



رقم 123/GE/2024

المسيلة في : 26 اغسطس 2024

شهادة إدارية

بخصوص مطبوعة الدروس الخاصة بالأستاذ
بلهوشات خالد

بناءً على محضر اللجنة العلمية لقسم الهندسة الكهربائية رقم 107/2024 المنعقد بتاريخ 04 مارس 2024 والمتضمن تعيين الخبراء: الأستاذ غماري زين أستاذ بجامعة المسيلة، الأستاذ بن دايدة عبد الملك أستاذ محاضر -أ- بجامعة المسيلة والأستاذ مداني لخضر أستاذ محاضر "أ" بجامعة سطيف 01 وذلك لتقييم مطبوعة الدروس الخاصة بالأستاذ بلهوشات خالد أستاذ محاضر "أ" بقسم الهندسة الكهربائية لجامعة المسيلة تحت عنوان "Heat transfer" وبعد إطلاع رئيس اللجنة العلمية ورئيس القسم على التقارير الواردة والتي كانت كلها ايجابية، وعليه فإن اللجنة لا ترى مانعا أن تتخذ سنداً في تدريس طلبة السنة الثانية ليسانس طاقات متجددة وبيئة، شعبة طاقات متجددة ميدان علوم و تكنولوجيا و أن تعتمد في أي تقييم للمسار العلمي للأستاذ المعني.

رئيس القسم
أ. د. زغلاش سمير

رئيس اللجنة العلمية
أ. د. بوقرة عبد الرحمان



ملاحظة: سلمت هذه الشهادة للمعني (ة) لاستعمالها في حدود ما يسمح به القانون

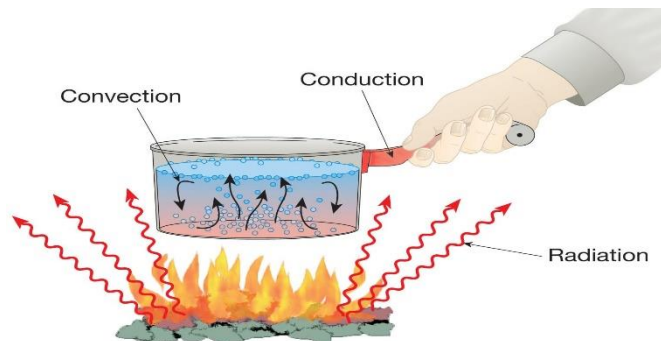
Pedagogical Support

License 2nd year L2-S3

Heat Transfer

Prepared by:

Dr. BELHOUCHE Khaled
Associate professor



Academic year: 2023/2024

Preface

This document is a teaching aid on the subject of heat transfer intended for Renewable Energy second-year, students.

Course Objectives:

1. Students will be able to derive the governing equations of heat transfer.
2. Students will be able to analytically and numerically solve problems of heat conduction and convection.
3. Students will be able to analytically solve problems of radiative heat transfer.

Dr. Khaled Belhouchet

Semester: 3**Year: 2nd year Science and Technology (ST).****Course: Heat transfer.****Responsible of the course: Dr. Khaled BELHOUCHE****Material:** Heat transfer**SHV:** 45h00 (Weekly timetable: 1h30, Tutorial: 1h30)**Coefficient:** 2**Teaching objectives:**

Master the three modes of heat transfer (conduction, convection and radiation) and introduction to heat exchangers.

Recommended prior knowledge:

Have some knowledge of thermodynamics.

Material content:**Chapter 1:** General information on heat transfer**Chapter 2:** Steady-state conduction heat transfer**Chapter 3:** Heat transfer by conduction in variable regime**Chapter 4:** Heat transfer by convection**Chapter 5:** Heat Transfer by Radiation**Assessment method:** Continuous assessment: 40%; Exam: 60%

Contents

General introduction.....	9
Heat Transfer: General	11
1.1. Introduction.....	11
1.2. The importance of the heat transfer study	12
1.3. Fundamental concepts	13
1.3.1. Concept of heat	13
1.3.2. Temperature fields	14
1.3.3. Isothermal surface	14
1.3.4. Temperature gradient.....	14
1.3.5. Heat flux and density	15
1.4. The different modes of heat transfer	16
1.4.1. Heat transmission by conduction.....	16
1.4.2. Heat transmission by convection	17
1.4.3. Heat transmission by radiation	18
1.5. Formulation of a heat transfer problem:	20
1.5.1. Energy balance:	20
Example 1.1	21
1.1. Solution	22
Example 1.2.....	22
Solution 1.2	23
Steady-state conduction heat transfer	25
2.1.Introduction.....	25
2.2.Fourier's Law	25
2.3. Thermal conductivity.....	26
2.4. Heat equation	27
2.4.1 In Cartesian coordinates	27
2.4.2. In cylindrical Cartesian coordinates:.....	30
2.4.3. In spherical coordinates:.....	31
2.5. Unidirectional conductive transfer:	31
2.5.1. Case of a simple wall	31
2.5.2. Case of a multi-layer wall.....	34

2.5.3. Case of a composite wall:.....	35
2.5.4. Case of a long hollow cylinder (tube)	36
2.5.5. Case of a multilayer hollow cylinder	38
2.5.7 Fin theory	39
2.6. Multi-directional conductive transfer	40
2.6.1. Form coefficient method.....	40
2.6.2. Numerical methods	41
2.1. Example	41
2.1. Solution	42
Heat transfer by conduction in variable regime.....	44
3.1. Introduction.....	44
3.1 Unidirectional conduction in variable regime without change of state:	44
3.1.1 Uniform temperature environment (zero gradient method):	46
3.1. Example	47
2.1 Solution	47
Heat transfer by convection	50
4.1. Introduction.....	50
4.1. Natural convection-forced convection.....	50
4.2. Newton's law	51
4.2 Convective exchange coefficient h	52
4.3. Natural convection	52
4.3.1 Principle of natural convection	52
4.4. Dynamic study	53
4.5. Study of the convection phenomenon.....	54
4.5.1 Boundary layers	54
4.5.2 Nature of the convective exchange coefficient h.....	57
4.5.3 Determination of the coefficient h by the dimensional analysis method.....	57
4.5.4 Dimensionless numbers	58
4.5.5 Thermal resistance	60
4.6. Forced convection:	61
4.6.1. Heat exchange along a flat plate (External forced convection $Re_c = 5 \times 10^5$):.....	61
4.6.2. Flow inside smooth cylindrical tubes (Internal forced convection $Re_c = 2300$):	61
4.6.3. Flow in annular spaces (Internal forced convection).....	61
4.7. Natural convection:.....	62

4.1. Example	64
4.1. Solution	65
Heat transfer by Radiation	68
5.1. Introduction.....	68
5.2. Principle of heat transfer by radiation	69
5.3. Preliminary definitions	69
5.4. Process of reception of radiation by a body.....	71
5.5. Laws of radiation:	73
5.5.1. Planck's law	73
5.5.2. Laws of Wien	74
5.5.3 Stefan-Boltzmann law	75
5.6. Radiation of real bodies.....	75
5.7. Gray body radiation	76
5.1. Example	77
5.1. Solution	78

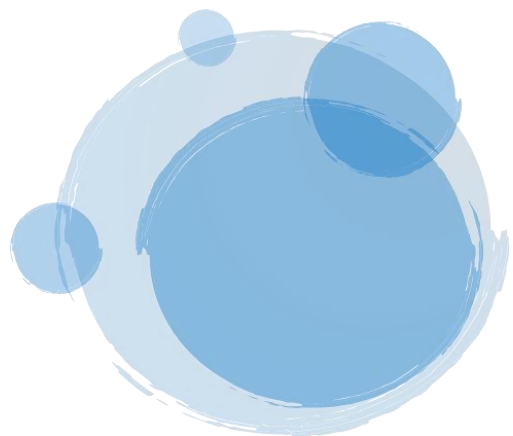
Figures List

Figure 1. 1. Orthogonality of the gradient and the isotherm	15
Figure 1. 2. Heating a metal bar at one of its ends	17
Figure 1. 3. Heat transfer by convection.....	18
Figure 1. 4. Heat transfer by radiation.....	19
Figure 1. 5. Illustration of the three modes of heat transfer	20
Figure 1. 6. Energy balance	20
Figure 2. 1. solid medium.	25
Figure 2. 2. Energy balance for an elementary system	27
Figure 2. 3. Cylindrical coordinate system	30
Figure 2. 4. Single wall.....	32
Figure 2. 5. Equivalent electrical diagram of a simple wall	33
Figure 2. 6. multi-layer wall	34
Figure 2. 7. Equivalent electrical diagram of a multi-layer wall.....	35
Figure 2. 8. Composite wall	36
Figure 2. 9. Equivalent electrical diagram of a composite wall	36
Figure 2. 10. Diagram of transfers in a hollow cylinder.....	37
Figure 2. 11. Equivalent electrical diagram of a hollow cylinder	38
Figure 2. 12. Diagram of transfers in a multilayer hollow cylinder.....	38
Figure 2. 13. Equivalent electrical diagram of a multilayer hollow cylinder.	39
Figure 2. 14. Examples of finned systems used in different application sectors	39
Figure 3. 1. Evolution of the temperature of a medium at uniform temperature.....	47

Figure 4. 1. Newton law	51
Figure 4. 2. Heat transfer by convection.....	52
Figure 4. 3. Dynamic study	53
Figure 4. 4. Boundary layers.....	54
Figure 4. 5. The speed and temperature diagrams	55
Figure 4. 6. The velocity and temperature profiles.....	55
Figure 4. 7. Flow in the annular space formed by the two pipes	62
Figure 5. 1. The electromagnetic spectrum	68
Figure 5. 2. Solid angle	70
Figure 5. 3. Diagram showing the luminance of surface element ds	71
Figure 5. 4 .Process of reception of radiation by a body.....	72
Figure 5. 5. Monochromatic emittance of the black body	74

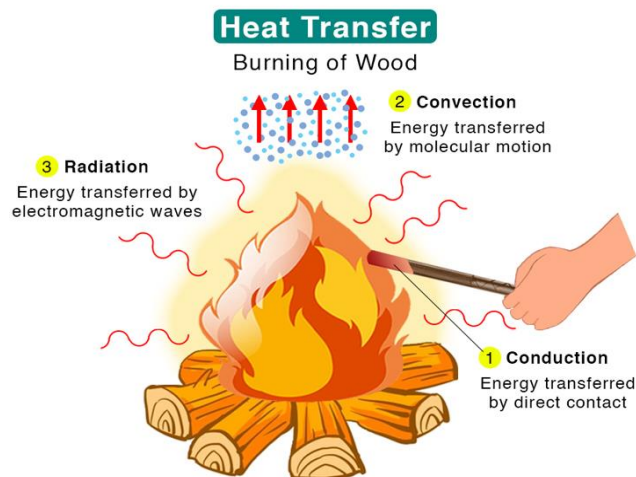
General introduction

Heat transfer is a fundamental science that deals with the rate of energy transfer thermal. In engineering practice, understanding the transfer mechanisms of heat is becoming increasingly important, as heat transfer plays a crucial role in the design of vehicles, power plants, refrigerators, electronic devices, buildings and bridges, among others. Even a chef must have an understanding intuitive heat transfer mechanism to cook food “properly” adjusting the heat transfer rate. We may not know it, but we already use the principles of heat transfer to seek thermal comfort. We isolate our body by putting on heavy coats in winter, and we minimize heat gain by radiation by staying in shaded places in summer. We accelerate the cooling hot food by blowing on it and keeps us warm in cold weather by huddling and thus minimizing the exposed surface area.



Chapter 1

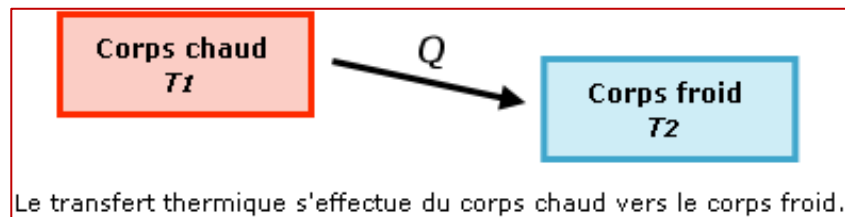
Heat transfer: General



Heat Transfer: General

1.1. Introduction

Heat transfer, which should be called thermal transfer, is defined as thermal energy in transit because of a temperature difference. Two bodies having the same temperature are said to be in thermal equilibrium. If their temperatures are different, the warmer body transfers thermal energy to the colder body, we say that there is heat transfer.



Heat transmission occurs through three basic mechanisms:

a) Conduction:

Thermal conduction, also called thermal diffusion, is a transfer of energy in a material medium without macroscopic movement, involving collisions of molecules (in fluids) or transfers of vibrations (in solids).

b) Convection:

Convection is a mode of heat transfer that involves the macroscopic movement of matter. This phenomenon occurs within flowing fluid media or between a solid wall and a moving fluid. There are two types of convection: natural and forced.

c) Radiation:

Every material body emits and absorbs energy in the form of electromagnetic radiation. The transfer of heat by radiation between two bodies separated by a

vacuum or a semi-transparent medium occurs via electromagnetic waves, therefore without material support.

1.2. The importance of the heat transfer study

Heat transfer is energy in transit due to a difference in temperatures within a body or between different bodies. Whenever there is a difference in temperatures, energy is transferred from the higher temperature region to the lower temperature. According to the concepts of thermodynamics, the energy transferred as a result of a temperature difference is heat. The laws of thermodynamics concern the transfer of energy, but they only apply to systems in equilibrium (they can be used to predict the amount of energy needed to change a system from one equilibrium state to another), but they are not used to predict how quickly (time) these changes may take place. Heat transfer complements thermodynamic principles, providing analytical methods that predict this rate of heat transfer.

Example:

Heating a steel bar immersed in hot water: thermodynamic principles can be used to predict the final temperatures once the two systems have reached equilibrium and the amount of energy transferred between states of initial and final equilibrium, but this tells us nothing about the rate of heat transfer or the temperature of the bar after a certain time, or the time it takes to obtain a certain temperature at a certain position of the bar. Performing a heat transfer analysis allows you to predict the rate of heat transfer from the water to the bar and from this information you can calculate the bar temperature as well as the water temperature as a function of time.

To carry out a complete analysis of heat transfer, it is necessary to consider three different mechanisms: conduction, convection and radiation.

The design of heat exchange systems and energy conversion requires some familiarity with each of these mechanisms, as well as their interactions.

1.3. Fundamental concepts

1.3.1. Concept of heat

Heat is considered a special form of energy. According to the first law of thermodynamics, heat is equivalent to work; hence, they have the same unit: the Joule. It spreads naturally from the hottest environment to that with the lowest temperature, while keeping the volume constant. This constitutes the second law of thermodynamics.

a) Sensible heat:

When a body receives or releases heat, heats up or cools down without changing state, we then speak of sensible heat. The quantity of heat received by a body of mass m with heat capacity c_p when it passes from an initial state at temperature T_1 to a final state at temperature T_2 is expressed by:

(1.1)

$$Q_{1 \rightarrow 2} = m \cdot c_p \cdot (T_2 - T_1)$$

b) Latent heat:

We speak of latent heat when the body that receives or transfers heat uses it to change state, without its temperature changing. The quantity of heat that must be supplied to a material of mass m so that, at constant temperature, it changes state (solid $\rightarrow$ liquid; liquid $\rightarrow$ gas) is given by:

$$Q = m \cdot L \quad (1.2)$$

Where L , is the Latent heat coefficient.

We then speak of "isothermal transformation" because the temperature of the system remains constant throughout the heat exchange process.

1.3.2. Temperature fields

We call temperature the physical quantity which measures the degree of heat of a medium, namely:

solids: it is the state of vibration of atoms inside a crystal lattice or movement of electrons for materials which have the ability to exchange electrons (metals for example);

fluids: this is the state of agitation of molecules. Temperature is expressed in degrees Kelvin (K), degrees Celsius ($^{\circ}\text{C}$) or degrees Rankine (R).

$$\begin{aligned} T(^{\circ}\text{K}) &= T(^{\circ}\text{C}) + 273.15 \\ T(^{\circ}\text{F}) &= 1.8 * T(^{\circ}\text{C}) + 32 \\ T(^{\circ}\text{R}) &= 1.8 * T(^{\circ}\text{C}) + 491.67 \end{aligned} \tag{1.3}$$

The regime is permanent or stationary if the temperature field is independent of time $T = f(x, y, z, t)$. Otherwise, it is said to be variable or transient.

1.3.3. Isothermal surface

The locus of points having the same temperature at each instant is called isothermal surface. Two isothermal surfaces cannot intersect because we would then have two different temperatures at the same point, which is physically impossible.

In steady state: Isothermal surfaces are invariable.

In variable regime: They are mobile and deformable.

1.3.4. Temperature gradient

If we bring together all the points in space that have the same temperature, we obtain a surface called an isothermal surface. The temperature variation per unit length is maximum along the normal to the isothermal surface. This variation is characterized by the temperature gradient:

$$\overrightarrow{\text{grad}}(T) = \vec{n} \cdot \frac{\partial T}{\partial n} \quad (1.4)$$

$\vec{n}$: Unit vector of the normal. : $\frac{\partial T}{\partial n}$ Derivative of the temperature along the normal.

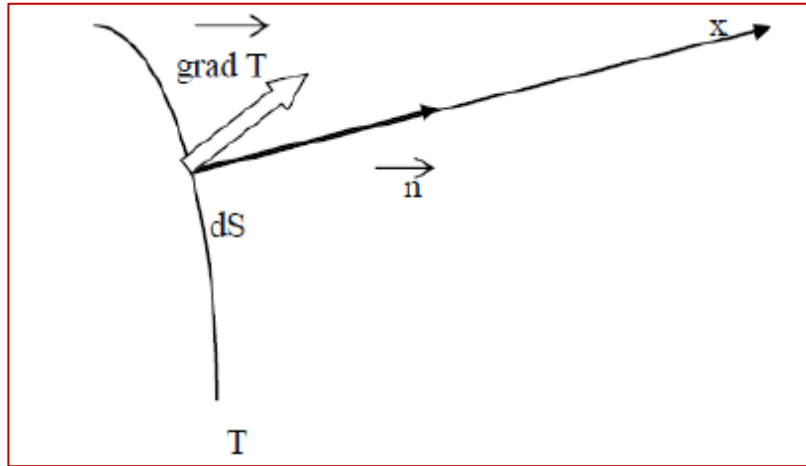


Figure 1. 1. Orthogonality of the gradient and the isotherm

1.3.5. Heat flux and density

The heat flow is the power exchanged per unit of time:

$$\Phi = \frac{dQ}{dt} \quad [W] \quad (1.5)$$

The heat flux density is the quantity of heat transmitted per unit of time and per unit of area of the isothermal surface:

$$\varphi = \frac{dQ}{S \cdot dt} = \frac{\Phi}{S} \quad [W/m^2] \quad (1.6)$$

The heat source is defined by the thermal power it produces. In the case of a chemical reaction, it is expressed:

$$Q' = A_0 e^{\alpha t} \quad (1.7)$$

Where A_0 and α are constants.

1.4. The different modes of heat transfer

1.4.1. Heat transmission by conduction

This transfer of thermal energy, which does not require macroscopic movement of matter, is created by:

- molecular agitation (in gases and liquids)
- vibrations of crystal lattices (in non-conductive solids)
- the movement of free electrons (in conductive metals)

Jean-Baptiste Joseph Fourier (1768-1830) proposed in 1822 the law of conduction now known as Fourier's law. This relationship indicates that the heat flow is proportional to the temperature gradient and is in the direction of decreasing temperatures.

In the case of conduction, the flow is calculated by:

$$\Phi_s = -\lambda \cdot S \cdot \text{grad}T \quad (1.8)$$

For unidirectional conduction:

$$\overrightarrow{\Phi_s} = -\lambda \cdot S \cdot \frac{dT}{dx} \quad (1.9)$$

The (-) sign is a consequence of the second law of thermodynamics, according to which heat must flow towards the lowest temperature zone. The temperature gradient is negative if the temperature decreases for increasing values of x , therefore the heat transferred from the positive direction must be of positive

magnitude, therefore a negative sign must be introduced in the right hand side of the equation former.

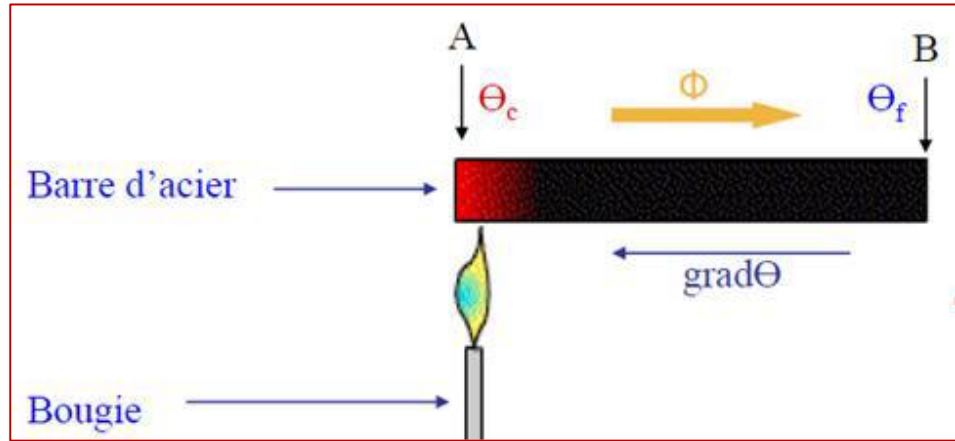


Figure 1. 2. Heating a metal bar at one of its ends

1.4.2. Heat transmission by convection

When a fluid at T_f comes into contact with a solid whose contact surface is at a temperature other than T_s , the process of thermal energy exchange is called convection. There are two types of convection.

a) **Free or natural convection** occurs when the driving force comes from the change in density in the fluid following contact with a surface at a different temperature, which gives rise to upward forces, with the fluid near the surface acquires a speed due only to this difference in density, without any external driving force.

b) **Forced convection**, occurs when an external driving force moves a fluid with a speed (v), over a surface which is found at a temperature T_s higher or lower than that of the fluid T_f , because the speed of the fluid in Forced convection is higher than natural convection, therefore, a greater quantity of heat is transferred for a given temperature.

Newton's law is used to calculate heat transfer by convection:

$$\Phi_S = h.S.(T_p - T_\infty) \quad (1.10)$$

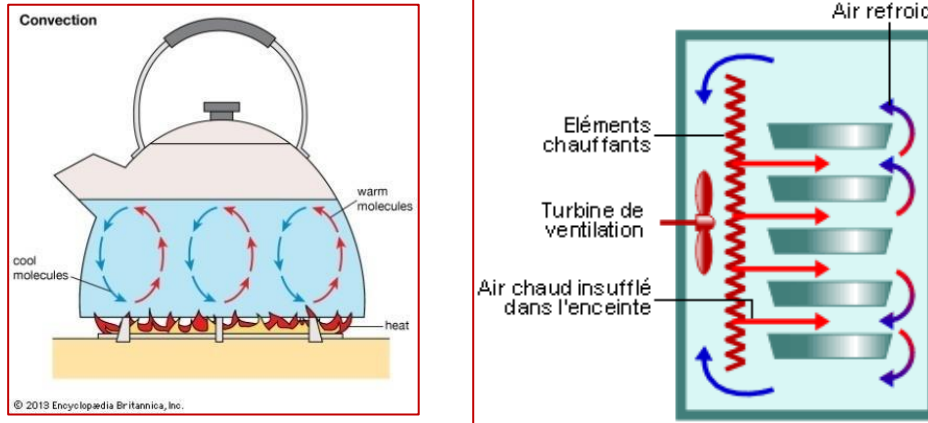


Figure 1. 3. Heat transfer by convection

1.4.3. Heat transmission by radiation

The maximum flux density emitted by a surface is given by the Stephan-Boltzmann law:

$$\Phi_{\max i} = \sigma.T_p^4 \quad (1.11)$$

σ : Stephan's constant = $5,669.10^{-8} \text{ W/m}^2.\text{K}^4$.

The maximum flux is obtained for an ideal surface (black body). However, real surfaces (gray bodies) have a certain emissivity (ϵ) which reduces the flux emitted by the surface:

$$\Phi_{\text{réel}} = \epsilon.\sigma.T_p^4 \quad (1.12)$$

In the case where this surface is surrounded by another surface at a temperature T_∞ , the net heat exchange is then:

$$\Phi_{\text{net}} = \sigma.\epsilon_p.S.(T_p^4 - T_\infty^4) \quad (1.13)$$

Example: heating of a wall by solar radiation during the day, and heat emitted by the wall at night.

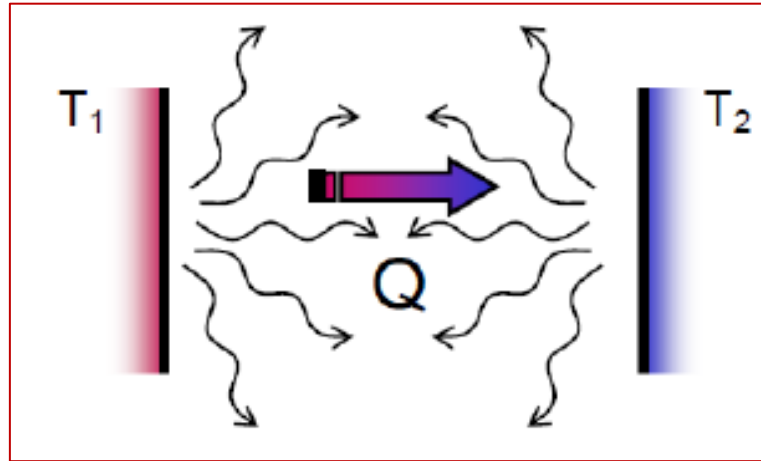


Figure 1. 4. Heat transfer by radiation

Noticed :

- ✓ In many thermal energy transformation problems, the three modes of heat transfer coexist but, generally, at least one of the three forms can be neglected, which will simplify the mathematical treatment of the transfer process;
- ✓ In the following figure, we can visualize the three modes of heat transfer at the same time.

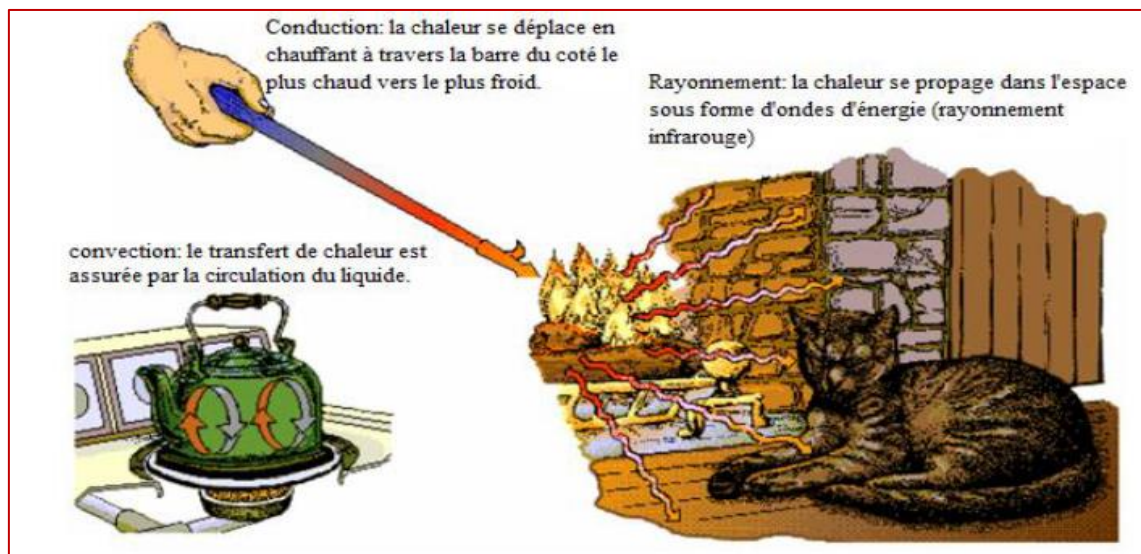


Figure 1. 5. Illustration of the three modes of heat transfer

1.5. Formulation of a heat transfer problem:

1.5.1. Energy balance:

Given a given control volume, at each instant, the energy conservation balance (mechanical energy + internal energy) on this volume is written:

$$(\text{What goes in}) - (\text{What goes out}) + (\text{What is generated}) = (\text{What stored})$$

$$E_e - E_s + E_g = E_{st} \quad (1.14)$$

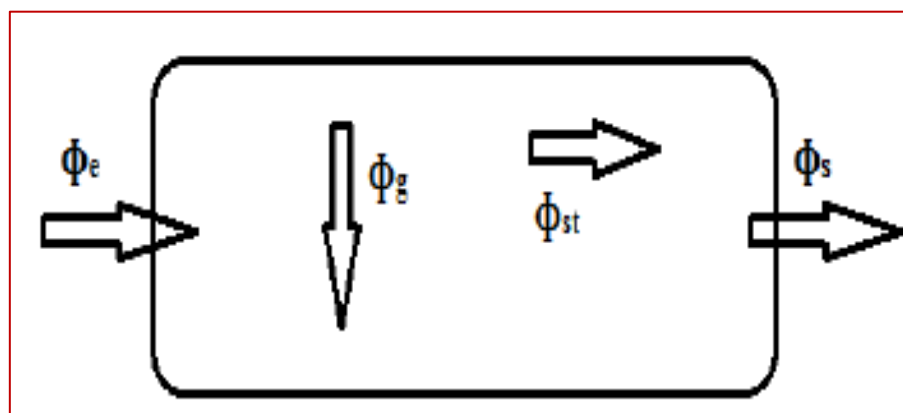


Figure 1. 6. Energy balance

The term "generated energy" should be considered in the broad sense: energy produced (in +) or consumed (in -). It is the same for the accumulation term (variation over time of the energy in the control volume). As part of the course, the energy balance will be limited to a thermal energy balance. The generation term, E_g , may appear in the following cases:

- exo or endothermic chemical reactions,
- nuclear reactions
- viscous dissipation
- Joule effect in an electrical resistance

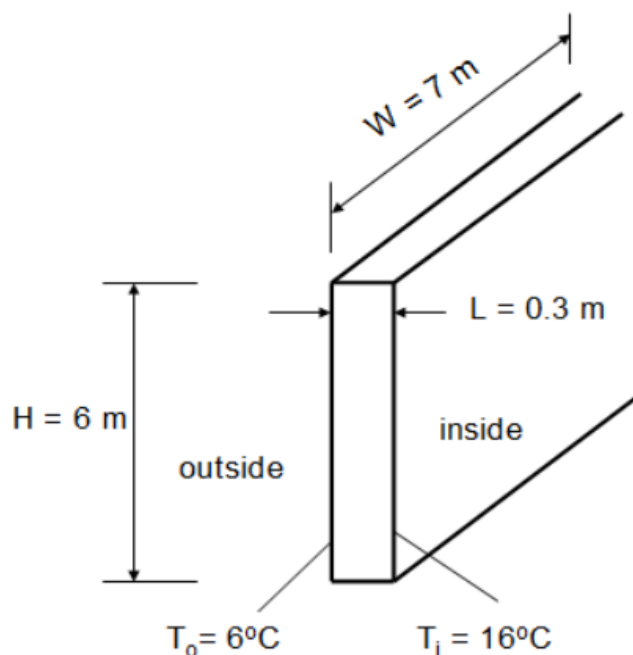
Across a surface, the balance is simply reduced to:

$$E_e - E_s = 0$$

(1.15)

Example 1.1

The wall of a house, 7 m wide and 6 m high is made from 0.3 m thick brick with $k = 0.6 \text{ W / m K}$. The surface temperature on the inside of the wall is 16°C and that on the outside is 6°C . Find the heat flux through the wall and the total heat loss through it.



1.1. Solution

For one-dimensional steady state conduction:

$$q = -k \frac{dT}{dx} = -\frac{k}{L}(T_i - T_o)$$

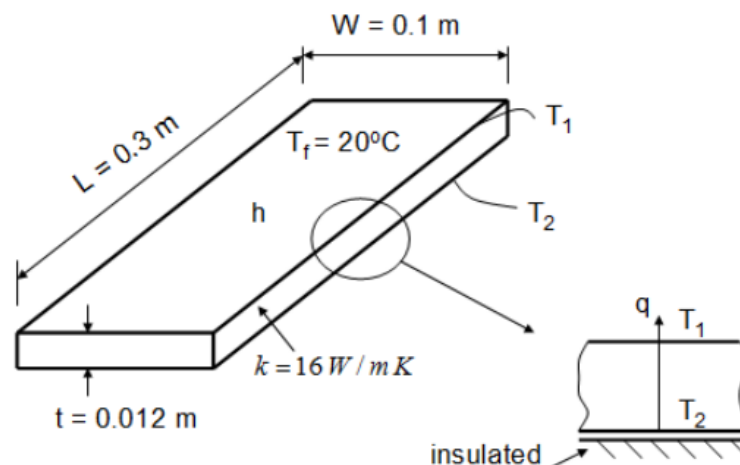
$$q = -\frac{0.6}{0.3}(16 - 6) = -20 \text{ W/m}^2$$

$$Q = qA = -20 \times (6 \times 7) = -840 \text{ W}$$

The minus sign indicates heat flux from inside to outside.

Example 1.2

A plate 0.3 m long and 0.1 m wide, with a thickness of 12 mm is made from stainless steel ($k = 16 \text{ W/mK}$), the top surface is exposed to an airstream of temperature 20°C . In an experiment, the plate is heated by an electrical heater (also 0.3 m by 0.1 m) positioned on the underside of the plate and the temperature of the plate adjacent to the heater is maintained at 100°C . A voltmeter and ammeter are connected to the heater and these read 200 V and 0.25 A, respectively. Assuming that the plate is perfectly insulated on all sides except the top surface, what is the convective heat transfer coefficient?



Solution 1.2

Heat flux equals power supplied to electric heater divided by the exposed surface area:

$$q = \frac{V \times I}{A} = \frac{V \times I}{W \times L} = \frac{200 \times 0.25}{0.1 \times 0.3} = 1666.7 \text{ W/m}^2$$

This will equal the conducted heat through the plate:

$$q = \frac{k}{t}(T_2 - T_1)$$

$$T_1 = T_2 - \frac{qt}{k} = 100 - \frac{(1666.7 \times 0.012)}{16} = 98.75^\circ\text{C} \quad (371.75 \text{ K})$$

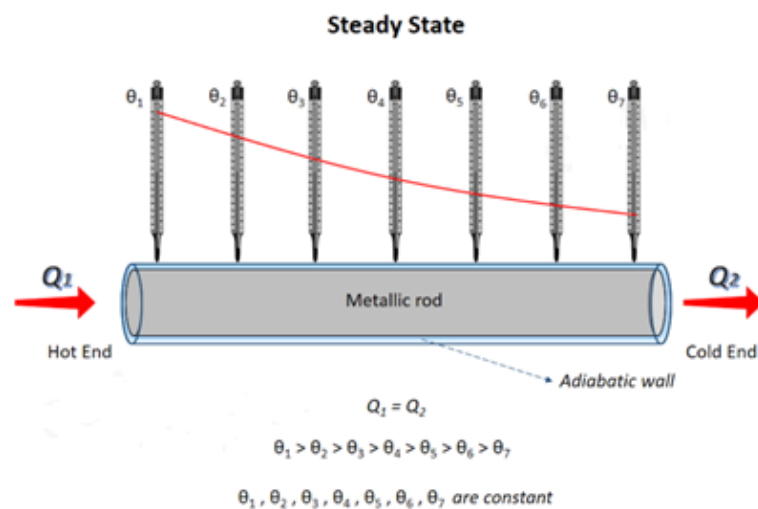
The conducted heat will be transferred by convection and radiation at the surface:

$$q = h(T_1 - T_f) + \sigma(T_1^4 - T_f^4)$$

$$h = \frac{q - \sigma(T_1^4 - T_f^4)}{(T_1 - T_f)} = \frac{1666.7 - 5.67 \times 10^{-8}(371.75^4 - 293^4)}{371.75 - 293} = 12.7 \text{ W/m}^2\text{K}$$

Chapter 2

Steady-state conduction heat transfer



Steady-state conduction heat transfer

2.1. Introduction

It is the transfer of heat within an opaque medium, without movement of material, under the influence of a temperature difference. The propagation of heat by conduction inside a body occurs according to two distinct mechanisms: transmission by the vibrations of atoms or molecules and transmission by free electrons.

2.2. Fourier's Law

A solid medium D and an elementary surface dS oriented by its unit normal $\vec{n}$ are considered in this case.

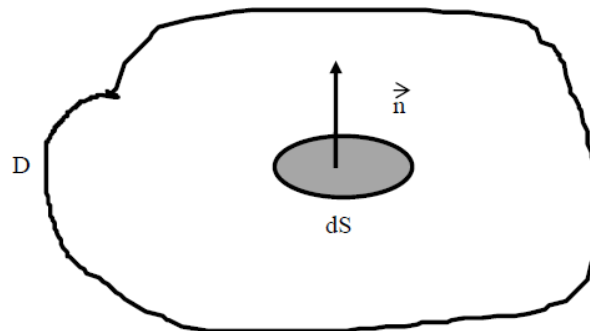


Figure 2. 1. solid medium.

FOURIER's law determines the quantity of heat d^2Q which passes through a surface dS in the direction of the normal $\vec{n}$ and over a time interval dt :

$$d^2Q = -\lambda \cdot \overrightarrow{\text{grad}T} \cdot \vec{dS} \cdot \vec{n} \cdot dt \quad (2.1)$$

With ;

$\overrightarrow{\text{grad}T}$: Temperature gradient calculated by:

$$\overrightarrow{\text{grad}T} \begin{cases} \frac{\partial T}{\partial x} \\ \frac{\partial T}{\partial y} \\ \frac{\partial T}{\partial z} \end{cases}$$

λ : Thermal conductivity of the medium ($\text{W.m}^{-1}.\text{K}^{-1}$) We also have:

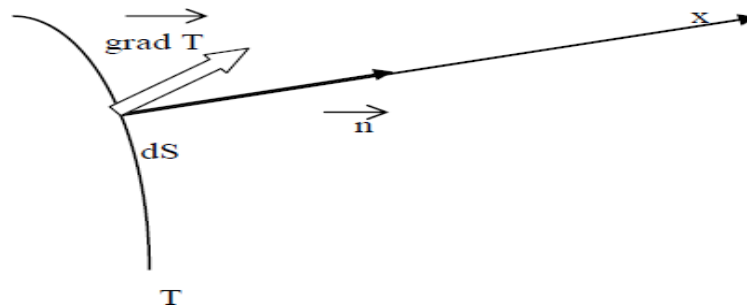
-The heat flow is:

$$d\Phi = \frac{d^2Q}{dt} = -\lambda \cdot \overrightarrow{\text{grad}T} \cdot dS \cdot \vec{n} \quad (2.2)$$

-The heat flux density is:

$$d\phi = \frac{d^2Q}{dt \cdot dS} = -\lambda \cdot \overrightarrow{\text{grad}T} \cdot \vec{n} \quad (2.3)$$

The (-) sign indicates that in terms of heat flow, the gradient from largest to smallest is negative. If the surface dS is on an isothermal surface, the vectors $\overrightarrow{\text{grad}T}$ and $\vec{n}$ are collinear, as follows:



$$d^2Q = -\lambda \cdot \frac{dT}{dx} \cdot dS \cdot dt \quad (2.5)$$

$$\text{Ou } d\Phi = -\lambda \cdot \frac{dT}{dx} \cdot dS \quad \text{et} \quad d\phi = -\lambda \cdot \frac{dT}{dx}$$

2.3. Thermal conductivity

The following table represents the usual thermal conductivity values.

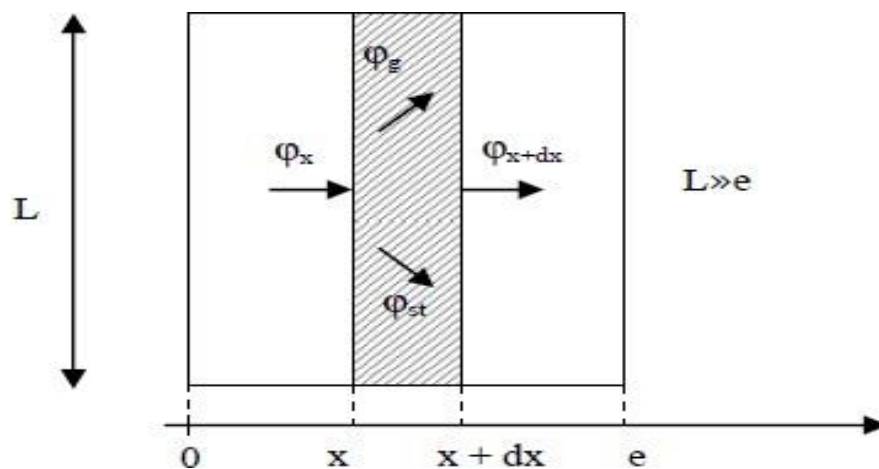
Table 2.1: thermal conductivity.

Substances	λ en W/ <u>m°C</u>
Gaz à la pression atmosphérique	0,006 - 0,15
Matériaux solides isolants (Laine de verre, polystyrène, liège, amiante...)	0,025 - 0,18
Liquides non métalliques	0,075 - 0,60
Matériaux non métalliques (brique, pierre à bâtir, bois, béton...)	0,10 - 2,2
Métaux liquides	7,5 - 67
Alliages métalliques	12 - 100
Métaux purs	45 - 365

2.4. Heat equation

2.4.1 In Cartesian coordinates

In the one-dimensional case, this equation expresses the unidirectional heat transfer through a plane wall:

**Figure 2. 2.** Energy balance for an elementary system

The energy balance on this system is written:

$$\Phi_x + \Phi_g = \Phi_{x+dx} + \Phi_{st} \quad (2.6)$$

Avec :

$$\Phi_x = -\left(\lambda S \frac{\partial T}{\partial x}\right)_x \quad \text{et} \quad \Phi_{x+dx} = -\left(\lambda S \frac{\partial T}{\partial x}\right)_{x+dx}$$

$$\Phi_g = \dot{q} \cdot dV = \dot{q} \cdot S \cdot dx$$

$$\Phi_{st} = \frac{\partial Q}{\partial t} = m \cdot c \cdot \frac{\partial T}{\partial t} = \rho \cdot V \cdot c \cdot \frac{\partial T}{\partial t} = \rho \cdot c \cdot S \cdot dx \cdot \frac{\partial T}{\partial t}$$

By reporting in the energy balance and dividing by dx, we obtain:

$$\frac{\left(\lambda S \frac{\partial T}{\partial x}\right)_{x+dx} - \left(\lambda S \frac{\partial T}{\partial x}\right)_x}{dx} + \dot{q} \cdot S = \rho \cdot c \cdot S \cdot \frac{\partial T}{\partial t} \quad (2.7)$$

From Taylor's development:

Soit ;

$$\left(\lambda S \frac{\partial T}{\partial x}\right)_{x+dx} = \left(\lambda S \frac{\partial T}{\partial x}\right)_x + \frac{\partial}{\partial x} \left(\lambda S \frac{\partial T}{\partial x}\right) dx$$

$$\frac{\partial}{\partial x} \left(\lambda S \frac{\partial T}{\partial x}\right) + \dot{q} \cdot S = \rho \cdot c \cdot S \cdot \frac{\partial T}{\partial t} \quad (2.8)$$

In the three-dimensional (3D) case, we obtain the following thermal equation:

$$\frac{\partial}{\partial x} \left(\lambda_x \frac{\partial T}{\partial x}\right) + \frac{\partial}{\partial y} \left(\lambda_y \frac{\partial T}{\partial y}\right) + \frac{\partial}{\partial z} \left(\lambda_z \frac{\partial T}{\partial z}\right) + \dot{q} = \rho \cdot c \cdot \frac{\partial T}{\partial t} \quad (2.9)$$

○ **Study of particular cases:**

a) If the medium is isotropic and homogeneous (The conductivity only depends on the temperature of the point considered):

The heat equation (2.9) can be put in the form:

$$\lambda \left(\frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial y^2} + \frac{\partial^2 T}{\partial z^2} \right) + \frac{\partial \lambda}{\partial T} \left[\left(\frac{\partial T}{\partial x} \right)^2 + \left(\frac{\partial T}{\partial y} \right)^2 + \left(\frac{\partial T}{\partial z} \right)^2 \right] + \dot{q} = \rho \cdot c \cdot \frac{\partial T}{\partial t} \quad (2.10)$$

b) The temperature or presents a negligible difference:

This is particularly important in the case of a homogeneous and isotropic material with a constant λ coefficient. Consequently, the previous sentence becomes:

$$\lambda \left(\frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial y^2} + \frac{\partial^2 T}{\partial z^2} \right) + \dot{q} = \rho.c. \frac{\partial T}{\partial t} \quad (2.11)$$

c) λ does not depend on the temperature and the internal heat release is negligible:

We have :

$$\lambda \left(\frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial y^2} + \frac{\partial^2 T}{\partial z^2} \right) = \rho.c. \frac{\partial T}{\partial t} \quad (2.12)$$

Expression that we usually put in the form (called Poisson equation):

$$a \nabla^2 T = \frac{\partial T}{\partial t} \quad (2.13)$$

The ratio $a = \lambda / \rho c$ is called thermal diffusivity ($\text{m}^2.\text{s}^{-1}$) which characterizes the speed of propagation of a heat flow through a material.

d) Constant temperature (does not depend on time):

It is the analysis of a steady state with or without heat release. If we assume that the conductivity λ is a constant and does not depend on temperature, we obtain:

✓ if the internal source of heat: $q \neq 0$

$$\lambda \left(\frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial y^2} + \frac{\partial^2 T}{\partial z^2} \right) + \dot{q} = 0 \quad (2.14)$$

✓ without internal heat release: $q = 0$, (we obtain the **Laplace** equation):

$$\nabla^2 T = 0$$

2.4.2. In cylindrical Cartesian coordinates:

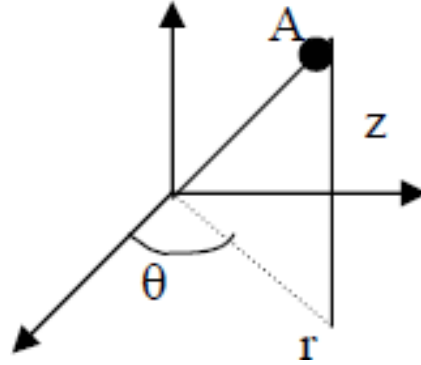


Figure 2. 3. Cylindrical coordinate system

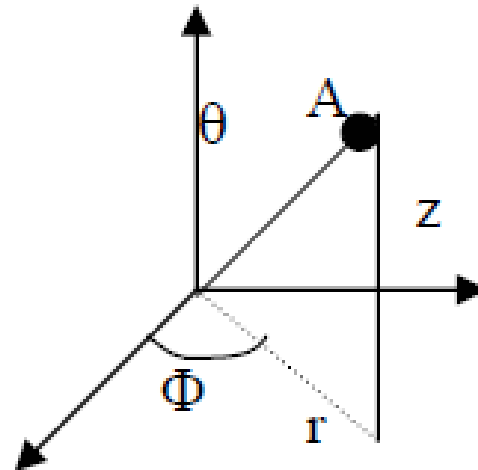
Either:

$$\begin{cases} x = r \cdot \cos \phi \\ y = r \cdot \sin \phi \\ z = z \end{cases}$$

$$\frac{\partial^2 T}{\partial r^2} + \frac{1}{r} \frac{\partial T}{\partial r} + \frac{1}{r^2} \frac{\partial^2 T}{\partial \theta^2} + \frac{\partial^2 T}{\partial z^2} + \frac{\dot{q}}{\lambda} = \frac{1}{a} \frac{\partial T}{\partial t} \quad (2.16)$$

If the case of a cylindrical symmetry problem where the temperature is only determined by r and t , equation (2.16) can be written as follows:

$$\frac{1}{r} \frac{\partial}{\partial r} \left(r \frac{\partial T}{\partial r} \right) + \frac{\dot{q}}{\lambda} = \frac{1}{a} \frac{\partial T}{\partial t} \quad (2.17)$$



2.4.3. In spherical coordinates:

Soient :

$$\begin{cases} x = r \cdot \cos \phi \cdot \sin \theta \\ y = r \cdot \sin \phi \cdot \sin \theta \\ z = r \cdot \cos \theta \end{cases}$$

$$\frac{1}{r} \left(\frac{\partial^2 (rT)}{\partial r^2} \right) + \frac{1}{r^2 \sin \theta} \frac{\partial}{\partial \theta} \left(\sin \theta \frac{\partial T}{\partial \theta} \right) + \frac{1}{r^2 \sin^2 \theta} \left(\frac{\partial^2 T}{\partial \phi^2} \right) + \frac{\dot{q}}{\lambda} = \frac{1}{a} \frac{\partial T}{\partial t} \quad (2.18)$$

2.5. Unidirectional conductive transfer:

2.5.1. Case of a simple wall

We assume that heat transfer is unidirectional and we neglect energy generation and storage.

Considering a homogeneous wall of thickness e , section S , thermal conductivity λ , whose faces are at temperatures T_1 and T_2 , the heat flux which passes through this wall is:

$$\Phi = -\lambda \cdot S \cdot \frac{dT}{dx} \quad (2.19)$$

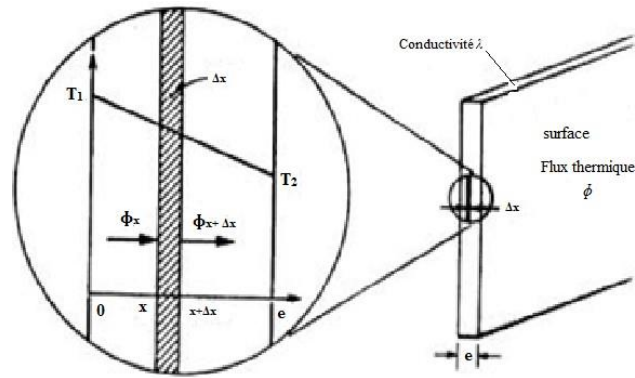


Figure 2. 4. Single wall

If we carry out an energy balance on the system (S) which contains a section of wall between the abscissa x and $x + dx$, it comes:

$$\Phi_x = \Phi_{x+dx} \Rightarrow -\lambda S \left(\frac{\partial T}{\partial x} \right)_x = -\lambda S \left(\frac{\partial T}{\partial x} \right)_{x+dx} \quad (2.20)$$

From where ;

$$\frac{\partial}{\partial x} \left(\frac{\partial T}{\partial x} \right) = 0$$

With the boundary conditions: $T(x = 0) = T_1$ and $T(x = e) = T_2$

D'où
$$T = T_1 - \frac{x}{e}(T_1 - T_2) \quad (2.21)$$

We find a linear profile for the temperature. The heat flow passing through the wall is calculated by:

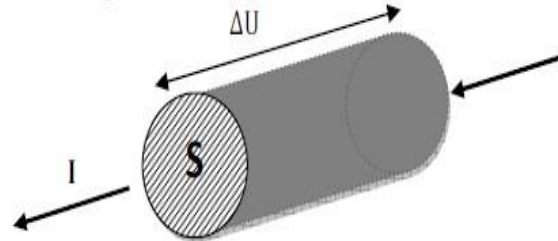
$$\Phi = -\lambda \cdot S \cdot \frac{dT}{dx}$$

$$\Phi = -\lambda \cdot S \cdot \frac{T_2 - T_1}{e} = \lambda \cdot S \cdot \frac{T_1 - T_2}{e} \quad (2.22)$$

- Concept of thermal resistance (electrical analogy):

An electric wire of length L , section S and thermal conductivity K_e subjected to a potential difference ΔU , lets an electric current pass such that:

$$\Delta U = U_0 - U_1 = \frac{L}{K_e \cdot S} \cdot I \text{ ou encore ; } \Delta U = R_e \cdot I$$



This equation is analogous to that giving the thermal potential:

$$\Delta T = T_1 - T_2 = (e / \lambda \cdot S) \cdot \Phi = R_{th} \cdot \Phi \quad (2.23)$$

We establish the following correspondence:

Table 2.2: Electrical-thermal analogy.

Thermique			électricité	
Loi de Fourier	$\Delta T = -(e / \lambda \cdot S) \cdot \Phi$	$\Leftrightarrow$	$\Delta U = R \cdot I$	Loi d'Ohm
Conductivité thermique	$\lambda(T)$	$\Leftrightarrow$	$\sigma(T)$	conductivité électrique
Température	T	$\Leftrightarrow$	V	potentiel électrique
Puissance thermique	Φ	$\Leftrightarrow$	I	intensité de courant
Résistance thermique	$R_{th} = e / \lambda \cdot S$	$\Leftrightarrow$	R	Résistance électrique

For the thermal problem, we establish its electrical equivalent:

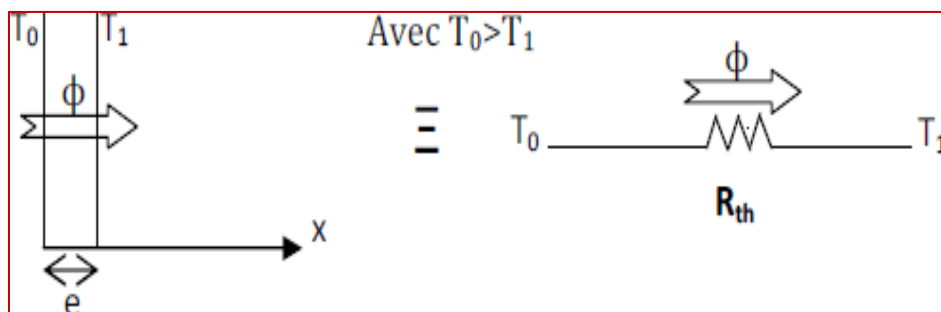


Figure 2. 5. Equivalent electrical diagram of a simple wall

In the case of a flat wall of thickness e , the thermal resistance is:

$$R_{th} = \frac{e}{\lambda S} \quad \text{en } ^\circ\text{C/W.} \quad (2.24)$$

The total thermal resistance is determined as in electricity, we know that:

- 1) Two thermal resistances are added in the case where they are in series:

$$R_{eq} = R_1 + R_2$$

- 2) The inverse of the thermal resistances are added in the case where they are in parallel:

$$\frac{1}{R_{eq}} = \frac{1}{R_1} + \frac{1}{R_2} \rightarrow R_{eq} = \frac{1}{\frac{1}{R_1} + \frac{1}{R_2}}$$

2.5.2. Case of a multi-layer wall

In steady state, the heat flow is conserved when crossing the wall and is written:

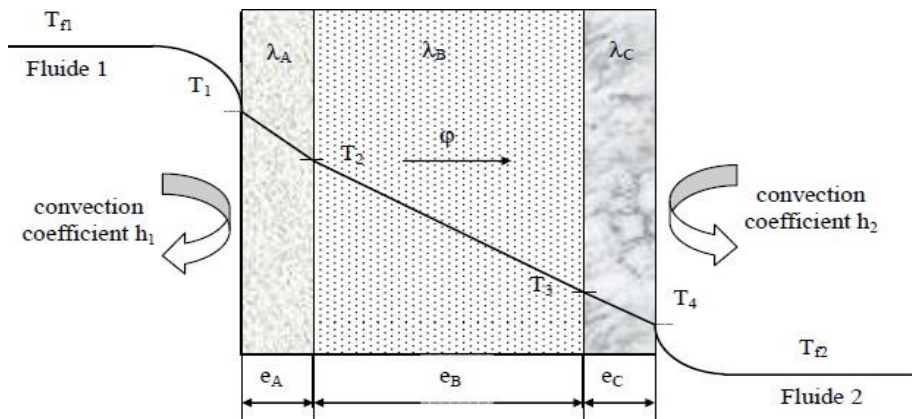


Figure 2. 6. multi-layer wall

$$\begin{aligned} \Phi &= h_1 S (T_{f1} - T_1) = \frac{\lambda_A S (T_1 - T_2)}{e_A} = \frac{\lambda_B S (T_2 - T_3)}{e_B} = \frac{\lambda_C S (T_3 - T_4)}{e_C} = h_2 S (T_4 - T_{f2}) \\ \Phi &= \frac{T_{f1} - T_{f2}}{\frac{1}{h_1 S} + \frac{e_A}{\lambda_A S} + \frac{e_B}{\lambda_B S} + \frac{e_C}{\lambda_C S} + \frac{1}{h_2 S}} \end{aligned} \quad (2.25)$$

The thermal resistance of a multi-layer wall is:

$$R_{th} = \frac{T_{f1} - T_{f2}}{\Phi} = \frac{1}{h_1 S} + \frac{e_A}{\lambda_A S} + \frac{e_B}{\lambda_B S} + \frac{e_C}{\lambda_C S} + \frac{1}{h_2 S} \quad (2.26)$$

The interactions between the different types of layers were considered perfect and there was no temperature discontinuity at the interfaces. Indeed, due to the roughness of the surfaces, a microlayer of air exists between the hollows of the surfaces in view, which contributes to the formation of thermal resistance (air is an insulator) called thermal contact resistance. The previous equation then becomes:

$$\Phi = \frac{T_{f1} - T_{f2}}{\frac{1}{h_1 S} + \frac{e_A}{\lambda_A S} + R_{AB} + \frac{e_B}{\lambda_B S} + R_{BC} + \frac{e_C}{\lambda_C S} + \frac{1}{h_2 S}} \quad (2.27)$$

The equivalent electrical diagram is shown in Figure 2.7.

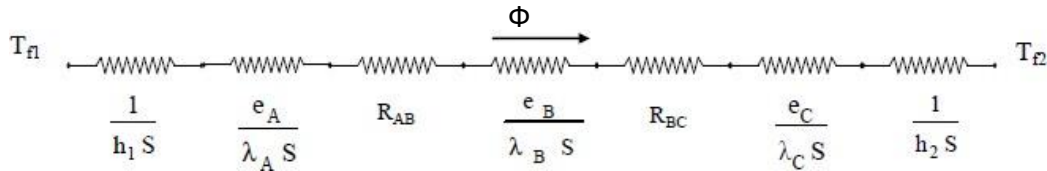


Figure 2. 7. Equivalent electrical diagram of a multi-layer wall.

2.5.3. Case of a composite wall:

The equivalent thermal resistance R of a portion of wall of width L and height using the laws of association of resistances in series and in parallel by the relationship (figure 2.8):

$$R_{th} = R_1 + R_2 + \frac{1}{\frac{1}{R_3} + \frac{1}{R_4} + \frac{1}{R_5}} + R_6 + R_7 \quad (2.28)$$

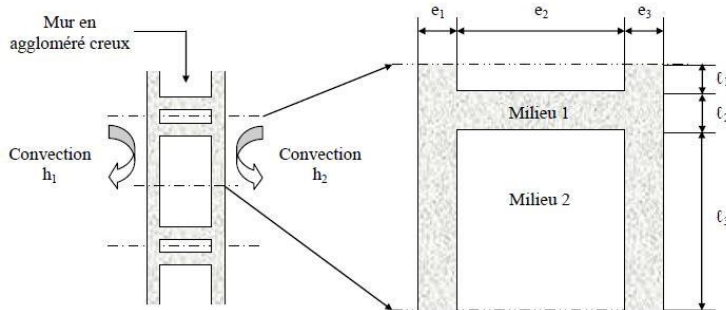


Figure 2. 8. Composite wall

With:

$$R_1 = \frac{1}{h_1 L} ; R_2 = \frac{e_1}{\lambda_1 L} ; R_3 = \frac{e_2}{\lambda_2 l_1 L} ; R_4 = \frac{e_2}{\lambda_1 l_2 L} ; R_5 = \frac{e_2}{\lambda_2 l_3 L} ; R_6 = \frac{e_3}{\lambda_1 L} ; R_7 = \frac{1}{h_2 L}$$

The equivalent electrical diagram is given in Figure 2.9.

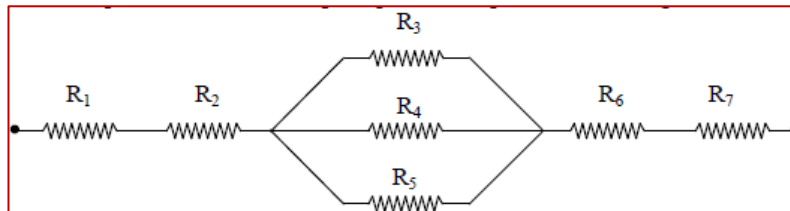


Figure 2. 9. Equivalent electrical diagram of a composite wall

2.5.4. Case of a long hollow cylinder (tube)

We assume a hollow cylinder with thermal conductivity λ interior and exterior of the cylinder, T_1 and T_2 are the temperatures of the internal and external walls (figure 2.10). The temperature gradient is considered to vary only radially.

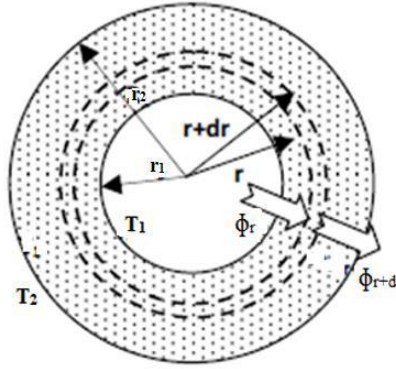


Figure 2. 10. Diagram of transfers in a hollow cylinder

Let us carry out the thermal balance of the system constituted by the part of the cylinder between the radii r and $r + dr$:

$$\Phi_r = \Phi_{r+dr} \quad (2.29)$$

$$\text{Avec :} \quad \Phi_r = -\lambda 2\pi r L \left(\frac{dT}{dr} \right)_r \quad \text{et} \quad \Phi_{r+dr} = -\lambda 2\pi (r + dr) L \left(\frac{dT}{dr} \right)_{r+dr}$$

$$\text{Soit :} \quad -\lambda 2\pi r L \left(\frac{dT}{dr} \right)_r = -\lambda 2\pi (r + dr) L \left(\frac{dT}{dr} \right)_{r+dr} \quad \text{d'où} \quad r \frac{dT}{dr} = C$$

$$\text{Avec les conditions aux limites :} \quad T(r_1) = T_1 \quad \text{et} \quad T(r_2) = T_2$$

$$\text{D'où :} \quad \frac{T(r) - T_1}{T_2 - T_1} = \frac{\ln\left(\frac{r}{r_1}\right)}{\ln\left(\frac{r_2}{r_1}\right)} \quad (2.30)$$

And by application of the relation $\Phi = -\lambda 2\pi r L \left(\frac{dT}{dr} \right)_r$ we obtain: This relation can also be put in the form:

$$\Phi = \frac{2\pi\lambda L (T_1 - T_2)}{\ln\left(\frac{r_2}{r_1}\right)} \quad (2.31)$$

$$\Phi = \frac{(T_1 - T_2)}{R_{12}} \quad \text{avec} \quad R_{12} = \frac{\ln\left(\frac{r_2}{r_1}\right)}{2\pi\lambda L}$$

This relationship can also be put in the form:

And be represented by the equivalent electrical diagram in Figure 2.11.

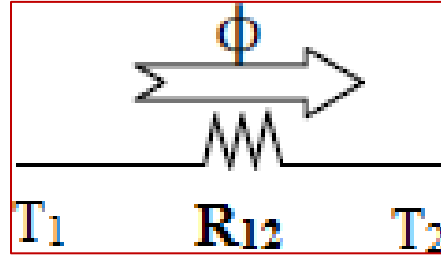


Figure 2. 11. Equivalent electrical diagram of a hollow cylinder

2.5.5. Case of a multilayer hollow cylinder

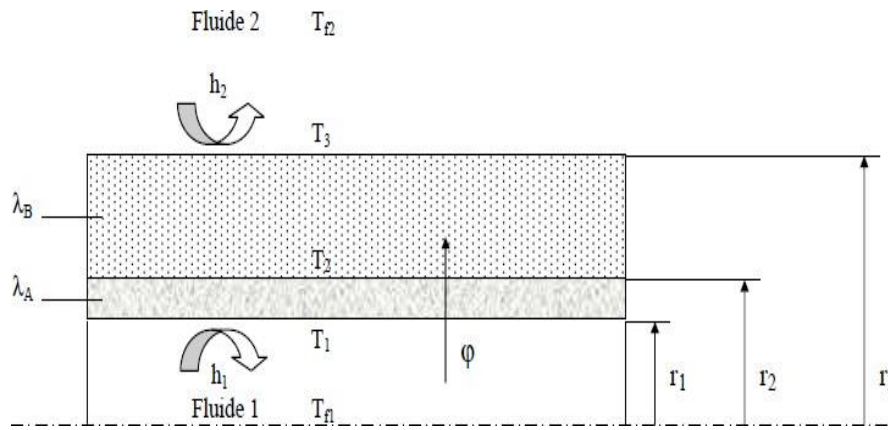


Figure 2. 12. Diagram of transfers in a multilayer hollow cylinder.

In steady state, the heat flow ϕ is preserved when crossing the different layers and is written (figure 2.12):

$$\Phi = h_1 2\pi r_1 L (T_{f1} - T_1) = \frac{2\pi \lambda_A L (T_1 - T_2)}{\ln\left(\frac{r_2}{r_1}\right)} = \frac{2\pi \lambda_B L (T_2 - T_3)}{\ln\left(\frac{r_3}{r_2}\right)} = h_2 2\pi r_3 L (T_3 - T_{f2}) \quad (2.32)$$

$$\text{D'où :} \quad \Phi = \frac{T_{f1} - T_{f2}}{\frac{1}{h_1 2\pi r_1 L} + \frac{\ln\left(\frac{r_2}{r_1}\right)}{2\pi \lambda_A L} + \frac{\ln\left(\frac{r_3}{r_2}\right)}{2\pi \lambda_B L} + \frac{1}{h_2 2\pi r_3 L}} \quad (2.33)$$

Which can be represented by the equivalent electrical diagram in Figure 2.13.

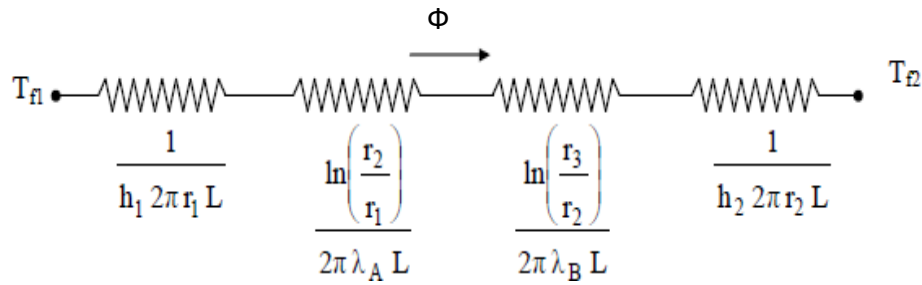


Figure 2. 13. Equivalent electrical diagram of a multilayer hollow cylinder.

- The thermal resistance of a multilayer hollow cylinder is:

$$R_{th} = \frac{T_{f1} - T_{f2}}{\Phi} = \frac{1}{h_1 2\pi r_1 L} + \frac{\ln\left(\frac{r_2}{r_1}\right)}{2\pi\lambda_A L} + \frac{\ln\left(\frac{r_3}{r_2}\right)}{2\pi\lambda_B L} + \frac{1}{h_2 2\pi r_3 L}$$

2.5.7 Fin theory

2.5.7.1 General

The fins are good conductors of heat with a greater surface area than the rest of the body. They are used to improve heat removal from a solid-state containment system with high flux densities.



Finned tube (radiator)



Heat sinks



Motorcycle engine.

Figure 2. 14. Examples of finned systems used in different application sectors

2.5.7.2. The bar equation

Performing the energy balance, the temperature profile in the bar is the solution of the following differential equation, called the bar equation:

$$\frac{d^2T}{dx^2} - \frac{h.P_e}{\lambda.S} \cdot (T - T_\infty) = 0$$

2.5.7.2.1. Flow extracted by a fin

A fin is a medium that is a good conductor of heat, one dimension of which is large compared to the others. By posing:

$$\omega^2 = \frac{h.P_e}{\lambda.S} \quad \text{et} \quad \theta = (T - T_\infty)$$

The previous equation becomes:

$$\frac{d^2\theta}{dx^2} - \omega^2 \cdot \theta = 0$$

2.6. Multi-directional conductive transfer

In the case where the heat diffusion does not take place in a single direction, two resolution methods can be applied:

2.6.1. Form coefficient method

In two-dimensional or three-dimensional systems where only two limiting temperatures T_1 and T_2 intervene, we show that the heat flow can be put in the form:

$$\varphi = \lambda.F(T_1 - T_2)$$

With :

λ : Thermal conductivity of the medium separating the surfaces S1 and S2 (W m⁻¹ K⁻¹)

T_1 : Surface temperature S_1 (K)

T_2 : Surface temperature S_2 (K)

F : Shape coefficient (m)

The shape coefficient F only depends on the shape, dimensions and relative position of the two surfaces S_1 and S_2 .

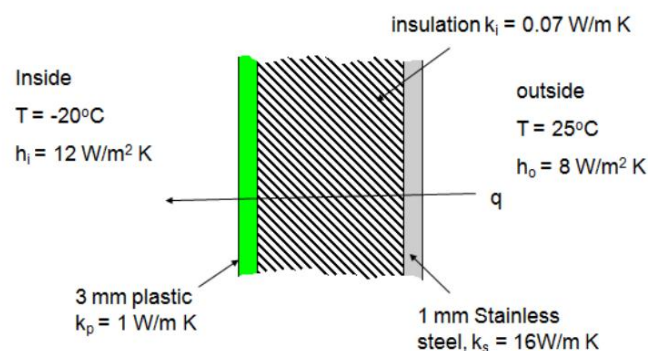
2.6.2. Numerical methods

In the case where the shape coefficient method cannot be applied (non-isothermal surfaces for example), the heat equation must be solved numerically:

- ❖ Finite difference method
- ❖ Finite element method.
- ❖ - Finite volume method...

2.1. Example

An industrial freezer is designed to operate with an internal air temperature of -20°C when the external air temperature is 25°C and the internal and external heat transfer coefficients are $12 \text{ W/m}^2 \text{ K}$ and $8 \text{ W/m}^2 \text{ K}$, respectively. The walls of the freezer are composite construction, comprising of an inner layer of plastic ($k = 1 \text{ W/m K}$, and thickness of 3 mm), and an outer layer of stainless steel ($k = 16 \text{ W/m K}$, and thickness of 1 mm). Sandwiched between these two layers is a layer of insulation material with $k = 0.07 \text{ W/m K}$. Find the width of the insulation that is required to reduce the convective heat loss to 15 W/m^2 .



2.1. Solution

$q = U\Delta T$ where U is the overall heat transfer coefficient given by:

$$U = \frac{q}{\Delta T} = \frac{15}{25 - (-20)} = 0.333 \text{ W / m}^2 \text{ K}$$

$$U = \left[\frac{1}{h_i} + \frac{L_p}{k_p} + \frac{L_i}{k_i} + \frac{L_s}{k_s} + \frac{1}{h_o} \right]^{-1} = 0.333$$

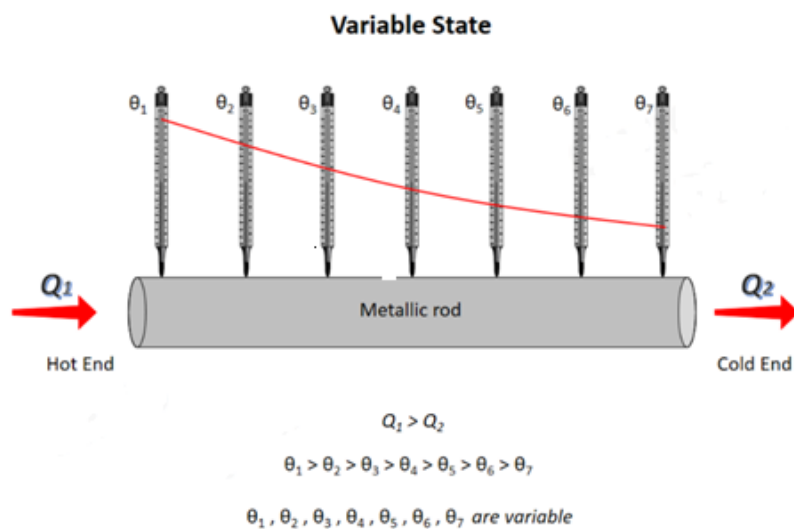
$$\left[\frac{1}{h_i} + \frac{L_p}{k_p} + \frac{L_i}{k_i} + \frac{L_s}{k_s} + \frac{1}{h_o} \right] = \frac{1}{0.333}$$

$$L_i = k_i \left\{ \frac{1}{0.333} - \left[\frac{1}{h_i} + \frac{L_p}{k_p} + \frac{L_s}{k_s} + \frac{1}{h_o} \right] \right\} = 0.07 \left\{ \frac{1}{0.333} - \left[\frac{1}{12} + \frac{0.003}{1} + \frac{0.001}{16} + \frac{1}{8} \right] \right\}$$

$$L_i = 0.195 \text{ m} \quad (195 \text{ mm})$$

Chapter 3

Heat transfer by conduction in variable regime



Heat transfer by conduction in variable regime

3.1. Introduction

In this chapter, we will present thermal conduction in variable regime, that is to say that the temperature varies, not only in space but also with time.

3.1 Unidirectional conduction in variable regime without change of state:

❖ Main problem

The general equation for heat is given by:

$$\text{div}(\lambda \overrightarrow{\text{grad}} T) + P = \rho C \frac{\partial T}{\partial t} \quad (3.1)$$

It requires an initial condition at any point T ($t=0$), and two boundary conditions. In the case where the thermal conductivity λ does not depend on the temperature, we obtain the Fourier equation:

$$\Delta T + \frac{P}{\lambda} = \frac{1}{a} \frac{\partial T}{\partial t} \quad (3.2)$$

- Dimensionless numbers:

The problem of thermal conduction (unidirectional case) is represented by the following system of equations:

$$\left\{ \begin{array}{ll} \frac{\partial^2 T}{\partial x^2} + \frac{P}{\lambda} = \frac{1}{a} \frac{\partial T}{\partial t} & \text{pour } 0 < x < l; \quad t > 0 \\ \frac{\partial T}{\partial x} = 0 & \text{pour } x = 0; \quad t > 0 \\ \lambda \frac{\partial T}{\partial x} = -h(T - T_\infty) & \text{pour } x = l; \quad t > 0 \\ T = T_0 & \text{pour } 0 < x < l; \quad t = 0 \end{array} \right. \quad (3.3)$$

The number of variables in a heat transfer problem can be reduced by the introduction of dimensionless numbers:

$$x^* = \frac{x}{l}; \quad \theta = \frac{T - T_\infty}{T_0 - T_\infty}; \quad G = \frac{Pl^2}{\lambda T_0 - T_\infty}$$

System (3.3) becomes:

$$\left\{ \begin{array}{ll} \frac{\partial^2 \theta}{\partial x^{*2}} + G = \frac{\partial \theta}{\partial F_0} & \text{pour } 0 < x^* < l; \quad F_0 > 0 \\ \frac{\partial \theta}{\partial x^*} = 0 & \text{pour } x^* = 0; \quad F_0 > 0 \\ \lambda \frac{\partial \theta}{\partial x^*} = -B_i \theta & \text{pour } x^* = l; \quad F_0 > 0 \\ \theta = 1 & \text{pour } 0 < x^* < l; \quad F_0 = 0 \end{array} \right. \quad (3.4)$$

In the variable regime, two dimensionless numbers are important:

- **The Biot number**: is the ratio between the internal thermal resistance and the external thermal resistance, l is the characteristic length of the medium ($l = r$ for a sphere).

$$B_i = \frac{hl}{\lambda} \quad (3.5)$$

It calculates the thermal thickness of the domain: a medium is thermally thin if the Biot number is less than 1 (i.e. the external resistance blocks the heat flow). We can also consider that the temperature is uniform depending on the dimension of l .

- **Fourier number**: is the ratio of flow through l^2 to the storage rate in l^3 , or the ratio of heat passing through to stored heat. The Fourier number describes the penetration of heat under varying conditions.

$$F_0 = \frac{\lambda \frac{\Delta T}{l} l^2}{\rho C l^3 \frac{\Delta T}{l}} = \frac{\alpha t}{l^2} \quad (3.6)$$

3.1.1 Uniform temperature environment (zero gradient method):

Heat transfer to a uniformly heated medium will be studied in this course, which is initially counterintuitive because a thermal gradient is necessary for heat transfer to occur. This proximity of medium at a constant temperature can however be justified in certain cases which we will specify.

For example, consider the quenching of a metal ball, which consists of immersing a ball initially at temperature T_i in a bath at constant temperature T_0 . If we assume that the temperature inside the ball is constant, which is more likely given its small size and its high thermal conductivity, we can write the heat balance of the ball between two times t and $t + dt$:

$$-h \cdot S \cdot (T - T_0) = \rho \cdot V \cdot c \frac{\partial T}{\partial t} \quad \text{soit ; } \frac{dT}{T - T_0} = -\frac{h \cdot S}{\rho \cdot V \cdot c} dt$$

D'où :

$$\frac{T - T_0}{T_i - T_0} = \exp\left(-\frac{h \cdot S}{\rho \cdot V \cdot c} t\right) \quad (3.7)$$

We notice that the grouping $\frac{h \cdot S}{\rho \cdot V \cdot c}$ is homogeneous at one time, we will call it

The constant of system time: $\tau = \frac{\rho \cdot V \cdot c}{h \cdot S}$

This quantity is fundamental insofar as it gives the order of magnitude of time of the physical phenomenon, we have in fact:

$$\frac{T - T_0}{T_i - T_0} = \exp\left(-\frac{t}{\tau}\right) \quad (3.8)$$

The definition of these two numbers (Biot number and Fourier number) allows us to write the expression for the temperature of the ball in the form:

$$\frac{T - T_0}{T_i - T_0} = \exp(-B_i \times F_o) \quad (3.9)$$

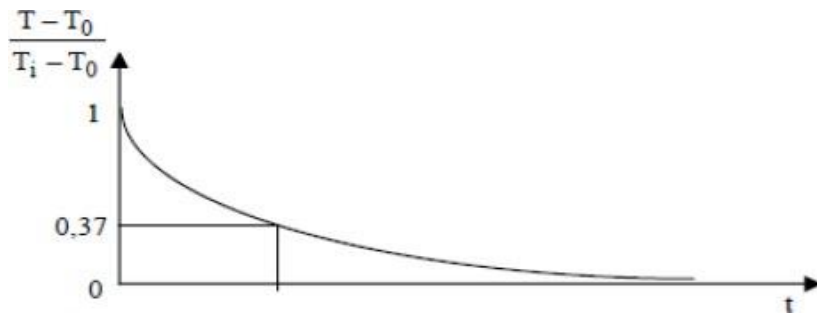


Figure 3. 1. Evolution of the temperature of a medium at uniform temperature

The product of the Biot and Fourier numbers can be used to calculate the evolution of the temperature of the sphere. The $Bi < 0.1$ criterion is generally called the “thermal accommodation” criterion because a device with $Bi < 0.1$ can be considered to be at uniform temperature.

3.1. Example

For the fin of example 4.5, a fan was used to improve the thermal performance, and as a result, the heat transfer coefficient is increased to $40 \text{ W/m}^2 \text{ K}$. Justify the use of the lumped mass approximation to predict the rate of change of temperature with time. Using the lumped mass approximation given below, calculate the time taken, τ , for the heat sink to cool from 60°C to 30°C .

$$(T - T_f) = (T_i - T_f) \exp\left(-\frac{hA_s\tau}{mC}\right)$$

2.1 Solution

Consider a single fin (the length scale L for the Biot number is half the thickness $t/2$)

$$B_i = \frac{hL}{k} = \frac{h \times t/2}{k} = \frac{40 \times 0.0005}{175} \approx 10^{-4}$$

Since $B_i \ll 1$, we can use the “lumped mass” model approximation.

$$\frac{(T - T_f)}{(T_i - T_f)} = \exp\left(-\frac{hA_s \tau}{mC}\right)$$

$$\tau = -\frac{mC}{hA_s} \ln\left(\frac{T - T_f}{T_i - T_f}\right)$$

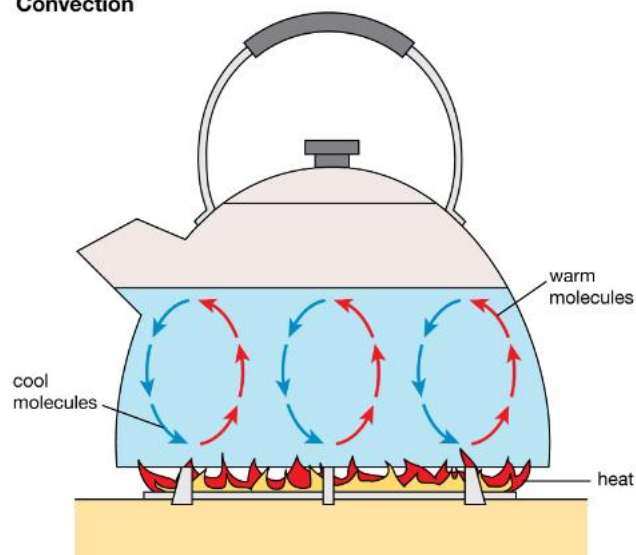
$$m = \rho A_s t / 2$$

$$\tau = -\frac{\rho C t}{2h} \ln\left(\frac{T - T_f}{T_i - T_f}\right) = -\frac{2700 \times 900 \times 0.001}{2 \times 40} \ln\left(\frac{30 - 20}{60 - 20}\right) = 42 \text{ seconds}$$

Chapter 4

Heat transfer by convection

Convection



Heat transfer by convection

4.1. Introduction

When heat transfer is accompanied by mass transfer, it is called convective transfer. This mode of heat exchange occurs in non-isothermal fluid environments or when a fluid circulates around a solid. Convection is a mode of transport of energy by the combined action of conduction, the accumulation of energy and the movement of the fluid medium. The study of heat transfer by convection essentially makes it possible to determine the heat exchanges occurring between a fluid and a wall. The quantity of heat exchanged per unit of time depends on several parameters:

the temperature difference between the wall and the fluid;

- fluid speed;
- the heat capacity of the fluid;
- the exchange surface;
- the surface condition of the solid;
- its size
- ...etc.

4.1. Natural convection-forced convection

Heat transmission by convection is designated, depending on the mode of fluid flow, by natural convection (also called free) and forced convection:

- When currents occur within the fluid simply due to temperature differences, we say that the convection is natural or free.
- On the other hand, if the movement of the fluid is caused by an external action, such as a pump or a fan, the process is called forced convection.

4.2. Newton's law

Newton's law gives the expression for the quantity of heat δQ exchanged between the surface of a solid at temperature T_s and the fluid at temperature T_f (often denoted T_∞).

The study of heat transfer by convection makes it possible to determine the heat exchanges occurring between a fluid and a wall.

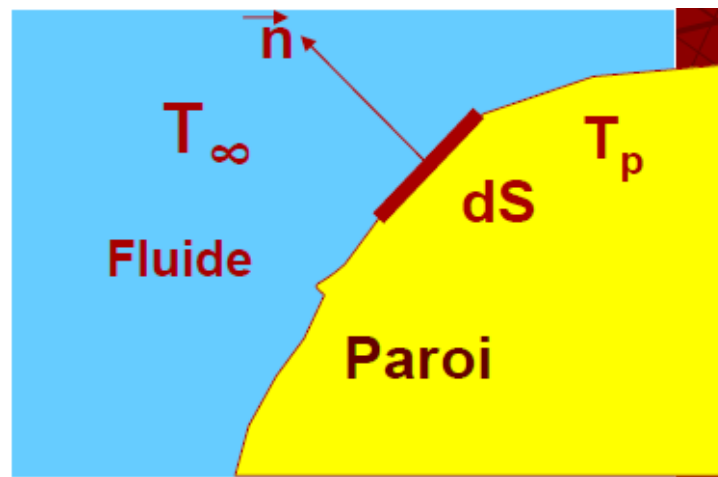


Figure 4. 1. Newton law

The quantity of heat δQ which passes through dS during the time interval dt can be written:

$$\delta Q = h dS (T_p - T_\infty) dt$$

- h is the convection exchange coefficient, it is expressed in $W/(m^2.K)$.
- δQ is expressed in Joules.
- $\delta Q/dt$ in Watts.

Whatever the type of convection (natural or forced) and whatever the flow regime of the fluid (laminar or turbulent), the transmitted heat flow is given by the relationship known as Newton's law (empirical relationship):

$$\phi = h dS (T_p - T_\infty)$$

- ϕ is the transmitted power (W)
- h is the exchange coefficient (in $W/(m^2.K)$)

- $T_p - T_\infty$ is the temperature difference between the body and the fluid (in K).
- dS is the exchange surface (in m^2)

4.2 Convective exchange coefficient h

The major problem to resolve before any calculation of the heat flow consists of determining the convective exchange coefficient h which depends on numerous parameters:

- Type of convection (natural or forced)
- fluid characteristics,
- nature of the flow,
- temperature,
- the shape of the exchange surface,
- ...etc.

4.3. Natural convection

4.3.1 Principle of natural convection

The hot particle starts moving and directly ensures the transfer of heat towards the colder medium: The regime becomes convective.

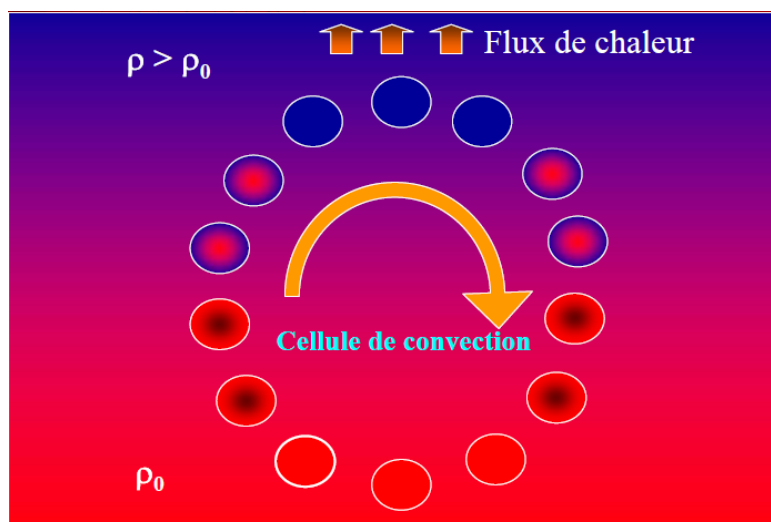


Figure 4. 2. Heat transfer by convection

4.4. Dynamic study

Consider a horizontal surface (S) at a temperature T_s in contact with a still fluid of density ρ at a temperature T_f .

A particle (P) of the fluid of volume V in contact with the surface (S) has a temperature close to T_s .

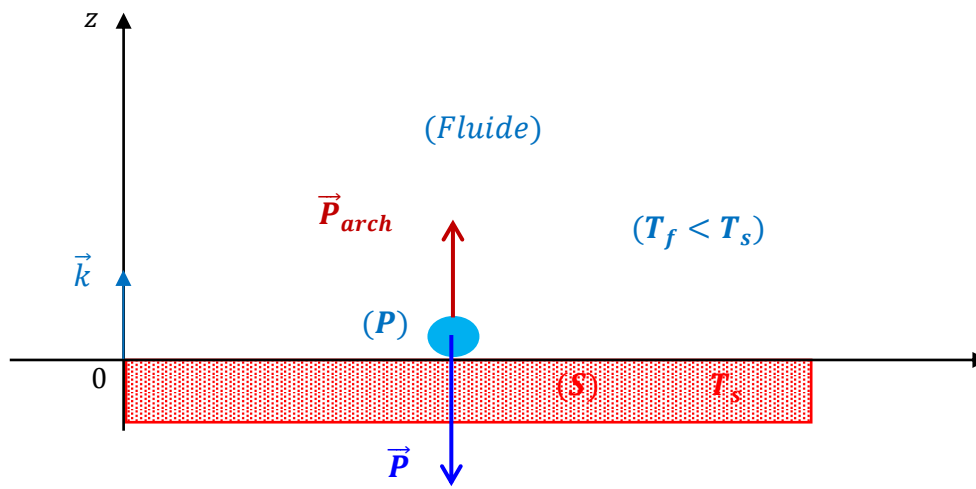


Figure 4. 3. Dynamic study

➤ Balance of forces acting on the particle (P):

- Archimedes' thrust: $\vec{P}_{arch} = \rho(T_f).V.g \vec{k}$
- The weight: $\vec{P} = -mg\vec{k} = -\rho(T_s).V.g \vec{k}$

As $T_s > T_f$, we of course have $\rho(T_f) > \rho(T_s)$ and consequently $\|\vec{P}_{arch}\| > \|\vec{P}\|$, which allows us to conclude that the densest parts are located below the least dense. Movements in the fluid will then be favored: this is the phenomenon of natural convection.

The fundamental principle of dynamics is written in this case:

$$\begin{aligned} \sum \vec{F}_{ext} &= m\vec{\gamma} \\ \Rightarrow \vec{P}_{arch} + \vec{P} &= \rho(T_s).V.\vec{\gamma} \\ \Rightarrow \rho(T_f).V.g - \rho(T_s).V.g &= \rho(T_s).V.(d^2 z)/(dt^2) \end{aligned}$$

The equation of movement of the particle (P) in the immediate vicinity of (S) is therefore written:

$$\Rightarrow (d^2 z)/(dt^2) = (\rho(T_f) - \rho(T_s))/(\rho(T_s)) \cdot g$$

4.5. Study of the convection phenomenon

4.5.1 Boundary layers

The study of flows near the walls is necessary for the determination of heat exchanges by convection between a solid and the fluid which surrounds it. Consider a fluid flowing along a surface S:

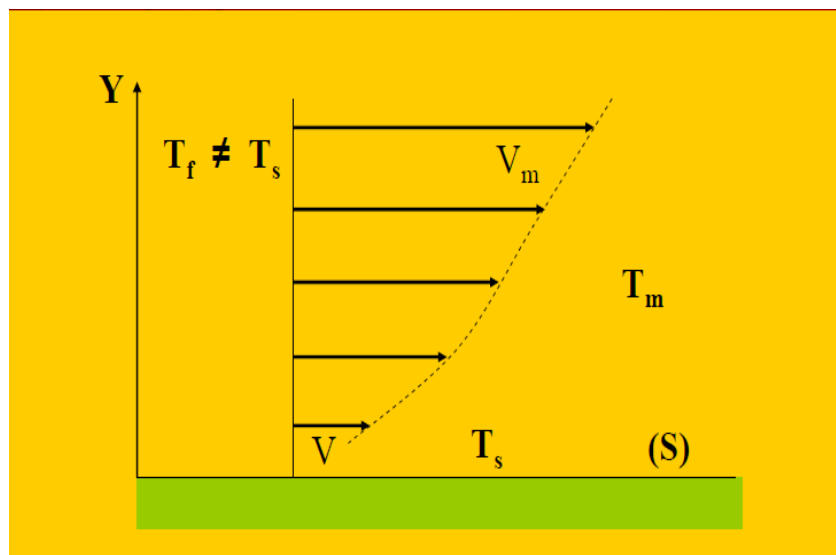


Figure 4. 4. Boundary layers

- Far from the surface, the fluid has an average speed V_m and an average temperature T_m.
- In the immediate vicinity of the surface, the temperature of the fluid is very close to that of the surface. The speed of the fluid is almost zero.

The speed and temperature diagrams, in the direction perpendicular to the surface (the Y direction), define a layer of fluid called the “boundary layer” whose temperature and speed have the appearance of the following curves:

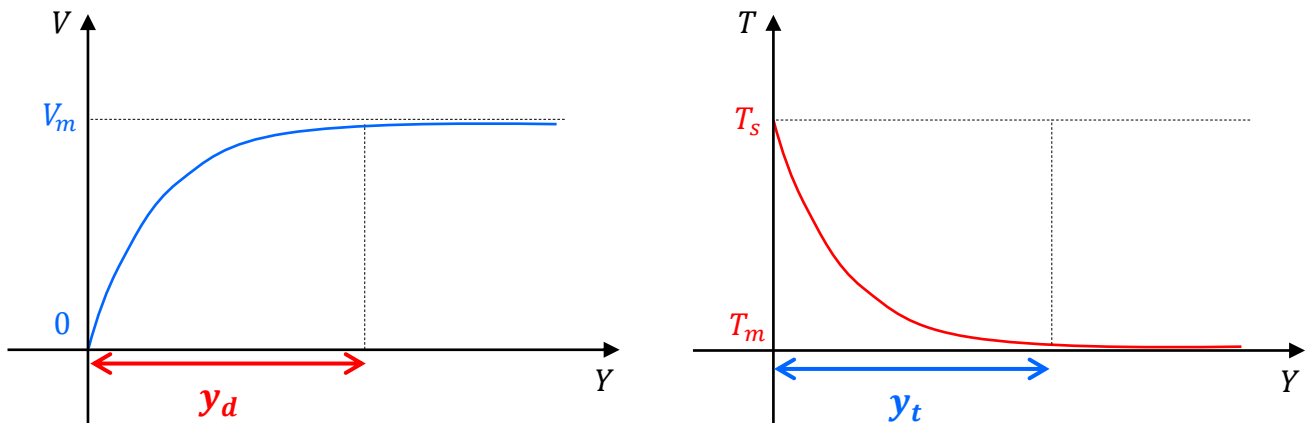


Figure 4. 5. The speed and temperature diagrams

We thus define two types of boundary layers:

- dynamic boundary layer that we note y_d
- thermal boundary layer that we note y_t

It should be noted that these two boundary layers generally have different thicknesses.

The velocity and temperature profiles developed within a flowing fluid can be represented like this:

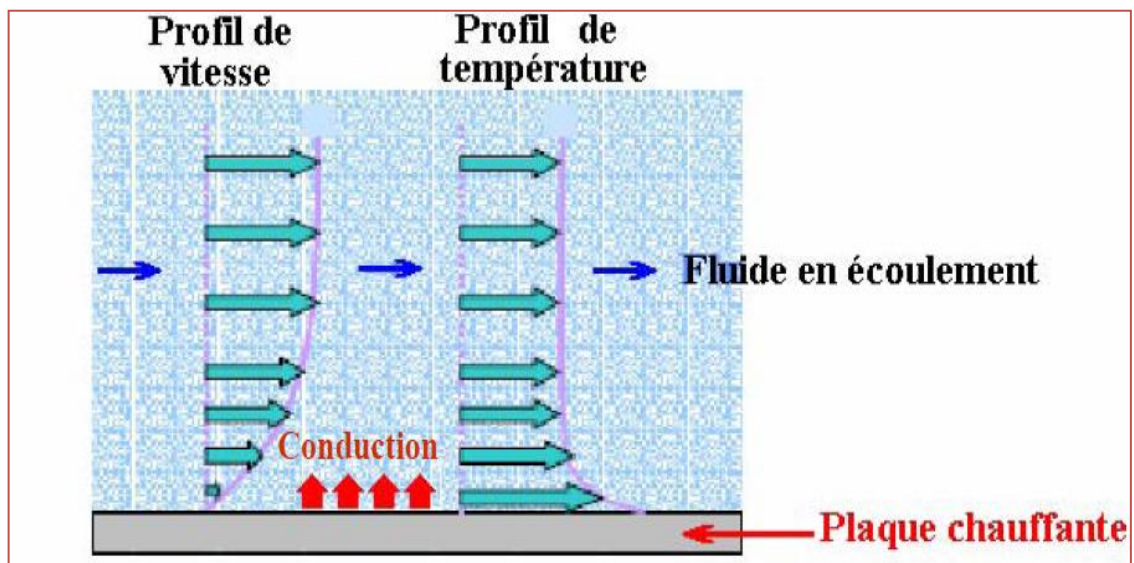


Figure 4. 6. The velocity and temperature profiles

Note that in the vicinity of the plate, dynamic and thermal boundary layers develop in which variations in speed and temperature are observed.

Heat transfer from the plate to the fluid results from two mechanisms:

- In the immediate vicinity of the surface, the transfer takes place by conduction.
- Far from the surface the transfer also results from the movement of the fluid.

In the boundary layer, if we admit that the heat transfer takes place essentially by conduction, therefore without transfer of matter in the y direction, we can write:

- The amount of heat through the surface (S):

$$\delta Q = -\lambda_s \cdot \frac{\partial T_s}{\partial y} \cdot dS \cdot dt \quad (1)$$

T_s is the surface temperature of the solid.

- The amount of heat through the boundary layer:

$$\delta Q = -\lambda_f \cdot \frac{\partial T_f}{\partial y} \cdot dS \cdot dt \quad (2)$$

T_f is the average temperature of the fluid quite far from the wall of the solid.

However, the variations of T_f and T_s as a function of y are generally not known to be able to deduce δQ from equalities (1) and (2). Newton's law allows this difficulty to be avoided by using only the temperature difference ($T_s - T_f$):

$$\delta Q = h \cdot (T_s - T_f) \cdot dS$$

4.5.2 Nature of the convective exchange coefficient h

The convective exchange coefficient, h , depends on several parameters and the heat exchange is all the more active (i.e. h greater) when:

- The fluid flow speed v is greater.
- Its specific heat c_p is greater.
- Its thermal conductivity λ (or its thermal diffusivity) is higher.
- Its kinematic viscosity $\nu = \mu/\rho$ is lower.

h can also depend on the dimensions of the wall, d , its nature and its shape. We can then write:

$$h = h(v, c_p, \lambda, \mu, d)$$

The large number of factors influencing heat transfer by convection explains the difficulty of any theoretical or even experimental study, especially if the coefficients which characterize the material vary with pressure and temperature. Hence the idea of thinking of an effective method allowing the determination of the expression of the coefficient h . The method using what is called “dimensional analysis” seems to be the easiest to implement for determining the expression of the convection coefficient h .

4.5.3 Determination of the coefficient h by the dimensional analysis method

Suppose that h is a function of the variables v, c_p, λ, μ and d

$$h = h(v, c_p, \lambda, \mu, d)$$

h is also an implicit function of the temperature T since c_p and λ depend on it.

Let's use the equations for the dimensions of each term:

$$* [\delta Q] = [energy] = [force][displacement] = M.L^2.t^{(-2)}$$

$$* [\lambda] = \frac{[\delta Q]}{L.T.t} = \frac{M.L^2.t^{-2}}{L.T.t} = M.L.T^{-1}.t^{-3}$$

$$* [c_p] = \frac{[\delta Q]}{M \cdot T} = \frac{M \cdot L^2 \cdot t^{-2}}{M \cdot T} = L^2 \cdot t^{-2} \cdot T^{-1}$$

$$* [\mu] = M \cdot L^{-1} \cdot t^{-1}$$

$$* [v] = L \cdot t^{-1}$$

$$* [d] = L$$

The equation with dimensions of h is obtained from Newton's law:

$$[h] = \frac{[\delta Q]}{[(T_s - T_f)] \cdot [dS] \cdot [dt]} = \frac{M \cdot L^2 \cdot t^{-2}}{T \cdot L^2 \cdot t} = M \cdot T^{-1} \cdot t^{-3}$$

By writing [h] in the form:

$$[h] = [c_p]^a \cdot [\lambda]^b \cdot [\mu]^c \cdot [d]^d \cdot [v]^e$$

or

$$[h] = (L^2 \cdot t^{-2} \cdot T^{-1})^a \cdot (M \cdot L \cdot T^{-1} \cdot t^{-3})^b \cdot (M \cdot L^{-1} \cdot t^{-1})^c \cdot (L)^d \cdot (L \cdot t^{-1})^e = M \cdot T^{-1} \cdot t^{-3}$$

Note that the fundamental quantities involved in the calculation of h are:

The mass M, the time t, the length L and the temperature T.

Identifying the exponents in the equation with dimensions of h provides a linear system of equations for calculating a, b, c, d and e:

- the exponent of M: $b+c=1$
- the exponent of T: $a+b=1$
- the exponent of L: $2a+b-c+d+e=0$
- the exponent of t: $2a+3b+c+e=3$

4.5.4 Dimensionless numbers

For convection problems, solving the dimensional equations reveals dimensionless numbers that are very useful in the study of fluid mechanics and in particular in convective phenomena. These numbers are in particular:

- ❖ The Reynolds number
- ❖ The Nusselt number
- ❖ The Prandtl number

a) The Reynolds number

The flow regime of a fluid can be laminar or turbulent. The transition from one regime to another is characterized by the Reynolds number:

$$Re = \frac{v \cdot d}{\nu}$$

In the case of a tube, d represents the internal diameter. Experience shows that for Re less than a critical value $Re_{cr}=2200$, the flow in a pipe is always laminar:

- If $Re < Re_{cr}$: the regime is said to be laminar
- If $Re > Re_{cr}$: the regime is said to be turbulent

b) The Nusselt number

The Nusselt number characterizes the importance of convection compared to conduction. It is the ratio of the quantity of heat exchanged by convection $h \cdot S \cdot \Delta T$ to a quantity of heat exchanged by conduction $\lambda \cdot S \cdot \Delta T / d$:

$$Nu = \frac{h \cdot S \cdot \Delta T}{\lambda \cdot S \cdot \frac{\Delta T}{d}} \Rightarrow Nu = \frac{h \cdot d}{\lambda}$$

Nu is a direct function of h , its knowledge makes it possible to determine the value of h .

c) The Prandtl number

It characterizes the speed distribution in relation to the temperature distribution (characterizes the thermal properties of the fluid):

$$Pr = \frac{\mu \cdot c_p}{\lambda}$$

d) The Grashof number:

The viscosity force of the fluid is characterized by the Grashof number:

$$Gr = \frac{g \cdot d^3 \cdot \beta_p \cdot \Delta T}{\nu^2}$$

d: characteristic dimension of the wall [m].

g: acceleration of gravity [m.s⁻²].

$\Delta T = T_p - T_f$: Characteristic temperature difference [°C].

β_p : Volume expansion factor of the fluid [°C⁻¹].

pour un gaz parfait : $\beta = \frac{1}{T}$

e) The Rayleigh number:

It is identical to the Reynolds number in natural convection.

$$Ra = Pr \cdot Gr$$

4.5.5 Thermal resistance

Consider a fluid of temperature T_1 which circulates in the vicinity of a wall of temperature T_2 . The exchanged heat flow is written:

$$\begin{aligned} \phi &= hS(T_2 - T_1) \\ (T_2 - T_1) &= \frac{1}{hS} \phi = R_{th} \phi \end{aligned}$$

The analogy with Ohm's law makes it possible to define the thermal resistance R_{th} of convection

$$R_{th} = \frac{1}{hS}$$

4.6. Forced convection:

In this section of the chapter we will see how to transfer heat by convection without phase change. The most common experimental correlations in forced convection are of the following type: $Nu = f(Re, Pr)$

For a given fluid temperature, the Prandtl number, which appears in the expression of the Nusselt number and characterizes the flowing fluid, must be determined. Alternatively, if there is heat exchange, the surface temperature and the fluid temperature would be different; the fluid properties would simply be taken for the average temperature T_m .

$$T_m = \frac{T_p + T_f}{2}$$

4.6.1. Heat exchange along a flat plate (External forced convection $Re \leq 5 \cdot 10^5$):

- Laminar speed:

$$Re \leq 5 \cdot 10^5$$

$$Nu_L = 0.66(Re_L)^{1/2} (Pr)^{1/3}$$

- Turbulent regime:

$$Re > 5 \cdot 10^5$$

$$Nu_L = 0.036(Re_L)^{4/5} (Pr)^{1/3}$$

4.6.2. Flow inside smooth cylindrical tubes (Internal forced convection $Re \geq 2300$):

Laminar speed: $Re \leq 2300$

Valid for : $Nu_L = Re \cdot Pr \times \frac{D}{L} < 100$

Turbulent regime $Re > 2300$

$$Nu_L = 0.023(Re)^{0.8} (Pr)^n$$

Valid for

$$n = 0.3 \quad si \quad T_f > T_p$$

$$n = 0.4 \quad si \quad T_f < T_p$$

4.6.3. Flow in annular spaces (Internal forced convection)

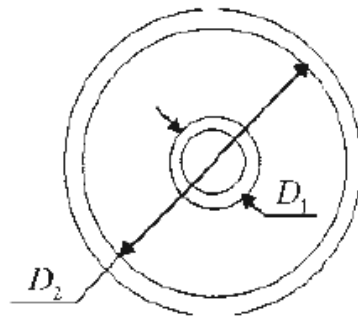


Figure 4. 7. Flow in the annular space formed by the two pipes

$$Nu_{Dh} = 0.023(Re_{Dh})^{0.8} (Pr)^n$$

$$\text{Avec ; } Re_{Dh} = \frac{U_m \cdot D_h}{\nu} \quad \text{et} \quad Nu_{Lh} = \frac{h \cdot D_h}{\lambda}$$

D_h : Diamètre hydraulique. Dans ce cas: $D_h = D_1 - D_2$

$n = 0,4$ pour chauffage

$n = 0,3$ pour refroidissement

4.7. Natural convection:

a) Practical method for calculating heat flow in natural convection:

The use of dimensional analysis reveals that the relationship between the flow of heat transported by convection and the factors on which it depends is found in the form of a relationship between three dimensionless numbers $Nu = f(Gr, Pr)$

The following steps are used to calculate a heat flux transmitted by natural convection:

- The determination of the following dimensionless numbers: Gr and Pr.
- According to the result of Gr, we choose an experimental correlation corresponding to the configuration studied.
- By applying this correlation for the calculation of the number Nu.

- Calculation of
$$h = \frac{\lambda \cdot Nu}{L}$$

- Using Newton's law to calculate the heat exchanged flow:

$$\Phi = h \cdot S (T_p - T_\infty)$$

Noticed :

The following should be taken into consideration when calculating h:

- The nature of the fluid (liquid or gas).
- The temperature of the fluid.
- Know the type of convection (natural or forced).
- Know the type of flow regime (laminar or turbulent).
- Know the type of contact between the fluid and surface (flat surface, or circulates between two flat surfaces, or circulates in a tube, etc.).

b) Different correlations for calculating h

We noticed that the relationships describing a natural convection problem can take the following form: $Nu = f(Gr, Pr)$

The relationship between these three dimensionless numbers cannot be established theoretically, but must be determined experimentally. Many scientific discoveries have been compiled in the literature. They are known as "experimental correlations." In this section, we will review the most common experimental correlations in natural convection.

The most common experimental correlations in natural convection are of the following type: $Nu = C \cdot Ra^n$ avec ; $Ra = Pr \cdot Gr$

The value of the coefficient C depends on the nature of the regime and the fluids. It is determined by calculating Ra, depending on the value found we choose the suitable values of c and n which are given in the following table:

Géométrie et orientation	Dimension caractéristique	C en convection laminaire $n = 1/4$	C en convection turbulente $n = 1/3$
Plaque verticale	Hauteur	0.59 $Ra \leq 10^9$	0.10 $Ra > 10^9$
Plaque horizontale chauffante vers le haut	Largeur	0.54 $10^4 \leq Ra \leq 10^7$	0.15 $10^7 < Ra \leq 10^{11}$
Plaque horizontale chauffante vers le bas	Largeur	0.27 $10^5 \leq Ra \leq 10^{10}$	0.54 $10^{10} < Ra \leq 10^{13}$
Cylindre horizontal	Diamètre extérieure	$C = 1.02$ et $n = 0.148$ $10^{-2} \leq Ra \leq 10^2$	0.135 $2 \times 10^7 < Ra \leq 10^{13}$
		0.54 $5 \times 10^2 \leq Ra \leq 2 \times 10^7$	

4.1. Example

Calculate the appropriate Reynolds numbers and state if the flow is laminar or turbulent for the following:

- A 10 m (water line length) long yacht sailing at 13 km/h in seawater $\rho = 1000 \text{ kg/m}^3$ and $\mu = 1.3 \times 10^{-3} \text{ kg/m s}$,
- A compressor disc of radius 0.3 m rotating at 15000 rev/min in air at 5 bar and 400°C and $\mu = \frac{1.46 \times 10^{-6} T^{3/2}}{(110 + T)} \text{ kg/m s}$
- 0.05 kg/s of carbon dioxide gas at 400 K flowing in a 20 mm diameter pipe. For the viscosity take $\mu = \frac{1.56 \times 10^{-6} T^{3/2}}{(233 + T)} \text{ kg/m s}$
- The roof of a coach 6 m long, travelling at 100 km/hr in air ($\rho = 1.2 \text{ kg/m}^3$ and $\mu = 1.8 \times 10^{-5} \text{ kg/m s}$)
- The flow of exhaust gas ($p = 1.1 \text{ bar}$, $T = 500^\circ\text{C}$, $R = 287 \text{ J/kg K}$ and $\mu = 3.56 \times 10^{-5} \text{ kg/m s}$) over a valve guide of diameter 10 mm in a 1.6 litre, four cylinder four stroke engine running at 3000 rev/min (assume 100% volumetric efficiency an inlet density of 1.2 kg/m^3 and an exhaust port diameter of 25 mm)

4.1. Solution

$$\text{a) } \text{Re} = \frac{\rho u L}{\mu} = \frac{10^3 \times \frac{13 \times 10^3}{3600} \times 10}{1.3 \times 10^{-3}} = 2.78 \times 10^7 \quad (\text{turbulent})$$

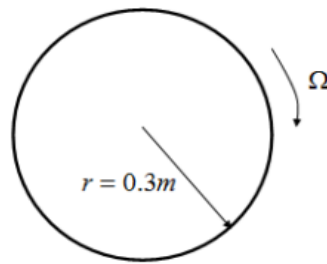
$$\text{b) } T = 400 + 273 = 673 \text{ K}$$

$$\mu = \frac{1.46 \times 10^{-6} \times 673^{3/2}}{(110 + 673)} = 3.26 \times 10^{-5} \text{ kg/m s}$$

$$\Omega = \frac{15000}{60} \times 2\pi = 1571 \text{ rad/s}$$

$$u = \Omega r = 1571 \times 0.3 = 471.3 \text{ m/s}$$

$$\rho = \frac{P}{RT} = \frac{100000}{287 \times 673} = 2.59 \text{ kg/m}^3$$



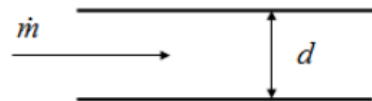
Characteristic length is r not D

$$\text{Re} = \frac{\rho u D}{\mu} = \frac{2.59 \times 471.3 \times 3}{3.26 \times 10^{-5}} = 1.12 \times 10^7 \quad (\text{turbulent})$$

$$\text{c) } \dot{m} = \rho u A = \rho u \times \frac{\pi D^2}{4}$$

$$u = \frac{4\dot{m}}{\rho \pi D^2}$$

$$\text{Re} = \frac{\rho u D}{\mu} = \frac{\rho \times 4\dot{m} D}{\rho \pi D^2 \mu} = \frac{4\dot{m}}{\pi D \mu}$$



$$\mu = \frac{1.56 \times 10^{-6} \times 400^{3/2}}{(233 + 400)} = 1.97 \times 10^{-5} \text{ kg/m s}$$

$$\text{Re} = \frac{4 \times 0.05}{\pi \times 0.02 \times 1.97 \times 10^{-5}} = 1.6 \times 10^5 \quad (\text{turbulent})$$

$$\text{d) } u = \frac{100 \times 10^3}{3600} = 27.8 \text{ m/s}$$

$$\text{Re} = \frac{\rho u L}{\mu} = \frac{1.2 \times 27.8 \times 6}{1.8 \times 10^{-5}} = 11.1 \times 10^7 \quad (\text{turbulent})$$

- e) Let $\dot{m}$ be the mass flow through the exhaust port

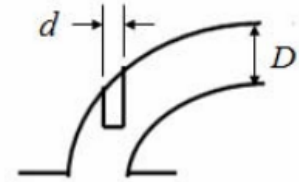
$\dot{m}$ = inlet density X volume of air used in each cylinder per second

$$\dot{m} = 1.2 \times \frac{1.6 \times 10^{-3}}{4} \times \frac{3600}{60} \times \frac{1}{2} = 0.012 \text{ kg/s}$$

$$u = \frac{4\dot{m}}{\pi D^2 \rho}$$

$$\text{Re}_d = \frac{\rho u d}{\mu}$$

$$\text{Re} = \frac{4 \times 0.01 \times 0.012}{\pi \times 3.56 \times 10^{-5} \times 0.025} = 6869 \quad (\text{laminar})$$

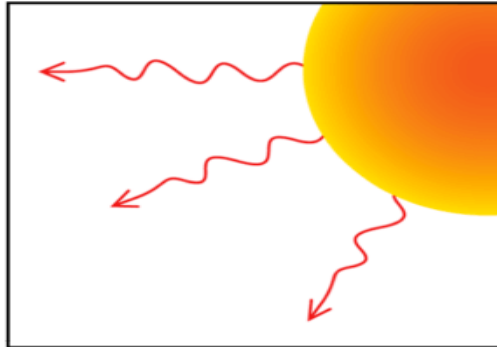


Chapter 5

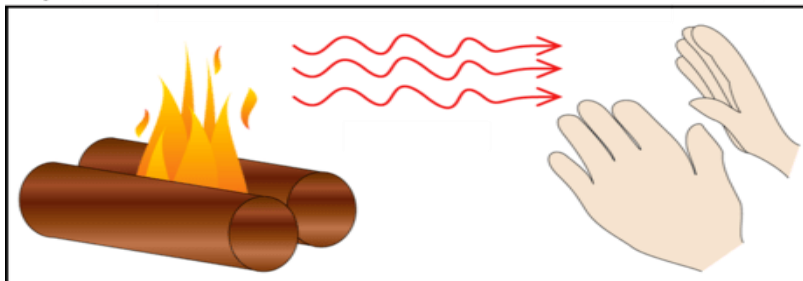
Heat transfer by Radiation

Thermal Radiation

A)



B)



Heat transfer by Radiation

5.1. Introduction

Thermal radiation is the energy emitted by matter that is at a finite temperature. It has a fundamental difference with respect to conduction and convection: substances that exchange heat do not need to be in contact, but can be separated by a vacuum. Radiation is a term applied generically to all kinds of phenomena related to electromagnetic waves.

We will focus our attention on radiation from solid surfaces, although this radiation can also come from liquids or gases. In thermal radiation, heat is transmitted by electromagnetic waves, like light, but of different wavelengths. Radiant energy depends on the characteristics of the surface and the temperature of the emitting body. By influencing a receptor, part of the energy passes to this other body, depending on its characteristics and its power of absorption. This energy results in an increase in the temperature of the second body. Heat transfer by radiation only involves the transport of energy, it does not need a material support, taking place even in a vacuum.

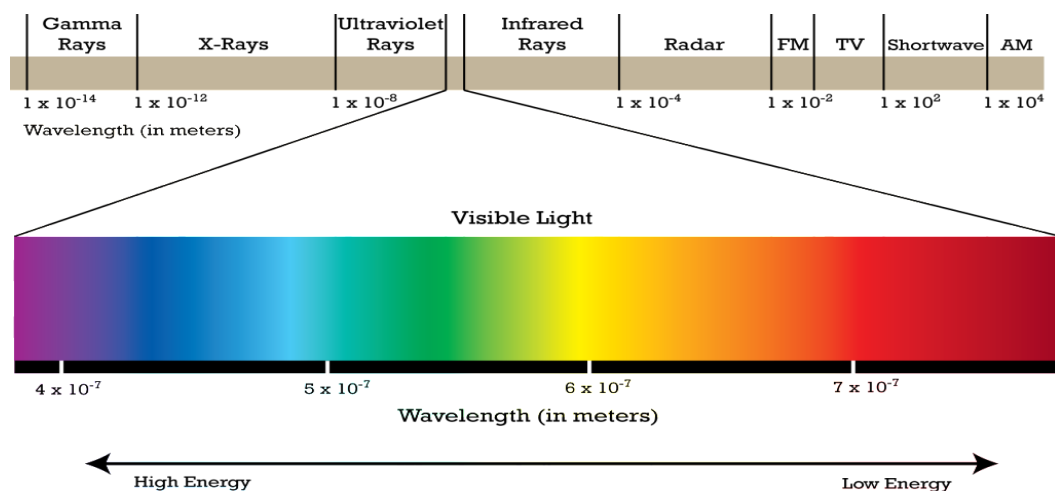


Figure 5. 1. The electromagnetic spectrum

5.2. Principle of heat transfer by radiation

It is a mode of heat (energy) exchange in the form of electromagnetic waves according to Planck's law ($E = h \cdot \nu$, such that: ν is the associated wave frequency and $h = 6,62 \cdot 10^{-34}$ J.s is Planck's constant). So it does not require any hardware support, it is analogous to the propagation of light. It propagates in a rectilinear manner at the speed of light ($c = 3 \times 10^8$ m/s). The thermal radiation emitted by bodies is located between wavelengths of $0.1 \mu\text{m}$ to $100 \mu\text{m}$. Practically, the three modes of heat transfer will coexist. But, this mode of transfer becomes predominant at temperatures higher than ordinary temperatures. Generally, all bodies (solid, liquid and gaseous) emit radiation of an electromagnetic nature. We can cite that, vacuum and simple gases like (O_2 , H_2 and N_2) represent perfectly transparent media but, compound gases like (CO_2 , H_2O , CO and CH_4) and certain liquids and solids like (glasses and polymers) are partially transparent. The majority of solids and liquids are opaque bodies since they stop the propagation of radiation just at their surfaces.

5.3. Preliminary definitions

The physical quantities will be designated according to the spectral composition or spatial distribution of the radiation:

- **Total magnitude:** it is relative to the entire spectrum;
 - **Monochromatic quantity:** it only concerns a narrow spectral interval ($d\lambda$), around a wavelength (λ);
 - **Hemispherical magnitude:** it is relative to all the directions of space;
 - **Directional quantity:** it characterizes a given direction of propagation.
- When studying the thermal equilibrium of a system, anybody must be considered as:
- **Emitter:** if it sends radiation linked to its temperature (unless it is perfectly transparent);
 - **Receiver:** If it receives, radiation emitted or reflected and diffused by the bodies that surround it.

- **Opaque body:** it is a body which does not transmit any radiation through itself, it stops the propagation of all radiation from its surface, it heats up by the absorption of the radiation;
- **Transparent body:** it is a body which transmits all the incident radiation;
- **Black body:** is the one that absorbs all the radiation it receives, it is characterized by an absorbent power ($\alpha_{\lambda T} = 1$). All black bodies radiate in the same way the same temperature, the black body radiates more than a non-black body.
- **Gray body:** is that whose absorbent power ($\alpha_{\lambda T}$) is independent of length wave (λ), it is characterized by ($\alpha_{\lambda T} = \alpha_T$). A gray body at high temperature for $\lambda < 3\mu\text{m}$, sun), a gray body at low temperature for ($\lambda > 3\mu\text{m}$, atmosphere).
- **Solid angle:** the elementary solid angle ($d\Omega$) under which the contour of a small surface (ds) is seen from a point (O) is given by:

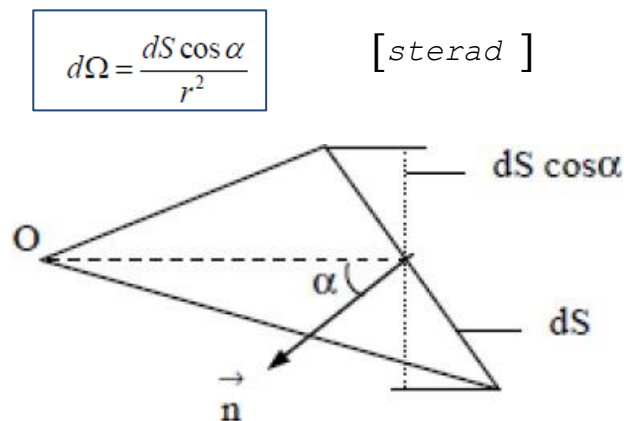


Figure 5. 2. Solid angle

Monochromatic energy emittance: the monochromatic emittance of a source at temperature (T) is given by:

$$M_{\lambda T} = d\varphi_{\lambda}^{\lambda+d\lambda} / dS.d\lambda \quad [W / m^2] \quad (5.1)$$

Such as; φ : is the energy flow emitted between the two wavelengths (λ) and ($\lambda+d\lambda$). Total energy emittance: defined as the flux density emitted by the elementary surface (dS) over the entire wavelength spectrum:

$$M_T = \int_{\lambda=0}^{\lambda=\infty} M_{\lambda T} d\lambda = d\varphi/dS \quad [W/m^2] \quad (5.2)$$

Energy intensity in one direction: the flux per unit of solid angle emitted by a surface (ds) under a solid angle ($d\Omega$) surrounding the direction (Ox):

$$I_x = d^2\varphi_x/d\Omega \quad (5.3)$$

Energy luminance in one direction: the energy intensity in the direction (Ox) per unit of apparent emitting surface (the projection of the surface (S) on the plane perpendicular to Ox):

$$L_x = I_x/dS_x = I_x/dS \cos \alpha = d^2\varphi_x/d\Omega dS \cos \alpha \quad (5.4)$$

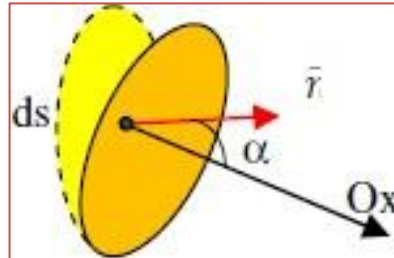


Figure 5. 3. Diagram showing the luminance of surface element ds .

Illumination (relative to a receiver): It is the flow received per unit of receiving surface, coming from all directions (Emittance).

5.4. Process of reception of radiation by a body

A heated equipment point emits electromagnetic radiation in all directions that are on the same side of the plane as the equipment point. When this radiation hits a body, some of the energy is reflected, some is transmitted through the

body, and the rest is absorbed quantitatively as heat. When incident radiation of energy ($\phi\lambda$) strikes a body (C) at temperature (T) (see figure opposite), we

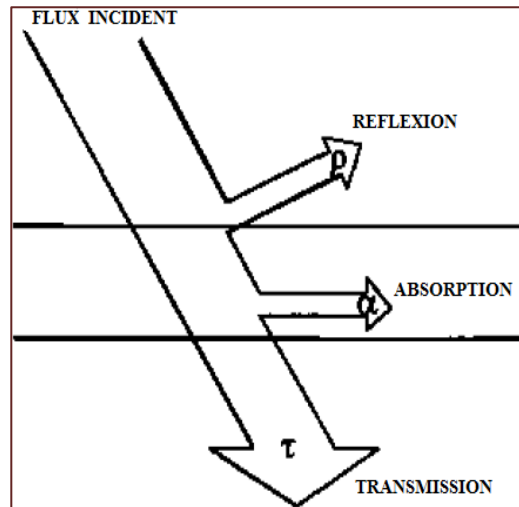


Figure 5. 4 .Process of reception of radiation by a body.

Notice that:

- Part of the energy ($\phi\lambda \cdot \rho_{\lambda T}$) is reflected by the surface (S) of the body;
 - Part of the energy ($\phi\lambda \cdot \alpha_{\lambda T}$) is absorbed by the body by heating it;
 - The rest of the energy ($\phi\lambda \cdot \tau_{\lambda T}$) is transmitted by continuing the path.
- Such as;
- From where;

$$\phi_{\lambda} = \phi_{\lambda} \cdot \rho_{\lambda T} + \phi_{\lambda} \cdot \alpha_{\lambda T} + \phi_{\lambda} \cdot \tau_{\lambda T} \quad (5.5)$$

$$\rho_{\lambda T} + \alpha_{\lambda T} + \tau_{\lambda T} = 1 \quad (5.6)$$

which respectively represent; the monochromatic reflective power ($\rho_{\lambda T}$), the monochromatic absorbing power ($\alpha_{\lambda T}$) and the monochromatic transmittance power ($\tau_{\lambda T}$). These powers depend on the nature of the body, its thickness, its temperature (T), the wavelength (λ), the incident radiation and the angle of incidence.

5.5. Laws of radiation:

5.5.1. Planck's law

For a black body, the monochromatic Emittance only depends on the wave length (λ) and the temperature (T):

$$E_{\lambda} = \frac{d\phi_{\lambda}}{dS} = \frac{C_1}{\lambda^5 \left(e^{\frac{C_2}{\lambda T}} - 1 \right)} \quad ; \quad [W / m^2 \cdot \mu m^{-1}] \quad (5.7)$$

Such as ;

T: temperature in °K

λ : The wavelength in μm

$C_1 = 2\pi h c^2 = 3,74 \times 10^8 W \cdot \mu^4 \cdot m^{-2}$: Planck constant

$C_2 = hc / k = 14400 \mu K$ k: Stefan-Boltzmann constant and c the speed of light

($c = 3 \times 10^8 m / s$).

Remarks

1. In the visible region (small wavelengths)

$$e^{\frac{C_2}{\lambda T}} \gg 1 \quad ; \quad \text{d'où} \quad E_{\lambda} = M_{\lambda,T}^0 = C_1 \lambda^{-5} e^{-\frac{C_2}{\lambda T}}$$

2. In the far infrared domain (long wavelengths), the development of $(e^{\frac{C_2}{\lambda T}})$ allows us to express $(M_{\lambda,T}^0)$ by:

$$E_{\lambda} = M_{\lambda,T}^0 = C_1 T / C_2 \lambda^4$$

Planck's law makes it possible to draw isothermal curves representing the variations of E_{λ} as a function of wavelength for various temperatures:

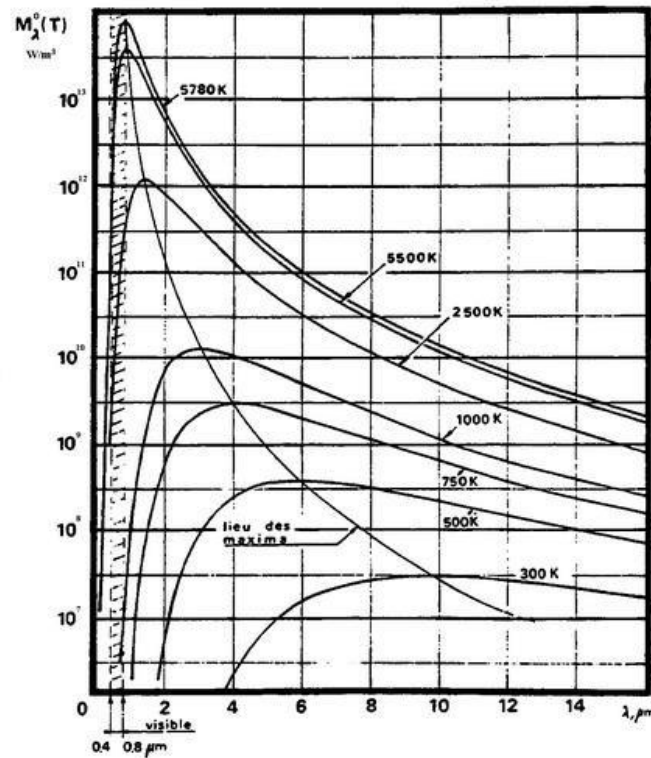


Figure 5. 5. Monochromatic emittance of the black body

5.5.2. Laws of Wien

5.5.2.1. Wien's first law

Wien's first law makes it possible to express or evaluate the wavelengths corresponding to the maximum monochromatic emittance (for which the radiation is maximum) as a function of temperature. For its derivation, it is enough to cancel the derivative of the emittance:

$$\frac{dM_{\lambda,T}^0}{d\lambda} = 0 \Leftrightarrow \lambda_m = \frac{2898}{T}; \quad [\mu m] \quad (5.8)$$

Noticed

- At room temperature ($T = 300 \text{ K}$, $\lambda_m = 9.6 \mu m$), a body emits long-wavelength infrared radiation which surrounds us but is not visible to our eye $[(0.36 \div 0.75) \mu m]$

- At the temperature of the sun ($T = 5790 \text{ K}$, $\lambda_m = 0.5 \text{ }\mu\text{m}$, maximum radiation), it is the visible yellow for which our eye has maximum luminous efficiency.

5.5.2.2. Wien's second law:

This law expresses the value of the maximum monochromatic emittance, it is enough to replace (λ_m) by its value in Planck's law to obtain:

With; (5.9)

$$M_{\lambda_m, T}^0 = B \cdot T^5$$

$$B = 1.287 \times 10^{-5} \text{ W} \cdot \text{m}^{-3} \cdot \text{K}^{-5} \text{ et } T \text{ en } ^\circ\text{K}$$

5.5.3 Stefan-Boltzmann law

It gives the total black body emittance, with the summation of all monochromatic emittances for all wavelengths or the integration of:

$$E = \frac{d\varphi}{dS} = \int_0^\infty E_\lambda d\lambda = M^0 = \int_0^\infty M_{\lambda, T}^0 d\lambda = \sigma \cdot T^4 \quad ; \quad [W / m^2] \quad (5.10)$$

Such as ;

$$\sigma = 5.67 \times 10^{-8} \text{ W m}^2 \cdot \text{K}^4 \quad \text{Stefan-Boltzmann constant}$$

5.6. Radiation of real bodies

The emissive properties of real bodies are defined relative to the emissive properties of black bodies in the same temperature and wavelength range, and they are distinguished using coefficients called emissivity factors. These monochromatic or total coefficients are defined as follows;

$$\varepsilon_{\lambda T} = \frac{M_{\lambda T}}{M_{0\lambda T}} \text{ et } \varepsilon_T = \frac{M_T}{M_{0T}}$$

According to Kirchhoff's law, we show that:

$$\alpha_{\lambda T} = \varepsilon_{\lambda T} \quad (5.11)$$

5.7. Gray body radiation

Gray bodies are characterized by $\alpha_{\lambda T} = \alpha_T$ so $\varepsilon_{\lambda T} = \varepsilon_T$ or

$M_T = \varepsilon_T \times M_{0T}$ we let us deduce the gray body emittance at temperature T:

$$M_T = \varepsilon_T \times \sigma \times T^4 \quad (5.12)$$

5.1. Example

Two adjacent compressor discs (Surfaces 1 and 2) each of 0.4 m diameter are bounded at the periphery by a 0.1 wide shroud (Surface 3).

- a) Given that $F_{12} = 0.6$, calculate all the other view factors for this configuration.
- b) The emissivity and temperature of Surfaces 1 and 2 are $\varepsilon_1 = 0.4$, $T_1 = 800\text{ K}$, $\varepsilon_2 = 0.3$, $T_2 = 700\text{ K}$ and Surface 3 can be treated as radiatively black with a temperature of $T_3 = 900\text{ K}$. Apply a grey body radiation analysis to Surface 1 and to Surface 2 and show that:

$$2.5 J_1 - 0.9 J_2 = 45545 \quad \text{W/m}^2$$

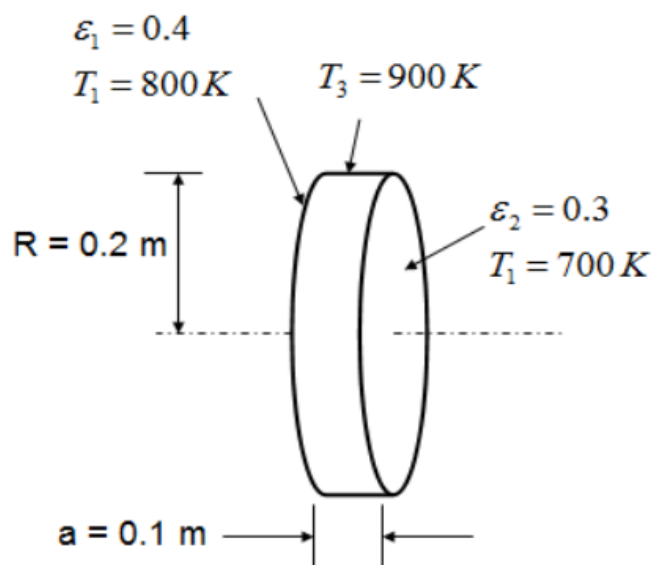
and

$$3.333 J_2 - 1.4 J_1 = 48334 \quad \text{W/m}^2.$$

The following equation may be used without proof:

$$\frac{E_{B,i} - J_i}{\frac{1 - \varepsilon_i}{\varepsilon_i}} = \sum_{j=1}^N F_{i,j} (J_i - J_j)$$

- c) Determine the radiative heat flux to Surface 2



5.1. Solution

a) $r_1 = r_2 = r = 0.2 \text{ m}$

$$a = 0.1 \text{ m}$$

$$\frac{r_2}{a} = \frac{0.2}{0.1} = 2$$

$$\frac{a}{r_1} = \frac{0.1}{0.2} = 0.5$$

$F_{12} = 0.6$ (Although this is given in the question, it can be obtained from appropriate tables with the above parameters)

$F_{11} = 0$ (As surface 1 is flat, it cannot see itself)

$F_{13} = 1 - 0.6 = 0.4$ (From the relation $\sum F_{ij} = 1$ in an enclosure)

$F_{21} = 0.6$ (Symmetry)

$F_{22} = 0$

$F_{23} = 0.4$

$$F_{31} = \frac{A_1}{A_3} F_{13} = \frac{\pi \times 0.2^2}{2 \times \pi \times 0.2 \times 0.1} \times 0.4 = 0.4$$

$F_{32} = 0.4$ (Symmetry)

$F_{33} = 1 - 0.4 - 0.4 = 0.2$

b)
$$\frac{\frac{E_{b,i} - J_i}{1 - \epsilon_i}}{\epsilon_i} = \sum_{j=1}^n (J_i - J_j) F_{ij}$$

Apply to surface 1, ($i = 1$)

Let $\frac{1 - \epsilon_1}{\epsilon_1} = \phi_1$

$$(E_{b,1} - J_1) = \phi_1 [F_{12}(J_1 - J_2) + F_{13}(J_1 - J_3)]$$

$$E_{b,1} = J_1 \{1 + \phi_1 F_{12} + \phi_1 F_{13}\} - \phi_1 F_{12} J_2 - \phi_1 F_{13} J_3$$

$$E_{b,1} = \sigma T_1^4$$

$$J_3 = \sigma T_3^4 \quad (\text{Radiatively black surface})$$

$$\phi_1 = \frac{1 - \varepsilon_1}{\varepsilon_1} = \frac{1 - 0.4}{0.4} = 1.5$$

$$\sigma T_1^4 = 2.5 J_1 - 0.9 J_2 - 0.6 \sigma T_3^4$$

$$56.7 \times 10^{-9} \times 800^4 = 2.5 \times J_1 - 0.9 \times J_2 - 0.6 \times 56.7 \times 10^{-9} \times 900^4$$

$$2.5 J_1 - 0.9 J_2 = 45545 \text{ W/m}^2 \quad (1)$$

Applying to surface 2 (i = 2)

$$E_{b,2} = J_2 \{1 + \phi_2 F_{21} + \phi_2 F_{23}\} - \phi_2 F_{21} J_1 - \phi_2 F_{23} J_3$$

$$E_{b,2} = \sigma T_2^4$$

$$\phi_2 = \frac{1 - \varepsilon_2}{\varepsilon_2} = \frac{1 - 0.3}{0.3} = 2.333$$

$$\sigma T_2^4 = 3.333 J_2 - 1.4 J_1 - 0.9333 \sigma T_3^4$$

$$3.333 J_2 - 1.4 J_1 = 48334 \text{ W/m}^2 \quad (2)$$

c) From (2):

$$J_1 = \frac{3.333J_2 - 48334}{1.4}$$

Substituting in (1)

$$2.5 \times \frac{3.333J_2 - 48334}{1.4} - 0.9J_2 = 45545 \text{ W/m}^2$$

$$J_2 = 26099 \text{ W/m}^2$$

The net radiative flux to surface 2 is given by

$$q_2 = \frac{\frac{E_{b,2} - J_2}{1 - \varepsilon_2}}{\varepsilon_2} = \frac{\frac{56.7 \times 10^{-9} \times 700^4 - 26099}{1 - 0.3}}{0.3} = -5.351 \times 10^3 \text{ W/m}^2$$

The minus sign indicates a net influx of radiative transfer as would be expected from consideration of surface temperatures.

Tutorials

Tutorial 01

Exercise 1:

One side of a 3 cm thick copper plate is maintained at 400°C and the other side at 100°C.

1. How much heat is transferred per unit area through the plate?

Data : copper thermal conductivity is 370 W/m²°C.

Exercise 2:

Air at 20°C blows on a 50 x 75 cm hot plate maintained at 250°C. The heat transfer coefficient by convection is 25 W/m² °C.

1. Calculate the heat transfer transmitted by convection.

Exercise 3:

Two infinite black plates at 800°C and 300°C exchange heat by radiation.

1. Calculate the heat transfer per unit area.

Exercise 4:

An electrical resistor is connected to a battery, as shown in the diagram. After a brief transient fluctuation, the resistance takes an almost uniform equilibrium temperature of 95°C, while the battery and connection cables remain at an ambient temperature of 25°C. The thermal and electrical resistance of the connection cables is negligible. If the electrical energy dissipates uniformly in the cylindrical resistance of diameter $D = 60$ mm and length $L_r = 25$ mm.

- 1) What is the heat flux generated per unit of volume g_{vol} in W/m³
- 2) What is the coefficient of convection that it should have to evacuate all the heat if the radiation resistance is neglected?

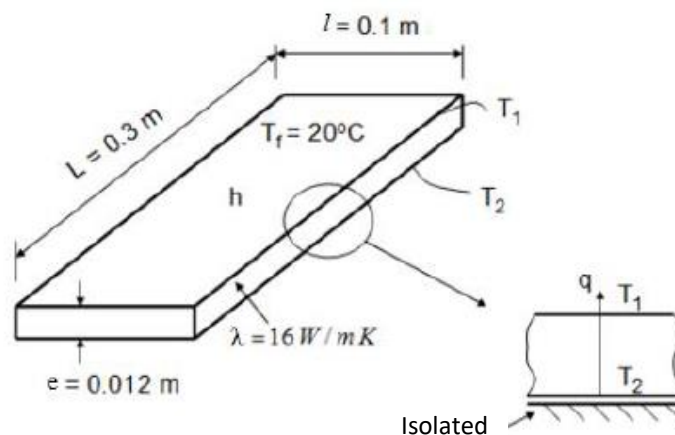
Data: $V_1 = 24V$, $I = 6A$, $L_r = 25mm$, $D_r = 60mm$, $T_w = 95$ °C, $T_\infty = 25$ °C

Exercise 5:

It is necessary to calculate the heat flux lost by a man in an environment where the temperature of the walls is $T_p = 27^\circ\text{C}$ and the environment is $T_\infty = 20^\circ\text{C}$ if the human being has a surface temperature of $T_h = 32^\circ\text{C}$ and a coefficient of heat transfer by convection between man and the environment and the emissivity of $h_h = 3\text{W/m}^2$ $^\circ\text{C}$, $\varepsilon = 0,9$ respectively, it is known that a normal human being has a body surface area of $S_h = 1,5\text{ m}^2$ neglecting the thermal resistance of clothing. Also calculate the energy lost in $t = 24$ hours.

Exercise 6:

A plate of 0.3 m in length, 0.1 m in width and 12 mm in thickness is made of stainless steel ($\lambda = 16\text{ W/m.K}$), the upper surface is exposed to a flow of air at a temperature of 20°C . In an experiment, the plate is heated by an electric heater (also 0.3 m x 0.1 m) positioned on the underside of the plate and the temperature of the plate adjacent to the radiator is maintained at 100°C . A voltmeter and ammeter are connected to the radiator and they display respectively 200 V and 0.25 A. Assuming that the plate is perfectly insulated from all sides except the top surface, what is the coefficient of heat transfer by convection?





Tutorial 02

Exercise 1

We consider a concrete wall with dimensions **3 m** in height, **5 m** in length, and **0.20 m** in thickness. The inner surface of the wall is at a temperature of **20°C**, while the outer surface is at **5°C**.

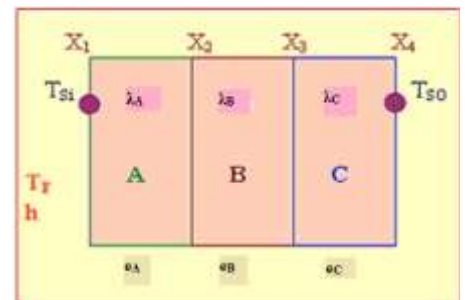
1. Calculate the heat flow that passes through this wall at a perpendicular angle, in steady state.
2. We now insulate this wall by adding a layer of glass wool **8 cm** thick and a plasterboard **2 cm** thick.
 - Calculate the new value of heat flow through the insulated wall.
 - Thermal conductivity of concrete: **0.92 W/ (m°C)**
 - Thermal conductivity of the plaster: **0.50 W/ (m. °C)**
 - Thermal conductivity of glass: **0.04 W/ (m°C)**

Exercise 2

A furnace wall is composed of three materials. **Material A** has a known thermal conductivity of $\lambda_A=20\text{W/mK}$ and a thickness of $e_A=0.3\text{m}$. **Material C** has a known thermal conductivity of $\lambda_C=50\text{W/mK}$ and a thickness of $e_C=0.15\text{m}$. **Material B**, which is located between materials A and C, has a thickness of $e_B=0.15\text{m}$, but its thermal conductivity λ_B is unknown. Under steady-state operating conditions, the measured temperatures are an external surface temperature of $T_{S0}=20^\circ\text{C}$, an internal surface temperature of $T_{Si}=600^\circ\text{C}$, and a surrounding temperature of $T_\infty=800^\circ\text{C}$. The internal convection heat transfer coefficient is $h=25\text{W/m}^2\cdot\text{K}$.

Determine the value of λ_B ?

Given Data: $\lambda_A = 20 \text{ W/m}\cdot\text{K}$, $\lambda_C = 50 \text{ W/m}\cdot\text{K}$, $\lambda_B = ?$, $e_A = 0.3 \text{ m}$, $e_B = 0.15 \text{ m}$, $e_C = 0.15 \text{ m}$, $T_F = 800^\circ\text{C}$, $T_{Si} = 600^\circ\text{C}$, $T_{S0} = 20^\circ\text{C}$, $h = 25 \text{ W/m}^2\cdot\text{K}$.



Exercise 3

A furnace wall with a surface area of **1m²** is composed of two layers:

- **First layer:** Refractory brick with a thickness of **L₁ = 0.20 m** and thermal conductivity **$\lambda_1 = 1.38 \text{ W/(m}\cdot^\circ\text{C)}$**
- **Second layer:** Insulating brick with a thickness of **L₂ = 0.10 m** and thermal conductivity **$\lambda_2 = 0.17 \text{ W/(m}\cdot^\circ\text{C)}$**

Calculate:

1. The thermal resistance of each layer
2. The total thermal resistance of the wall
3. The inner wall temperature, given that the outer temperature is **30°C** and the heat loss is **1000 W**



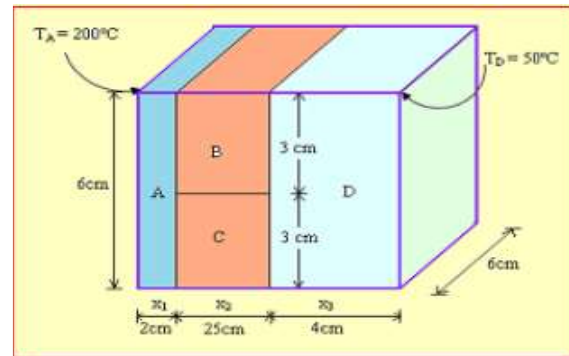
Tutorial 03

Exercise 1:

A composite wall section with the dimensions shown in the figure below. The uniform temperatures of 200 °C and 50 °C in the left and right surfaces respectively. If the thermal conductivity of wall materials is:

$$\lambda_A = 70 \text{ W/mK} ; \lambda_B = 60 \text{ W/mK} ;$$
$$\lambda_C = 40 \text{ W/mK} ; \lambda_D = 20 \text{ W/mK}.$$

Data: $S_A = S_D = 36 \times 10^{-4} \text{ m}^2$
 $S_B = S_C = 18 \times 10^{-4} \text{ m}^2$



- 1) Give the equivalent electrical diagram and calculate the equivalent thermal resistance.
- 2) Determine the heat transfer flow through this section of the wall and the temperatures at the contact surfaces.

Exercise 2:

The water flows through a cast steel pipe ($\lambda = 50 \text{ W/mK}$) with an outside diameter of 104 mm and a wall thickness of 2 mm.

- 1) Calculate the heat flow lost per unit length, in case of an uninsulated pipe when water temperature is 15 °C, outside air temperature is -10 °C, the water-side heat transfer coefficient is 30 kW/m²K and the external heat transfer coefficient is 20 W/m²K.
- 2) Calculate the heat flux lost when the pipe is covered with insulation having an outside diameter of 300 mm and a thermal conductivity of $\lambda = 0.05 \text{ W/mK}$.

Exercise 3:

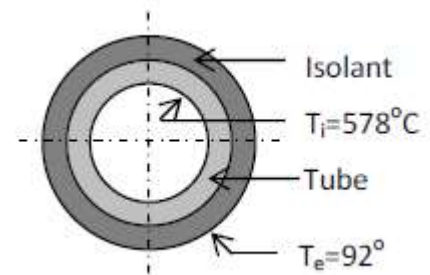
Water at 80°C is pumped through 100 m of stainless steel pipes, $\lambda = 16 \text{ W/m.K}$ of 47 mm inner and 50 mm outer radii respectively. The coefficient of heat transfer due to water is 2000 W/m².K. The surface of the pipe loses heat by convection in air at 20°C and the heat transfer coefficient is 200 W/m².K.

- 1) Calculate the heat flow through the pipe.
- 2) Also calculate the heat flow through the pipe when an insulation layer, $\lambda = 0.1 \text{ W/m.K}$ and a radial thickness of 50 mm is wound around the pipe.

Exercise 4:

A stainless steel tube ($\lambda = 19 \text{ W/m.C}$) with 3cm of inner diameter and 5cm of outer diameter is insulated by an asbestos layer ($\lambda = 0.2 \text{ W/m.C}$) of 2.5cm thickness (see figure below). Knowing that the tube inner wall is maintained at 578 °C and the insulation outer wall at 92°C.

- 1) Calculate the heat flow lost per meter of length.
- 2) Give the equivalent electrical diagram
- 3) What is the importance of using the analogy between Thermal and electrical quantities?



Tutorial 04

Exercise 1:

Calculate the time required for a small 16°C cast aluminum plate to be heated to 510°C using blast furnace gases at 1204°C .

We give: $C = V/S = 15 \text{ cm}$, $h = 85 \text{ W/m}^2\text{K}$, $\lambda = 210 \text{ W/m.K}$, $\rho_{\text{al}} = 2700 \text{ Kg/m}^3$ and $C_{\text{al}} = 940 \text{ J/Kg.K}$.

Exercise 2:

To warm the baby's milk, the mother pours it into a small glass wall with a diameter of 6 cm. The height of the milk in the glass is 7 cm. She then places the glass in a large pot filled with hot water (60°C). The milk is constantly stirred, ensuring that its temperature remains constant at all times. Determine how long it would take for the milk to warm up from 3°C to 38°C if the heat transfer coefficient between water and glass is $120 \text{ W/m}^2 \cdot \text{C}$. Assume that the properties of milk are the same as those of water.

- Why can milk be treated as a small thermal environment under these conditions?

Data: The thermal conductivity, density and specific heat of water are :

$\lambda = 0.607 \text{ W/m} \cdot ^{\circ}\text{C}$, $\rho = 998 \text{ kg/m}^3$ and $C_p = 4.182 \text{ kJ/kg} \cdot ^{\circ}\text{C}$.

Exercise 3:

In a cylinder of 4 cm diameter, air flows at an average speed of 26.5 m/s.

Calculate the heat transfer coefficient h **knowing that:**

$\rho = 1,2 \text{ kg/m}^3$, $C_p = 0,24 \text{ kcal/kg} \cdot ^{\circ}\text{C}$, $\eta = 1,9 \cdot 10^{-5} \text{ Pa.s}$, $\lambda = 6,2 \cdot 10^{-6} \text{ kcal/m.s} \cdot ^{\circ}\text{C}$, $\text{Nu} = 0,023 \cdot \text{Re}^{0,8} \cdot \text{Pr}^{0,4}$



Exercise 4:

A full copper bar of **1cm** in diameter and a length of **10cm** is cooled by sweeping it by a current of Helium, previously cooled to 77K and which strikes it perpendicularly with an average flow rate of 54m/s. The wall temperature of the copper rod is then set at 80K.

We deduce the heat release (in W/g) which occurs in the copper bar.. The characteristics of helium at the temperature considered:

- $\rho = 0,65 \text{ kg/m}^3$ $\lambda = 0,06 \text{ W/ (m.K)}$
- $\mu = 8,5 \cdot 10^{-6} \text{ kg/(m.s)}$ $C_p = 5300 \text{ J / (kg.K)}$

1. Calculate the Reynolds number and the Nusselt number.
2. Calculate the coefficient of convection h and the convective heat flux.
3. Calculate the copper mass and heat in W/g.



Tutorial 05

Exercise 01:

To heat a room, a cylindrical radiator with a diameter $D=2$ cm and length $L=0.5$ m is used. The radiator behaves as a blackbody and emits a power of 1 kW. Convection and conduction heat transfers are negligible.

1. Calculate the temperature of the radiator.
2. Determine the wavelength at which the spectral energy density emitted by the radiator is maximum.
3. What should the radiator's temperature be for this wavelength to be $2\text{ }\mu\text{m}$? What would then be the power emitted?

Given: Stefan-Boltzmann constant $\sigma = 5.67 \times 10^{-8} \text{ W} \cdot \text{m}^{-2} \cdot \text{K}^{-4}$.

Exercise 02:

After sunset, a person standing nearby can sense the energy radiated by a brick wall. Such a wall often has a temperature of 45°C and an emissivity of approximately 0.9.

1. What is the radiated thermal flux per unit area from such a wall?
2. What is the wavelength corresponding to the maximum radiated energy?
3. What type of radiation is this?

Heat Transfer Course Summary

This document serves as a pedagogical support for the course **Heat Transfer**, designed for **second-year students in Renewable Energy**. The aim of this course is to provide students with a comprehensive understanding of the mechanisms, principles, and applications of heat transfer in various energy systems. As heat transfer plays a fundamental role in renewable energy technologies—such as solar thermal systems, geothermal energy, biomass conversion, and energy storage—mastering its theoretical and practical aspects is essential for future engineers and researchers in this field.

The course introduces students to the three fundamental modes of heat transfer: **conduction**, **convection**, and **radiation**. Through this teaching aid, learners will be guided to:

1. **Derive and interpret the governing equations** that describe the behavior of heat transfer in different physical situations, ensuring a strong theoretical foundation.
2. **Develop analytical and numerical solutions** for problems involving **heat conduction and convection**, emphasizing both mathematical rigor and practical engineering relevance.
3. **Analyze and solve radiative heat transfer problems** using analytical approaches, allowing students to understand thermal interactions between surfaces and within enclosures.

In addition to theoretical content, this document includes examples, exercises, and problem-solving strategies aimed at reinforcing conceptual understanding and enhancing problem-solving skills. It is intended to serve both as a **reference material** during lectures and as a **self-study resource**, helping students connect fundamental heat transfer concepts to renewable energy applications and real-world systems.

Keywords: Heat transfer; Renewable energy; Conduction; Convection; Radiation; Solar thermal; Geothermal; Biomass; Energy storage; Thermal analysis.
