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TITLE

Asymptotic behaviour of the wave equation in domains with moving boundary

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Introduction

A partial differential equation (PDE) is an equation that contains one or more partial derivatives of an unknown function that depends on at least two variables. Hyperbolic PDEs arise in many physical applications as models of waves, such as acoustic, elastic, electromagnetic, quantum mechanics,... .The prototypical example of a hyperbolic PDE is the wave equation in one dimension describing the small vibration of a string of finite length. To begin with, consider the one-dimensional wave equation in $\mathbb{R}$,if $u(x,t)$ denotes the vertical displacement of any point of a vibrating string at position x at time t,and c is the speed of propagation of the wave, then

$$u_{tt} = c^2 u_{xx}.$$

As is well known, there exists a wide variety of literature for this problem in a cylindrical domain, i.e. both of end points of the string are fixed, see for instance see [7, 11, 12]. In practical situations, many processes evolve in domains whose boundary has moving parts, a simple model, i.e. is a heat process in a combustion chamber where a piston is attached. Part of the boundary moves with the motion of the piston. In contrast with the cylindrical case, there are few works on the wave equations defined in non-cylindrical domains, see for instance [1, 2, 8] and the references therein.

The main purpose of this work is to study the asymptotic behavior in time of the energy for the wave equation in a one-dimensional in the interval $]0, \alpha_k(t)[$ where $\alpha_k(t)$ is an increasing function. We obtain the decay of the energy using the multiplier method (energy integrals). When $\alpha_k(t) = 1 + kt$, we detail the paper of H SUN,H LI, L LU [10] then we generalize their result by considering a wave equation with a speed $c = c(t)$ depending in time. Moreover we consider also the case where $\alpha_k(t)$ is not linear.

This work contains fours chapters:

In the first chapter we derive the model of vibrating string then recall some results related to the wave equation in cylindrical and non cylindrical domains. In the second chapter we study the energy of the wave equation in $\Omega_t =]0, 1 + kt[$ then consider a wave equation with variable speed. In next chapter we consider a more general variable domains without transforming the problem to a cylindrical one as it is the case in [5]. In the last chapter, the theoretical results will be illustrated through some examples and their corresponding graphs. The numerical results are given using the penalty method due to LIONS [7].

Finally we conclude with a conclusion and some references on the subject.

Conclusion

In this work, we studied the vibrating string with a variable length which is described by the following wave equation in a non-cylindrical domain

$$\begin{cases} u_{tt} - c^2 u_{xx} = 0 & & \text{in } \hat{Q}_T^{\alpha}, \\ u(0,t) = 0 & u(\alpha(t),t) = 0 & \text{on } (0,T), \\ u(x,0) = u^0, & u_t(x,0) = u^1 & \text{in } (0,1). \end{cases}$$

Where $\alpha(t)$ is a increasing smooth function such that $0 < \alpha'(t) < c$. We show the decay of the energy of the wave with time in both cases when α is linear or not and also when the speed of the wave is varying with time, i.e. $c = c\,(t)$. Moreover, some numerical results illustrating our theoretical results was obtained.

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الملخص:

في هذا العمل نهتم بدراسة السلوك المقارب بالنسبة للزمن لإهتزاز حبل يملك طرفا ثابتا وآخر متحرك . باستخدام طريقة الجداءات (تكاملات الطاقة) ،نبين أن طاقة الحبل تتناقص مع الزمن ، كما أخذنا في الاعتبار الحالات التي تكون فيها سرعة الموجة ثابتة أومتغيرة ،من أجل أنواع مختلفة من الحركة للطرف غير الثابت للحبل، بالإضافة الى ذلك تحصلنا على بعض النتائج العددية باستخدام طريقة المعاقبة .

الكلمات المفتاحية: معادلة الموجة، مجال غير اسطواني ، طريقة الجداءات ،المعاقبة .

Résumé :

Ce travail est consacré à l'étude du comportement asymptotique dans le temps d'une corde vibrante avec une extrémité fixe et une autre variable avec le temps. En utilisant la méthode de multiplication (les intégrales d'énergie), on montre que l'énergie est décroissante en temps. Nous avons examiné les cas d'une vitesse constante ou variable de l'onde pour différents types de mouvement de l'extrimité variable de la corde. De plus quelques résultats numériques sont obtenus en utilisant la méthode de pénalisation.

Mots clés: ***Equation d'onde, domaine non-cylindrique,solutions sous forme de séries.***

Abstract:

This work is devoted to the study the asymptotic behavior in time of a vibrating string with a fixed end-point and the other is a moving one. Using the multiplier method (energy integrals) we show that the energy of the string decay in time. We have considered both cases of waves with constant and variable speed and different motions of the variable end-point. Moreover some numerical results are obtained using the penalization method.

Keywords: ***wave equation, non-cylindrical domains, multiplier method, penalization.***