.L)(T_0 - T_\infty) \qquad (I.63)$$

II.8.5. Efficiency of a fin

The efficiency of a fin is defined as the ratio between the heat flow removed by the fin φ_p and the heat flow that would be removed without the fin φ_{max}:

The flow exchanged by this ideal fin would be:

$$\varphi_{max} = h.S.(T_0 - T_\infty) \qquad (I.64)$$

- In the case of the "infinite length" fin, the efficiency is written as:

$$\varphi_p = \sqrt{h.p.\lambda.S}(T_0 - T_\infty) \qquad (I.65)$$

$$\varepsilon = \frac{\varphi_p}{\varphi_{max}}$$

$$\varepsilon = \frac{\sqrt{h.p.\lambda.S}(T_0 - T_\infty)}{h.S.(T_0 - T_\infty)}$$

$$\Rightarrow \varepsilon = \sqrt{\frac{\lambda.p}{h.S}} \qquad (I.66)$$

- In the case of the "finite length L" fin,

$$\varphi_{max} = h.p.L.(T_0 - T_\infty) \qquad (I.67)$$

Efficiency is written as:

$$\varepsilon = \frac{\varphi_p}{\varphi_{max}} = \frac{m.\lambda.S.\tanh(m.L)(T_0 - T_\infty)}{h.p.L.(T_0 - T_\infty)} = \frac{\sqrt{\frac{h.p}{\lambda.S}}.\lambda.S.\tanh(mL)}{h.p.L} = \frac{\sqrt{h.p.\lambda.S}}{h.p.L}.\tanh(mL)$$

$$\varepsilon = \frac{\tan h(m.L)}{m.L} \qquad (I.68)$$

The effectiveness of a fin is established if $\varepsilon \geq 1$. Thus, the efficiency of the fin is improved by:

- ✓ The choice of a high-conductivity material.
- ✓ The choice of the fin geometry, such as high $\frac{P}{s}$ (use of thin fins).
- ✓ The choice of a "relatively" low convective heat transfer coefficient (while ensuring a high φ_p evacuated flow).

Thus, the use of fins will be more justified in cases where the flowing fluid is a gas rather than a liquid, and when heat transfer occurs by natural convection.

Exercises

Exercise 01:

Calculate the thermal flux as well as the thermal flux density through a flat and homogeneous plate with a thickness of 50mm if it is:

• In stainless steel (λa=16W/m.K) with dimensions 3m x 2m.

• In concrete (λ_b=0.92W/m.K) with dimensions 30m x 20m.

In both cases, the temperatures at the surfaces of the plate are maintained constant and equal to: Tp_1=100°C, Tp_2=90°C.

Solution:

-In stainless steel (λa=16W/m.K) with dimensions 3m x 2m.

$$\varphi_a = \frac{\lambda_a}{e}.S.(T_1 - T_2) = \frac{16}{50.10^{-3}}.(3\,.2)(100 - 90) = \mathbf{19200W}$$

The heat flux density

$$\phi_a = \frac{\varphi_a}{S} = \frac{19200}{3\,.2} = 3200W/m^2$$

-In concrete (λ_b=0.92W/m.K) with dimensions 30m x 20m.

$$\varphi_b = \frac{\lambda_b}{e}(T_1 - T_2) = \frac{0{,}92}{50.10^{-3}}(30.20)(100 - 90) = \mathbf{110400W}$$

The heat flux density

$$\phi_b = \frac{\varphi_b}{S} = \frac{110400}{30\,.20} = \mathbf{184W/m^2}$$

Exercise 02:

-Determine the heat flux through a flat wall with a thickness of e=10cm and a conductivity of λ=8.5W/m.K. The temperatures on the limiting faces of the wall are respectively equal to 100°C and 30°C. The surface area of the wall S=3m².

- Find the temperature gradient in the direction of flow.

-Calculate the depth of the wall where the temperature is 60°C.

Solution:

$$\varphi = \frac{\lambda}{e}.S.(T_1 - T_2) = \frac{8,5}{10.10^{-2}}.(3\,)(100 - 30) = \mathbf{17850W}$$

$$\varphi = -\lambda.S.\frac{dT}{dx}$$

$$\Rightarrow \frac{dT}{dx} = \frac{-\varphi}{\lambda.S} = \frac{-17850}{8,5.3} = \mathbf{-700K/m}$$

$$T(x) = \frac{(T_2 - T_1)}{e}x + T_1 \Rightarrow T(x) = -700.x + 100$$

$$x = \frac{100 - T(x)}{700} = \frac{100 - 60}{700} = 0{,}057m = \mathbf{5,7cm}$$

Exercise 03:

The heat flux density Φ is 5000 $W.m^{-2}$ at the surface of an electric heating element. The temperature of this same element is 110°C when it is cooled by forced convection in air with a temperature of 60°C.

-What is the average exchange coefficient h?

-What will be the temperature of the heating element if the flux density is reduced to 2000 $W.m^{-2}$?

Solution:

Using the expression for the total exchanged flux density, one can determine the average exchange coefficient:

$$\bar{h} = \frac{\varphi}{\Delta T} = \frac{5000}{110 - 60} = \mathbf{100W.m^{-2}.K^{-1}}$$

If the flow is reduced, the forced convection coefficient remains the same, so the temperature difference will be:

$$\Delta T = T_{element} - 60 = \frac{\varphi}{h} = \frac{2000}{100} = \mathbf{20°C}$$

So the temperature

$$T_{element} = \Delta T + 60 = 20 + 60 = \mathbf{80°C}$$

Exercise 04:

A pipe transports steam through a large room where the air and walls are at a temperature of 25°C. The external diameter of the pipe is 70mm and the surface temperature is 200°C. Its emissivity is 0.8.

1. What is the power emission flux density of the surface of the duct?

2. What is the density of the radiation power flux from this surface?

3. The convection coefficient from the surface to the ambient air is 15W/m²·K and the surface is considered gray.

-Calculate the heat transfer flux density of this surface per unit length of the pipe?

Solution:

1.Emission power flux density:

$$q_{surface} = \varepsilon.\sigma.T_p^4 = 0.8x5.67x10^{-8}x(200 + 273.15)^4 = 2270W/m^2$$

2. Radiation power flux density:

$$q_{max} = \sigma.T^4 = 5.67x10^{-8}x(25 + 273.15)^4 = 447W/m^2$$

3. Heat transfer flux from this surface:

$$Q_{tot} = Q_{ray} + Q_{conv} = h.S.(T_P - T_\infty) + \varepsilon.S.\sigma.(T_p^4 - T_\infty^4)$$

$$S = \pi x D x L = \pi x 70x10^{-3} x L$$

$$Q_{tot} = \pi x D x L x\left[h.(T_P - T_\infty) + \varepsilon.\sigma.(T_p^4 - T_\infty^4)\right]$$

Heat transfer flux density of this surface per unit length of the pipe:

$$q_{tot}\frac{Q_{tot}}{L} = \pi xDxLx\left[h.(T_P - T_\infty) + \varepsilon.\sigma.(T_p^4 - T_\infty^4)\right]$$

$$= \pi x70x10^{-3}x[15.(200 - 25) + 0.8x5.67x10^{-8}((200 + 273.15)^4 - (25 + 273.15)^4)]$$

$$= 998W/m$$

Exercise 05:

A 15 cm thick concrete wall separates a room at a temperature of Ti= 20 °C from the outside where the temperature is Te= 5 °C.

We are given: hi = 9.1 W.m-2.K^{-1}, he = 16.7 W.m-2.K^{-1}, λ = 1.74 W.m^{-1}.K^{-1}.

Calculate:

- The total thermal resistance.

- The flux density.

- The internal and external temperatures of the wall.

Solution:

$$\varphi = h_i(T_i - T_{ip}) = \frac{\lambda}{e}(T_{ip} - T_{ep}) = h_e(T_{ep} - T_e) = \frac{(T_i - T_e)}{R}$$

With

$$R = \frac{1}{h_i} + \frac{e}{\lambda} + \frac{1}{h_e} = \frac{1}{9{,}1} + \frac{0{,}15}{1{,}74} + \frac{1}{0{,}06} = \mathbf{0{,}2562 m^2.K.W^{-1}}$$

$$\varphi = \frac{(T_i - T_e)}{R} = \frac{20-5}{0{,}2562} = \mathbf{58{,}546 W.m^2}$$

From the first equality:

$$T_{ip} = T_i - \frac{\varphi}{h_i} = 20 - 58{,}546\,.\,0{,}11 = \mathbf{13{,}6°C}$$

In the same way

$$T_{ep} = T_e + \frac{\varphi}{h_e} = 5 + 58{,}546\,.\,0{,}06 = \mathbf{8{,}5°C}$$

Exercise 06:

Calculate the flux passing through a glass pane with an area of 1 m² and a thickness of 3.5 mm. The temperature of the inner surface of the glass is 10°C, while that of the outer surface is 5°C.

1-Deduce the thermal resistance of the glass, knowing that the thermal conductivity of the glass is: λg= 0.7 $W.m^{-1}.K^{-1}$

2-For the same wall temperatures, calculate the flux passing through 1 m² of a 26 cm thick brick wall.

3-Deduce the thermal resistance, knowing that the thermal conductivity of the bricks is: λ_b= 0.52 $W.m^{-1}.K^{-1}$.

Solution:

-Flux passing through 1m² of glass:

$$\varphi = \frac{\lambda.}{e}.S.(T_1 - T_2) = \frac{0{,}7}{3{,}5\,.\,10^{-3}}.1.(10-5) = \mathbf{1000\ W}$$

-Thermal resistance of 1m^2 of glass:

$$R_{glass} = \frac{(T_1 - T_2)}{\varphi} = \frac{10-5}{1000} = \mathbf{5.10^{-3}}\ \frac{\mathbf{°C}}{\mathbf{W}}$$

Or

$$R_{verre} = \frac{e}{\lambda.S} = \frac{3{,}5\,.\,10^{-3}}{0{,}7\,.\,1} = \mathbf{5\,.\,10^{-3}}\ \frac{\mathbf{°C}}{\mathbf{W}}$$

-Flux passing through 1m² of brick wall:

$$\varphi = \frac{\lambda.}{e}.S.(T_1 - T_2) = \frac{0{,}52}{0{,}26}.1.(10 - 5) = \mathbf{10\ W}$$

-Thermal resistance of 1m² of brick wall:

$$R_{brick} = \frac{(T_1 - T_2)}{\varphi} = \frac{10 - 5}{100} = \mathbf{0,5\ \frac{°C}{W}}$$

Or

$$R_{brick} = \frac{e}{\lambda.S} = \frac{0{,}26}{0{,}52\ .\ 1} = \mathbf{0,5\ \frac{°C}{W}}$$

Exercise 07

A double glazing consists of two glass panes separated by a layer of still dry air. The thickness of each glass pane is 3.5 mm and that of the air layer is 12 mm.

The thermal conductivity of glass is equal to 0.7 $W.m^{-1}.°C^{-1}$ and that of air is 0.024 $W.m^{-1}.°C^{-1}$ in the studied temperature range. For a temperature drop of 5°C between the two extreme faces of the double glazing, calculate the heat losses for a 1m² window.

Compare these thermal losses to those that would be obtained with a single pane of glass with a thickness of 3.5 mm.

Solution:

-Double glazing consists of three thermal resistances in series.

$$R_{tot} = R_v + R_a + R_v$$

The flux passing through this double glazing is given by:

$$\varphi_{double\ glazing} = \frac{T_{int} - T_{ext}}{R_{tot}} = \frac{T_{int} - T_{ext}}{R_g + R_a + R_g} = \frac{T_{int} - T_{ext}}{\frac{e_g}{\lambda_g S} + \frac{e_a}{\lambda_a . S} + \frac{e_g}{\lambda_g . S}} = \frac{(T_{int} - T_{ext}).S}{2\frac{e_g}{\lambda_g} + \frac{e_a}{\lambda_a .}}$$

$$\varphi_{double\ glazing} = \frac{5\ .\ 1}{2.\frac{3{,}5\ .\ 10^{-3}}{0{,}7} + \frac{12\ .\ 10^{-3}}{0{,}024}} = \mathbf{9,8W}$$

Let's compare the flux passing through double glazing to that passing through a single glass pane for the same surface area and the same temperature difference.

$$\varphi_{1\ \text{single pane}} = \frac{T_{int} - T_{ext}}{R_g} = \frac{\lambda_g . S}{e_g}(T_{int} - T_{ext})$$

$$\varphi_{1\ \text{single pane}} = \frac{0,7\ .\ 1}{3,5\ .\ 10^{-3}}\ .\ 5 = \mathbf{1000}\ \boldsymbol{W}$$

Exercise 08:

Calculate the flux passing through the 50 m² facade of a house. The wall is made of bricks 26 cm thick. The facade is pierced with 4 windows of 2 m² in surface area and 3.5 mm in thickness and a wooden door of 2 m² and 42 mm in thickness.

It is assumed that the internal wall temperature is equal to 10°C for all the materials constituting the facade, likewise, the external wall temperature is 5°C.

Thermal conductivity of glass: $\lambda g = 0.7\ W.m^{-1}.K^{-1}$

Thermal conductivity of bricks: $\lambda b = 0.52\ W.m^{-1}.K^{-1}$

Thermal conductivity of wood: $\lambda_{wood} = 0.21\ W.m^{-1}.K^{-1}$

Solution:

The facade can be likened to 3 resistances in parallel: that of the windows, that of the door, and that of the brick wall. We therefore calculate each of these resistances to deduce the equivalent resistance and finally to calculate the flux passing through the facade.

-Thermal resistance of windows:

$$R_{glass} = \frac{e_{glass}}{\lambda_g . S_{glass}} = \frac{3,5\ .10^{-3}}{0,7.\ 4.\ 2} = \mathbf{0,625\ .10^{-3}\ °C/W}$$

-Thermal resistance of the door:

$$R_{door} = \frac{e_{door}}{\lambda_{wood} . S_{door}} = \frac{0,042}{0,21\ .2} = \mathbf{0,1\ °C/W}$$

-Thermal resistance of the wall:

$$R_{wall} = \frac{e_{wall}}{\lambda_b . S_{wall}} = \frac{0,26}{0,52.(50 - 4.2 - 2)} = \mathbf{0,0125\ °C/W}$$

-Equivalent resistance of the facade:

$$\frac{1}{R} = \frac{1}{R_{vitre}} + \frac{1}{R_{porte}} + \frac{1}{R_{mur}} = \frac{1}{0{,}625\,.10^{-3}} + \frac{1}{0{,}1} + \frac{1}{0{,}0125} = 1690 W/°C$$

$$R = \frac{1}{1690} = \mathbf{0{,}592\,.10^{-3}\ °C/W}$$

So the flow passing through the facade is:

$$\varphi_{facade} = \frac{(T_1 - T_2)}{R} = 5.\ 1690 = \mathbf{8450\ W}$$

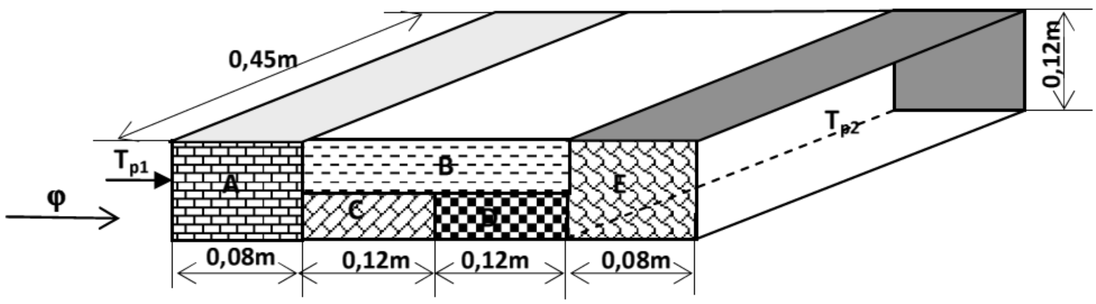
Exercise 09:

Let's consider a wall composed of several layers of different materials illustrated below with the dimensions in the three directions. Assuming one-dimensional conduction and knowing the temperatures of the left and right surfaces, respectively, Tp_1 and Tp^2, as well as the thermal conductivities of these different layers.

-Calculate the heat flux per unit area through this wall.

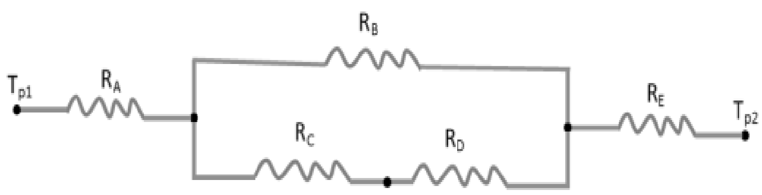
Data:

T_{p1} =200°C, T_{p2} =50°C, k_A =70 W/m.K, k_B =60 W/m.K, k_C =40 W/m.K, k_D =30 W/m.K, k_E =20 W/m.K



Solution:

To simplify the calculation, we must represent the equivalent diagram of the wall in question, of course based on the analogy existing between thermal and electrical quantities.



Equivalent electrical diagram

The heat flux is therefore given by:

$$\varphi = \frac{T_{P1} - T_{p2}}{R_A + R_{éq} + R_E}$$

such as;

$$\begin{cases} R_A = \dfrac{L_A}{k_A.S_A} = \dfrac{0{,}08}{70\ .\ 0{,}12\ .\ 0{,}45} = 0{,}02116K/W \\ R_B = \dfrac{L_B}{k_B.S_B} = \dfrac{0{,}24}{60\ .\ 0{,}06\ .\ 0{,}45} = 0{,}1481K/W \\ R_C = \dfrac{L_C}{k_C.S_C} = \dfrac{0{,}12}{40\ .\ 0{,}06\ .\ 0{,}45} = 0{,}1111K/W \\ R_D = \dfrac{L_D}{k_D.S_D} = \dfrac{0{,}12}{30\ .\ 0{,}06\ .\ 0{,}45} = 0{,}1481K/W \\ R_E = \dfrac{L_E}{k_E.S_E} = \dfrac{0{,}08}{20\ .\ 0{,}12\ .\ 0{,}45} = 0{,}0740K/W \end{cases}$$

$$\frac{1}{R_{éq}} = \frac{1}{R_B} + \frac{1}{R_C + R_D} = \frac{R_B + R_C + R_D}{R_B.(R_C + R_D)}$$

$$\Rightarrow R_{éq} = \frac{R_B.(R_C + R_D)}{R_B + R_C + R_D}$$

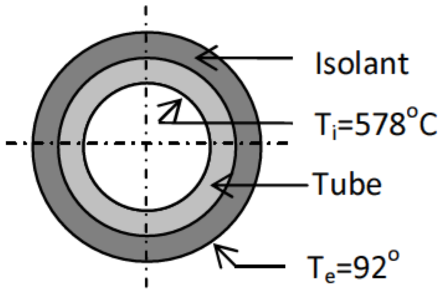
$$R_{éq} = \frac{0{,}1481.(0{,}1111 + 0{,}1481)}{0{,}1481 + 0{,}1111 + 0{,}1481} = \mathbf{0,09425K/W}$$

$$\varphi = \frac{(T_{P1} - T_{P2})}{R_A + R_{éq} + R_E} = \frac{(200 - 50)}{0{,}02116 + 0{,}09425 + 0{,}0740} = \mathbf{791,94W}$$

Exercise 10:

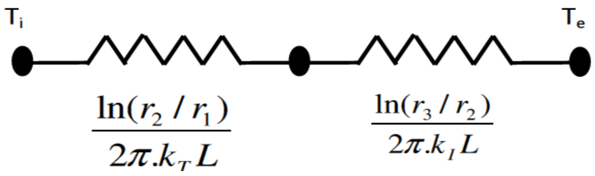
A stainless steel tube (k=19W/m.°C) with a 3cm inner diameter and a 5cm outer diameter is insulated by a layer of asbestos (k=0.2 W/m.°C) with a thickness of 2.5cm. Knowing that the temperature of the inner wall of the tube is maintained at 578°C and that of the outer wall of the insulation is at 92°C.

1. Provide the equivalent electrical diagram.
2. Calculate the heat loss per meter of length.



Solution:

1. Equivalent electrical diagram:



2. The heat flux lost per meter of length:

$$\varphi = \frac{\Delta T}{\sum R_i} = \frac{T_i - T_e}{R_T + R_I}$$

$$R_T = \frac{\ln(r_2/r_1)}{2\pi.k_T.L}$$

$$R_I = \frac{\ln(r_3/r_2)}{2\pi.k_I.L}$$

$$\varphi = \frac{2\pi.L(T_i - T_e)}{\frac{\ln(r_2/r_1)}{k_T} + \frac{\ln(r_3/r_2)}{k_I}}$$

$$\Rightarrow \frac{\varphi}{L} = \frac{2\pi(T_i - T_e)}{\frac{\ln(r_2/r_1)}{k_T} + \frac{\ln(r_3/r_2)}{k_I}}$$

$$\frac{\varphi}{L} = \frac{2\pi(578 - 92)}{\frac{\ln(2{,}5/1{,}5)}{19} + \frac{\ln(5/2{,}5)}{k_I}} = \mathbf{874{,}3087W/m}$$

Exercise 11:

Let's take a stainless steel spoon (λ= 15 W/m.°C), partially immersed in boiling water at 93°C in a kitchen at 24°C. The handle of the spoon has a cross-section of 0.2cm*1.3cm and extends 18cm into the air above the water's surface. If the heat transfer coefficient at the surface of the spoon exposed to air is h=17 W/m².°C, determine the temperature difference at the surface of the spoon's handle. Indicate your hypotheses.

Solution:

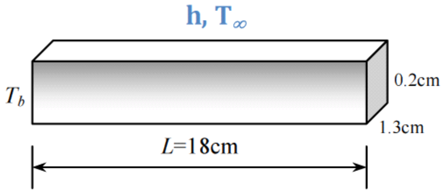
Hypotheses:

1. The temperature of the submerged part of the spoon is equal to the temperature of the water.
2. The temperature of the spoon varies along the spoon T (x).
3. The heat transfer from the tip of the spoon is negligible.
4. The heat transfer coefficient is constant and uniform across the entire surface of the spoon.
5. The thermal properties of the spoon are constant.
6. The heat transfer by radiation is assumed to be negligible.

Noting that the cross-section of the spoon is constant and x has its origin at the free surface of the water. The temperature variation along the spoon can be expressed as:

$$\frac{T(x) - T_\infty}{T_b - T_\infty} = \frac{\cosh a(L - x)}{\cosh aL}$$

Where ;



$$P = 2.(0{,}002 + 0{,}013) = 0{,}030\text{m}$$

$$S_c = (0{,}002)(0{,}013) = 0{,}000026\text{m}^2$$

$$a = \sqrt{\frac{h.p}{\lambda.S_C}} = \sqrt{\frac{17\,.\,0{,}03}{15\,.\,0{,}000026}} = 36\text{m}^{-1}$$

The temperature of the tip of the spoon is determined by:

$$T(L) = T_\infty + (T_b - T_\infty)\frac{\cosh a(L - L)}{\cosh aL}$$

$$T(L) = 24 + (93 - 24)\frac{\cosh a(0)}{\cosh a(36\,.\,0{,}18)} = \mathbf{24{,}2°}\boldsymbol{C}$$

Therefore, the temperature difference across the entire exposed handle of the spoon is:

$$\Delta T = T_b - T(L) = (93 - 24{,}2) = \mathbf{68{,}8°C}$$

Chapiter II
Heat Transfer by Convection

II.1. Introduction

Thus far we have focused on heat transfer by conduction and have considered convection only to the extent that it provides a possible boundary condition for conduction problems. We used the term convection to describe energy transfer between a surface and a fluid moving over the surface. Convection includes energy transfer by both the bulk fluid motion (advection) and the random motion of fluid molecules (conduction or diffusion). In our treatment of convection, we have two major objectives. In addition to obtaining an understanding of the physical mechanisms that underlie convection transfer, we wish to develop the means to perform convection transfer calculations.

This chapter is developed primarily to achieving the former objective. Physical origins are discussed, and relevant dimensionless parameters, as well as important analogies, are developed.

II.2. Definitions

Convection is defined as the mode of heat transfer between a solid surface and a fluid (liquid or gas) at different temperatures. It is therefore a transfer of heat accompanied by speed.

In convection, the transfer of heat at the solid surface occurs only through conduction. But, in the parts of the fluid that surround the surface, two simultaneous phenomena occur: conduction and mass diffusion through movement at both the molecular and macroscopic levels. Thanks to this movement, the transferred heat flow is greater. The higher the speed, the greater the heat transfer. We can summarize the mechanism of heat transfer by convection as follows: the fluid in contact with the solid surface receives heat from it through conduction and then transfers it to the rest of the fluid that is not in direct contact with the surface through diffusion, thanks to the movement of the fluid.

We can consider two types of convection, depending on the causes that produce the movement of the fluid: forced convection and free or natural convection.

•*Forced convection*: occurs when the movement of the fluid is a consequence of imposed external actions (pump, fan, wind, etc.). In this case, the temperature field is convected by an imposed external flow.

• ***Free or natural convection***: is produced by a movement of the fluid, caused by a difference in density between the cold and hot parts of the fluid. The distribution of temperature generates its own movement by creating rotational Archimedean forces.

• When both types of convection exist simultaneously without one being negligible compared to the other, the convection is said to be ***mixed.***

II.3. Physical mechanism of convection

Convection heat transfer is complicated by the fact that it involves fluid motion as well as heat conduction. The fluid motion enhances heat transfer, since it brings warmer and cooler chunks of fluid into contact, initiating higher rates of conduction at a greater number of sites in a fluid. Therefore, the rate of heat transfer through a fluid is much higher by convection than it is by conduction. In fact, the higher the fluid velocity, the higher the rates of heat transfer.

Experience shows that convection heat transfer strongly depends on the fluid properties dynamic viscosity m, thermal conductivity λ, density ρ, and specific heat cp, as well as the fluid velocity U. It also depends on the geometry and the roughness of the solid surface, in addition to the type of fluid flow (such as being streamlined or turbulent). Thus, we expect the convection heat transfer relations to be rather complex because of the dependence of convection on so many variables. This is not surprising, since convection is the most complex mechanism of heat transfer.

Despite the complexity of convection, the rate of convection heat transfer is observed to be proportional to the temperature difference and is conveniently expressed by Newton’s law of cooling as

$$\varphi = h.S.(T_S - T_\infty) \qquad \text{(II.1)}$$

Where
h : convection heat transfer coefficient, $W/m^2 \cdot K$
S : heat transfer surface area, m^2
T_s: temperature of the surface, °C
T_∞: temperature of the fluid sufficiently far from the surface, °C.

Fluid flow is often confined by solid surfaces, and it is important to understand how the presence of solid surfaces affects fluid flow.

All experimental observations indicate that a fluid in motion comes to a complete stop at the surface and assumes a zero velocity relative to the surface. That is, a fluid in direct contact with a solid "sticks" to the surface due to viscous effects, and there is no slip. This is known as the no-slip condition. A fluid layer adjacent to a moving surface has the same velocity as the surface.

A consequence of the no-slip condition is that all velocity profiles must have zero values with respect to the surface at the points of contact between a fluid and a solid surface(Figure II.1).

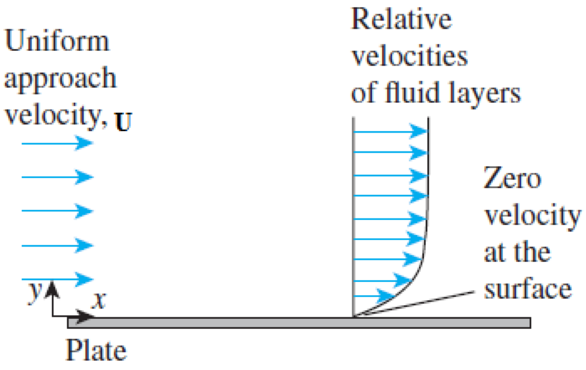


Figure II.1. A fluid flowing over a stationary surface comes to a complete stop at the surface because of the no-slip condition

An implication of the no-slip condition is that heat transfer from the solid surface to the fluid layer adjacent to the surface is by pure conduction, since the fluid layer is motionless, and can be expressed as

$$\varphi_{conv} = \varphi_{cond} = -\lambda_{fluid} . S . \left.\frac{\partial T}{\partial y}\right]_{y=0} \qquad (II.2)$$

Where T represents the temperature distribution in the fluid and $\left.\frac{\partial T}{\partial y}\right]_{y=0}$ is the *temperature gradient* at the surface. Heat is then *convected away* from the surface as a result of fluid motion. Note that convection heat transfer from a solid surface to a fluid is merely the conduction heat transfer from the solid surface to the fluid layer adjacent to the surface. Therefore, we can equate Eqs. II.1 and II.2 for the heat flux to obtain

$$h = \frac{-\lambda_{\text{fluid}} \cdot \left.\frac{\partial T}{\partial y}\right|_{y=0}}{(T_S - T_\infty)} \qquad (II.3)$$

II.4. Classification of fluid flows

Convection heat transfer is closely tied with fluid mechanics, which is the science that deals with the behavior of fluids at rest or in motion, and the interaction of fluids with solids or other fluids at the boundaries. There is a wide variety of fluid flow problems encountered in practice, and it is usually convenient to classify them on the basis of some common characteristics to make it feasible to study them in groups. There are many ways to classify fluid flow problems, and here we present some general categories.

II.4.1.Viscous versus Inviscid Regions of Flow

When two fluid layers move relative to each other, a friction force develops between them and the slower layer tries to slow down the faster layer. This internal resistance to flow is quantified by the fluid property *viscosity*, which is a measure of internal stickiness of the fluid. Viscosity is caused by cohesive forces between the molecules in liquids and by molecular collisions in gases. There is no fluid with zero viscosity, and thus all fluid flows involve viscous effects to some degree. Flows in which the frictional effects are significant are called ***viscous flows***. However, in many flows of practical interest, there are *regions* (typically regions not close to solid surfaces) where viscous forces are negligibly small compared to inertial or pressure forces. Neglecting the viscous terms in such ***inviscid flow regions*** greatly simplifies the analysis without much loss in accuracy.

II.4.2. Internal versus External Flow

A fluid flow is classified as being internal or external, depending on whether the fluid is forced to flow in a confined channel or over a surface. The flow of an unbounded fluid over a surface such as a plate, a wire, or a pipe is ***external flow***. The flow in a pipe or duct is ***internal flow*** if the fluid is completely bounded by solid surfaces. Water flow in a pipe, for example, is internal flow, and airflow over a ball or over an exposed pipe during a windy day is external flow. The flow of liquids in a duct is called *open-channel flow* if the duct is only partially filled with the liquid and there is a free surface. The flows of water in rivers and irrigation ditches are examples of such flows. Internal flows are dominated by the influence of viscosity throughout the flow field. In external flows the viscous effects are limited to boundary layers near solid surfaces and to wake regions downstream of bodies.

II.4.3. Compressible versus Incompressible Flow

A flow is classified as being compressible or incompressible, depending on the level of variation of density during flow. Incompressibility is an approximation, and a flow is said to be incompressible if the density remains nearly constant throughout. Therefore, the volume of every portion of fluid remains unchanged over the course of its motion when the flow (or the fluid) is incompressible.

II.4.4. Laminar versus Turbulent Flow

To address a convection problem, it is important to determine the flow regime of the fluid. The convective heat transfer coefficient depends heavily on the flow regime. Some flows are smooth and orderly while others are rather chaotic. The highly ordered fluid motion characterized by smooth layers of fluid is called ***laminar***. The word laminar comes from the movement of adjacent fluid particles together in "laminates." The flow of high-viscosity fluids such as oils at low velocities is typically laminar. The highly disordered fluid motion that typically occurs at high velocities and is characterized by velocity fluctuations is called ***turbulent*** (Figure II.2). The flow of low-viscosity fluids such as air at high velocities is typically turbulent. The flow regime greatly influences the required power for pumping. A flow that alternates between being laminar and turbulent is called transitional.

The transition between laminar and turbulent flow can be determined by calculating the critical Reynolds number. In the case of a flat wall:

$$\mathrm{Re_c} = \frac{\mathrm{U_\infty . x}}{\nu} \qquad (\mathrm{II.4})$$

U_∞ is the free stream velocity far from the solid obstacle and ν is the kinematic viscosity in m^2/s:

$$\nu = \frac{\mu}{\rho}$$

μ is the dynamic viscosity.

In the case of flow in a cylindrical tube of diameter D, the Reynolds number is:

$$Re_c = \frac{U_\infty . D}{\nu} = \frac{\rho . U_\infty . D}{\mu} \qquad (II.5)$$

The Reynolds number represents the ratio of inertial forces to viscous forces.

The critical Reynolds number depends on the surface roughness and the level of turbulence of the free flow. It is generally on the order of 10^5 to $3x10^6$. In the cylinders, it is generally equal to 2300. It characterizes the transition from laminar flow to turbulent flow, that is to say:

If $Re < Re_c$ the flow is laminar

If $Re > Re_c$ the flow is turbulent.

In general, the representative value of the critical Reynolds number is:

$Re_c = 5x10^5$ for flat plates

$Re_c = 2300$ for cylinders

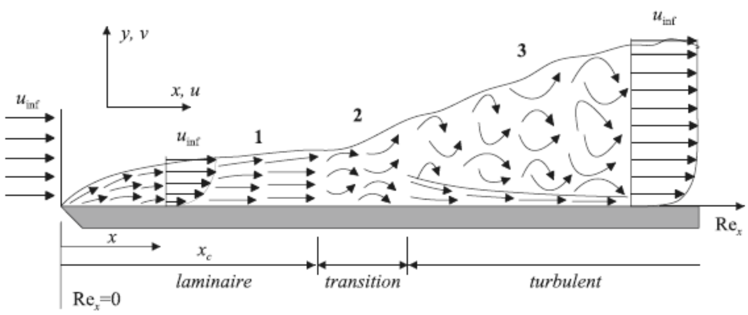


Figure II.2. Development of boundary layer on a long flat surface
Three flow regimes are visible: laminar (1), transitional (2), and turbulent. (3).

II.4.5. Natural (or Unforced) versus Forced Flow

A fluid flow is said to be natural or forced, depending on how the fluid motion is initiated. In forced flow, a fluid is forced to flow over a surface or in a pipe by external means such as a pump or a fan. In natural flows, any fluid motion is due to natural means such as the buoyancy effect, which manifests itself as the rise of the warmer (and thus lighter) fluid and the fall of cooler (and thus denser) fluid.

II.4.6. Steady versus Unsteady Flow

The term ***steady*** implies no change at a point with time. The opposite of steady is **unsteady**.

The term uniform implies no change with location over a specified region. These meanings are consistent with their everyday use (steady girlfriend, uniform distribution, etc.).

The terms unsteady and transient are often used interchangeably, but these terms are not synonyms. In fluid mechanics, unsteady is the most general term that applies to any flow that is not steady, but transient is typically used for developing flows. The term periodic refers to the kind of unsteady flow in which the flow oscillates about a steady mean.

II.5. Significance of the Boundary Layers

II.5.1. Velocity Boundary Layer

To introduce the concept of a boundary layer, consider flow over the flat plate of Figure II.5. When fluid particles make contact with the surface, their velocity is reduced significantly relative to the fluid velocity upstream of the plate, and for most situations it is valid to assume that the particle velocity is zero at the wall.

These particles then act to retard the motion of particles in the adjoining fluid layer, which act to retard the motion of particles in the next layer, and so on until, at a distance $y=\delta$ from the surface, the effect becomes negligible. This retardation of fluid motion is associated with *shear stresses* τ acting in planes that are parallel to the fluid velocity (Figure II.3). With increasing distance y from the surface, the x velocity component of the fluid, u, must then increase until it approaches the free stream value u_∞. The subscript ∞ is used to designate conditions in the *free stream* outside the boundary layer.

The quantity δ is termed the *boundary layer thickness*, and it is typically defined as the value of y for which $u = 0.99.u_\infty$. The *boundary layer velocity profile* refers to the manner in which u varies with y through the boundary layer. Accordingly, the fluid flow is characterized by two distinct regions, a thin fluid layer (the boundary layer) in which velocity gradients and shear stresses are large and a region outside the boundary layer in which velocity gradients and shear stresses are negligible. With increasing distance from the leading edge, the effects of viscosity penetrate farther into the free stream and the boundary layer grows (δ increases with x).

The significance of velocity boundary layer stems from its relation to the surface shear stress τ_s, and hence to surface frictional effects. For external flows it provides the basis for determining the local *friction coefficient.*

$$C_f = \frac{\tau_s}{\rho . u_\infty^2/2} \qquad \text{(II. 6)}$$

Assuming a *Newtonian fluid*, the surface shear stress may be evaluated from knowledge of the velocity gradient at the surface.

$$\tau_s = \mu \left.\frac{\partial u}{\partial y}\right|_{y=0} \qquad \text{(II. 7)}$$

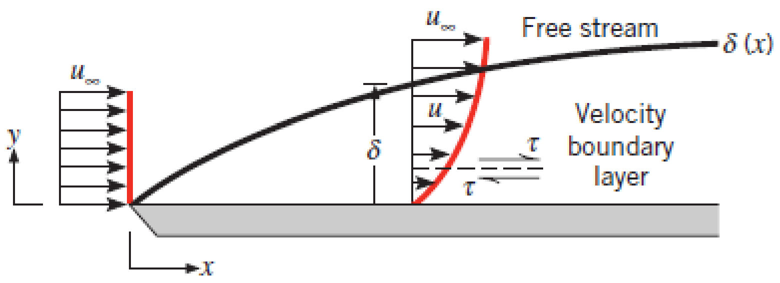


Figure II.3. Velocity boundary layer development on a flat plate.

II.5.2. Thermal Boundary Layer

Just as a velocity boundary layer develops when there is fluid flow over a surface, a thermal boundary layer must develop if the fluid free stream and surface temperatures differ. Consider flow over an isothermal flat plate (Figure II.4). At the leading edge the temperature profile is uniform, with $T(y) = T_\infty$. However, fluid particles that come into contact with the plate achieve thermal equilibrium at the plate's surface temperature. In turn, these particles exchange energy with those in the adjoining fluid layer, and temperature gradients develop in the fluid. The region of the fluid in which these temperature gradients exist is the thermal boundary layer, and its thickness δ_t is typically defined as the value of y for which the ratio $[(Ts - T)/(Ts - T\infty)] = 0.99$. With increasing distance from the leading edge, the effects of heat transfer penetrate farther into the free stream and the thermal boundary layer grows.

The relation between conditions in this boundary layer and the convection heat transfer coefficient may readily be demonstrated. At any distance x from the leading edge, the local surface heat flux may be obtained by applying Fourier's law to the fluid at y=0. That is,

$$\Phi = -\lambda_f.\frac{\partial T}{\partial y}\bigg|_{y=0} \qquad (II.8)$$

This expression is appropriate because, at the surface, there is no fluid motion and energy transfer occurs only by conduction. Recalling Newton's law of cooling, we see that

$$\Phi = h.(T_S - T_\infty) \qquad (II.9)$$

And combining this with Equation (II.8), we obtain

$$h = \frac{-\lambda_f.\frac{\partial T}{\partial y}\Big|_{y=0}}{(T_S - T_\infty)} \qquad (II.10)$$

Hence, conditions in the thermal boundary layer, which strongly influence the wall temperature gradient $\partial T/\partial y|_{y=0}$, determine the rate of heat transfer across the boundary layer.

Since (T_s-T_∞) is a constant, independent of *x*, while δ_t increases with increasing *x*, temperature gradients in the boundary layer must decrease with increasing *x*. Accordingly,

The magnitude of $\partial T/\partial y|_{y=0}$ decreases with increasing *x*, and it follows that ϕ and *h* decrease with increasing *x*.

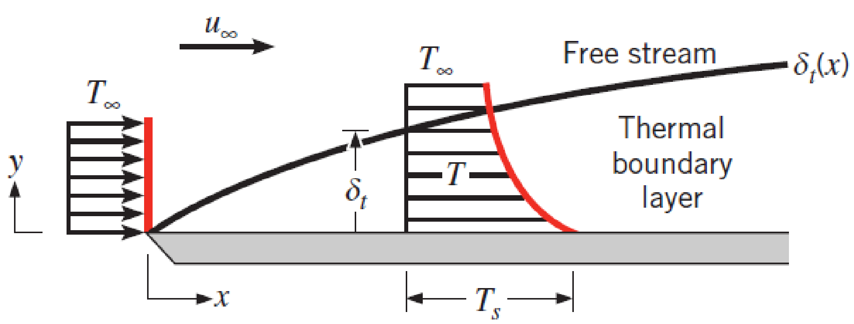


Figure II.4. Thermal boundary layer development on an isothermal flat plate.

II.6. Methods for solving a convection problem

The methods for solving a convection problem can be classified into different categories based on the approach used to analyze the phenomena. Here is a detailed description of the four methods you mentioned:

II.6.1. Analytical method

The analytical method involves the use of differential equations to model and solve convection problems. This approach allows for exact solutions, as long as it is possible to formulate and solve the equations governing fluid dynamics and heat transfer. The equations generally used include the Navier-Stokes equations for fluid dynamics and the heat equation for thermal transfer.

✓ ***Advantages***

-*Precise results*: it allows obtaining exact solutions under certain idealized assumptions (such as simple geometries and well-defined boundary conditions).

-*Experimental verification*: it provides a basis for validating experimental results.

-*Theoretical analysis*: it allows for understanding the fundamental mechanisms behind the phenomenon of convection.

✓ ***Limits***

-*Difficulty of application:* the application of this method quickly becomes complex, even impossible, for geometries or complex boundary conditions.

-*Simplifying assumptions:* this method often requires assumptions that simplify reality (laminar flows, steady-state regimes, etc.).

II.6. 2. Dimensional method

The dimensional method, also called dimensional analysis, is based on the use of dimensionless numbers (such as the Rayleigh number, the Nusselt number, or the Prandtl number) to establish relationships between the different physical quantities that characterize the convection problem.

✓ ***Advantages***

- *Simplification*: it reduces the complexity of the equations by expressing them in a dimensionless form, which helps to identify the dominant parameters of the problem.

-*Experimental guidance:* it directs the experiments and allows the definition of experimental conditions based on simple relationships between physical quantities.

-*Identification of trends:* it helps to generalize results from a specific case.

✓ ***Limits***

-*Approximations:* although it identifies general trends, it does not always provide an exact solution to the problem.

-*Dependent on experience:* experimental validations are often necessary to corroborate the results.

II.6. 3. Analogical method

The analogical method consists of transposing solutions from one mode of energy transfer to another. For example, one can draw an analogy between heat transfer and mass transfer, or between heat transfer by conduction and that by convection, using physical or mathematical similarities.

✓ ***Advantages***

-*Extension of solutions:* allows the use of known solutions for a phenomenon (for example, conduction) to deduce results in another field (for exemple, convection).

-*Ease of use*: it provides an intuitive approach based on similar phenomena.

✓ ***Limits:***

-*Restricted validity*: the analogy may not apply correctly to all regimes or flow conditions.

-*Approximations:* the solutions obtained by analogy may be less precise than those obtained by methods specific to the phenomenon studied.

II.6.4. Empirical or experimental method:

This method is based on experimental measurements and direct observations of the convection phenomenon. Based on the collected data, empirical relationships are established between the different quantities involved. These relationships are often guided by the results of analytical and dimensional methods.

✓ ***Advantages:***

-*Realistic approach:* it takes into account real phenomena without requiring simplifying assumptions.
-*General validity:* empirical relationships can apply to a wide range of conditions and configurations.
-*Practice*: it allows testing complex cases or those not analytically soluble.

✓ ***Limits:***

-*Cost and time*: experiments can be expensive and time-consuming to set up.
-*Deduction of laws:* it is sometimes difficult to generalize results from empirical relationships.

These four methods for solving a convection problem are often used in conjunction with each other. The analytical method and the dimensional method provide a theoretical understanding and general trends, while the experimental method allows for testing and validating results under real conditions. Finally, the analogical method can simplify certain problems by using solutions derived from similar phenomena.

II.7. Dimensional analysis

We can express physical quantities in terms of a limited number of fundamental dimensions.

Examples: Speed: $L.T^{-1}$; dynamic viscosity: $M.L\text{-}1.T^{-1}$; force: $M.L.T^{-2}$

In these examples, we see that the number of fundamental dimensions is 3: Mass M, Length L, Time T.

These three fundamental dimensions are not always sufficient. For heat transfer problems, it is necessary to add a 4th dimension: temperature θ and, when the exchange of energy between mechanical quantities and thermal quantities is not measurable, we will add the amount of heat Q which will be considered a 5th dimension.

The method of dimensional analysis, which is based on the principle of dimensional homogeneity of the terms in an equation, is known as the Vaschy-Buckingham theorem or the π-group theorem.

So all physical quantities are expressed in the following fundamental dimensions:

• The mass (kg) M

• Length (m) L

• Time (s) T

• Temperature (K) θ

The basic variables of heat transfer problems and their dimensions are given in Table II.1.

Table II.1. Units and dimensions of variables in heat transfer by convection.

Quantities	Symbol	S.I. Unit	Dimensional Equation
Speed	U	m/s	$L.T^{-3}$
Characteristic length	D	m	L.
Fluid density	ρ	Kg/m^3	$M.L^{-3}$
Dynamic viscosity of the fluid	μ	Kg/(m.s)	$M.L^{-1}.T^{-1}$
Specific heat capacity	C	J/(kg.K)	$L^2.T^2.\theta^{-1}$
Thermal conductivity of the fluid	λ	W/(m.K)	$M.L.T^{-3}.\theta^{-1}$
Convective heat transfer coefficient	h	$W/(m^2.K)$	$M.T^{-3}.\theta^{-1}$
Temperature difference	T_P-T_∞	K	θ

The VASCHY-BUCKINGHAM theorem allows us to predict that the most general form of the physical law describing the studied phenomenon will be written as:

$$F(\pi_1, \pi_2, \pi_3, \pi_4) = 0$$

Where the πi are dimensionless groups of the form:

$$\pi_i = D^a.\lambda^b.U^c.\rho^d.\mu^e.C^f.h^g.(T_P - T_\infty)^i \qquad (II.11)$$

The number of independent variables in the convection heat transfer problem (D, λ, U, ρ, μ, C, h, Tp-T∞) is n = 8.

According to the BUCKINGHAM theorem, we can formulate N = n - r dimensionless variables, where (r) represents the rank of the matrix defined by the exponents of the dimensions of the variables given in Table II.2.

Table II.2. Equations with dimensions of the 8 quantities.

	D	U	P	μ	λ	C	h	Tp-T∞
Length L	1	1	-3	-1	1	2	0	0
Mass M	0	0	1	1	1	0	1	0
Time T	0	-1	0	-1	-3	-2	-3	0
Temperature θ	0	0	0	0	-1	-1	-1	1

The matrix defined in Table II.1 has rank r = 4. According to BUCKINGHAM, we can therefore formulate

N = 8 - 4 = 4 dimensionless parameters for heat transfer problems by convection.

Each parameter (πi) will be formulated by potential expressions of the basic variables.

$$\pi = (L)^{a}.(L^{+1}.M^{+1}.T^{-3}.\theta^{-1})^{b}.(L^{+1}.T^{-1})^{c}.(L^{-3}.M^{+1})^{d}.(L^{-1}.M^{+1}.T^{-1})^{e}.(L^{+2}.T^{-2}.\theta^{-1})^{f}.(M^{+1}.T^{-3}.\theta^{-1})^{g}.(\theta)^{i}$$

Or with the grouped dimensions

$$[\pi] = [M]^{(b+d+e+g)}.[L]^{(a+b+c-3d+e+2f)}.[T]^{(-3b-c-e-2f-3g)}.[\theta]^{(-b-f-g+i)} \qquad (II.12)$$

For π to remain dimensionless, it is necessary to eliminate the dimensions; the exponents in parentheses must have a value of zero, which defines the following system of linear equations.

$$\begin{cases} b+d+e+g=0 \\ a+b+c-3d-e+2f=0 \\ -3b-c-e-2f-3g=0 \\ -b-f-g+i=0 \end{cases} \qquad (II.13)$$

In order to determine the dimensionless parameters πi, we must solve the system of linear equations (II.1). For the definition of each parameter πi, we can choose four coefficients arbitrarily (usually set to zero) and calculate the other four coefficients using the equation (II.1).

We obtain the following dimensionless parameters for thermal convection using the dimensional analysis method:

- **Nusselt number**

g = 1 To obtain a law of the form h = f (. . .)

c = d = 0 The group π found will not depend on the kinetic energy of the fluid ρU^2

i = 0 The found π group will not depend on the temperature difference T_p - T_∞.

Resolution of the system determining the first dimensionless group π

With g=1 and c=d=i=0

$$\begin{cases} b+d+e+g=0 \\ a+b+c-3d-e+2f=0 \\ -3b-c-e-2f-3g=0 \\ -b-f-g+i=0 \end{cases} \Rightarrow \begin{cases} b+e=-1 \\ a+b-e+2f=0 \\ -3b-e-2f-3g=3 \\ -b-f=1 \end{cases} \qquad (II.14)$$

$$\Rightarrow a=1, b=-1, e=0, f=0$$

$\pi_i = D^a.\lambda^b.U^c.\rho^d.\mu^e.C^f.h^g.(T_P - T_\infty)^i$

With: a=1, b=-1, c=0, d=0, e=0, f=0, g=1, i=0

$$\Rightarrow \pi_1 = Nu = \frac{h.D}{\lambda} \qquad (II.15)$$

- **Reynolds number**

$$\pi_i = D^a.\lambda^b.U^c.\rho^d.\mu^e.C^f.h^g.(T_P - T_\infty)^i$$

4 out of the 8 parameters can be chosen arbitrarily

b=0, f=0, g=0, i=0

$$\begin{cases} b+d+e+g=0 \\ a+b+c-3d-e+2f=0 \\ -3b-c-e-2f-3g=0 \\ -b-f-g+i=0 \end{cases} \Rightarrow \begin{cases} d+e=0 \\ a+c-3d-e=0 \\ -c-e=0 \\ i=0 \end{cases} \qquad (II.16)$$

$\Rightarrow$ d=1, c=1, a=1, e=-1
Then

a=1, b=0, c=1, d=1, e=-1, f=0, g=0, i=0

$$\Rightarrow \pi_2 = Re = \frac{\rho.U.D}{\mu} \qquad (II.17)$$

- **Prandtl number Pr**

$$\pi_i = D^a.\lambda^b.U^c.\rho^d.\mu^e.C^f.h^g.(T_P - T_\infty)^i$$

4 out of the 8 parameters can be chosen arbitrarily

a=0, c=0, g=0, i=0

In such a way as to retain only the characteristics of the fluid: ρ, μ, λ, C

With a=c=g=i=0

$$\begin{cases} b+d+e+g=0 \\ a+b+c-3d-e+2f=0 \\ -3b-c-e-2f-3g=0 \\ -b-f-g+i=0 \end{cases} \Rightarrow \begin{matrix} b+d+e=0 \\ b-3d-e+2f=0 \\ -3b-e-2f=0 \\ -b-f=0 \end{matrix} \qquad (II.18)$$

$\Rightarrow$b=-1, d=0, e=1, f=1

$$\Rightarrow \pi_3 = Pr = \frac{\mu.C}{\lambda} \qquad (II.19)$$

II.8. Interpretation of Dimensionless Numbers

- **Reynolds number:** The ratio of inertial forces to viscous forces characterizes the type of flow in a pipeline.

$$Re = \frac{\rho . U . D}{\mu} \qquad (II.20)$$

ρ: density of the fluid [kg/m³],

U: average velocity of the fluid [m/s],

D: smallest geometric dimension of the problem, diameter Dh for a pipe in [m], width L for a plate,

μ: dynamic viscosity of the fluid [Pa.s].

D_h: hydraulic diameter, $D_h = \frac{4.S}{P}$ (S: surface, P: perimeter)

-Rectangular tube:

$$D_h = \frac{4.a.b}{2.(a+b)} = \frac{2.a.b}{(a+b)}$$

-Annular space:

$$D_h = \frac{4.[\tau.(D_2^2 - D_1^2]}{4.(\tau.D_1 + \tau.D_2)} = D_2 - D_1$$

-Space between two planes: $D_h = 2.b$

- **Nusselt number:**

The ratio of the amount of heat exchanged by convection to the amount of heat exchanged by conduction.

$$Nu = \frac{h.D}{\lambda} \qquad (II.21)$$

h: convective heat transfer coefficient in [W/m².K],

λ: thermal conductivity of the fluid in [W/m.K].

Prandtl number:

This number compares the fluid's ability to diffuse momentum through its viscosity to its ability to diffuse heat through its thermal diffusivity.

$$Pr = \frac{\mu . C_p}{\lambda} = \frac{\nu}{a} \qquad (II.22)$$

Cp: specific heat capacity of the fluid in [J/kg.K].

Stanton or Margoulis number:

Ratio of heat flux to a reference heat flux by convection.

$$St = Ma = \frac{h}{\rho . U . Cp} = \frac{Nu}{Re . Pr} \qquad (II.23)$$

Grashof number:

Characterizes the flow in natural convection (replaces Re), an increase in Gr indicates an increase in the intensity of natural convection.

$$Gr = \frac{\beta . g . \Delta T . \rho^2 . D^3}{\mu^2} \qquad (II.24)$$

β: fluid expansibility in [K^{-1}],

ΔT: temperature difference between fluid and wall: $\Delta T = T_{wall} - T_{fluid}$

Rayleigh number:

Characterizes the flow in natural convection (remplace Re)

$$Ra = Pr . Gr = \frac{g . \beta . \Delta T . D^3}{a . \nu} \qquad (II.25)$$

a = λ / (ρ .Cp) : thermal diffusivity [m²/s],

ν = μ / ρ : kinematic viscosity of the fluid [m²/s].

II.9. Practical correlations for calculating the thermal convection coefficient

The knowledge of the convection coefficient h is mediated by the Nusselt number, whose expression, yet to be determined, is a function only of the Reynolds number and the Prandtl number (forced convection) or the Grashof number and the Prandtl number. (Free convection).

Nu = f(Re, Pr) Forced convection

Nu = f(Gr, Pr) Free convection(natural)

Due to the difference in flow characteristics for laminar and turbulent regimes, we present correlations specific to each case.

II.9.1. Natural Convection

a)- Vertical flat plate

The relations accounting for experimental studies of heat transfer in natural convection are generally of the form:

$$\mathrm{Nu_L} = \mathrm{C}(\mathrm{Gr_L}.\mathrm{Pr})^{\mathrm{n}} = \mathrm{C}(\mathrm{Ra_L})^{\mathrm{n}} \qquad (\mathrm{II}.26)$$

The physical quantities involved in the Grashof and Prandtl numbers must be calculated for the average temperature $\frac{T_P+T_\infty}{2}$

The exponent n will take the following values:

n=1/4 when the convection is laminar

n=1/3 when the convection is turbulent.

The value of the coefficient C depends on the convection regime as well as the geometry and inclination of the wall. This value is given in Table II.3.

Table II.3: Constants C and n.

Geometry and orientation of the wall	Characteristic dimension L	C	
		Laminar convection n=1/4	Convection turbulente n=1/3
Vertical plate	Height	0.59 $10^4<Ra_L<10^9$	0.13 $10^9<Ra_L<10^{13}$
Cylinder	Outer diameter	0.53 $10^3<Ra_D<10^9$	0.10 $10^9<Ra_D<10^{13}$
Upper surface of a heated plate or lower surface of a cooled plate	Width Or S/P	0.54 $10^5<Ra_L<2x10^7$	0.14 $2x10^4<Ra_L<3x10^{10}$
Lower surface of a heated plate or upper surface of a cooled plate	Width Or S/P	0.27 $3x10^5<Ra_L<3x10^{10}$	0.07 $3x10^{10}<Ra_L<10^{13}$

-**Churchill and Chu** offer for the entire Ra_L range:

$$\overline{Nu_L} = \left(0.825 + \frac{0.387Ra_L^{1/6}}{1 + (0.492/Pr)^{8/27}}\right)^2 \qquad (II.27)$$

This correlation can be applied to a vertical cylinder if:

$$\frac{D}{L} \geq \frac{35}{Gr_L^{1/4}}$$

Churchill and Chu also propose a more precise correlation in the case of laminar flows:

$$\overline{Nu_L} = 0.68 + \frac{0.67Ra_L^{\frac{1}{4}}}{\left[1 + \left(\frac{0.492}{Pr}\right)^{\frac{9}{16}}\right]^{\frac{4}{9}}} \qquad (II.28) \qquad Ra_L \leq 10^9$$

b) - Inclined plane

Archimedes' buoyancy has two components: normal and parallel to the surface of the plate. The reduction of the parallel component leads to the reduction of convection. The convection coefficient can be approximately calculated using vertical plate correlations by replacing g with g.cos(θ) for $0 \leq \theta \leq 60°$.

c)- Cylindre horizontal

The average Nusselt number based on the cylinder diameter:

$$\overline{Nu_D} = \frac{\bar{h}.D}{\lambda} = C.Ra_D^n \qquad (II.29)$$

C and n are constants whose values are given in Table II.4.

Table II.4. Constants C and n

Ra_D	C	n
10^{-10}-10^{-2}	0.675	0.058
10^{-2}-10^{2}	1.02	0.148
10^{2}-10^{4}	0.850	0.188
10^{4}-10^{7}	0.480	0.250
10^{7}-10^{12}	0.125	0.333

For a horizontal cylinder, the average Nusselt number for a wide range of Rayleigh numbers has been correlated by **Churchull and Chu** (1975) as follows:

$$\overline{Nu}_D = \left(0.60 + \frac{0.387 Ra_D^{1/6}}{[1 + (0.559/Pr)^{9/16}]^{8/27}}\right)^2 \qquad (II.30) \qquad Ra_D \leq 10^{12}$$

d)- Spheres

For spheres, by extrapolating the results of Churchill and Chu for cylinders, the expression for the average Nusselt number for $Pr \geq 0.7$ and $Ra_D \leq 10^{11}$ is given by Churchill (1983):

$$\overline{Nu}_D = 2 + \frac{0.589 Ra_D^{1/4}}{[1 + (0.469/Pr)^{9/16}]^{4/9}} \qquad (II.31)$$

e) Vertical channels

For a vertical channel with constant and identical wall temperatures, Elenbaas determined that:

$$\overline{Nu}_e = \frac{1}{24} . Ra_e \left(\frac{e}{L}\right) \left\{1 - \exp\left[-\frac{35}{Ra_e (e/L)}\right]\right\}^{3/4} \qquad (II.32)$$

This correlation is semi-empirical and valid for symmetrically heated channels where:

$$\overline{Nu}_e = \left(\frac{q/S}{T_P - T_\infty}\right)\frac{e}{\lambda} \qquad \text{et} \;\; Ra_e = \frac{g\beta(T_P - T_\infty)e^3}{a.\nu}$$

Are the average Nusselt and Rayleigh numbers based on the spacing "e" between the plates.

-For channels with isoflux plates, the local Nusselt numbers are defined as follows:

$$\overline{Nu}_{e,L} = \left(\frac{\varphi_P}{T_{P,L} - T_\infty}\right)\frac{e}{\lambda} \qquad (II.33)$$

L refers to the condition x=L where the temperature of the plate is maximum.

-Pour les canaux avec les plaques isoflux symétriques dans le régime complètement développé :

$$Nu_{e,L} = 0.144 \left[Ra_e^* \left(\frac{e}{L}\right)\right]^{1/2} \qquad (II.34)$$

And the modified Rayleigh number by:

$$Ra_e^* = \frac{g\beta\varphi_p e^4}{\lambda.a.\nu}$$

And for axisymmetric conditions with an isolated surface at the boundary:

$$Nu_{e,L} = 0.204\left[Ra_e^*\left(\frac{e}{L}\right)\right]^{1/2} \qquad (II.35)$$

f)- Inclined channels

Azevedo and Sparrow conducted experiments in inclined channels filled with water. For the conditions of isothermal plates and isothermal-insulated plates for $0 \leq \theta \leq 45°$ within the limit of $Ra_e(e/L) > 200$, the average Nusselt number is given by:

$$Nu_e = 0.645\left[Ra_e\left(\frac{e}{L}\right)\right]^{1/4} \qquad (II.36)$$

And the properties of the fluid are evaluated at the film temperature. : $T_f = \frac{T_P + T_\infty}{2}$

g). Rectangular cavities

$$Ra_e = \frac{g\beta(T_1 - T_2)L^3}{a.\nu} > 1708$$

Lorsque les surfaces sont verticales

- For low aspect ratios [Catton]

$$\overline{Nu_L} = 0.22\left[Ra_L\left(\frac{Pr}{0.2 + Pr}\right)\right]^{0.28}\left[\frac{H}{L}\right]^{-1/4} \qquad (II.37) \qquad \begin{cases} 2 < \frac{H}{L} < 10 \\ Pr < 10^5 \\ 10^3 < Ra_L < 10^{10} \end{cases}$$

- For very low aspect ratios [Catton

$$]\overline{Nu_L} = 0.18\left[Ra_L\left(\frac{Pr}{0.2+Pr}\right)\right]^{0.29} \qquad (II.38) \qquad \begin{cases} 1 < \frac{H}{L} < 2 \\ 10^{-3} < Pr < 10^5 \\ 10^3 < \frac{Ra_L.Pr}{0.2+Pr} \end{cases}$$

- Pour de grands rapports de forme [McGregor]

$$\overline{Nu_L} = 0.42.Ra_L^{1/4}.Pr^{0.012}\left[\frac{H}{L}\right]^{-0.3} \qquad (II.39) \qquad \begin{cases} 10 < \frac{H}{L} < 40 \\ 1 < Pr < 2x10^4 \\ 10^4 < Ra_L < 10^7 \end{cases}$$

- Pour une large plage de rapports de forme[McGregor]

- $\overline{Nu_L} = 0.046.Ra_L^{1/3}$ $\qquad (II.40) \qquad \begin{cases} 1 < \frac{H}{L} < 40 \\ 1 < Pr < 20 \\ 10^6 < Ra_L < 10^9 \end{cases}$

-For all these correlations $T = \frac{T_1 + T_2}{2}$

II.9.2. Forced Convection

a)- Laminar regime

In the fully developed region, the Nusselt number Nu is characterized by:

$$\begin{cases} Nu_D = 3.66 & \text{uniform temperature } T_p = Cte \\ Nu_D = 4.36 & \text{uniform flux density } \varphi_p = Cte \end{cases}$$

In the entry region, the energy equation is more difficult to solve, and only two analytical solutions have been obtained. In the thermal entry region, the average Nusselt number Nu_D is given by Edwards' correlation.(1979)

$$Nu_D = 3.66 + \frac{0.065.Re_D.Pr.\frac{D}{L}}{1+0.04(Re_D.Pr.\frac{D}{L})^{2/3}}.T_P = Cte \qquad (II.41)$$

We can verify that for $L\rightarrow\infty$, Nu_D=3.66

For a developed laminar flow, Sieder & Tate (1936) propose

$$Nu_D = 1.86\left(Re_D.Pr.\frac{D}{L}\right)^{1/3}.\left(\frac{\mu}{\mu_P}\right)^{0.14} \qquad 0.48 < Pr < 16700 \qquad (II.42)$$

b) - Turbulent regime

Turbulence homogenizes the temperature at the core of the flow, and a strong thermal gradient is observed near the walls. Empirical correlations are quite numerous.

We will only mention:

-Colburn correlations:

$$Nu_D = 0.023.Re_D^{0.8}.Pr^{\frac{1}{3}} \qquad (II.43)$$

$$\begin{cases} 0.7 \le Pr \le 160 \\ Re_D > 400 \\ x/D \ge 10 \end{cases}$$

-Dittus–Boelter correlation(1933)

$$Nu_D = 0.023.Re_D^{0.8}.Pr^{n} \qquad (II.44)$$

$$\begin{cases} 0.7 \leq Pr \leq 160 \\ Re_D \geq 10000 \\ \dfrac{x}{D} \geq 10 \end{cases}$$

With $\begin{cases} n = 0.3 \quad \text{cooling} \quad T_m > T_P \\ n = 0.4 \quad \text{heating} \quad T_m < T_P \end{cases}$

In the entrance region of a fully developed turbulent flow (x/D<10), the Colburn correlation must be corrected as follows to account for the variation in the velocity profile:

$$Nu_D = 0.023.Re_D^{0.8}.Pr^{1/3}\left[1 + \left(\frac{D}{x}\right)^{0.7}\right] \qquad (II.45)$$

c)- Non-circular tubes

If the cross-section of the tube is not circular, a first approximation can be used by replacing the tube's diameter with its hydraulic diameter (D_h), and it is this diameter that is used to determine Re_D and Nu_D.

$$D_h = \frac{4.S}{P}$$

S is the passage section and P the wetted perimeter.

Example: rectangular duct with sides a and b

$$D_h = \frac{4.a.b}{2(a+b)} = \frac{2a.b}{(a+b)}$$

Exercises

Exercise 01:

During the flow of air at T∞= 20°C over a plate surface maintained at a constant temperature of Ts= 160°C, the dimensionless temperature profile within the air layer over the plate is determined to be

$$\frac{T(y)-T_\infty}{T_S-T_\infty}=e^{-ay}$$

Where a=3200 m^{-1} and y is the vertical distance measured from the plate surface in m.
-Determine the heat flux on the plate surface and the convection heat transfer coefficient.

Solution:

Airflow over a flat plate has a given temperature profile. The heat flux on the plate surface and the convection heat transfer coefficient are to be determined.

Assumptions

1 The given nondimensional temperature profile is representative of the variation of temperature over the entire plate.

2 Heat transfer by radiation is negligible.

Properties

The thermal conductivity of air at the film temperature of

Tf =(*Ts* + *T*`)/2 = (160°C + 20°C)/2 =90°C is *k* = 0.03024 W/m·K.

Analysis

Noting that heat transfer from the plate to air at the surface is by conduction, heat flux from the solid surface to the fluid layer adjacent to the surface is determined from

$$\dot{q}=\dot{q}_{cond}=-k_{fluid}.\left.\frac{\partial T}{\partial y}\right|_{y=0}$$

Where the temperature gradient at the plate surface is

$$\left.\frac{\partial \mathrm{T}}{\partial \mathrm{y}}\right|_{y=0}=(\mathrm{T_S}-\mathrm{T_\infty})\mathrm{a}[\mathrm{e}^{-\mathrm{ay}}]_{\mathrm{y}=0}=(\mathrm{T_S}-\mathrm{T_\infty})(-\mathrm{a})=(160-20)(-3200)$$

$$=-4.48\mathrm{x}10^{5}\mathrm{°C/m}$$

Substituting, the heat flux is determined to be

$$\dot{q}=-0.03024x(-4.48x10^5)=\mathbf{1.35x10^4W/m^2}$$

Then the convection heat transfer coefficient becomes

$$h = \frac{-k_{fluid}\left(\frac{\partial T}{\partial y}\right)_{y=0}}{T_S - T_\infty} = \frac{-0.03024x(-4.48x10^5)}{160-20} = \mathbf{96.8W/m^2.K}$$

Exercise 02:

By following the approach used in forced convection, perform a dimensional analysis to derive the expression for the Grashof number in natural convection.

Solution:

The quantities related to natural convection are:

• Fluid characteristics: λ_f, ρ, μ, C p, β, g, ΔT

•Temperature difference in the boundary layer ΔT

•Geometric characteristic of the wall: length L

We therefore have n=7 physical quantities, which are expressed using k=4 fundamental units: M, L, T, θ (mass, length, time, and temperature).

By applying the Vaschy–Buckingham theorem, with n-k=3 we write the relationship with the dimensionless groups π_1, π_2, π_3:

$$\psi(\pi_1, \pi_2, \pi_3) = 0$$

By choosing 4 base quantities λf, ρ, μ, L, these three groupings are expressed:

$$\begin{cases} \pi_1 = \lambda_f^{a1}.\rho^{b1}.\mu^{c1}.L^{d1}.h \\ \pi_2 = \lambda_f^{a2}.\rho^{b2}.\mu^{c2}.L^{d2}.\beta.g.\Delta T \\ \pi_3 = \lambda_f^{a3}.\rho^{b3}.\mu^{c3}.L^{d3}.Cp \end{cases}$$

a) The dimensional equations of the different parameters are:

$$[L] = L,\ \ [\mu] = Pa.s = M.L^{-1}.T^{-1}, [\lambda_f] = W.m^{-1}.K^{-1} = M.L^{-1}.T^{-3}.\theta^{-1},$$

$$[Cp] = J.kg^{-1}.K^{-1} = L^2.T^{-2}.\theta^{-1}, [h] = W.m^{-2}.K^{-1} = M.T^{-3}.\theta^{-1}$$

$$[\rho] = M.L^{-3}, [g\beta\Delta T] = L.T^{-2}$$

By writing the dimensional equation of the group $\pi2$ (the one that involves βgΔT), we obtain:

$$[\pi_2] = (M.L^{-1}.T^{-3}.\theta^{-1})^{a2}(M.L^{-3})^{b2}.(M.L^{-1}.T^{-1})^{c2}.(L)^{d2}.(L.T^{-2})$$

$$\pi_2 = \left[M^{a2+b2+c2}.L^{a2-3b2-c2-d2+1}.T^{-3a2-c2-2}.\theta^{-a2}\right] = 1$$

The exponents are the solution to the system:

$$\begin{cases} a2 + b2 + c2 = 0 \\ a2 - 3b2 - c2 - d2 + 1 = 0 \\ -3a2 - c2 - 2 = 0 \\ a2 = 0 \end{cases}$$

We find: a_2=0, b_2=2, c_2=-2, d_2=3. This therefore leads to:

$$\pi_2 = \lambda_f^0.\rho^2.\mu^{-2}.L^3.\beta.g.\Delta\theta = \frac{\beta.g.\Delta T.\rho^2.L^3}{\mu^2}$$

With the kinematic viscosity of the fluid ν=μ/ρ :

$$\boldsymbol{\pi_2 = \frac{\beta.g.\Delta T.L^3}{\nu^2}}$$

The grouping π_2 is called the Grashof number:

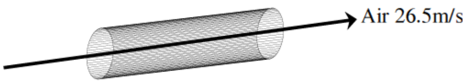
$$\mathbf{Gr} = \frac{\beta.g.\Delta T.L^3}{\nu^2}$$

Exercice 03 :

In a cylinder with a diameter of 4 cm, air flows at an average speed of 26.5 m/s.

-Calculate the heat transfer coefficient h given that:
$\rho = 1,2\ kg/m^3$, $Cp = 0,24\ kcal/kg°C$, $\eta = 1,9\ 10^{-5} Pa.s$, $k = 6,2\cdot10^{-6} kcal/m\ s\ °C$

$Nu = 0,023\ Re^{0,8}.Pr^{0,4}$



Solution:

$$Pr = \frac{\mu.C_P}{k} = \frac{1,9\ .\ 10^{-5}.\ 0,24}{6,2\ .\ 10^{-6}} = 0,735$$

$$Re = \frac{\rho.u_m.d}{\mu} = \frac{26,5\ .\ 0,04\ .1,2}{1,9\ .\ 10^{-5}} = 66947$$

$Nu = 0,023\ Re^{0,8}.Pr^{0,4} = 147,5$

$$Nu = \frac{h.d}{k}$$

$$\Rightarrow \mathbf{h = 2,286\ .10^{-2}\ [kcal.m^{-2}.s^{-1}.°C^{-1}]}$$

Exercice 04 :

A thin plate with a length of 3m and a width of 1.5m is subjected to an air flow at a speed of 2.0m/s and a temperature of 20°C, in the longitudinal direction. The temperature of the plate surfaces is 84°C. It is requested to calculate:

1. The heat transfer coefficient by convection along the length (for Pr=0.71);
2. The heat flux transmitted by the plate to the air.

The characteristics of the air at 20°C are:

ρ=1.175 kg/m³, μ=1.8×10^{-5} kg/m·s, k=0.026 W/m·K, and Cp=1006 J/kg·K.

Solution :

1. The convective heat transfer coefficient h:

-The nature of the flow:

$$\mathrm{Re} = \frac{\rho.\mathrm{L.u}}{\mu} = \frac{1{,}175\ .3\ .2}{1{,}80\ \ .10^{-5}} = 391666{,}67 < 500.000$$

$\Rightarrow$ *The flow is therefore laminar, we apply in this case*:

$$\mathrm{Nu} = \frac{\mathrm{h.L}}{\mathrm{k}} = 0{,}66.\mathrm{Re}^{0{,}5}.\mathrm{Pr}^{0{,}33} \Rightarrow \mathrm{h} = \frac{0{,}66.\mathrm{Re}^{0{,}5}.\mathrm{Pr}^{0{,}33}.\mathrm{k}}{\mathrm{L}}$$

$$\mathrm{h} = \frac{0{,}66.(391666{,}67)^{0{,}5}.(0{,}71)^{0{,}33}.0{,}026}{3} = 3{,}1972\mathrm{W/m^2.K}$$

1. The heat flux transmitted by the plate (the latter has two walls, lower and upper) to the air is given by:

$$\varphi = 2.\mathrm{h.S}(\mathrm{T_p} - \mathrm{T_a}) = 2\ .3{,}1972.(3\ .1{,}5).(357 - 293) = 1841{,}587\mathrm{W}$$

Exercice 05 :

Calculate the amount of heat transmitted by the flow of water moving in a forced manner through a coil made of a tube with a diameter of 20mm. The water flow rate is 0.28 kg/s and its temperature is 120°C. The temperature of the inner wall of the pipe, which is 4 m long, is considered constant and equal to 95°C.

The characteristics of water at 120°C are:

ρ=945.3 kg/m^3, μ=2.34×10^{-4} kg/m.s, k=0.68 W/m.K, and C p =4250 J/kg.K.

Solution

The convective heat transfer coefficient h:

The nature of the flow:

$$\mathrm{Re} = \frac{\rho.\mathrm{u}.\mathrm{D}}{\mu}$$

$$Q = \rho.S.u \Rightarrow u = \frac{Q}{\rho.S} = \frac{0,28}{945,3\ .\pi.(0,01)^2} = 0,943m/s$$

$$Re = \frac{\rho.\mathrm{u}.\mathrm{D}}{\mu} = \frac{945,3\ .\ 0,943\ .0,02}{2,34\ .\ 10^{-4}} = 76189,56 > 2300$$

The flow regime is therefore turbulent

$$\frac{\mathrm{L}}{\mathrm{D}} > 60\ , \qquad 1000 < \mathrm{Re} < 120000\ \ \mathrm{et}\ \mathrm{Pr} > 0,7$$

$$1000 < Re < 120000;\ \frac{L}{D} = \frac{4}{0,02} = 200 > 60$$

$$\mathrm{Pr} = \frac{\mu.\mathrm{Cp}}{\mathrm{k}} = \frac{2,34.10^{-4}.\ 4250}{0,685} = 1,45 > 0,7$$

$$\mathrm{Nu} = \frac{\mathrm{h}.\mathrm{D}}{\mathrm{k}} = 0,023.\mathrm{Re}^{0,8}.\mathrm{Pr}^{0,33} \Rightarrow \mathrm{h} = \frac{0,023\ .\mathrm{Re}^{0,8}\ .\mathrm{Pr}^{0,33}.\mathrm{k}}{\mathrm{D}}$$

$$\mathrm{h} = \frac{0,023\ .(76189,56)^{0,8}.(1,45)^{0,33}.0,685}{0,02} = 7164,01\mathrm{W/m^2.K}$$

The heat flux transmitted by the water (to the inner wall of the tube) is given by:

$$\varphi = \mathrm{h.S.}\left(\mathrm{T_e} - \mathrm{T_p}\right) = \mathrm{h.\pi.D.L.}\left(\mathrm{T_e} - \mathrm{T_p}\right) = 7164,01\ .\pi.(0,024).(393 - 368)$$

$$\boldsymbol{\varphi = 45,012\mathrm{kW}}$$

Exercice 06 :

The wall of a building is 6 m high and 10 m long. Under the heating due to the sun, its outside temperature reaches Tm=40°C. The ambient outside temperature is Tair = 20°C.

We provide the following physical properties of air at a temperature of 30°C:

•Density: ρ_{air} = 1.149 kg m–3

•Thermal conductivity: λ_{air} = 0.0258 W $m^{-1}K^{-1}$

•Dynamic viscosity: μ_{air} = 18.4×10−6 Pa s

•Specific heat capacity: Cp_{air} = 1006 J kg−1K−1

Calculate the heat flux exchanged by convection between the wall and the air.

Solution :

In natural convection, such an exchange is calculated by experimental correlation:

$$Nu = C(Gr.Pr)^n$$

The Grashof number is:

$$Gr_L = \frac{\beta.g.\Delta T.\rho_{air}^2.L^3}{\mu_{air}^2}$$

With : $\beta_{air} = \frac{1}{(273+30)} = 0{,}0029K^{-1}, g = 9{,}81m.s^{-1}, \Delta T = 20K, \quad \rho_{air} = 1{,}149kg.m^{-3}$,

$\mu_{\,air} = 18{,}4\,.\,10^{-6}$ Pa . s , L = 6m

We obtain:

$$Gr_L = \frac{0{,}0033\,.\,9{,}81\,.\,20\,.\,(1{,}149)^2(6)^3}{(18{,}4\,.\,10^{-6})^2} = \mathbf{5{,}45\,.\,10^{11}}$$

The Prandtl number

$$Pr = \frac{\mu_{air}.Cp_{air}}{\lambda_{air}} = \frac{18{,}4\,.\,10^6.1006}{0{,}0258} = \mathbf{0{,}718}$$

We then calculate the product Gr_L. Pr=Ra_L which determines the natural convection regime (laminar or turbulent), the critical value being 10^9:

$Gr_L.Pr = Ra_L = 5{,}45\,.\,10^{11}.0{,}718 = 3{,}91\,.\,10^{11}$

We are therefore in a regime of turbulent natural convection and we use the coefficients C=0.10 and n=1/3 in the previous correlation. We deduce the value of the Nusselt number:

$$Nu_L = \frac{h.L}{\lambda_{air}} = C(Gr_L.Pr)^n = 0{,}10\,.\,(3{,}91\,.\,10^{11})^{1/3} = \mathbf{731}$$

Thus, the convective heat transfer coefficient h is:

$$h = \frac{\lambda_{air}.Nu}{L} = \frac{0{,}0258.731}{6} = \mathbf{3{,}14\,W.m^{-2}.K^{-1}}$$

So the heat flux exchanged over the entire surface S is:

$$\phi = h.S.(T_m - T_{air})$$

Let

$\phi = 3{,}14\,.\,6.\,10\,.\,(40 - 20) = \mathbf{3768W}$

Exercice 07 :

A horizontal pipe with an outer diameter of 0.1m, used for transporting high-pressure steam, runs through a large room where the air and wall temperatures are 23°C.

The outer surface of the pipe at a temperature of 16.5°C and an emissivity of 0.85.

-Estimate the thermal flux released by the pipe per unit length.

Solution :

Air : $T_f = \frac{165+23}{2} = 94°\ ,\ \lambda = 0{,}0313\ W.m^{-1}\ \ ,\nu = 22{,}8\ m^2.s^{-1}, a = 32{,}8\,.\,10^{-6}m^2.s^{-1}$

and $Pr = 0{,}697$

$$Ra_D = \frac{g\beta(T_S - T_\infty)D^3}{a.\nu} = \frac{9{,}81\,.\,\frac{1}{367}(165-23)0{,}1^3}{32{,}8\,.\ 10^{-6}\ .\,22{,}8\,.\,10^{-6}} = \mathbf{5{,}073\,.\,10^6}$$

$$Nu_D = \left(0{,}6 + \frac{0{,}387\,.\,(5{,}073\,.\,10^6)^{1/6}}{[1 + (0{,}559/Pr)^{9/16}]^{8/27}}\right)^2 = \mathbf{23{,}3}$$

$$h = \frac{Nu_D.\lambda}{D} = \frac{23{,}3\ .\ 0{,}0313}{0{,}1} = \mathbf{7{,}29W.m^{-2}.K^{-1}}$$

$\varphi_{conv} = h.\pi.D.(T_S - T_\infty)\ = 7{,}29.\pi.0{,}1.(165-23) = \mathbf{325W.m^{-1}}$

$\varphi_{rad} = \varepsilon.\sigma.\pi.D.(T_S^4 - T_\infty^4) = 0{,}85\,.\,5{,}67\,.\,10^{-8}.\pi.0{,}1.((165+273)^4 - (23+273)^4)$

$= \mathbf{441W.m^{-1}}$

$$\varphi = \varphi_{conv} + \varphi_{rad} = 325 + 441 = \mathbf{766\ W.}$$

Chapiter III
Heat Transfer by Radiation

III.1. Introduction

Any heated body emits electromagnetic radiation from its outer surface, the power of which depends on its temperature; this is thermal radiation. The spectral analysis of this radiation shows a majority of short wavelengths at very high temperatures and a majority of long wavelengths for temperatures below 500 K.

III.2. Nature of Radiation

All bodies, regardless of their state: solid, liquid, or gas, emit radiation of an electromagnetic nature. This emission of energy occurs at the expense of the internal energy of the emitting body.

Radiation propagates in a straight line at the speed of light, and it consists of radiations of different wavelengths as demonstrated by William Herschel's experiment:

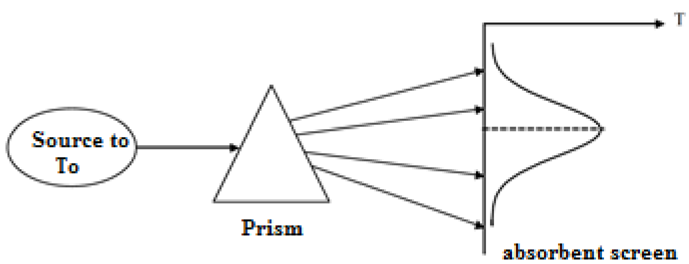


Figure III.1. Principle of William Herschel's experiment.

Passing through a prism, the radiations are more or less deviated according to their wavelength. We therefore send the radiation emitted by a source at temperature T_0 onto a prism and project the deflected beam onto an absorbing (blackened) screen, thus obtaining the decomposition of the total incident radiation into a spectrum of monochromatic radiations..

If a thermometer is moved along the screen, it measures the temperature Te characterizing the energy received by the screen at each wavelength. By constructing the curve $T_e = f(\lambda)$, we obtain the spectral distribution of the radiated energy for the temperature T_0 of the source. It is then observed that:

- The emitted energy is maximum for a certain wavelength λ_m that varies with T_0.

- Energy is emitted only over a wavelength interval [λ_1, λ_2] characterizing thermal radiation.

The different types of electromagnetic waves and their corresponding wavelengths are represented in Figure III.2. It should be noted that the thermal radiation emitted by bodies ranges between 0.1 and 100 μm. It should also be noted that radiation is perceived by humans:

- By the eye: for 0.38 μm < λ < 0.78 μm visible radiation.
- Through the skin: for 0.78 μm < λ < 314 μm infrared radiation (IR).

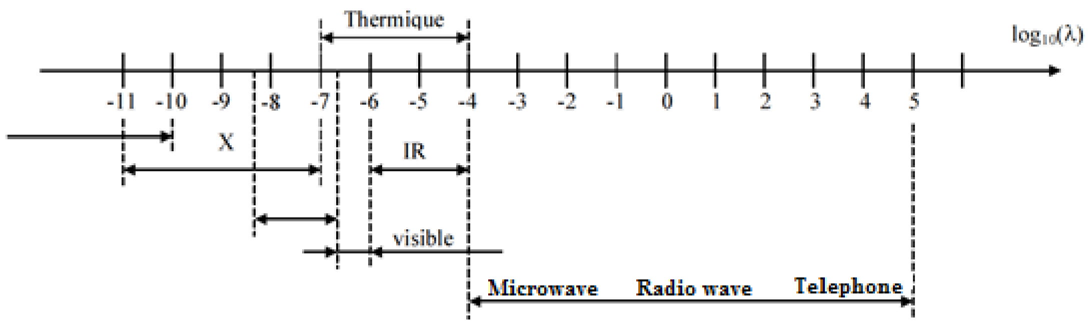


Figure III.2. Spectrum of electromagnetic waves (λ in m).

III.3. Classification of bodies subjected to radiation

Depending on the nature of the body and the wavelength of the incident radiation, one of three phenomena reflection, transmission, and absorption may be predominant.

III.3.1. Transparent Bodies

The propagation of thermal radiation occurs in a vacuum in a straight line, and at the speed of light ($c=3x10^8$ m/s), without any decrease in the transported energy. It is said, therefore, that a vacuum is a perfectly transparent medium. When radiation does not undergo any attenuation while passing through a medium, it is said that the medium is transparent to that radiation; this is also the case for certain simple gases (O_2, H_2, N_2) in the visible and infrared ranges.

III.3.2. Opaque Bodies

The vast majority of solids and liquids are said to be opaque because they stop the propagation of any radiation right at their surface: these bodies heat up by absorption or radiation.

III.3.3. Semi-transparent bodies

On the other hand, some bodies are partially transparent because the electromagnetic wave can propagate through the medium in question. The propagation is accompanied by electromagnetic absorption that increases the energy of the medium traversed.

III.4. Law of Conservation of Energy

Let ϕ_i be the incident flux, ϕ_r the reflected flux, ϕt the transmitted flux, and ϕa the absorbed flux, the conservation of energy is written as:

$$\phi_i = \phi_r + \phi_a + \phi_t \qquad \text{(III. 1)}$$

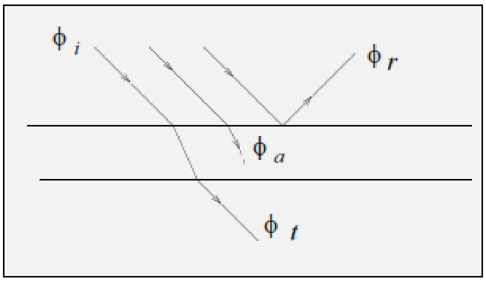


Figure III.3. Decomposition of the incident flux.

-Thermal absorption coefficient

Let's ask:

$$\rho = \frac{\phi_r}{\phi_i} \quad \text{reflection factor}$$

$$\alpha = \frac{\phi_a}{\phi_i} \quad \text{absorption factor}$$

$$\tau = \frac{\phi_t}{\phi_i} \quad \text{transmission factor}$$

The conservation of energy is written as: $\rho + \alpha + \tau = 1$. These parameters characterize the behavior of a body in relation to the received radiation. The coefficient is important in thermodynamics: it measures the proportion of conversion of incident electromagnetic radiation into thermal energy.

The coefficient α is low for polished and non-oxidized metal surfaces. It increases for bodies that appear black but always remains less than one.

III.5. Definition of Energy Quantities

III.5.1. Energy Flux

The flux of a source S is called the power radiated, denoted φ by S, in all the space surrounding it, across all wavelengths. The flux φ is expressed in W.

- The flux sent by a surface element dS into an elementary solid angle dΩ is denoted $d^2\varphi$
- The flux sent throughout space by an elementary surface dS is denoted dφ.
- The flux sent by a surface S in the solid angle dΩ surrounding the Ox direction is denoted $d\varphi_x$.

We therefore have the following relationships:

$$d\varphi = \int d^2\varphi \quad \text{et} \quad \varphi = \int_S d\varphi = \int_\Omega d\varphi_x \qquad (\text{III.}2)$$

III.5.2. Energy Emittance

It is the total flux emitted per unit area of the source:

$$M = \frac{d\phi}{dS}\left(\frac{W}{m^2}\right) \qquad (\text{III.}3)$$

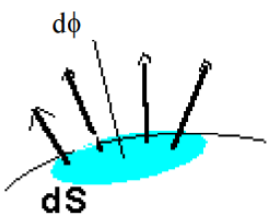


Figure III.4. Emittance of a source, in W/m².

-Emittance Monochromatic:

A surface element dS emits a certain flux of energy by radiation in all directions of the half-space. This flux is distributed over a range of wavelengths. If we consider the energy flux $d\varphi_\lambda^{\lambda+d\lambda}$ emitted between the two wavelengths λ and λ+dλ, the monochromatic emissivity of a source at temperature T is defined by:

$$M_{\lambda T} = \frac{d\phi^{2}{}_{\lambda}^{\lambda+d\lambda}}{dS.\,d\lambda} \qquad (III.4)$$

-Total Emittance:

It is the heat flux density emitted by radiation by dS over the entire spectrum of wavelengths. It is no longer a function of the temperature T and the nature of the source:

$$M_T = \int_{\lambda=0}^{\lambda=\infty} M_{\lambda T}\, d\lambda = \frac{d\varphi}{dS} \qquad (III.5)$$

III.5.3.Solid angle

The solid angle dΩ characterizes directions originating from a point and contained within a portion of space. He is to space what the angle is to the plane. The solid angle under which a surface Σ is seen from a given point O is equal to the area cut out on a unit sphere by the cone with vertex O surrounding the surface Σ.

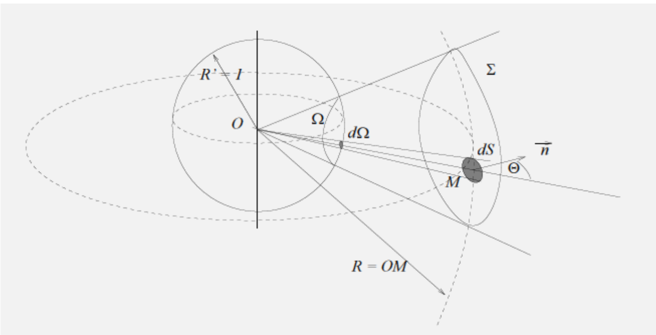


Figure III.5. Construction of the solid angle Dω.

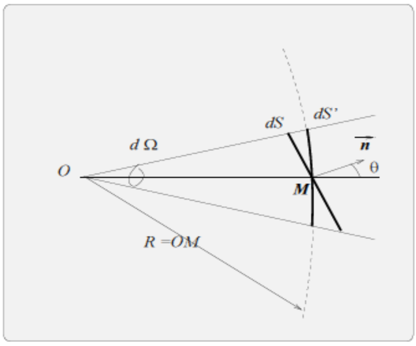


Figure III.6. Projection of any surface onto a sphere of radius R.

To evaluate dΩ, let's construct the sphere centered at O with a radius of R=OM. The projection of the surface element dS onto the sphere of radius R=OM cuts out a spherical cap dS'. The solid angle is equal to the surface projected from the surface dS onto the unit sphere.

$$d\Omega = \frac{dS'}{R^2} = \frac{dS.\cos\theta}{R^2} \quad (\text{Stéradian Sr}) \qquad (III.6)$$

III.5.4. Intensity of a source in a direction

Let there be a direction $\vec{n}$ that makes an angle θ with the normal to the surface of an emissive body. If dϕn is the fraction of flux radiated in the elemental angle dΩ, the total energy intensity of a source in the direction $\vec{n}$ is called the flux radiated per unit solid angle in that direction. It is expressed in Watts per Steradian. ($W.Sr^{-1}$).

$$I_n = \frac{d\phi_n}{d\Omega} \qquad (III.7)$$

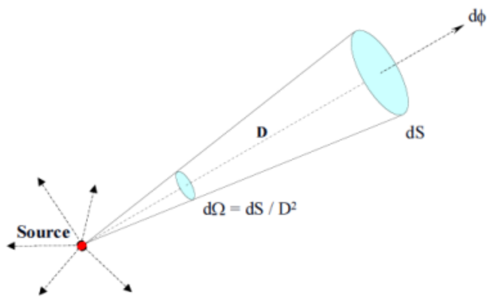


Figure III.7. Intensity of a thermal source

III.5.5. Luminance of a source in a direction

We define the luminance L_n of a surface area dS source, in the direction $\vec{n}$, as the quotient of the intensity I_n of the source in that direction, by the apparent surface area dS' of the source in the same direction.

$$L_n = \frac{I_n}{dS'} = \frac{I_n}{dS.\cos\theta} \qquad (III.8)$$

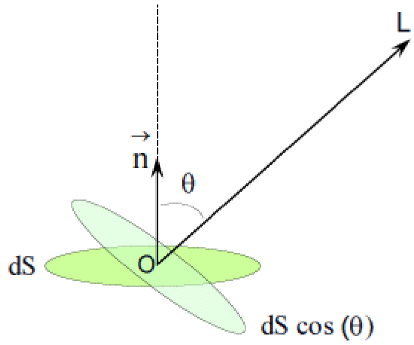


Figure III.8. Luminance of a source in a direction

En effet, vu de la direction $\vec{n}$, tout se passe comme si le flux était émis par la surface projetée dS'=dS.cosθ. En introduisant la définition de l'intensité (I_n), la luminance s'exprime par la relation :

Indeed, from the direction $\vec{n}$, everything happens as if the flux were emitted by the projected surface dS'=dS.cosθ. By introducing the definition of intensity (In), luminance is expressed by the relation:

$$L_n = \frac{\frac{d\phi_n}{d\Omega}}{dS.\cos\theta} = \frac{d^2\phi_n}{d\Omega.dS.\cos\theta} \qquad (III.9)$$

III.5.6.Illuminance (Irradiation)

a)-Spectral Illuminance (Irradiation)

The concept of emittance is replaced, for incident radiation, by the illumination of the receiving surface. The spectral illuminance E_λ ($W.m^{-2}.\mu m^{-1}$) is the rate of transfer of radiant energy received at the wavelength λ in all directions per unit of receiving surface area, per unit of wavelength dλ around λ.

b)-Eclairement(Irradiation) total

This is how we refer to the total flux received by the receiving surface unit:

$$E = \int_0^\infty E_\lambda(\lambda)d\lambda = \frac{d\phi}{dS} \qquad (III.10)$$

Illuminance E is expressed in $W.m^{-2}$.

It is therefore the flux density arriving on the unit receiving surface dS, coming from the half-space visible from this surface.

III.6. Lambert's Law

It is said that a source follows Lambert's law (or that it is diffusely emitting) if its luminance does not depend on the emission direction. Most emissive bodies verify this property.

$$L_n = L$$

III.7.Thermal radiation of a black body

III.7.1.Definition of a black body

A black body is an object that will absorb all incident radiation striking it, without reflecting or allowing any fraction to escape, regardless of the wavelengths and directions of propagation. Similarly, a black body will be capable of radiating at each wavelength the maximum amount of thermal energy theoretically storable in this frequency band at a determined temperature level T. Such a body, thermally ideal, does not exist in nature.

In all that follows, the quantities related to the black body will be assigned a superscript "0". Thus, for example, the monochromatic emittance of the black body at the wavelength λ will be denoted M_0^λ.

III.7.2.Laws of thermal radiation of a black body

III.7.2.1.Planck's Law

This law relates the monochromatic emittance of the black body M_0^λ to the wavelength λ and its absolute temperature T. She expresses herself in the form:

$$M_\lambda^0 = \frac{2\pi.h.C^2.\lambda^{-5}}{e^{\frac{hC}{k\lambda T}} - 1} \qquad (III.11)$$

With:

$C = C_0/n$: n is the refractive index of the medium, and $C_0 = 2.9979 \times 10^{-8}$ m/s

• h: is Planck's constant, $h = 6.6255 \times 10^{-34}$ J.s

• k: is Boltzmann's constant, $k = 1.3805 \times 10^{-23}$ J/K.

When radiation propagates in a medium whose refractive index is equal to one, which is strictly the case for a vacuum, and for air in the first approximation, PLANCK's law can be expressed in the following simplified form, which is the one that will be used in everyday practice:

$$M_\lambda^0 = \frac{C_1\lambda^{-5}}{e^{C_2/\lambda T} - 1} \qquad (III.12)$$

C_1 and C_2 are two physical constants whose values are given, in SI units, in the following table III.1:

Table III.1. The values of physical constants.

T	λ	$C_1 = 2\pi h C_0^2$	$C_2 = \frac{hC_0}{k}$	M_λ^0
K	m	$3.741\ .10^{-16}$ W.m^2	0.014388 m.K	W/m^3
K	µm	$3.741\ .10^{8}$ W.µm^4/ m^2	14.388 µm.K	W/(m^2.µm)

Figure III.9 below represents the curves of variation of the monochromatic emissivity of a black body, calculated using relation (III.12), for various values of the absolute temperature of this black body. Each curve shows a maximum at a certain abscissa λm, which is more pronounced the higher the temperature T.

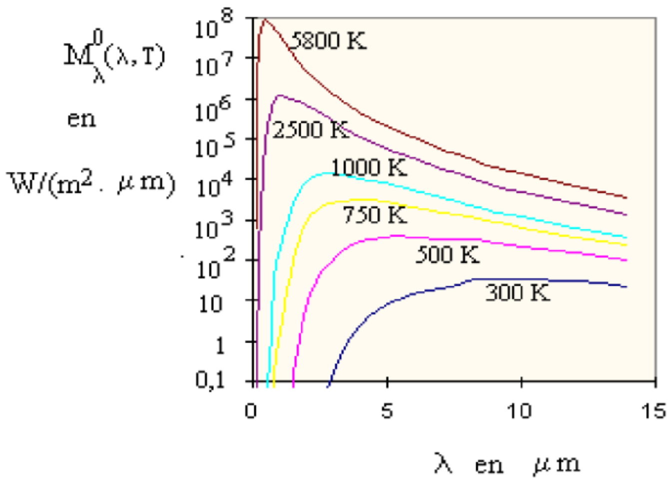


Figure III.9. Monochromatic Emittance of the Black Body.

III.7.2.2. Wien's Laws

- 1^{st}Wien's Law or laws of displacement

The abscissa λm of the maximum of M_0^{λ} shifts towards shorter wavelengths as the temperature increases. This is what the "displacement law" of WIEN expresses:

$$\lambda_m .T = 2898\ \mu m.K \qquad (III.13)$$

- 2^{nd} Law of WIEN

This law provides the value of the maximum $M_{\lambda m}^0$ as a function of T. She expresses herself in the following way:

$$M_{\lambda m}^0 = B.T^5 \qquad (III.14)$$

The constant B is given in the following table III.2:

Table III.2. The values of constant B

T	**λ**	**B**	$\mathbf{M_{\lambda m}^0}$
K	m	$1.287\ .10^{-5}\ W/(m^3.K^5)$	W/m^3
K	µm	$1.287\ .10^{-11}\ W/(m^2.\mu m.K^5)$	$W/(m^2.\mu m)$

III.7.2.3. Stefan-Boltzmann Law

This law provides the total emissivity of black body radiation in a vacuum, as a function of its absolute temperature. We can establish it by integrating over the entire spectrum, the relation (III.11) expressing Planck's law.

The Stefan-Boltzmann law is expressed by the relation:

$$M^0 = \sigma . T^4 \qquad (III.15)$$

σ is the Stefan-Boltzmann constant, whose expression and numerical value are given in the following table III.3:

Table III.3. The value of the Stefan-Boltzmann constant.

$\sigma = \frac{2\pi^5}{15} \frac{k^4}{C_0^2 h^3}$	T	M^0
$5.67.10^{-8}$ W/(m².K⁴)	K	W/m²

III.8. Thermal Radiation of Real Bodies

The evaluation of the emissive properties of real substances is done in relation to those of a black body placed under the same temperature and wavelength conditions, using coefficients called emissivities, total or monochromatic, hemispherical or directional. Thus, the total and monochromatic emittance of a given real surface will be provided by relations of the type:

$$M = \varepsilon . M^0 \quad \text{et} \quad M_\lambda = \varepsilon_\lambda . M_\lambda^0$$

Relations in which ε is the hemispherical emissivity of the body, and ε_λ its monochromatic emissivity at the wavelength λ.

As for the luminances L and L_λ of the body, they will be related to those of the black body by the relations:

$$L_{Ox} = \varepsilon_{Ox} . L^0 = \varepsilon_{Ox} . \frac{M^0}{\pi} \quad \text{et} \quad L_{Ox,\lambda} = \varepsilon_{Ox,\lambda} . \frac{M_\lambda^0}{\pi}$$

In which ε_{Ox} is the total directional emissivity of the body, and $\varepsilon_{Ox,\lambda}$ its monochromatic directional emissivity.

III.8.1. Notion of gray body

Simplifying assumption: all surfaces present will have emissivity independent of the emission direction and wavelength, just like a black body, but at lower energy levels. Such bodies can then be classified as gray, and will be characterized in terms of their radiative properties by the following relations: $\varepsilon_{Ox} = \varepsilon_{Ox,\lambda} = \varepsilon =$ Constant.

The total emissivity of a gray body will then be deduced from that of a black body given by the Stefan-Boltzmann law:

$$M = \varepsilon.\sigma.T^4 \quad en\,\frac{W}{m^2} \qquad (III.16)$$

Table III.4 below provides some values of emissivity ε.

Table III.4. The emissivity values.

Nature of the substance	**Emissivity ε**
Refractory	0.8
Reflective paint	0.3
Polished Iron	0.15 (à20°C) à 0.35 (à900°C)
Polished Aluminum	0.05
Oxidized Aluminum	0.15

III.8.2.Kirchhoff's Law

This law establishes a relationship between the emissive and absorptive properties of a body. To demonstrate this, we consider a body placed in a closed enclosure whose walls possess the properties of a black body. The entire system is in thermal equilibrium at a uniform temperature T, and the body in question therefore neither gains nor loses heat.

Under these conditions, a surface element dS of the body emits in a solid angle element dΩ surrounding a direction Ox inclined at an angle β to the normal a monochromatic flux:

$$\left\lfloor d^2\varphi_{Ox,\lambda}\right\rfloor_{émis} = \varepsilon_{Ox,\lambda}\; L_\lambda^0\, dS\, \cos\theta\, d\Omega \qquad (III.17)$$

Simultaneously, the surface dS receives, in the same solid angle dΩ and at the same wavelength λ, a flux emitted by the black body which has the value :

$$L_\lambda^0\, dS\, \cos\theta\, d\Omega \qquad (III.18)$$

The body in question will therefore absorb part of it:

$$\left\lfloor d^2\Phi_{Ox,\lambda}\right\rfloor_{absorbed} = \alpha_{Ox,\lambda}\, L_\lambda^0\, dS\, \cos\theta\, d\Omega \qquad (III.19)$$

The thermal equilibrium of the body implies the equality of emitted and absorbed fluxes, hence the following relationship that expresses Kirchhoff's Law:

$$\varepsilon_{Ox,\lambda} = \alpha_{Ox,\lambda}$$

In the case where the radiations emitted and received by the body are perfectly distributed over all directions of the hemispherical space (case of diffuse emission and illumination), the previous Kirchhoff law is also applicable to the hemispherical monochromatic properties:

$$\varepsilon_\lambda = \alpha_\lambda$$

It is generally not possible to extend Kirchhoff's law to the total radiation emitted and absorbed by any body, and therefore to consider that $\alpha = \varepsilon$.

Indeed, the total emissivity $\varepsilon(T)$ of a body is defined by the relation:

$$\varepsilon(T) = \frac{M(T)}{M^0(T)} = \frac{\int_0^\infty \varepsilon_\lambda\, M_\lambda^0(T)\, d\lambda}{\sigma\, T^4} \qquad (III.20)$$

This function $\varepsilon(T)$ is a characteristic property of a single emitting body, depending on its monochromatic emissivity ε_λ, and varying with its temperature T. On the other hand, the total absorption coefficient of the same body is the fraction α absorbed by the body, over all the incident wavelengths. If E_λ is the monochromatic illumination falling on the body, we have:

$$\alpha = \frac{\int_0^\infty \alpha_\lambda\, E_\lambda\, d\lambda}{\int_0^\infty E_\lambda\, d\lambda} = \frac{\int_0^\infty \alpha_\lambda\, E_\lambda\, d\lambda}{E} \qquad (III.21)$$

The absorption coefficient α therefore also depends on the body in question, through the α_λ, but also on the spectral composition E_λ of the received radiation, and thus ultimately on the nature and temperature of the body that emitted the absorbed radiation.

That is why the total absorption coefficient α cannot be an intrinsic characteristic of a body, as its total emissivity ε is.(T).

Generally speaking, we will therefore have: $\alpha \neq \varepsilon$. There are two important exceptions to this situation:

-The black body, defined by the properties: $\varepsilon_\lambda = 1$ and $\alpha_\lambda = 1$ for all λ

It immediately follows that: $\varepsilon = \alpha$

-Grey bodies, defined by the property: $\varepsilon_\lambda = \varepsilon$ for any λ

The relation: $\varepsilon_\lambda = \alpha_\lambda$ therefore leads to: $\alpha_\lambda = \varepsilon$ for any λ,

That is to say: $\varepsilon = \alpha$.

III.9.Radiative exchanges between surfaces

In general, in a real case, several bodies are placed in mutual interaction. Each body emits radiation in all directions of space and receives complex radiation that is the result of direct emissions from the bodies surrounding it and a large number of reflections depending on the considered geometry.

Although an exact and precise calculation is generally difficult, standard geometries can be considered to analyze radiative exchanges in order to determine the net exchange flux. These particular cases also require having information on the behavior of the surfaces and volumes considered with respect to radiation: black body, opaque, gray, or transparent.

II.9.1. Radiative exchanges between black surfaces

The equation:

$$d^2\phi_{1\to 2} = \mathrm{L}^0_{\mathrm{T}1}\,\mathrm{dS}_1\,\cos\beta_1\,\mathrm{d}\Omega_{1\to 2} \qquad (\mathrm{III}.22)$$

describes the total flux emitted by a surface element dS_1 of a black body in the solid angle $d\Omega_{1\to 2}$(direction D).

With:

$$\mathrm{d}\Omega_{1\to 2} = \frac{\mathrm{dS}_2\,\cos\beta_2}{\mathrm{d}^2} \qquad (\mathrm{III}.23)$$

So

$$d^2\phi_{1\to 2} = \frac{\mathrm{M}^0_{\mathrm{T}1}}{\pi}\frac{\mathrm{dS}_1\,\cos\beta_1\ \mathrm{dS}_2\cos\beta_2}{d^2} \qquad (\mathrm{III}.24)$$

Let there be a second black body whose surface element dS_2 intercepts the radiation emitted by $d\Omega_2 \to 1$. So when body number 2 is a black body, this flux is completely absorbed. Simultaneously, dS_2 (at the temperature $2T_2$) emits towards dS_1:

$$\mathrm{d}^2\phi_{2\to 1} = \frac{\mathrm{M}^0_{\mathrm{T}2}}{\pi}\frac{\mathrm{dS}_1\,\cos\beta_1\ \mathrm{dS}_2\cos\beta_2}{\mathrm{d}^2} \qquad (\mathrm{III}.25)$$

Le bilan de l’échange est :

$$\begin{aligned}\mathrm{d}^2\phi_{12} = \mathrm{d}^2\phi_{21} &= \mathrm{d}^2\phi_{1\to 2} - \mathrm{d}^2\phi_{2\to 1}\\ &= \sigma(\mathrm{T}_1^4 - \mathrm{T}_2^4)\frac{\mathrm{dS}_1\,\cos\beta_1\ \mathrm{dS}_2\cos\beta_2}{\pi.\mathrm{d}^2}\end{aligned} \qquad (\mathrm{III}.26)$$

By integration, one obtains The total flow exchanged between S_1 and S_2:

$$\phi_{12} = \sigma(T_1^4 - T_2^4) \iint\limits_{S_1\ S_2} \frac{dS_1 \cos\beta_1 \ dS_2 \cos\beta_2}{\pi . d^2} \qquad (III.27)$$

$$\phi_{12} = \sigma(T_1^4 - T_2^4). S_1 F_{12} = \sigma(T_1^4 - T_2^4). S_2 F_{21} \qquad (III.28)$$

With

$$S_1 F_{12} = S_2 F_{21} = \iint\limits_{S_1\ S_2} \frac{dS_1 \cos\beta_1 \ dS_2 \cos\beta_2}{\pi . d^2} \qquad (III.29)$$

F_{12} and F_{21} are purely geometric and dimensionless quantities.

F_{12}: Form factor under which S_1 sees S_2

F_{21}: Form factor under which S_2 sees S_1

As a result, the problem of calculating exchanges is reduced solely to the calculation of these shape factors.

III.9.1.1. The Form Factors

We define form factors by:

$$S_i F_{ij} = S_j F_{ji} = \iint\limits_{S_i\ S_j} \frac{dS_i \cos\beta_i \ dS_j \cos\beta_j}{\pi . d^2} \qquad (III.30)$$

The form factor Fij is also the fraction of the hemispherical flux that reaches Sj from Si:

$$F_{i,j} = \frac{\phi_{i \to j}}{\phi_i} \qquad (III.31)$$

II.9.1.2. Relationship between form factors

- Addition relation

Let a surface Sj be decomposable into two surfaces (S_{j1} and S_{j2}), the integral operator being a linear operator, we obtain the relation:

$$S_j = S_{j1} + S_{j2} \Rightarrow F_{ij} = F_{ij1} + F_{ij2} \qquad (III.32)$$

-Reciprocal relationship

The symmetry of the expressions of F_{12} and F_{21} leads to the reciprocity of the form factors.

$$S_1 . F_{12} = S_2 . F_{21} \qquad (III.33)$$

-Relation of enclosure (additivity relation or complementarity relation)

In a closed enclosure, the flux emitted ϕ_i by a surface i is likely to be received by the n surfaces that constitute the surface (the surface i understand).

$$\Phi_i = \Phi_{i1} + \Phi_{i2} + \cdots \ldots \ldots \ldots \ldots \ldots + \Phi_{ii} + \cdots \ldots \ldots \ldots \ldots \ldots \ldots + \Phi_{in} \qquad (III.34)$$

$$\Rightarrow \Phi_i = \sum_{j=1}^{n} \Phi_{ij} = \sum_{j=1}^{n} F_{ij}\Phi_i = \phi_i \sum_{j=1}^{n} F_{ij} \qquad (III.35)$$

From where, $\sum_{j=1}^{n} F_{ij} = 1$

III.9.1.3. Estimation of form factors

There are several calculation methods to estimate the desired form factor. For example, we cite:

- Use of reciprocity and enclosure relationships. This method is only possible for simple cases.

- Use of formulas and charts(see Appendix R.3 , R.4, R5).

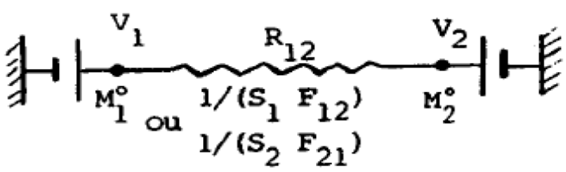
III.9.1.4. Representation of radiative exchanges between black surfaces by electrical analogy

The relationship expressing the net flux exchanged between two black surfaces

$$\phi_{1,2} = \sigma(T_1^4 - T_2^4)S_1.F_{12} = (M_1^0 - M_2^0)S_1.F_{12} \qquad (III.36)$$

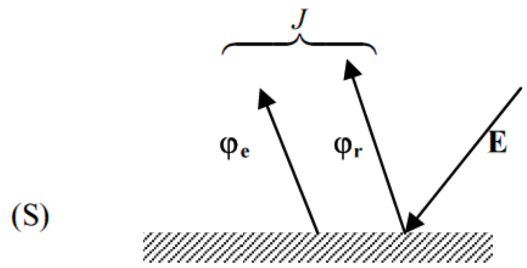
From where:

$$(M_1^0 - M_2^0) = \frac{\phi_{1,2}}{S_1 F_{12}} \qquad (III.37)$$



III.9.2. Radiative exchanges between opaque gray surfaces separated by a perfectly transparent medium.

Radiative exchanges between gray surfaces are more complex because the surfaces reflect by radiation, leading to multiple reflections in a closed enclosure. A new concept is being used: it is the radiosity J.



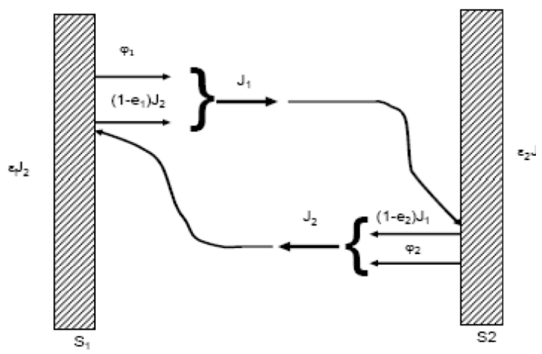
The radiosity of the surface is the sum of the emitted flux and the reflected flux per unit area.

$$J = \varphi_e + \varphi_r \qquad (III.38)$$

From where:

$$\varphi_r = \rho E = (1-\alpha)E = (1-\varepsilon)E \qquad \text{(For the gray body: } \alpha = \varepsilon\text{)}$$

Case of two infinite parallel planes (total influence):



For S_1 we have:

$$\varphi_e = \varphi_1 = \varepsilon_1.\sigma.T_1^4 \qquad (III.39)$$

$$\varphi_r = (1-\varepsilon_1).J_2$$

J_2 constitutes the illumination of S_1, J_1 constitutes the illumination of S_2:

$$J_1 = \varphi_1 + J_2.(1-\varepsilon_1) \qquad (III.40)$$

$$J_2 = \varphi_2 + J_1.(1-\varepsilon_2) \qquad (III.41)$$

The final expression of the illuminances is:

$$J_1 = \frac{\varphi_1 + (1-\varepsilon_1)\varphi_2}{1-(1-\varepsilon_1)(1-\varepsilon_2)} \qquad (III.42)$$

$$J_2 = \frac{\varphi_2 + (1-\varepsilon_2)\varphi_1}{1-(1-\varepsilon_1)(1-\varepsilon_2)} \qquad (III.43)$$

The flux φ_{12} is written as:

$$\phi_{12} = S_1\varphi_{12} = S_1(J_1 - J_2) = S_1\frac{\varepsilon_1\varepsilon_2}{1-(1-\varepsilon_1)(1-\varepsilon_2)}\sigma(T_1^4 - T_2^4) \qquad (III.44)$$

$$\frac{\phi_{12}}{S} = (J_1 - J_2) = \frac{\varepsilon_1 \varepsilon_2}{1 - (1-\varepsilon_1)(1-\varepsilon_2)} \sigma(T_1^4 - T_2^4) \qquad \text{(III.45)}$$

With: $S_1 = S_2 = S$

III.9.3. Radiation of Partially Transparent Bodies

For this type of body, we must introduce the transmission τ, the balance becomes:

$$\alpha + \rho + \tau = 1 \qquad \text{(III.46)}$$

When a radiation φ_0 penetrates a semi-transparent medium, it gradually diminishes through absorption; between x and x + dx, it will have lost a certain percentage of its value at x. If this percentage is proportional to dx, we will have:

$$\frac{d\phi}{\phi(x)} = -kdx \qquad \text{(III.47)}$$

And after integration

$$\phi(x) = \phi_0 e^{-kx} \qquad \text{(III.48)} \qquad \text{(Beer-Lambert law)}$$

In fact, the problem can be decomposed into a sum $\sum \phi_\lambda(x)$ with a coefficient K_λ for each wavelength:

$$\phi_\lambda(x) = \phi_{0\lambda} e^{-k_\lambda x} \qquad \text{(III.49)}$$

-Gases as radiation receivers

A gas layer of thickness L will therefore absorb (not to be confused with L the luminance):

$$\phi_{abs} = \phi_0 - \phi_L = \phi_0\left[1 - e^{-kL}\right] = \phi_0 . \alpha_L \qquad \text{(III.50)}$$

$\alpha_L = 1 - e^{-KL}$ is therefore the absorption coefficient of the L layer. The transmission factor will therefore be:

$$\tau_L = e^{-kL} = 1 - \alpha_L \qquad \text{(III.51)}$$

-Gases as radiation emitters

The extension of Kirchhoff's law for gases of thickness L is written as

$$\varepsilon_{\lambda L} = \alpha_{\lambda L} = 1 - e^{k_\lambda L} \qquad \text{(III.52)}$$

Gases are not gray bodies: $\varepsilon_L \neq \alpha_L$

Indeed, ε_L depends on the temperature of the gas, whereas α_L depends on the temperature of the absorbed radiation.

Exercises

Exercise 01:

For a black body maintained at 115°C, determine:

1. The total emissive power
2. The wavelength at which the maximum spectral emissive power occurs.
3. The maximum spectral emissive power.

Solution:

$$M^0 = \sigma . T^4 = 5{,}67 . 10^{-8} . 388^4 = 1285 W. m^{-2}$$

$$\lambda_{max} . T = 2897{,}6 \Rightarrow \lambda_{max} = \frac{2897{,}6}{388} = 7{,}47 \mu m$$

$$\left(M_\lambda^0\right)_{max} = \frac{3{,}742 . 10^8 . \lambda^{-5}}{\exp\left(\frac{1{,}439 . 10^4}{\lambda T}\right) - 1} = \frac{3{,}742 . 10^8 . 7{,}47^{-5}}{\exp\left(\frac{1{,}439 . 10^4}{7{,}47 . 388}\right) - 1} = 113{,}06 \ W. m^{-2} . \mu m^{-1}$$

Exercise 2:

A 100 W lamp is powered at 220 V. It consists of a tungsten filament placed at the center of a spherical bulb with a diameter of 8 cm, inside which a vacuum is created. For the light to be sufficiently white, the temperature of the filament must be 2600 K.

1) Determine the diameter and length of the filament if the total hemispherical emissivity factor of tungsten is 0.3 (tungsten resistivity 88 μΩcm).

2) Determine the power radiated in the visible range (between 0.4 and 0.7 μm) if the hemispherical spectral emission factor is 0.45 in this range.

3) What is the power absorbed by the bulb assuming the glass is perfectly transparent up to 2.7 μm and behaves like a black body beyond that, given that the hemispherical spectral emission factor of tungsten is 0.2 in the entire range beyond 2.7 μm?

4) Determine the temperature of the bulb if the total hemispherical emissivity factor of the glass is 0.93, neglecting heat losses due to natural convection.

Solution:

1) Determine the diameter and length of the filament if the total hemispherical emission factor of tungsten is 0.3 (tungsten resistivity 88 μΩcm).
2)

$M_T^0 = \varepsilon_{Tu}.\sigma.T^4 = 0{,}3\ .\ 5{,}67.10^{-8}.2600^4 = 7{,}77\,.10^5\ \ W/m^2$

$\varphi = S.M_T^0 = 100W$

$$S = \pi.D.L = \frac{100}{M_T^0} = \frac{100}{7{,}77\,.10^5}$$

$D.L = 4\,.10^{-5}\,m^2$

The electrical resistance of the wire is:

$$R_e = \frac{V^2}{P} = \frac{220^2}{100} = 484\Omega$$

$$R_e = \rho\frac{L}{\frac{\pi D^2}{4}} = 484\Omega$$

$$\frac{L}{D^2} = 43197\ .10^4\ \ m^{-1}$$

$\mathbf{L = 0{,}898m\ ,D = 4{,}56\ \ 10^{-5}m}$

2) the power radiated in the visible range (between 0.4 and 0.7 μm) if the hemispherical spectral emission factor is 0.45 in this range:

$$P_{visible} = S.\int_{0,4}^{0,7} M_{\lambda,T}^0 d\lambda = S.\varepsilon_{visible}\left(\int_0^{0,7} M_{\lambda,T}^0 d\lambda - \int_0^{0,4} M_{\lambda,T}^0 d\lambda\right)$$

$$= S.\varepsilon_{visible}.\left(F_{0-0,7.T} - F_{0-0,4.T}\right).\sigma.T^4$$

$\lambda_1 = 0{,}4\mu m\ \ \text{et}\ \ T = 2600K \rightarrow \lambda_1.T = 0{,}4\ \ .\ 2600 = 1040\mu m.K \rightarrow F_{0-0,4T} = 0{,}0005$

$\lambda_2 = 0{,}7\mu m\ \ \text{et}\ \ T = 2600K \rightarrow \lambda_2.T = 0{,}7\ \ .\ 2600 = 1820\mu m.K \rightarrow F_{0-0,7T} = 0{,}0418$

$P_{visible} = \pi.D.L.\varepsilon_{visible}(0{,}0418 - 0{,}0005).5{,}67\,.10^{-8}\ .\ 2600^4$

$P_{visible} = \pi\ \ .4{,}56\ .10^{-5}.0{,}898.\ 0{,}45\,.(0{,}418 - 0{,}0005).5{,}67\,.10^{-8}\ .2600^4 = 6{,}2W$

The calculation can be done differently:

$P_{Total} = \varepsilon\ .\ S.\sigma.T^4 = P = 100W$

The total power radiated by the wire is:

$$S\,.\,\sigma\,.\,T^4 = \frac{P}{\varepsilon} = \frac{100}{0{,}3}$$

$$P_{visible} = S.\,\varepsilon_{visible}.\left(F_{0-0{,}7T} - F_{0-0{,}4T}\right).\,\sigma.\,T^4 = P.\frac{\varepsilon_{visible}}{\epsilon}\left(F_{0-0{,}7T} - F_{0-0{,}4T}\right)$$

3) the power absorbed by the bulb assuming that the glass is perfectly transparent up to 2.7 µm and behaves like a black body beyond, assuming that the hemispherical spectral emission factor of tungsten is 0.2 throughout the domain beyond 2.7 µm:

$$P_{inf} = S.\,\varepsilon_{visible}.\left(1 - F_{0-2{,}7.T}\right).\,\sigma.\,T^4 = P.\frac{\varepsilon_{inf}}{\varepsilon}\left(1 - F_{0-2{,}7.T}\right)$$

$$\lambda_3 = 2{,}7\mu m \quad \text{et} \quad T = 2600K \Rightarrow \lambda_3.\,T = 2{,}7\ .\ 2600 = 7020\mu m.K$$

$$\Rightarrow F_{0-2{,}7.T} = \frac{80{,}97}{100} = 0{,}8097$$

$$P_{inf} = P.\frac{\varepsilon_{inf}}{\varepsilon}\left(1 - F_{0-2{,}7.T}\right) = 100.\frac{0{,}2}{0{,}3}(1 - 0{,}8097) = 12{,}7W$$

4) Steady state: the power absorbed by the glass is equal to the power radiated by the glass to the outside, hence:

$$S_{glass}.\,\varepsilon_{glass}\,.\,\sigma.\,T^4_{glass} = P_{inf} = \mathbf{12{,}3W}$$

With ε_{Glass}, the total hemispherical emission factor of the glass.

$$T^4_{glass} = \frac{12{,}3}{S_{glass}.\,\varepsilon_{glass}.\,\sigma} = \frac{12{,}3}{0{,}93\ .\ 5{,}67\,.\,10^{-8}\,.\ 4\ .\,\pi.\,(4\,.\,10^{-2})^2}$$

$$T^4_{glass} = 1{,}16\ .\,10^{10}K\,K \Rightarrow T_{glass} = \mathbf{328K}$$

Exercise 3:

A cylindrical furnace with a diameter of 75 mm and a height of 150 mm is open at the top to an ambient temperature of 27°C. The lateral and bottom surfaces (assumed to be black bodies) are electrically heated and maintained at temperatures of 1350°C and 1650°C, respectively.

- What is the radiative heat flux lost by the furnace upwards?What is the radiative heat flux lost by the furnace upwards?

Solution:

$$\varphi = \varphi_{13} + \varphi_{23} = S_1.F_{13}.\sigma.(T_1^4 - T_3^4) + S_2.F_{23}.\sigma.(T_2^4 - T_3^4)$$

$$S_1 = \pi.D.L = \pi.0{,}075\,.0{,}15 = 0{,}035m^2$$

$$S_2 = \frac{\pi.D}{4} = \frac{\pi(0{,}075)^2}{4} = 0{,}0044m^2$$

$$\left.\begin{array}{r} R_1 = R_2 = \frac{r}{h} = \frac{37{,}5}{150} = 0{,}25 \\ X = 1 + \frac{1+R_2^2}{R_1^2} = 1 + \frac{1+0{,}25^2}{0{,}25^2} = 18 \end{array}\right\} \rightarrow F_{23} = \frac{1}{2}\left[X - \sqrt{X^2 - 4\left(\frac{R_2}{R_1}\right)^2}\right] = 0{,}056$$

$$F_{21} + F_{23} = 1 \rightarrow F_{21} = 1 - F_{23} = 1 - 0{,}056 = 0{,}944$$

$$F_{12} = \frac{s_2}{s_1}.F_{21} = \frac{0{,}0044}{0{,}035}0{,}944 = 0{,}118 = F_{13}$$

$$\varphi_{13} = S_1F_{13}.\sigma.(T_1^4 - T_3^4) = 0{,}035\,.0{,}118\,.5{,}67\,.10^{-8}[(1923)^4 - (300)^4] = 1639W$$

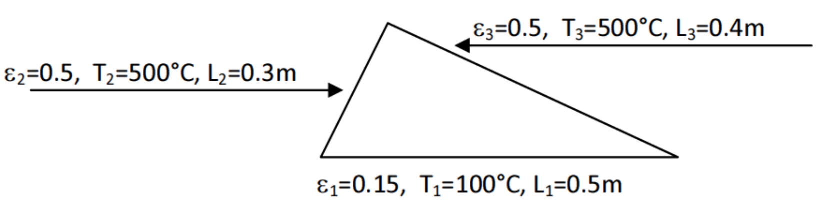
$$\varphi_{23} = S_2F_{23}.\sigma.(T_2^4 - T_3^4) = 0{,}0044\,.0{,}056.5{,}67\,.10^{-8}[(1923)^4 - (300)^4] = 205W$$

$$\varphi = 1639 + 205 = \mathbf{1844W}$$

Exercise 4:

Let there be three infinite gray flat surfaces (1, 2, and 3) with semi-infinite dimensions (see figure):

- Calculate the following shape factors (see figure): F_{11}, F_{12}, F_{13}, F_{21}, F_{22}, F_{23}, F_{31}, F_{32}, F_{33}.
- Provide the radiosity of each surface.
- Provide the definition of the net flux of surface 1.
- Calculate the net flux of surface 1.



Solution:

Calculate the following shape factors (see figure):

F 11 , F 12 , F 13 , F 21 , F 22 , F 23 , F 31 , F 32 , F 33 .

F 11 = F 22 = F 33 =0 flat surfaces.

Application of Hottel's Rule:

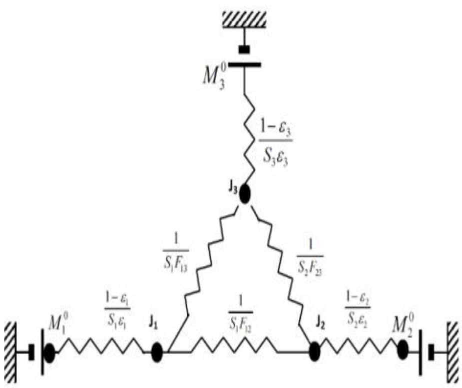
$$F_{12}=\frac{S_1+S_2-S_3}{2S_1}$$

$$F_{12}=\frac{S_1+S_2-S_3}{2S_1}=\frac{L_1+L_2-L_3}{2L_1}=\frac{0{,}5+0{,}3-0{,}4}{2\ .0{,}5}=\frac{0{,}4}{1}=0{,}4$$

$$F_{13}=\frac{S_1+S_3-S_2}{2S_1}=\frac{L_1+L_3-L_2}{2L_1}=\frac{0{,}5+0{,}4-0{,}3}{2\ .0{,}5}=\frac{0{,}6}{1}=0{,}6$$

$$F_{23}=\frac{S_2+S_3-S_1}{2S_2}=\frac{L_2+L_3-L_1}{2L_2}=\frac{0{,}3+0{,}4-0{,}5}{2\ .0{,}3}=\frac{0{,}2}{0{,}6}=\frac{1}{3}$$

$$S_2.F_{23}=S_3.F_{32}\Rightarrow F_{32}=\frac{L_2}{L_3}.F_{23}=\frac{0{,}3}{0{,}4}\frac{1}{3}=0{,}25$$



$$S_2.F_{21}=S_1.F_{12}\Rightarrow F_{21}=\frac{L_1}{L_2}.F_{12}=\frac{0{,}5}{0{,}3}0{,}4=\frac{2}{3}$$

$$S_3.F_{31}=S_1.F_{13}\Rightarrow F_{31}=\frac{L_1}{L_3}.F_{13}=\frac{0{,}5}{0{,}4}0{,}6=0{,}75$$

•Radiosity of different surfaces:

$$J_1=\varphi_1+(1-\varepsilon_1)[F_{11}.J_1+F_{12}.J_2+F_{13}.J_3]$$

$$J_2=\varphi_2+(1-\varepsilon_2)[F_{21}.J_1+F_{22}.J_2+F_{23}.J_3]$$

$$J_3=\varphi_3+(1-\varepsilon_3)[F_{31}.J_1+F_{32}.J_2+F_{33}.J_3]$$

•Definition of the net flux of surface 1:

The net flux of surface 1 is equal to the difference between the emitted flux and the absorbed flux.

$$\phi_{1,nette}=\frac{S_1\varepsilon_1}{1-\varepsilon_1}(M_1^0-J_1)$$

•Calculation of the net flux of surface 1.

$$\begin{pmatrix} -\varphi_1 \\ -\varphi_2 \\ -\varphi_3 \end{pmatrix} = \begin{bmatrix} -1 & (1-\varepsilon_1)F_{12} & (1-\varepsilon_1)F_{13} \\ (1-\varepsilon_2)F_{21} & -1 & (1-\varepsilon_2)F_{23} \\ (1-\varepsilon_3)F_{31} & (1-\varepsilon_3)F_{32} & -1 \end{bmatrix} \begin{pmatrix} J_1 \\ J_2 \\ J_3 \end{pmatrix}$$

$$\begin{pmatrix} -\varepsilon_1.\sigma.T_1^4 \\ -\varepsilon_2.\sigma.T_2^4 \\ -\varepsilon_3.\sigma.T_3^4 \end{pmatrix} = \begin{bmatrix} -1 & 0{,}34 & 0{,}51 \\ \frac{1}{3} & -1 & \frac{1}{6} \\ 0{,}375 & 0{,}125 & -1 \end{bmatrix} \begin{pmatrix} J_1 \\ J_2 \\ J_3 \end{pmatrix}$$

$$\begin{pmatrix} 164 \\ 16466 \\ 16466 \end{pmatrix} = \begin{bmatrix} 1 & -0{,}34 & -0{,}51 \\ -\frac{1}{3} & 1 & -\frac{1}{6} \\ -0{,}375 & -0{,}125 & 1 \end{bmatrix} \begin{pmatrix} J_1 \\ J_2 \\ J_3 \end{pmatrix}$$

$$[K] = [M].[J]$$

$$[M]^{-1}.[K] = [M]^{-1}.[M].[J] = [J]$$

After inverting the matrix:

$$\begin{bmatrix} J_1 = 25536 \\ J_2 = 29934 \\ J_3 = 29785 \end{bmatrix}$$

$$\frac{\phi_{1,nette}}{S_1} = \frac{\varepsilon_1}{1-\varepsilon_1}(M_1^0 - J_1) = -4477 W.m^{-2}$$

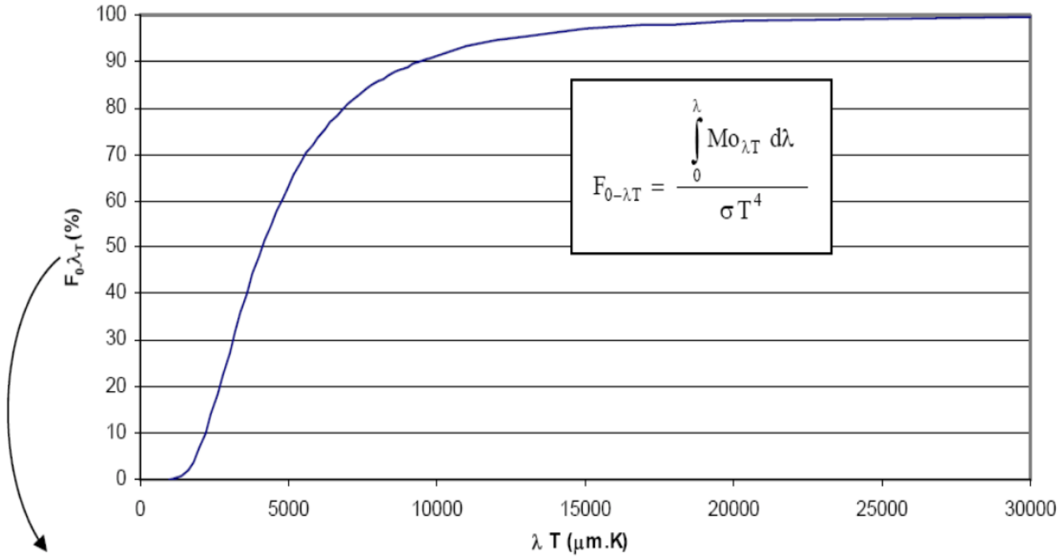
Surface 1 receives heat.

Appendix

Appendix R.1 : Solar radiative properties of materials

Description/composition	Solar Absorptivity, α_s	Emissivity, ε, at 300 K	Ratio, α_s/ε	Solar Transmissivity, τ_s
Aluminum				
Polished	0.09	0.03	3.0	
Anodized	0.14	0.84	0.17	
Quartz-overcoated	0.11	0.37	0.30	
Foil	0.15	0.05	3.0	
Brick, red (Purdue)	0.63	0.93	0.68	
Concrete	0.60	0.88	0.68	
Galvanized sheet metal				
Clean, new	0.65	0.13	5.0	
Oxidized, weathered	0.80	0.28	2.9	
Glass, 3.2-mm thickness				
Float or tempered				0.79
Low iron oxide type				0.88
Marble, slightly off-white (nonreflective)	0.40	0.88	0.45	
Metal, plated				
Black sulfide	0.92	0.10	9.2	
Black cobalt oxide	0.93	0.30	3.1	
Black nickel oxide	0.92	0.08	11	
Black chrome	0.87	0.09	9.7	
Mylar, 0.13-mm thickness				0.87
Paints				
Black (Parsons)	0.98	0.98	1.0	
White, acrylic	0.26	0.90	0.29	
White, zinc oxide	0.16	0.93	0.17	
Paper, white	0.27	0.83	0.32	
Plexiglas, 3.2-mm thickness				0.90
Porcelain tiles, white (reflective glazed surface)	0.26	0.85	0.30	
Roofing tiles, bright red				
Dry surface	0.65	0.85	0.76	
Wet surface	0.88	0.91	0.96	
Sand, dry				
Off-white	0.52	0.82	0.63	
Dull red	0.73	0.86	0.82	
Snow				
Fine particles, fresh	0.13	0.82	0.16	
Ice granules	0.33	0.89	0.37	
Steel				
Mirror-finish	0.41	0.05	8.2	
Heavily rusted	0.89	0.92	0.96	
Stone (light pink)	0.65	0.87	0.74	
Tedlar, 0.10-mm thickness				0.92
Teflon, 0.13-mm thickness				0.92
Wood	0.59	0.90	0.66	

Appendix R.2: Fraction of energy F0-λT radiated by a black body between 0 and λ



a \ b	0	40	80	120	160
1 000	0,03	0,05	0,08	0,11	0,16
1 200	0,21	0,29	0,38	0,49	0,62
1 400	0,78	0,96	1,17	1,41	1,68
1 600	1,97	2,30	2,66	3,06	3,48
1 800	3,94	4,42	4,94	5,49	6,07
2 000	6,68	7,31	7,97	8,65	9,36
2 200	10,09	10,84	11,61	12,40	13,21
2 400	14,03	14,86	15,71	16,57	17,44
2 600	18,32	19,20	20,09	20,99	21,89
2 800	22,79	23,70	24,61	25,51	26,42
3 000	27,33	28,23	29,13	30,03	30,92
3 200	31,81	32,70	33,58	34,45	35,32
3 400	36,18	37,03	37,88	38,71	39,54
3 600	40,36	41,18	41,98	42,78	43,56
3 800	44,34	45,11	45,87	46,62	47,36
4 000	48,09	48,81	49,53	50,23	50,92
4 200	51,60	52,28	52,94	53,60	54,25
4 400	54,88	55,51	56,13	56,74	57,34
4 600	57,93	58,51	59,09	59,65	60,21
4 800	60,66	61,30	61,83	62,35	62,87
5 000	63,38	63,88	64,37	64,85	65,33
5 200	65,80	66,26	66,72	67,16	67,60
5 400	68,04	68,46	68,88	69,30	69,70
5 600	70,11	70,50	70,89	71,27	71,65
5 800	72,02	72,38	72,74	73,09	73,44
6 000	73,78	74,12	74,45	74,78	75,10
6 200	75,41	75,72	76,03	76,33	76,63
6 400	76,92	77,21	77,49	77,77	78,05
6 600	78,32	78,59	78,85	79,11	79,36
6 800	79,61	79,86	80,10	80,34	80,58
7 000	80,90	81,04	81,26	81,47	81,70
7 200	81,92	82,13	82,34	82,55	82,75
7 400	82,95	83,15	83,34	83,53	83,72
7 600	83,91	84,09	84,27	84,45	84,62

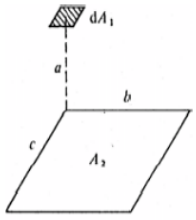
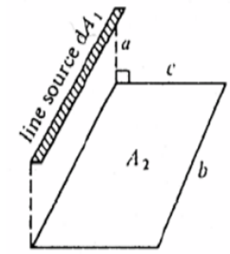
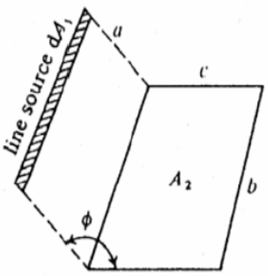
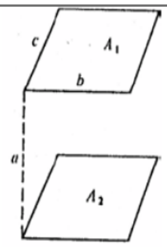
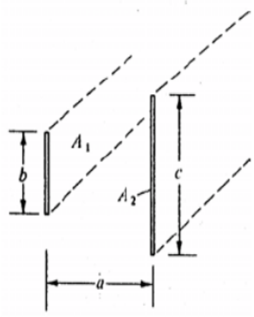
a \ b	0	40	80	120	160
7 800	84,80	84,97	85,14	85,30	85,47
8 000	85,63	85,78	85,94	86,10	86,25
8 200	86,40	86,55	86,69	86,83	86,98
8 400	87,12	87,25	87,39	87,52	87,66
8 600	87,80	87,92	88,04	88,17	88,29
8 800	88,41	88,53	88,65	88,77	88,88
9 000	88,89	89,11	89,22	89,33	89,44
9 200	89,55	89,65	89,76	89,86	89,96
9 400	90,06	90,16	90,26	90,35	90,45
9 600	90,54	90,63	90,72	90,81	90,90
9 800	90,99	91,08	91,16	91,25	91,33
10 000	91,42				

a \ b	0	200	400	600	800
10 000	91,42	91,81	92,19	92,54	92,87
11 000	93,18	93,48	93,76	94,02	94,27
12 000	94,50	94,73	94,94	95,14	95,33
13 000	95,51	95,68	95,84	96,00	96,14
14 000	96,29	96,42	96,54	96,67	96,78
15 000	96,89	97,00	97,10	97,19	97,29
16 000	97,37	97,46	97,54	97,62	97,69
17 000	97,77	97,83	97,90	97,96	98,02
18 000	98,08	98,14	98,19	98,24	98,29
19 000	98,34	98,38	98,43	98,47	98,51
20 000	98,55				
30 000	99,53				
40 000	99,78				
50 000	99,89				
60 000	99,93				
70 000	99,96				
80 000	99,97				
90 000	99,98				
100 000	99,98				

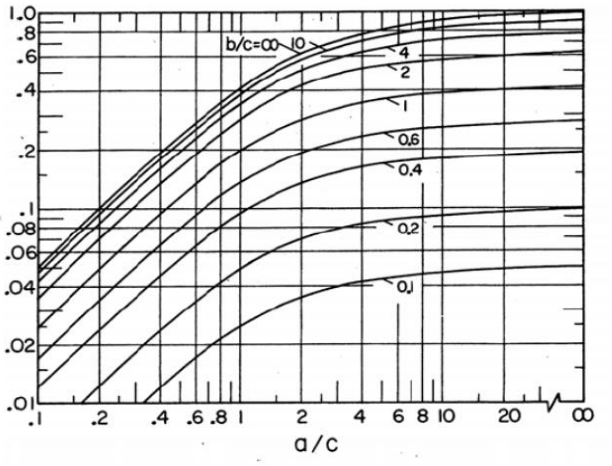
Utilisation :
$\lambda T = a + b$

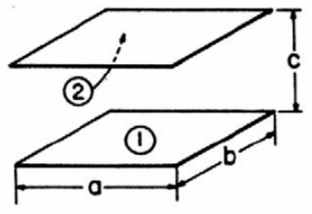
Exemple : λT = 2720 μm.K
se lit à 2600 + 120
d'où : $F_{0\text{-}\lambda T}$ = 20,99 %

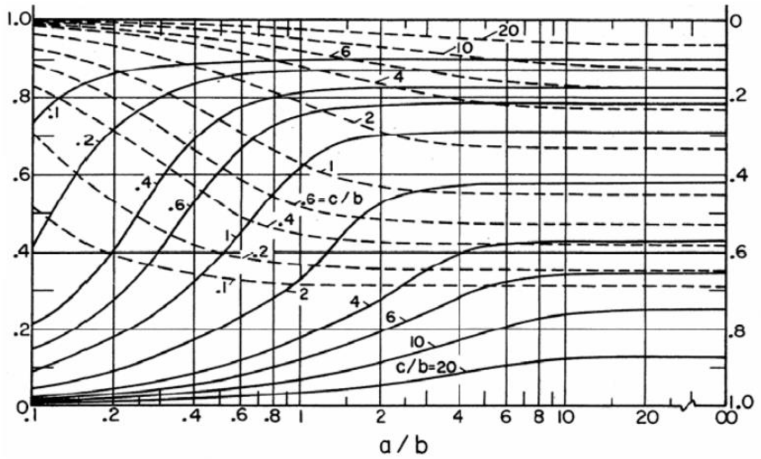
Appendix R.3: Geometric Radiation Shape Factors

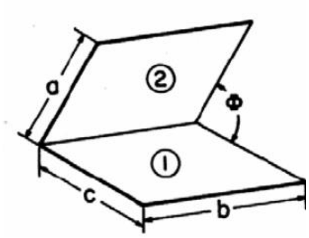
Configuration	Schema	Form factor value
Surface dS parallel to a rectangular plane		
Linear source parallel to a rectangular plane		$F_{dA_1-A_2} = \frac{1}{\pi B}\left[\sqrt{1+B^2}\tan^{-1}\left(\frac{C}{\sqrt{1+B^2}}\right) - \tan^{-1} C + \frac{BC}{\sqrt{1+C^2}}\tan^{-1}\left(\frac{B}{\sqrt{1+C^2}}\right)\right]$ $B = \frac{b}{a}, \quad C = \frac{c}{a}$
linear source parallel and rectangular plane intersecting at an angle ϕ		$F_{dA_1-A_2} = \frac{1}{\pi}\left\{\tan^{-1} B + \frac{\sin^2\phi}{2B}\ln\left[\frac{B^2+X^2}{(1+B^2)X^2}\right] - \frac{\sin 2\phi}{2B}\left[\frac{\pi}{2} - \phi + \tan^{-1}\left(\frac{C-\cos\phi}{\sin\phi}\right)\right] + \frac{Y}{B}\left[\tan^{-1}\left(\frac{C-\cos\phi}{Y}\right) + \tan^{-1}\left(\frac{\cos\phi}{Y}\right)\right] \times \cos\phi + \frac{C\cos\phi - 1}{X}\tan^{-1}\left(\frac{B}{X}\right)\right\}$ $B = \frac{b}{a}, \quad C = \frac{c}{a}, \quad X = \sqrt{C^2 - 2C\cos\phi + 1},$ $Y = \sqrt{B^2 + \sin^2\phi}$
Two parallel rectangular planes of the same area		$F_{A_1-A_2} = \frac{1}{\pi}\left[\frac{1}{BC}\ln\left(\frac{XY}{X+Y-1}\right) + \frac{2\sqrt{X}}{B}\tan^{-1}\frac{C}{\sqrt{X}} + \frac{2\sqrt{Y}}{C}\tan^{-1}\frac{B}{\sqrt{Y}} - \frac{2}{C}\tan^{-1} B - \frac{2}{B}\tan^{-1} C\right]$ $B = \frac{b}{a}, \quad C = \frac{c}{a}, \quad X = 1 + B^2, \quad Y = 1 + C^2$
Two infinite parallel bands of different widths		$F_{A_1-A_2} = \frac{1}{2B}\left[\sqrt{(B+C)^2+4} - \sqrt{(C-B)^2+4}\right]$ $F_{A_2-A_1} = \frac{1}{2C}\left[\sqrt{(B+C)^2+4} - \sqrt{(B-C)^2+4}\right]$ $B = \frac{b}{a}, \quad C = \frac{c}{a}$ $F_{A_1-A_2} = F_{A_2-A_1} = \frac{1}{B}\left[\sqrt{B^2+1} - 1\right]$ for $b = c$

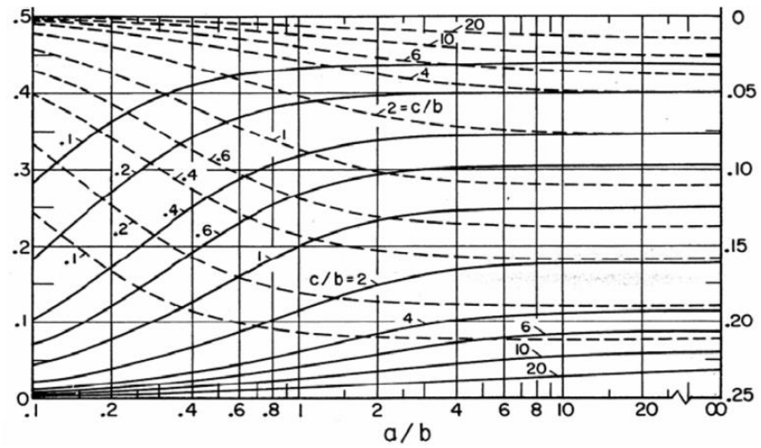
Appendix R.4: Geometric Radiation Shape Factors



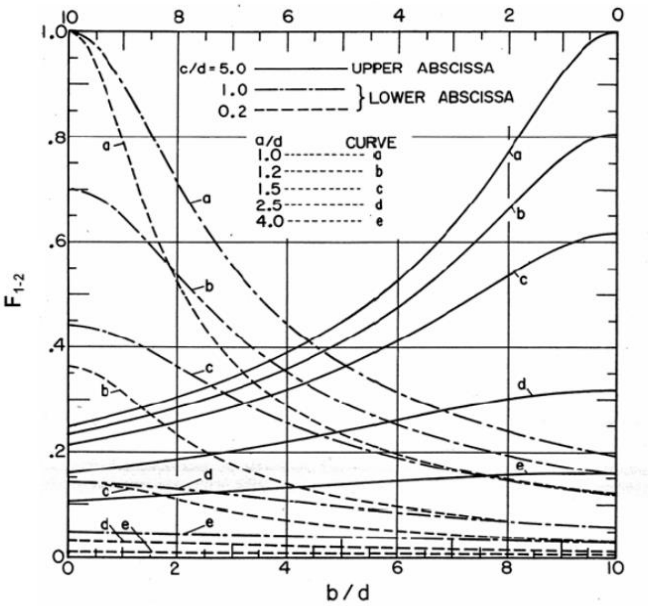


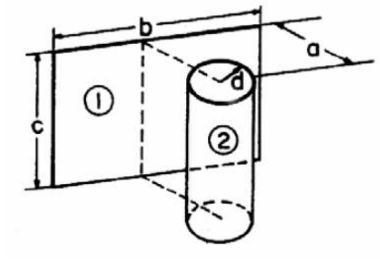


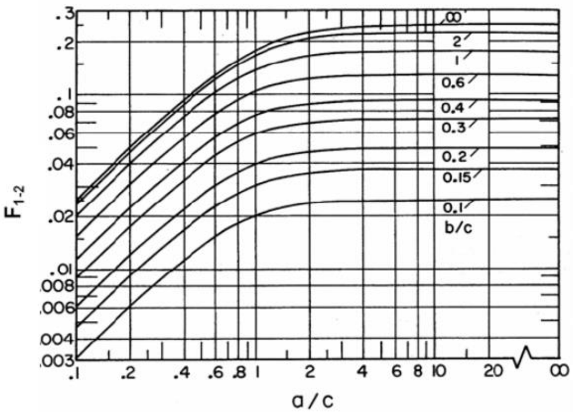


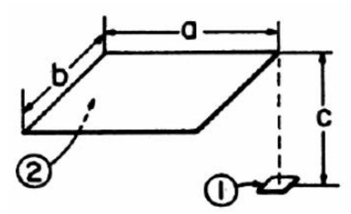


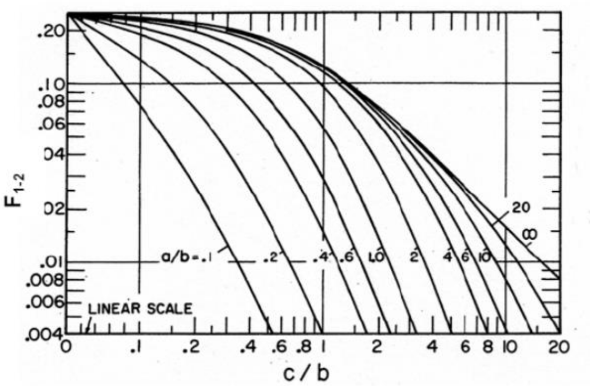
Appendix R.5: Geometric Radiation Shape Factors

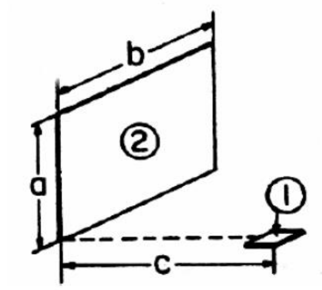












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