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Numerical treatment of the nonlinear Fredholm integral equation by Newton-Kantorovich method

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Dedication

I dedicate this humble work to my dearest mother in the first place *Nassira Brabri*

to my dear father

to my dear and only brother Yaakoub

to my husband

to all the family Haffaf and to all the family Brabri

I thank everyone who encouraged and helped me and was not stingy

List of symbols

X, Y	Banach spaces
(X, d)	Metric space
$\mathbb{R}^n$	Set of n-tuples $x = (x_1, x_2, ..., x_n)$
A	Integral operator
φ	Unknown function in the integral equation
$K\left(x, y, \varphi\left(y\right)\right)$	Kernel of the integral equation
$C\left(\left[a, b\right], \mathbb{R}^n\right)$	Set continuously differentiable functions $\varphi : [a, b] \to \mathbb{R}^n, ([a, b] \subset \mathbb{R}^n\text{open})$
$NFIE$	Nonlinear Fredholm integral equation
NK	Newton-Kantorovich method
$NKMS$	combination of Newton-Kantorovich and modified Simpson method

Introduction

Nonlinear equation appears in many scientific areas such as physics, fluid mechanics, population models, chemical kinetics, economic systems, and medicine and can be modeled by nonlinear integral equation. The difficulty lies in finding the exact solution for such system. Alternatively, the approximte or numerical solutions can be sought. One of the well known approximate method is Newton-Kantorovich method which reduces the nonlinear into sequence of linear integral equations. The approximate solution is then obtained by processing the convergent sequence. In 1939, Kantorovich presented an iterative method for functional equation in Banach space and derived the convergence theorem for Newton method. In 1948, Kantorovich proved a semilocal convergence theorem for Newton method.

An integral equation is defined as an equation in which the unknown function φ to be detetermined appear under the integral sign. Many initial and boundary value problems associated with ordinary differential equation (ODE) and partial differential equation (PDE) can be transformed into problems of solving some approximate integral equations.

A general form of an integral equation in $\varphi\left(x\right)$ is of the form

$$\varphi\left(x\right)=f\left(x\right)+\lambda\int_{\alpha(x)}^{\beta(x)}K\left(x,y,\varphi\left(y\right)\right)dy$$

where $K(x,y)$ is called the kernel of this integral equation, $\alpha\left(x\right)$ and $\beta\left(x\right)$ are the limits of integration. It is to be noted here that both the kernel $K\left(x,y\right)$ and the function $f\left(x\right)$ in the integral equation are given functions, and λ is a constant parameter.

The aim of this work is to solve numerically the nonlinear Fredholm integral equation by Newton-Kantorovich and modified Simpson method.

The first chapter will present few basic concepts from general theoretical framework, such as compactness in Banach space and different theory of fixed point.

The second chapter contains some classification of linear and nonlinear integral equations, and the existence and uniqueness theorem of the nonlinear Fredholm integral equation.

The last chapter present the aim of this work, which is studying nonlinear Fredholm integral equation, we well use what we had seen in the previous chapters, then we approximate the solution using Newton-Kantorovich method, also we give some examples solving by this method and modified Simpson method. For more details, see (([1]) , ([3]) , ([4]) , ([5]) , ([6])) and others.

Chapter 1

Basic concepts

The purpose of this chapter is to present some notations that will be used througout this work, and we state some difinitions and results from the literature that will be required later. We recall some notions of compactness in Banach space, and we mention some fundamental theorems such that the Arzela-Ascoli theorem and fixed point theorems.

1.1 Compactness

Definition 1.1. ***(Compact operator).***

Let X, Y be two banach space and let S be a subset of X. We say that $T{:}S \to Y$ is compact, if it is continuous and for every bounded set $B \subseteq S$, the set $\overline{T(B)}$ is compact in Y.

Definition 1.2. *(Completely Continuous Operator)*

Let X be a Banach space. An operator $T : X \longrightarrow X$ is called totally bounded if for every bounded subset S of X, $T\left(S\right)$ is compact. Moreover, T is said to be completely continuous over X if it is continuous and totally bounded over X.

Definition 1.3. *Let X and* **Y** *be a Banach space. An operator $T \in \mathcal{L}(X, Y)$ is called comact (or completely continous) if T takes bounded sets in X into precompact sets in Y. equivalently T is compact if and only if for every bonded sequence $(x_n) \subset X T\left(x_n\right)$ has a subsequence convergent in Y.*

Definition 1.4. *(Nonlinear integral operator)*

A nonlinear integral operator T is an operator that admits a formulation of the form

$$\left(T\varphi\right)x = \int_a^b K\left(x, y, \varphi\left(y\right)\right)dy$$

The fonction K called the kernel of the operator.

Remark 1.5. *if K is a continuous function of $[a, b] \times [a, b]$, the operator T is called an integral operator with continuous kernel K.*

Theorem 1.6. *[4](Arzela-ascoli)*

$G \subset C\left(\Omega\right)$ then G is relatively compact if G is bounded and G is equicontinous set

1. *G bounded: $\forall \varphi \in G$, $\forall x \in \Omega$, $\exists M > 0$ such that $\left|\varphi\left(x\right)\right| \leq M$*

2. *G equicontinous: $\forall \epsilon > 0$, $\forall \varphi \in G$, $\forall x, y \in \Omega$, $\exists \delta > 0$ such that*

$$|x - y| < \delta \Longrightarrow \left|\varphi\left(x\right) - \varphi(y)\right| < \epsilon$$

Theorem 1.7. *[4]The integral operator A defined from $C\left(\Omega\right)$ into $C\left(\Omega\right)$*

$$A\varphi\left(x\right) \ = \int_{\Omega} K(x,y)\varphi\left(y\right) \ dy \qquad x,y\in\Omega$$

with continous kernel $K\left(x,y\right)$ is a compact operator.

Proof. [4]Let G be a bounded set of $C\left(\Omega\right)$ then for each $\varphi\in G$, there existe $M>0$, such that

$$\left\Vert \varphi\right\Vert \leq M,$$

besides, for all $x\in\Omega$ and $\varphi\in G$ we get

$$\begin{aligned} A\varphi\left(x\right) &= \left|\int_{\Omega} k\left(x,y\right)\varphi\left(y\right)dy\right| \\ &\leq \max\left|K(x,y)\right| M mes\left(\Omega\right) \end{aligned}$$

it follows that $A\left(G\right)$ is bounded
By assumption the kernel $K\left(x,y\right)$ is continous over the compact $\Omega\times\Omega$
Thus it is uniformly continous and there for
$\forall\epsilon>0,\ \exists\delta>0,\ \forall x,\ y,\ z\in\Omega,\ \left|x-y\right|<\delta\Longrightarrow\left|k\left(x,z\right)-k\left(y,z\right)\right|<\frac{\epsilon}{Mmes(\Omega)}$.
Hence, for each $\varphi\in G$ and $x,\ y\in\Omega$, with $\left|x-y\right|<\delta$

$$\begin{aligned} \left|A\varphi\left(x\right)-A\varphi\left(y\right)\right| &= \left|\int_{\Omega}\left(k\left(x,z\right)-K\left(y,z\right)\right)\varphi\left(z\right)dz\right| \\ &< \frac{\epsilon}{Mmes\left(\Omega\right)} Mmes\left(\Omega\right)=\epsilon \end{aligned}$$

This relation expresses that $A\left(G\right)$ is equicontinous. Hence $A\left(G\right)$ is relatively compact, so by Arzela-Ascoli theorem A is compact. □

Theorem 1.8. *[6]A compact linear operator is a bounded operator, the converse false.*

Definition 1.9. *[6](Relatively compact)*
Let X be a metric space, $\Omega\subseteq X$ is relatively compact in X, if $\overline{\Omega}$ is compact in X.

Theorem 1.10. *[6]Any set bounded and finite dimension of a normed space is relatively compact.*

1.2 Fixed point theorem

This section contains basic theory of fixed point we will recall some important and different theorems such that Banach fixed point theorem, Brouwer, Schauder, Schaefer, and Krasnoselskii theorems, these theorems are very useful.

Many nonlinear equations are naturally formulated as fixed-point problems

$$\varphi = T\left(\varphi\right) \tag{1.1}$$

where T is fixed point operator, may be nonlinear. A solution $\varphi^{*}$of (1.1) is called a fixed point of the map T.

Theorem 1.11. *[5]Let (X, d) be a complete metric space and let $T : X \to X$ be a contraction with Lipschitzian constant L. Then T has a unique fixed point $\varphi^{*} \in X$. Furthermore, for any $\varphi \in X$ we have*

$$\lim_{n\to\infty} T^{n}\left(\varphi\right) = \varphi^{*},$$

with

$$d\left(T^{n}\left(\varphi\right), \varphi\right) \leq \frac{L^{n}}{1-L} d\left(T\left(\varphi\right), \varphi\right).$$

Theorem 1.12. *[10]Let F be a closed subset in a Banach space and let $T : F \to F$ a contraction application, then*

(a) *The equation $Tx = x$, has an unique solution*

(b) *The unique solution x can be obtained by the limit of the sequence (x_n) of F defined by*

$$x_{n+1} = Tx_{n},\ \ n = 1, 2, ...$$

where x_0 is an arbitary element of F

$$x = \lim_{n\to\infty} T^{n}x_{0}$$

Theorem 1.13. *[6](Banach's fixed point theorem)*

Let be A a contraction on a Banach space X. Then A has a unique fixed point.

Proof. Banach's fixed point theorem implies that there exists a unique fixed point for A^N. Call this element φ_0. Now just note that

$$\begin{aligned} \|A(\varphi_0) - \varphi_0\| &= \left\|A^N(A(\varphi_0)) - A^N(\varphi_0)\right\| \\ &\leq c\,\|A(\varphi_0) - \varphi_0\| \end{aligned}$$

implies that $A(\varphi_0) = \varphi_0$ since $0 < c < 1$. The uniqueness is clear since a fixed point for A is also a fixed point for A_N. □

Theorem 1.14. *[5](Schauder's fixed point theorem)*

Let X be a Banach space and let $S \subset X$ be bounded,closed,and convex. Assumet $T : S \to S$ is a completely continuous operator. Then T has at least one fixed point in the set S.

The finite-dimensional version of Schauder's theorem, namely Brouwer's fixed point theorem.

Theorem 1.15. *[5](Brouwer's fixed point theorem)*

Let $S \subset \mathbb{R}^n$ be a nonempty convex compact set and let $T : S \to S$ be a continuous mapping. Then there existe at least one $\varphi \in S$ with $T(\varphi) = \varphi$.

Theorem 1.16. *[7](Schaefer's Fixed-point Theorem)*

Let X be a Banach space and let $T : X \to X$ be a completely continuous operator. Then either:

(i) *The operator equation $\varphi = \lambda T\varphi$ has a solution for $\lambda = 1$.*

or

(ii) *The set $\varepsilon = \{\varphi \in X;\ \varphi = \lambda T\varphi,\ \lambda \in]0,1[\}$ is unbounded.*

Theorem 1.17. *[5](Krasnoselskii's fixed point theorem)*

Assume that S is a closed bounded convex subset of a Banach space X. Furthermore assume that T_1 and T_2 are mappings from S into X such that

- *$T_1(\varphi) + T_2(v) \in S$ for all φ, $v \in S$,*
    - *T_1 is a contraction,*
    - *T_2 is continuous and compact.*

Chapter 2

Linear and nonlinear integral equation

In this chapter we present the theory of linear and nonlinear integral equations and we illustrate different criterions of classification of theses equation, also we discuss existence of solution of some kind of integral equations.

2.1 Classification

Integral equations appear in many types. The types mainly on the limits of integration and the kernel of the equation. In this section we will be concerned on the following types of integral equations.

2.1.1 Linear integral equation

1) Fredholm integral equation

For Fredholm integral equation, the limits of integration are fixed. Moreover the unknown function $\varphi(x)$ may appear only inside integral equation in the form:

$$f(x) = \int_a^b K(x,y)\,\varphi(y)\,dy.$$

This is called Fredholm integral equation of the first kind. However, for Fredholm integral equation of the second kind, the unknown function $\varphi(x)$ appears inside and outside the integral sign. The second kind is represented by the form:

$$\varphi(x) = f(x) + \lambda \int_a^b K(x,y)\,\varphi(y)\,dy.$$

2) Volterra integral Equation

In Volterra integral equations, at least one of the limits of integration is a variable. For the first kind Volterra integral equations, the unknown function $\varphi(x)$ appears only inside integral sign in the form:

$$f(x) = \int_0^x K(x,y)\,\varphi(y)\,dy.$$

However, Volterra integral equations of the second kind, the unknown function $\varphi(x)$ appears inside and outside the integral sign. The second kind is represented by the form:

$$\varphi(x) = f(x) + \lambda \int_0^x K(x,y)\,\varphi(y)\,dy.$$

3) Volterra-Fredholm integral Equation

The Volterra-Fredholm integral equation arise from parabolic boundary value problems, from the mathematical modelling of the spatio-temporal development of an epidemic, and from various physical and biological models. The Volterra-Fredholm integral equation appear in the literature in two forms, namely

$$\varphi(x) = f(x) + \lambda_1 \int_a^x K_1(x,y)\,\varphi(y) + \lambda_2 \int_a^b K_2(x,y)\,\varphi(y)\,dy, \tag{2.1}$$

and

$$\varphi(x,y) = f(x,y) + \lambda \int_0^y \int_\Omega F(x,y,\xi,\tau,\varphi(\xi,\tau))\, d\xi d\tau, (x,y) \in \Omega \times [0,T] \qquad (2.2)$$

where $f(x,y)$ and $F(x,y,\xi,\tau,\varphi(\xi,\tau))$ are analytic functions on $D = \Omega \times [0,T]$, and Ω is a closed subset of $\mathbb{R}^n$, $n = 1,2,3$. It is interesting to note that (2.1) contains Volterra disjoint and Fredholm integral equations, whereas (2.2) contains mixed Volterra and Fredholm integral equations. Moreover, the unknown functions $\varphi(x)$ and $\varphi(x,y)$ appear inside and outside the integral signs.

4) Sigular integral equations

Volterra integral equations of the first kind

$$f(x) = \lambda \int_{g(x)}^{h(x)} K(x,y)\,\varphi(y)\,dy,$$

or of the second kind

$$\varphi(x) = f(x) + \int_{g(x)}^{h(x)} K(x,y)\,\varphi(y)\,dy,$$

are called singular if one of the limits of integration $g(x)$, $h(x)$ or both are infinite. Moreover, the previous two equations are called singular if the kernel $K(x,y)$ becomes unbounded at one or more points in the interval of integration. In this section we will focus our concern on equations of the form:

$$f(x) = \int_0^x \frac{1}{(x-y)^\alpha}\varphi(y)\,dy, 0\langle\alpha\langle 1.$$

or of the second kind:

$$\varphi(x) = f(x) + \int_0^x \frac{1}{(x-y)^\alpha}\varphi(y)\,dy, 0 < \alpha < 1.$$

The last two standard forms are called generalized Abel's integral equation and weakly singular integral equations respectively. For $\alpha = \frac{1}{2}$, the equation:

$$f(x) = \int_0^x \frac{1}{\sqrt{x-y}}\varphi(y)\,dy,$$

is called the Abel's singular integral equation. It is to be noted that the kernel in each equation become infinity at the upper limit $y = x$.

5) Fredholm integro-differential Equations

Fredholm integro-differential equations appear when we convert differential equations to integral equations. The Fredholm integro-differential contains the unknown function $\varphi(x)$ and one of its derivatives $\varphi^{(n)}(x)$, $n \geq 1$ inside and outside the integral sign respectively. The limits of integration in this case are fixed as in the Fredholm integral equations. The equation is labeled as integro-differential because it contains differential and integral operators in the some equation. It is important to note that initial conditions shoud be given for Fredholm integro-differential equation appears in the form:

$$\varphi^{(n)}(x) = f(x) + \lambda \int_a^b K(x,y)\,\varphi(y)\,dy,$$

where $\varphi^{(n)}$ indicates the nth derivative of $\varphi(x)$. Other derivatives of less order may appear with $\varphi^{(n)}$ at the left side.

2.1.2 Nonlinear integral equations

1) Fredholm integral equations

The nonlinear Fredholm integral equations of the second kind are characterized by fixed limits of integration of the form

$$\varphi(x) = f(x) + \lambda \int_a^b K(x,y,\varphi(y))\,dy, \tag{2.3}$$

where the unknown function $\varphi(x)$ occurs inside and outside the integral sign, λ is a parameter, and a and b are constants. For this type of equation the kernel $K(x,y)$ and the function $f(x)$ are given real valued function.

The nonlinear Fredholm integral equation of the first kind is given by the form:

$$f(x) = \int_a^b K(x,y,\varphi(y))\,dy,$$

where the kernel $K(x,y)$ and the function $f(x)$ are given real valued functions.

2) Volterra integral equations

The nonlinear Volterra equation are characterized by at least one variable limit of integration. In the nonlinear Volterra integral equations of the second kind, the unknown function $\varphi(x)$ appears inside and outside the integral sign, and it is represented by the form

$$\varphi(x) = f(x) + \int_0^x K(x,y,\varphi(y))\,dy.$$

However, the nonlinear Volterra integral equation of the first kind is expressed in the form

$$f\left(x\right)=\int_{0}^{x}K\left(x,y,\varphi\left(y\right)\right)dy.$$

For these two kinds, the kernel $K\left(x,y\right)$ and the function $f\left(x\right)$ are given real valued functions.

3) Fredholm Integro-Differetial Equations

The nonlinear Fredholm integro-differential equation of the second kind is of the form

$$\varphi^{(n)}\left(x\right)=f\left(x\right)+\lambda\int_{0}^{1}K\left(x,y\right)\varphi\left(y\right)dy,$$

where $\varphi^{(n)}\left(x\right)$ is the nth derivative of $\varphi\left(x\right)$. The kernel $K\left(x,y\right)$ and the function $f\left(x\right)$ are given real-valued functions.

Definition 2.1. *(Uryson and Hammerstein integral equation)*

1. *An equation of the form*

$$\varphi\left(x\right)-\int_{\Omega}F\left(x,y,\varphi\left(y\right)\right)=h\left(x\right),\ y\in\Omega$$

   is called Uryson integral equation, F and h are arbitary functions.

2. *An equation of the form*

$$\varphi\left(x\right)-\int_{\Omega}k\left(x,y\right)f\left(y,\varphi\left(t\right)\right)=h\left(x\right),\ y\in\Omega$$

   is called Hammerstein integral equation.

Remark 2.2. *Hammerstein's integral equation is a special case of Uryson's integral equation.*

2.2 Existence and uniqness theorem

In this section we study nonlinear Fredholm integral equation in Banach space, we well use all what we had seen in the previous chapter, such that fixed point theorems in order to prove existence and uniqueness of solution of this kind of equations.

Theorem 2.3. *[5]Suppose that $K(x, y, \varphi)$ is defined and continuous on the set $[a, b] \times [a, b] \times \mathbb{R}$ and that it satisfies a Lipshits condition of the form*

$$|K(x, y, \varphi_1) - K(x, y, \varphi_2)| < C|\varphi_1 - \varphi_2|$$

suppose further that $f \in C[a, b]$. Then the nonlinear Urysohn integral equation

$$\varphi(x) = \lambda \int_a^b K(x, y, \varphi(y))\, dy + f(x)$$

has a unique solution on the interval $[a, b]$ whenever $|\lambda| < \frac{1}{C(b-a)}$.

Theorem 2.4. *[1](existence theorem)*

We first rewrite the nonlinear Fredholm integral equation of the second kind by

$$\varphi(x) = f(x) + \lambda \int_a^b K(x, y, \varphi(y))\, dy,$$

the specific conditions under which a solution exists for the nonlinear Fredholm integral equation are:

(i) *The function $f(x)$ is bounded, $|f(x)| < R$, in $a \leq x \leq b$.*

(ii) *The function $K(x, y, \varphi(y))$ is integrable and bounded where*

$$|K(x, y, \varphi(y))| < k$$

in $a \leq x, y \leq b$.

(iii) *The function $K(x, y, \varphi(y))$ satisfies the Lipschitz condition*

$$|K(x, y, z_1) - K(x, y, z_2)| < M|z_1 - z_2|.$$

Theorem 2.5. *[3](Existence and uniqueness of solution)*

Suppose

- $\left\| \int_a^b K(x, y, \varphi(y))\, dy \right\| \leq M\|\varphi(y)\|$,
- $|K(x, y, z_1) - k(x, y, z_2)| \leq N(x, y)|z_1 - z_2|$ *for all x, y, z_1, $z_2 \in [a, b]$.*

- $\int_a^b \int_a^b |N(x,y)|^2 \, dxdy = K^2 < \infty$,

  then the nonlinear Fredholm equation

$$\varphi(x) = \alpha \int_a^b K(x, y, \varphi(y)) \, dy + f(x) \tag{2.4}$$

has a unique solution in $L^2([a,b])$ *provided* $|\alpha| K < 1$, K *is continous and* $f \in L^2([a,b])$

Proof. consider the operator

$$T\varphi = \alpha A\varphi + f,$$

where

$$(A\varphi)(x) = \int_a^b K(x, y, \varphi(y)) \, dy,$$

then

$$\begin{aligned} \|T\varphi_1 - T\varphi_2\| &= |\alpha| \, \|K(x,y,\varphi_1(y)) - K(x,y,\varphi_2(y)) \, dy\| \\ &\leq |\alpha| \left(\int_a^b \left(\int_a^b K(x,y,\varphi_1(y)) - K(x,y,\varphi_2(y)) \right)^2 dx \right)^{\frac{1}{2}} \\ &\leq |\alpha| \left(\int_a^b \left(\int_a^b N(x,y) \, |\varphi_1(y) - \varphi_2(y)| \, dy \right)^2 dx \right)^{\frac{1}{2}} \\ &\leq |\alpha| \, K \, \|\varphi_1 - \varphi_2\|, \end{aligned}$$

clearly, if $|\alpha| K < 1$, T is contraction operator, so that it has a unique fixed point and that fixed point is a solution of the non linear equation (**??**). □

Chapter 3

Numerical methods and examples

In this chapter, we will use numerical method based on the Newton-Kantorovich method and modified Simpson method has been proposed to approximate the solution of nonlinear Fredholm integral equation, then we given an examples to tested this method.

3.1 Newton-Kantorovich method

The Newton-Kantorovich method is a well-known method for solving nonlinear integral equation. This method attempts to solve a sequence of linear integral equations. we develop a combination of the Newton-Kantorovich and modified Simpson method. This method solves the nonlinear integral equations of the Urysohn form.

Definition 3.1. *A proper functional $F : U \to V$ is said Fréchet differentiable(or strongly differentiable) at a point $\varphi \in D$, where D is an open set in U, if there is a linear operator $L : U \to U^*$ such that, for any $\varphi + h \in D$,*

$$F(\varphi + h) = F(\varphi) + \langle L(\varphi), h \rangle + \omega(\varphi, h)$$

and

$$\lim_{\|h\| \to 0} \frac{\omega(\varphi, h)}{\|h\|} = 0.$$

The quantity $\langle L(\varphi), h \rangle$ is called Fréchet differential(or strong differential) and $L(\varphi) = F'(\varphi)$ is called the Fréchet derivative(or strong derivative) of the functional F at a point φ.

Now, we will consider Newton's method for solving the equation

$$F(\varphi) = 0 \tag{3.1}$$

where $F : U \to V$ is Fréchet differentiable and U, V be two Banach spaces. This method provides a powerful tool for the theoretical as well as the numerical investigation of nonlinear operator equations. In this section will present the theory of Newton's method, together with some of its application to important types of nonlinear equation. Kantorovich proposed to solve a function equation (3.1) where F is defined and Fréchet differentiable on some open convex set of a Banach space X, with a rang in a Banach space Y.

One way to approach this problem (3.1) would be to find a linear equation

$$L\varphi = \phi \tag{3.2}$$

which approximates (3.1) in some sense, at least in a neighborhood of exact solution φ^*. It is easy to construct a linear equation (3.2) which approximates the nonlinear equation (3.1) if the operator F is differentiable. By the first two terms of Taylor's formula

$$\begin{aligned} 0 &= F(\varphi^*) \\ &= F(\varphi_n) + F'(\varphi_n)(\varphi^* - \varphi_n) + O(|\varphi^* - \varphi_n|) \\ &\approx F(\varphi_n) + F'(\varphi_n)(\varphi^* - \varphi_n). \end{aligned}$$

This appoximate solution is said to be obtained from (3.1) by the process of linearization by differentiation, sometimes called quasilinearization or the tangent method. Thus,

$$\varphi^* \approx \varphi_n - \left[F'\left(\varphi_n\right)\right]^{-1} F\left(\varphi_n\right).$$

This leads to the well-known Newton method for solving the equation (3.1):

$$\varphi_{n+1} = \varphi_n - \left[F'\left(\varphi_n\right)\right]^{-1} F\left(\varphi_n\right)$$

choose an initial guess $\varphi_0 \in U$; *for* $n = 0, 1, ...$

Theorem 3.2. *[5](Kantorovich) Suppose that :*

1) $F : D(F) \subset U \to V$ *is differentiable on an open convex set* $D(F)$, *and the derivative is Lipschitz continuous:*

$$\left\|F'(\varphi) - F'(\upsilon)\right\| \leq L \left\|\varphi - \upsilon\right\| \forall \varphi,\ \upsilon \in D(F).$$

2) *For some* $\varphi_0 \in D(F)$, $\left[F'\left(\varphi_0\right)\right]^{-1}$ *exists and is a continuous operator from* V *to* U, *and such as* $h = abL \leq \frac{1}{2}$ *for some* $a \geq \left\|\left[F'\left(\varphi_0\right)\right]^{-1}\right\|$ *and* $b \geq \left\|\left[F'\left(\varphi_0\right)\right]^{-1} F\left(\varphi_0\right)\right\|$.

Denote

$$t^* = \frac{1 - (1 - 2h)^{1/2}}{aL}, t^{**} = \frac{1 + (1 - 2h)^{1/2}}{aL}.$$

3) φ_0 *is chosen so that* $\overline{B}\left(\varphi_1, r\right) \subset D(F)$, *where* $r = t^* - b$.

Then the equation (3.1) has a solution $\varphi^* \in B\left(\varphi_1, r\right)$ *and the solution is unique in*$\overline{B}\left(\varphi_0, t^{**}\right) \subset D(F)$; *the sequence* $\{\varphi_n\}$ *converges to* φ^*, *and we have the error estimate*

$$\left\|\varphi_n - \varphi^*\right\| \leq \frac{\left[1 - (1 - 2h)^{1/2}\right]^{2n}}{2^n aL}, n = 0, 1, ...$$

Let us apply the Newton-Kantrovich method to solve a nonlinear Fredholm integral equations of the second kind (2.3). We obtain the following iteration process

$$\begin{cases} \varphi_n(x) = \varphi_{n-1}(x) + \epsilon_{n-1}(x), n = 1, 2, ... \\ \epsilon_{n-1}(x) = \xi_{n-1}(x) + \int_a^b \frac{\partial K}{\partial \varphi}\left(x, y, \varphi_{n-1}(y)\right) \varphi_{n-1}(y)\, dy \\ \xi_{n-1}(x) = f(x) + \int_a^b K\left(x, y, \varphi_{n-1}(y)\right) dy - \varphi_{n-1}(x). \end{cases} \tag{3.3}$$

we can to write (3.3) as

$$\begin{cases} \varphi_n(x) = \varphi_{n-1}(x) + \epsilon_{n-1}(x), n = 1, 2, ..., \\ \epsilon_{n-1}(x) = f(x) + \int_a^b K(x, y, \varphi_{n-1}(y))\, dy - \varphi_{n-1}(x) + \int_a^b \frac{\partial K}{\partial \varphi}(x, y, \varphi_{n-1}(y))\, \varphi_{n-1}(y)\, dy. \end{cases} \tag{3.4}$$

The last algorithmis (3.4) based on the solution of the linear integral equation for the correction $\epsilon_{n-1}(x)$ with the kernel and right-hand side that vary from step to step. This process has a high rate of convergence, but it is rather complicated because we must solve a new equation at each step of iteration. To simplify the problem, we can replace the second equation of the algorithmis (3.4) by the equation

$$\epsilon_{n-1}(x) = f(x) + \int_a^b K(x, y, \varphi_{n-1}(y))\, dy - \varphi_{n-1}(x) + \int_a^b \frac{\partial K}{\partial \varphi}(x, y, \varphi_0(y))\, \epsilon_{n-1}(y)\, dy. \tag{3.5}$$

or by the equation

$$\epsilon_{n-1}(x) = f(x) + \int_a^b K(x, y, \varphi_{n-1}(y))\, dy - \varphi_{n-1}(x) + \int_a^b \frac{\partial K}{\partial \varphi}(x, y, \varphi_m(y))\, \epsilon_{n-1}(y)\, dy. \tag{3.6}$$

Whose kernels do not vary. In the equation (3.5), φ_0 is the initial solution, and in the equation (3.6) m is fixed and satisfies the condition $m < n - 1$.

3.2 Modified Simpson method

In this section, we shall describe the modified Simpson method for the approximate solution of nonlinear integral equations of the second kind with continuous kernels. The goal of the modified Simpson method is to approximate the definite integral of $f(x)$ over the interval $[a, b]$ by evaluating $f(x)$ at a finite number of sample points

Modified Simpson rule's

Consider $G = [a, b]$, let $t_0 = a < x_1 < ... < x_{2j-1} < x_{2j} < ... < x_{2n} = b$ be an equidistant subdivision of a step $h = x_{2j+1} - x_{2j}$, for $j = 0, 1, 2, ..., n$. Our objective, it's to approximate the solution of the Fredholm integral equation to the nodes of even indice (at the point t_{2j}), then the modified Simpson have the form

$$\int_{x_{2j}}^{x_{2j+2}} f(x)\,dx \simeq \frac{x_{2j+1}-x_{2j}}{3}\left[f(x_{2j})+4f(x_{2j+1})+f(x_{2j+1})\right].$$

Now, we approximate the two integrals on the right-hand side of the algorithme by modified Simpson method

$$\begin{aligned}
\epsilon(x_{2j}) &= f(x_{2j})+\int_a^b K(x_{2j},y,\varphi(y))\,dy-\varphi(x_{2j})+\int_a^b \frac{\partial K}{\partial \varphi}(x_{2j},y,\varphi_{n-1}(y))\,\epsilon(y)\,dx\\
&= f(x_{2j})-\varphi(x_{2j})+\sum_{i=0}^{n-1}\int_{x_{2i}}^{x_{2i+2}}\left(K(x_{2j},y,\varphi(y))+\frac{\partial K}{\partial \varphi}(x_{2j},y,\varphi(x))\,\epsilon(y)\right)dx
\end{aligned}$$

by the numerical integration formulas of modified Simpson method, so we get

$$\begin{aligned}
\epsilon_{2j} &= f_{2j}-\varphi_{2j}+\sum_{i=0}^{n-1}\frac{h}{3}\left[K_{2j,2i,2i}+4K_{2j,2i+1,2i+1}+K_{2j,2i+2,2i+2}\right] \qquad (3.7)\\
&\quad+\sum_{i=0}^{n-1}\frac{h}{3}\left[K'_{2j,2i,2i}\varphi_{2i}+4K'_{2j,2i+1,2i+1}\varphi_{2i+1}+K'_{2j,2i+2,2i+2}\varphi_{2i+2}\right],
\end{aligned}$$

where $\epsilon(x_{2j}) = \epsilon_{2j}$, $f(x_{2j}) = f_{2j}$, $\varphi(x_{2j}) = \varphi_{2j}$, $K(x_{2j},y_{2i},\varphi(y_{2i})) = K_{2j,2i,2i}$, and $\frac{\partial K}{\partial \varphi}(x_{2j},y_{2i},\varphi(y_{2i}))\,\epsilon(y_{2i}) = K'_{2j,2i,2i}\epsilon_{2i}$.

Since h sufficiently small, we approximate φ_{2i+1} by $\frac{\varphi_{2i}+\varphi_{2i+2}}{2}$, the equation (3.7) becomes

$$\begin{aligned}
\epsilon_{2j} &= G_{2j}-\varphi_{2j}+\sum_{i=0}^{n-1}\frac{h}{3}\left[K'_{2j,2i,2i}\varphi_{2i}+4K'_{2j,2i+1,2i+1}\varphi_{2i+1}+K'_{2j,2i+2,2i+2}\epsilon_{2i+2}\right]\\
&= G_{2j}-\varphi_{2j}+\sum_{i=0}^{n-1}\frac{h}{3}\left[K'_{2j,2i,2i}\varphi_{2i}+4K'_{2j,2i+1,2i+1}\left(\frac{\varphi_{2i}+\varphi_{2i+2}}{2}\right)+K'_{2j,2i+2,2i+2}\epsilon_{2i+2}\right]\\
&= G_{2j}-\varphi_{2j}+\sum_{i=0}^{n-1}\frac{h}{3}\left[\left[K'_{2j,2i,2i}+2K'_{2j,2i+1,2i+1}\right]\epsilon_{2i}+\left[2K'_{2j,2i+1,2i+1}+K'_{2j,2i+2,2i+2}\right]\epsilon_{2i+2}\right]\\
&= G_{2j}-\varphi_{2j}+\frac{h}{3}\left[\sum_{i=0}^{n-1}\left[K'_{2j,2i,2i}+2K'_{2j,2i+1,2i+1}\right]\epsilon_{2i}+\sum_{i=0}^{n-1}\left[2K'_{2j,2i+1,2i+1}+K'_{2j,2i+2,2i+2}\right]\epsilon_{2i+2}\right]\\
&= G_{2j}-\varphi_{2j}+\frac{h}{3}\left[\sum_{i=0}^{n-1}\left[K'_{2j,2i,2i}+2K'_{2j,2i+1,2i+1}\right]\epsilon_{2i}+\sum_{i=1}^{n}\left[2K'_{2j,2i-1,2i-1}+K'_{2j,2i,2i}\right]\epsilon_{2i}\right]\\
&= G_{2j}-\varphi_{2j}+\frac{h}{3}\left[2K'_{2j,2n-1,2n-1}+K'_{2j,2n,2n}\right]\epsilon_{2n}+\frac{h}{3}\left[2K'_{2j,0,0}+K'_{2j,1,1}\right]\epsilon_0\\
&\quad+\frac{2h}{3}\sum_{i=1}^{n-1}\left[K'_{2j,2i-1,2i-1}+K'_{2j,2i,2i}+K'_{2j,2i+1,2i+1}\right]\epsilon_{2i}
\end{aligned}$$

Finally

$$\begin{aligned}\epsilon_{2j} &= \frac{h}{3}\left[2K'_{2j,2n-1,2n-1}+K'_{2j,2n,2n}\right]\varphi_{2n}+\frac{h}{3}\left[2K'_{2j,0,0}+K'_{2j,1,1}\right]\epsilon_0 \\ &+\frac{2h}{3}\sum_{i=1}^{n-1}\left[K'_{2j,2i-1,2i-1}+K'_{2j,2i,2i}+K'_{2j,2i+1,2i+1}\right]\epsilon_{2i}+G_{2j}-\varphi_{2j}\end{aligned} \tag{3.8}$$

with

$$G_{2j}=f_{2j}+\sum_{i=1}^{n}\frac{h}{3}\left[K_{2j,2i,2i}+4K_{2j,2i+1,2i+1}+K_{2j,2i+2,2i+2}\right]$$

The evaluation of (3.8) on the x_{2j} gives a system of algebraic equation of the form

$$\phi = D$$

where the vectors ϕ and D respectively defined the coponeets of ϵ_{2j} and the right-hand side of the equations (3.8) , for $j=0,1,...,n$.

And the initial approximation $Y^{(0)}=0$, $\phi = F-Y^{(0)}$, and F defined the components of f_{2j}

$$\begin{cases} Y^{(k+1)}=Y^{(k)}+\phi^{(k)} \\ \phi^{(k+1)}=D \end{cases} \quad k=0,1,2,...$$

and by recurrence, we can to calculate the vector of solution Y in all points x_{2j} for $j=0,1,...,n$.

3.3 Numerical examples

In this section, implementation of the methods mentioned in this paper will be done. This implementation consists of computer programmes written in MATLAB, which approximates solutions to some example nonlinear Fredholm integral equation of the second kind.

Example 3.3. *[5]Consider the following nonlinear Fredholm integral equation*

$$\varphi(x)-\frac{1}{2}\int_0^1 xy^2\sin(\varphi(y))\,dy = x^3+\frac{x}{6}(\cos(1)-1),$$

where the solution exact

$$\varphi(x)=x^3.$$

Example 3.4. *[9]Consider the nonlinear Fredholm integral equation*

$$\varphi(x) = \frac{7}{8}x + \frac{1}{2}\int_0^1 xy\varphi^2(y)\,dy,$$

with exact solution

$$\varphi(x) = x.$$

Example 3.5. *[13]Let the integral equation*

$$\varphi(x) = \exp(x) - \frac{x}{192}\left(\exp(2)+1\right) + \frac{1}{48}\int_0^1 xy\varphi^2(y)\,dy$$

with exact solution

$$\varphi(x) = \exp(x)$$

Example 3.6. *[1]Let the integral equation*

$$\varphi(x) = \cos(x) - \frac{\pi}{112} + \frac{1}{156}\int_0^1 \varphi^2(y)\,dy$$

with exact solution

$$\varphi(x) = \cos(x)$$

nodes	E solution	A solution	Error
0	$0.0000e+00$	$0.0000e+00$	$0.0000e+00$
0.1	$1.0000e-03$	$8.8119e-03$	$7.8119e-03$
0.2	$8.0000e-03$	$2.3624e-02$	$1.5624e-02$
0.3	$2.7000e-02$	$5.0436e-02$	$2.3436e-02$
0.4	$6.4000e-02$	$9.5245e-02$	$3.1245e-02$
0.5	$1.2500e-01$	$1.6404e-01$	$3.9041e-02$
0.6	$2.1600e-01$	$2.6278e-01$	$4.6784e-02$
0.7	$3.4300e-01$	$3.9738e-01$	$5.4380e-02$
0.8	$5.1200e-01$	$5.7368e-01$	$6.1678e-02$
0.9	$7.2900e-01$	$7.9755e-01$	$6.8554e-02$
1	$1.0000e+00$	$1.0752e+00$	$7.5225e-02$

Table 3.1: Comparison of resultants, absolute error, for K=1 NFIE (example1)

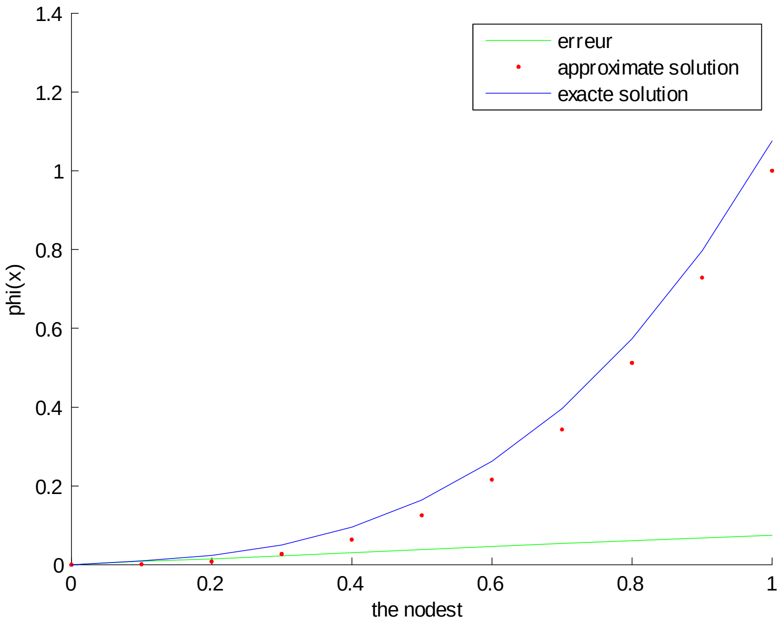

Figure 3.1: Comparison of resultants, absolute error, for K=1 NFIE (example1)

nodes	E solution	A solution	Error
0	$0.0000e + 00$	$0.0000e + 00$	$0.0000e + 00$
0.1	$1.0000e - 03$	$8.5082e - 03$	$7.5082e - 03$
0.2	$8.0000e - 03$	$2.3016e - 02$	$1.5016e - 02$
0.3	$2.7000e - 02$	$4.9525e - 02$	$2.2525e - 02$
0.4	$6.4000e - 02$	$9.4033e - 02$	$3.0033e - 02$
0.5	$1.2500e - 01$	$1.6254e - 01$	$3.7541e - 02$
0.6	$2.1600e - 01$	$2.6105e - 01$	$4.5049e - 02$
0.7	$3.4300e - 01$	$3.9556e - 01$	$5.2557e - 02$
0.8	$5.1200e - 01$	$5.7207e - 01$	$6.0066e - 02$
0.9	$7.2900e - 01$	$7.9657e - 01$	$6.7574e - 02$
1	$1.0000e + 00$	$1.0751e + 00$	$7.5082e - 02$

Table 3.2: Comparison of resultants, absolute error, for K=4 NFIE (example1)

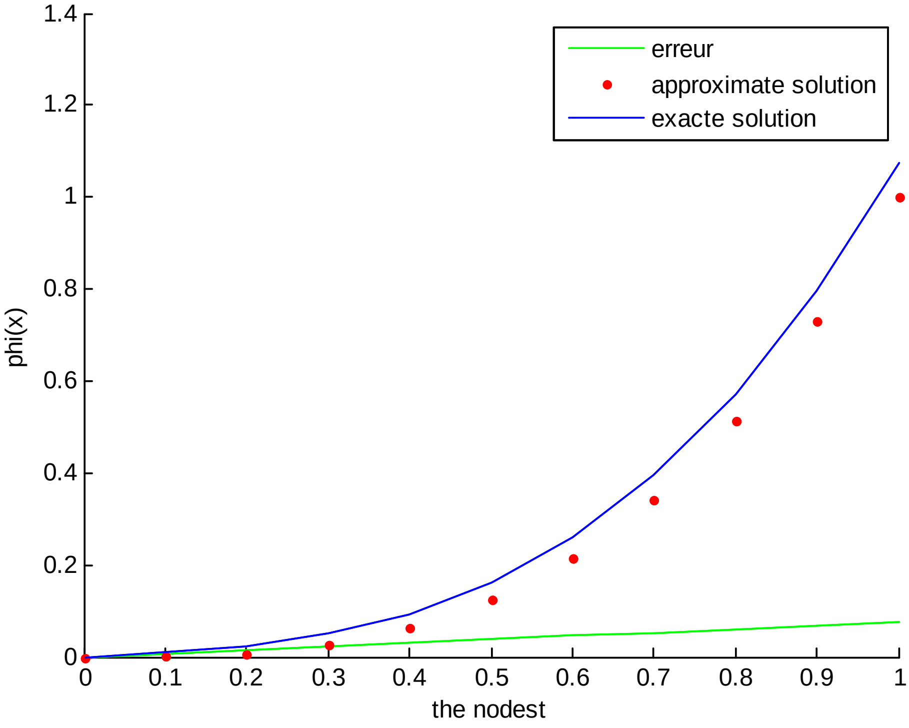

Figure 3.2: Comparison of resultants,absolute error, for K=4 NFIE (example1)

nodes	E solution	A solution	Error
0	$0.0000e+00$	$0.0000e+00$	$0.0000e+00$
0.1	$1.0000e-01$	$1.0710e-01$	$7.0964e-03$
0.2	$2.0000e-01$	$2.1423e-01$	$1.4228e-02$
0.3	$3.0000e-01$	$3.2153e-01$	$2.1530e-02$
0.4	$4.0000e-01$	$4.2921e-01$	$2.9208e-02$
0.5	$5.0000e-01$	$5.3748e-01$	$3.7479e-02$
0.6	$6.0000e-01$	$6.4652e-01$	$4.6517e-02$
0.7	$7.0000e-01$	$7.5640e-01$	$5.6397e-02$
0.8	$8.0000e-01$	$8.6702e-01$	$6.7022e-02$
0.9	$9.0000e-01$	$9.7805e-01$	$7.8045e-02$
1	$1.0000e+00$	$1.0888e+00$	$8.8793e-02$

Table 3.3: Comparison of resultants, absolute error, for K=1 NFIE (example2)

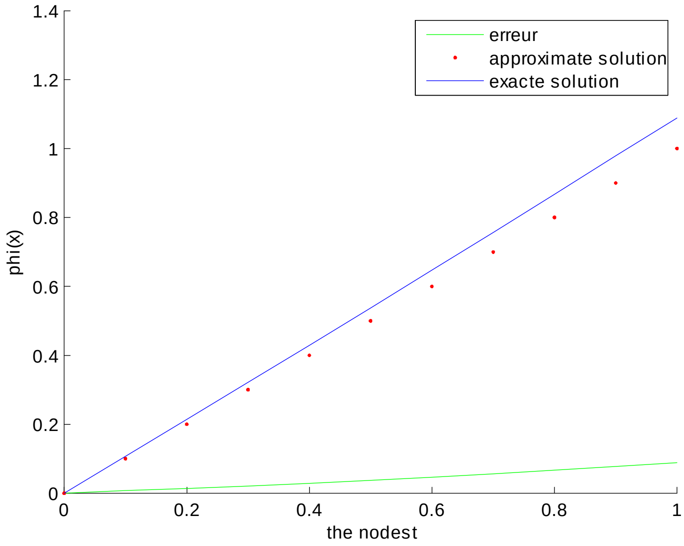

Figure 3.3: Comparison of resultants, absolute error, for K=1 NFIE (example2)

nodes	E solution	A solution	Error
0	$0.0000e+00$	$0.0000e+00$	$0.0000e+00$
0.1	$1.0000e-01$	$1.0898e-01$	$8.9809e-03$
0.2	$2.0000e-01$	$2.1796e-01$	$1.7962e-02$
0.3	$3.0000e-01$	$3.2694e-01$	$2.6943e-02$
0.4	$4.0000e-01$	$4.3592e-01$	$3.5924e-02$
0.5	$5.0000e-01$	$5.4490e-01$	$4.4904e-02$
0.6	$6.0000e-01$	$6.5389e-01$	$5.3885e-02$
0.7	$7.0000e-01$	$7.6287e-01$	$6.2866e-02$
0.8	$8.0000e-01$	$8.7185e-01$	$7.1847e-02$
0.9	$9.0000e-01$	$9.8083e-01$	$8.0828e-02$
1	$1.0000e+00$	$1.0898e+00$	$8.9809e-02$

Table 3.4: Comparison of resultants, absolute error, for K=4 NFIE (example2)

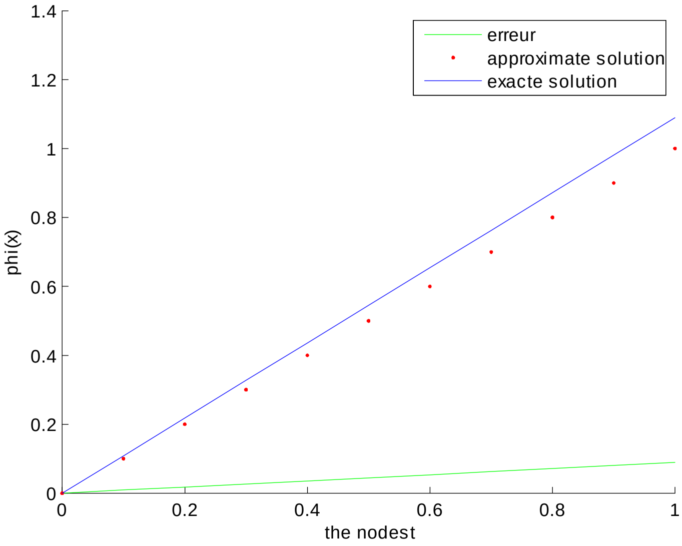

Figure 3.4: Comparison of resultants, absolute error, for K=4 NFIE (example2)

nodes	E solution	A solution	Error
0	$1.0000e+00$	$1.0000e+00$	$0.0000e+00$
0.1	$1.1052e+00$	$1.1007e+00$	$4.5065e-03$
0.2	$1.2214e+00$	$1.2124e+00$	$9.0131e-03$
0.3	$1.3499e+00$	$1.3363e+00$	$1.3520e-02$
0.4	$1.4918e+00$	$1.4738e+00$	$1.8026e-02$
0.5	$1.6487e+00$	$1.6262e+00$	$2.2532e-02$
0.6	$1.8221e+00$	$1.7951e+00$	$2.7038e-02$
0.7	$2.0138e+00$	$1.9822e+00$	$3.1543e-02$
0.8	$2.2255e+00$	$2.1895e+00$	$3.6047e-02$
0.9	$2.4596e+00$	$2.4191e+00$	$4.0550e-02$
1	$2.7183e+00$	$2.6732e+00$	$4.5048e-02$

Table 3.5: Comparison of resultants,absolute error, for K=1 NFIE (example3)

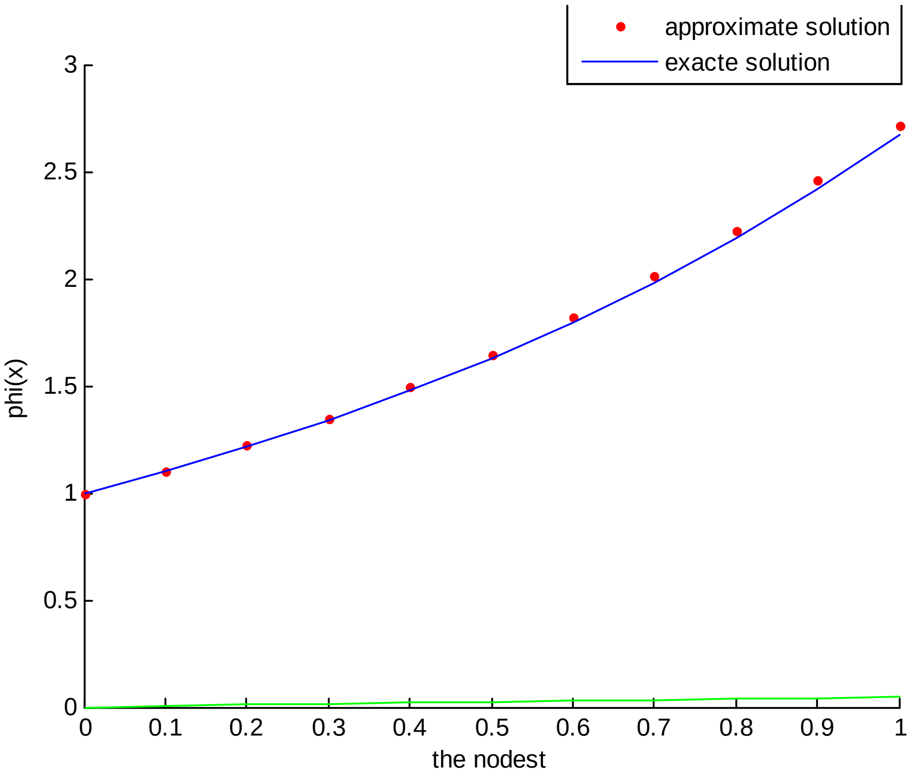

Figure 3.5: Comparison of resultants,absolute error, for K=1 NFIE (example3)

nodes	E solution	A solution	Error
0	$1.0000e+00$	$1.0000e+00$	$0.0000e+00$
0.1	$1.1052e+00$	$1.1007e+00$	$4.5044e-03$
0.2	$1.2214e+00$	$1.2124e+00$	$9.0089e-03$
0.3	$1.3499e+00$	$1.3363e+00$	$1.3513e-02$
0.4	$1.4918e+00$	$1.4738e+00$	$1.8018e-02$
0.5	$1.6487e+00$	$1.6262e+00$	$2.2522e-02$
0.6	$1.8221e+00$	$1.7951e+00$	$2.7027e-02$
0.7	$2.0138e+00$	$1.9822e+00$	$3.1531e-02$
0.8	$2.2255e+00$	$2.1895e+00$	$3.6035e-02$
0.9	$2.4596e+00$	$2.4191e+00$	$4.0540e-02$
1	$2.7183e+00$	$2.6732e+00$	$4.5044e-02$

Table 3.6: Comparison of resultants,absolute error, for K=4 NFIE (example3)

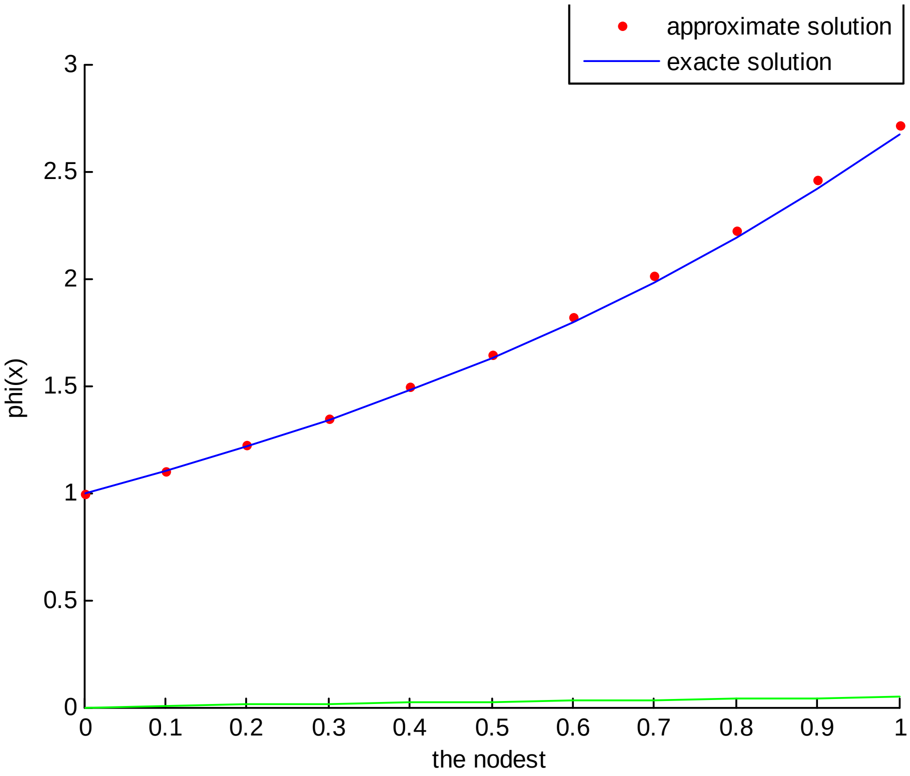

Figure 3.6: Comparison of resultants, absolute error, for K=4 NFIE (example3)

nodes	E solution	A solution	Error
0	$1.0000e+00$	$9.8884e-01$	$1.1162e-02$
0.1	$9.9500e-01$	$9.8384e-01$	$1.1162e-02$
0.2	$9.8007e-01$	$9.6891e-01$	$1.1160e-02$
0.3	$9.5534e-01$	$9.4418e-01$	$1.1158e-02$
0.4	$9.2106e-01$	$9.0991e-01$	$1.1154e-02$
0.5	$8.7758e-01$	$8.6644e-01$	$1.1148e-02$
0.6	$8.2534e-01$	$8.1420e-01$	$1.1139e-02$
0.7	$7.6484e-01$	$7.5372e-01$	$1.1127e-02$
0.8	$6.9671e-01$	$6.8560e-01$	$1.1112e-02$
0.9	$6.2161e-01$	$6.1052e-01$	$1.1092e-02$
1	$5.4030e-01$	$5.2923e-01$	$1.1068e-02$

Table 3.7: Comparison of resultants, absolute error, for K=1 NFIE (example4)

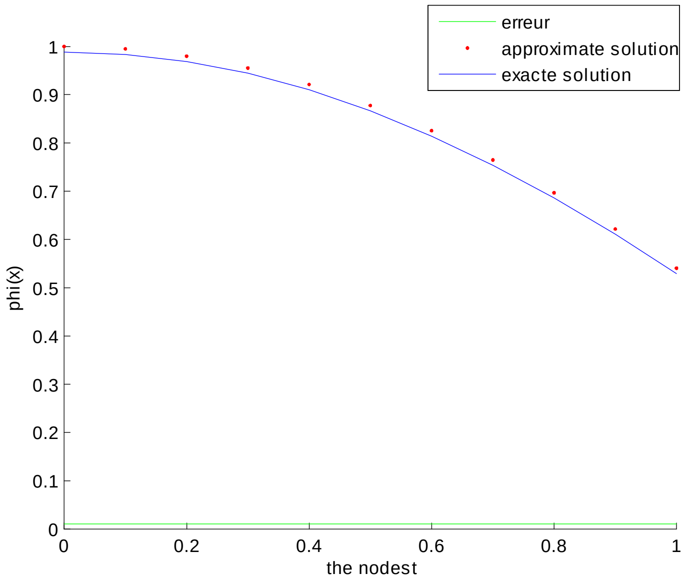

Figure 3.7: Comparison of resultants, absolute error, for K=1 NFIE (example4)

nodes	E solution	A solution	Error
0	$1.0000e+00$	$9.8895e-01$	$1.1054e-02$
0.1	$9.9500e-01$	$9.8395e-01$	$1.1054e-02$
0.2	$9.8007e-01$	$9.6901e-01$	$1.1054e-02$
0.3	$9.5534e-01$	$9.4428e-01$	$1.1054e-02$
0.4	$9.2106e-01$	$9.1001e-01$	$1.1054e-02$
0.5	$8.7758e-01$	$8.6653e-01$	$1.1054e-02$
0.6	$8.2534e-01$	$8.1428e-01$	$1.1054e-02$
0.7	$7.6484e-01$	$7.5379e-01$	$1.1054e-02$
0.8	$6.9671e-01$	$6.8565e-01$	$1.1054e-02$
0.9	$6.2161e-01$	$6.1056e-01$	$1.1054e-02$
1	$5.4030e-01$	$5.2925e-01$	$1.1054e-02$

Table 3.8: Comparison of resultants, absolute error, for K=4 NFIE (example4)

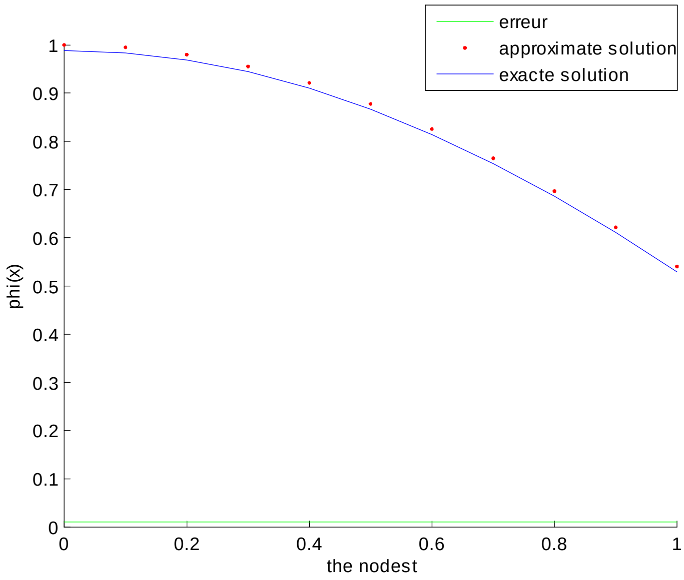

Figure 3.8: Comparison of resultants, absolute error, for K=4 NFIE (example4)

Conclusion

There are various numerical methods to solve nonlinear integral equations. In this our work, we had present a one numerical method for solving nonlinear Fredholm integral equation, based on Newton-Kantorovich and modified Simpson method. We have presented numerical method as algorithm and applied these algorithm to both Fredholm integral equation using Matlab Software. we tested this method by using some different examples. This method was effective.

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Abstract

In this work, we apply the Newton-Kantorovich method to solve the nonlinear Fredholm integral equation, we look for an approximate solution for nonlinear Fredholm integral equation of the second kind using a combination between Newton-Kantorovich method and modified Simpson method, then we give numerical results of some exampls which are solving by this combination.

Keywords: fixed point theorems, modified Simpson method, Newton-Kantorovich method, Fredholm integral equation.

Résumé

Dans ce travail, nous appliquons la methode de Newton-Kantorovich pour résoudre l'équation intégrale non linéaire de Fredholm, nous cherchons une solution approchée pour ces équation de seconde espèce en utilisant une combinaison entre la méthode de Newton-Kantorovich et la méthode de Simpson modifiée, puis nous donnons des résultats d'exemples qui sont résolus par ces combinaisons

Mots clés: Théoremes de point fixe, Méthode de Simpson modifiée, Méthode de Newton-Kantorovich, Equation intégrale non linear de Fredholm.

الملخص

في هذا العمل نطبق طريقة نيوتن كانتوروفيش لحل معادلة فريدهولم التكاملية غير الخطية من النوع الثاني و نبحث عن حل تقريبي لها باستخدام تركيبة بين طريقة نيوتن كانتوروفيش و طريقة سمبسون المعدلة ثم نعطي النتائج العددية لبعض الأمثلة التي يتم حلها من خلال هذه التركيبة.

الكلمات المفتاحية: نظريات النقطة الصامدة، طريقة نيوتن كانتوروفيش، معادلة فريدهولم التكاملية.