



Order N°:.....

MOHAMED BOUDIAF UNIVERSITY- M'SILA
MATHEMATICS AND COMPUTING FACULTY

Computer Science Department

DISSERTATION

Submitted in Partial Fulfillment of The Requirements
for The Degree of MASTER

Field: Computer Sciences

Major: Networks

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Theme

Rigidity Test Using The Pebble Game
Distributed Implementation

Discussed publicly on 31st may 2016 by the jury:

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June 2016

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The Rigidity theory has a long history. It begun since the 18th century, we found some scientists talked about the closed spatial figure[12]. In the 19th century, Maxwell [23] was studying whether structures are stable or deformable, and a host of other engineers who worked on bridges and mechanical linkages [10].

Rigidity theory for plane bar and joint frameworks (2D) have known a mile stone in the second half of the 20th century. Henneberg introduced important inductive technique so as to construct planar bar and joint frameworks [17]. Henneberg's techniques are still widely used, particularly to prove and analyze rigidity and flexibility of such structure [36].

The Rigidity theory in modern era has known a remarkable advancement with G.Laman who introduced a theorem in 1970 [19] which brought the rigidity problem completely to the set of combinatorial problem.

In this combinatorial characterization of rigidity, one can simply count the vertices and edges in a graph in order to determine its flexibility or rigidity. Even though the ancient scientists and other mechanical engineers [12], were counting vertices and edges to evaluate rigidity of a given graph.

Since that time, there has been an evolving interest in this fascinating subject, and our understanding and characterization of rigid and flexible structures has remarkably grown over the past four decades.

Researchers are continuing to apply rigidity theory in many field such as computer-aided design (CAD), molecular biology, glass network flexibility predictions, sensor networks (localization, communication), and linkages in mechanical engineering [26,28], and host of others.

Throughout our work in this research paper, we will focus on testing the combinatorial graph rigidity of the structures using the famous pebble game algorithm in totally decentralized systems. So, the asynchronous decentralization of this algorithm.

The aim of this dissertation is the distributed implementation of the pebble game algorithm in order to evaluate rigidity of a distributed system using threads, inter-process communications techniques in asynchronous message passing.

To achieve this aim we have to put an outline. So, our work will be divided into three major chapters.

The first chapter contains general overview and background of rigidity theory, including the basic notions of graphs, frameworks, rigidity theory, the pebble game algorithm, some theorems for counting the combinatorial properties, and some details about the sufficient and

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necessary condition for 2D and 3D, as we will talk about the famous open problem of Double Banana in 3D.

The second chapter will show the interest of rigidity theory in various application such as Architecture, Protein, Computer-Aided Design, Bridges, Glass frameworks, Chemistry, as we will focus on some examples of the needle towers.

The third chapter contains a detailed description of the distributed implementation of the pebble game algorithm using java threads. As we will mention some detail about java thread operations and synchronization, and at last we will mention the detailed execution steps of a given graph.

At the end, we will conclude a general conclusion about the whole work, then we will mention the list of references used in our research.

CONCLUSION

In this dissertation, we have presented the famous Rigidity Theory and its importance, its applications, and the adequate algorithm for evaluating rigidity and flexibility.

The notion of Flexibility is as important as Rigidity in some areas such as protein, both notions are important, when evaluating rigidity, rigidity is evaluating simultaneously.

We showed the major application areas of this notion such as Biology, Architecture and various CAD applications. We shed light on some important applications.

For evaluating rigidity quickly in 2D, we have presented the $2|V|-3$ pebble game algorithm in its two versions, the centralized and the distributed ones, which counts $2|V|-3$ independent edges to be rigid. It was an important task to offer detailed illustrations of the basic pebble operation which is covering any edge in the structure to be studied.

The pebble game algorithm gives us one of the most important information which is the Degree of Freedom (DOF), that can be concluded from the number of free pebbles remained once the rigidity have been evaluated, i.e. at the end of the execution of this algorithm, the DOF equals the remaining Free Pebbles.

We have mentioned several important theorems which will increase our knowledge and understanding of the pebble game algorithm. It is very important to understand the details of the pebble game algorithm and know some additional properties and useful extensions for having a closer look, and be able to correct or propose versions.

The overall strategy and objective of this dissertation was to implement the distributed pebble game algorithm for testing the combinatorial rigidity theory, with the underlying motivation that these algorithms can be used to address important applications and problems in many fields.

The decentralization of the aforementioned algorithm (the centralized version of the pebble game for 2D proposed in 1997 by D. J. Jacobs and B. Hendrickson) proposed by Ryan K. Williams et al is based on asynchronous inter-agent message-passing and a distributed memory architecture, coupled with algorithm auction for electing leaders to lead during covering a specific edge.

In the auction algorithm we used, we proposed a combination of two politics, the first one, storing the neighbors of the current leader in FIFO order, then we elect the neighbor who has the maximum number of neighbors from this list, in fact it is logic, to be elected as leader you have to be the most supported.

The decentralization of the pebble game algorithm is based on seven algorithms: the auction algorithm, the initialization of the agent in the constructor of the thread, the leader run, the message request algorithm, the message found and the not found message algorithm.

CONCLUSION

Our implementation was done using Java threads, which is the ideal simulation of the distributed algorithms, car in one hand, fault tolerance and message alteration is null, in other hand, the communication between the threads was done using inter-communication processes politics in asynchronous messaging mode. i.e shared handler.

During this research, we proved that the pseudo algorithm published in the used article is not detailed enough to be implemented. It needs more efforts.

We found that this problem is one of the most curious domain, beside to its open problems such as the double banana in 3D. So, the gate is still open to the research and towards the application of this algorithm in many fields of research. Here is the entrance to another researcher.

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ملخص

لقد تساءل جيمس كلارك ماكسويل ما إذا كان حساب عدد العقد وعدد الروابط في شبكة ما كافٍ لمعرفة تماسك البنية أم ليوثتها. بعدها أجابه ليمان وأكد ذلك بأن هذا المشكل في المستوي هو مشكلة حسابية محضة بمعنى الحساب كافٍ. شرط حساب $2|V|-3$ في المستوي يمكن حسابه بسرعة باستعمال خوارزم لعبة الحجيرات الذي اقترح له هاندريكسون وجايبكوس نسخته الممركزة وبعده اقترح مجموعة من الباحثين النسخة الموزعة للخوارزم. في هذه المذكرة طبقنا النسخة الموزعة لخوارزم لعبة الحجيرات باستعمال التواصل غير المتواقت للرسائل وباستعمال برنامج البرمجة جافا واستعملنا لذلك العمليات الجزئية وطرق التواصل بينها. لنصل في الأخير إلى الهدف المنشود.

الكلمات المفتاحية : Rigidity, Flexibility, Laman graph, pebble game algorithm.

Abstract

James Clark Maxwell wondered: counting vertices and edges in a given framework is enough to make predictions about its rigidity and flexibility ?

Laman in 1970 confirmed that the 2d generic bar and joint frameworks is completely combinatorial problem. i.e counting is enough to decide.

The $2|V|-3$ counting condition for 2d structures verified by the fast pebble game algorithm which tracks quickly this count. Hendrickson and Jacobs proposed the centralized pebble game version in 1997. In 2014, group of researchers proposed the decentralized version in IEEE.

In this thesis, we implement the asynchronous, distributed algorithm of the pebble game for testing the rigidity in 2d framework, using java threads and interprocess communications protocols.

We have also introduced several rigidity theory applications, we will apply our algorithm on some 2d structures. The results obtained in this Master thesis showed that our distributed implementation is well done and worked.

Keywords: Rigidity, Flexibility, Laman graph, pebble game algorithm.

Résumé

James Clark Maxwell pose une question : Compter le nombre des arcs et des sommets d'un certain graphe est suffisant pour tester sa rigidité ?

Laman en 1970 affirma que en 2d, une structure bar-joint générique est totalement un problème combinatoire i.e compter est suffisant pour évaluer.

La condition $2|V|-3$ pour les structures 2d vérifiée par l'algorithme de pebble game qui est le plus rapide testeur. Hendrickson et Jacobs proposeront la version centralisée de cet algorithme en 1997. En 2014 un ensemble de chercheurs proposeront la version décentralisée.

En cette mémoire, nous implémentons la version décentralisée et asynchrone de l'algorithme de pebble game pour tester la rigidité dans un système complètement distribué en utilisant Java threads et IPC les protocoles de la communication entre-processus.

Nous avons aussi mentionné les différents domaines d'applications de la rigidité, et enfin nous avons réalisé notre objectif.

Les mots clés: Rigidity, Flexibility, Laman graph, pebble game algorithm.