DEMOCRATIC AND POPULAR REPUBLIC OF ALGERIA
MINISTRY OF HIGHER EDUCATION AND SCIENTIFIC RESEARCH
UNIVERSITY MOHAMED BOUDIAF - M'SILA

FACULTY : TECHNOLOGY
DEPARTMENT : ELECTRICAL ENGINEERING
N° : Rob-04

FIELD: SCIENCE AND TECHNOLOGY
SECTOR: AUTOMATIC
OPTION : ROBOTICS

Thesis presented for obtaining
From the Academic Master's degree

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Titled:

Nonlinear control of a manipulator arm

Supported on 12/06/2024, before the jury composed of:

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Academic year: 2023/2024

APPRECIATION

We would like first of all to thank ALLAH the almighty and merciful, who gave us the strength and patience to do this modest work.

I would like to express my gratitude to my thesis director, Mr. **BOUGUERRA Abderrahmen**. I thank her for mentoring, guiding, helping and advising me.

I also want to thank my co-director **LOUKAL Keltoum** and all the teachers who helped us during the study period at the university.

I would also like to thank the professors of Department of Electrical Engineering **University Mohamed Boudiaf of M'sila**, who provided me with the tools to succeed in my university studies and the jury members for the interest they have shown in our work by agreeing to review our work.

Finally, I would like to express my gratitude to all those who contributed, directly or indirectly, to the realization of this work.

Dedications

إلى من كلل العرق جبينه ومن علمني أن النجاح لا يأتي إلا بالصبر و الإصرار ، إلى النور الذي أنار دربي و السراج الذي لا ينطفي نوره أبدا , من بذل الغالي والنفيس و استمديت منه قوتي و اعتزازي بذاتي والدي العزيز"عمار" إلى من جعل الجنة تحت أقدامها و سهلت لي الشدائد بدعائها إلى الإنسانة العظيمة التي لطالما تمنت أن تقر عينها لرؤيتي في يوم كهذا أمي العزيزة"سعاد".

إلى ضلعي الثابت و أمان أيامي إلى من شددت عضدي بهم فكانوا لي ينابيع أرتوي منها ، إلى خيرة أيامي و صفوتها إلى قرة عيني إلى أخواتي و أخي الغاليين "لينة" "رقية"و "علي".

لكل من كان عونا وسندا في هذا الطريق، للأصدقاء الأوفياء ورفقاء السنين لأصحاب الشدائد و الأزمات، إلى من أفاضني بمشاعره ونصائحه المخلصة.

و إليكم عائلتي

أهديكم هذا الانجاز و ثمرة نجاحي الذي طالما تمنيته ، ها أنا اليوم اكملت و أتممت أول ثمراته

بفضله سبحانه و تعالى

فالحمد لله على ما وهبني و أن يجعلني مباركا و يعينني أينما كنت، فمن قال أنا لها نالها فنالها و إن أبت رغما عنها أتيت بها فالحمد لله شكرا و حبا و امتنانا على البدء و الختام.

زرواق أمل

﴿وَآخِرُ دَعْوَاهُمْ أَنِ الْحَمْدُ لِلَّهِ رَبِّ الْعَالَمِينَ﴾

الحمد لله الذي ما تم جهد ولا ختم سعي إلا بفضله، و ما تخطيت هذه العقبات و الصعوبات إلا بتوفيقه فالحمد والشكر لله ربي العالمين.أود أن أقدم هذا العمل المتواضع

إلى اعز الناس و أقربهم إلى قلبي إلى سندي وضلعي الثابت أبي "نور الدين " إلى جنتي في الأرض حبيبتي أمي "ربيعة" اللذان كانا عونا لي وكان لدعائها المبارك أعظم اثر في ما أنا عليه الآن .

إلى زهراتي أخواتي :" اميمة" " راضية" "ابتسام" وصغيرتنا " سلسبيل" دون ان انسى حلوتي" تسنيم ".

إلى من ساندني وتحمل الكثير معي زوجي" زاكي ".

إلى جميع صديقاتي ومنهم صديقة الطفولة "ياسمين " ولن انسي من رفعت ضغطي وعلمتني الصبر "أمل".

إلى جدي "الطاهر" و جدتي "زينب" إلى جميع عائلتي أخوالا وخالات أعماما وعمات صغيرهم وكبيرهم

إلى عائلتي الثانية عائلة زوجي .

إلى جميع استاذاتي و اساتذتي الى معلمتي الحنونة " بن وجاح رزيقة " الى كل من دعا لي في ظهر الغيب بالنجاح و التوفيق.

إلى كل هؤلاء اهدي هذا العمل . و اسأل الله العلي القدير ان ينفعنا به و يمدنا بتوفيقه.

عوينة هبة الله بثينة

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General Introduction

A robot is a scientific field that refers to a machine which may automatically conduct tasks and manipulate items based on a program. They tend to be used as replacements for humans in instances when they are unable to carry out the work, such as dangerous, high-precision, or repetitive tasks. it's an interdisciplinary branch of engineering and science, integrates mechanical engineering, electrical engineering, computer science, and other fields.

Today's robotics is a science concerned with the intelligent movement of various robot devices. The most commonly encountered robot manipulators are serial robot mechanisms, and today, the most useful and efficient robotic systems are the industrial robot manipulators, which can replace human employees in hard or repetitive jobs, or where a human would otherwise be faced with risky conditions [1]. As automation and artificial intelligence evolve and the presence of robots in various sectors, including manufacturing, healthcare, military, and everyday life. Robots are obliged to perform precise, repetitive, and hazardous tasks, thereby has to enhance productivity, safety, and efficiency.

The complexity of robotic systems necessitates advanced control techniques to ensure their accurate and efficient operation. Traditional linear control methods, while effective in many cases often fall short when dealing with the inherent nonlinearity present in robotic systems. Nonlinearities in robots arise from various factors, such as joint friction, dynamics of motors and kinematic limits. Consequently, nonlinear control strategies have gained important role in solving these challenges because it's enable more precise manipulation, better handling of uncertainties and improved robustness against external disturbances.

In this thesis, we delve into the realm of nonlinear control applied to robotic systems, such as sliding mode,feedback linearization control and assessing their applicability and effectiveness in real-world robotic applications.

Theoretical studies have proven the effective use of sliding mode control, with robotics and electricity being the primary domains of application. Because of its resilience to model uncertainties and disturbances, this type of control is useful and crucial. Nevertheless, these achievements come with a price, like the phenomenon of chattering or reluctance brought on by this control's discontinuous portion, which may be harmful to the motors[2].

Feedback linearization transforms nonlinear system dynamics into a (fully or partly) linear form using exact state transformations and feedback, allowing the application of linear control techniques. Unlike conventional Jacobian linearization, it doesn't rely on linear approximations. This method simplifies system dynamics and aids in developing robust or adaptive nonlinear controllers. Successfully applied to helicopters, high-performance aircraft,industrial robots and biomedical devices, feedback linearization continues to evolve, though [3].

The objective of this work is

- A glimpse about robots and manipulator arm with 2DOF.
- Study of nonlinear control and synthesis of control laws.
- Application of nonlinear control on a 2DOF manipulator arm.
- Comparative study between the different proposed commands.

This paperdivided into three chapters is organized as follows:

Chapter 1: We have presented in this chapter some general information on 2DOF manipulating robots and open loop simulation plus the PID controller.

Chapter 2: will focus on the theory of sliding mode control and we will present the application of the principle of this non-linear method on the model of a manipulator arm with 2DOF.

Chapter3: In this chapter we studied another non-linear control method which is the feedback linearization and wepresented the theory and all the notions and laws necessary to understand this technique, then we applied it on our 2DOF manipulator armand in the end we added a comparative study of all the controls mentioned in the previous chapters which is carried out on the basis of the simulation results.

1.1 Introduction

Robotics is a set of technical disciplines (mechanical, electronic, automatic, computer and control engineering to design, build, and operate robots) articulated around a common objective and object [5]. this chapter delves into the definitions and types of robots, explores their movement and modeling, and demonstrates the simulation of an open-loop system. It further explains the implementation of PID control, followed by simulations and results that showcase the improvements in robot performance. Through this comprehensive examination, we will gain a deeper understanding of how robots are designed, controlled, and optimized for various applications

1.2Historical

1947: First teleoperated electric manipulator
1954:First programmable robot.
1961:Use of an industrial robot, marketed by the company UNIMATION (USA), on a General Motors assembly line.
1961:First robot with effort control.
1963:Using vision to control a robot.
1973: First wheeled mobile robot[4].

1.3Definition of a robot

According to the generic definition, a robot is a physical mechanism that transforms its surroundings in order to accomplish its intended purpose: the desired task[2]. It is an automatic device capable of manipulating objects or executing operations according to a fixed or modifiable program. A robot is often defined as an automatic manipulator with a programmable cycle and it is a mechanical system equipped with actuators and controlled by a computer that is intended to perform a wide variety of tasks. There are a variety of robots depending on the structure.

1.4Type of robot

There are two types of robots:
• Manipulating robot.

• Mobile robot.

1.4.1 Manipulator robot

Robot manipulators are serial robot mechanisms.The mechanical configuration of it comprises a series of interconnected rigid bodies or links, called robot segments which is connected by joints[5]A robot manipulator is commonly defined as an automatic device capable of manipulating objects or carrying out operations in accordance with a fixed or modifiable program. It can manipulate materials, parts, tools, and specialized devices during variable and programmed motions to perform a variety of functions[4]. This robotic system is typified by an articulated arm facilitating mobility, a wrist mechanism enabling dexterity, and an effector module dedicated to task execution. Operating within a fixed spatial domain, such robots are commonly deployed for specific, repetitive tasks within industrial environments. Their structural design and interlinking articulations afford them requisite flexibility andprecision essential for efficient task performance.

Figure1.1: manipulator robot [6].

1.4.1.1 Applications of manipulating robots

In the realm of manufacturing enterprises, intricate and repetitive tasks traditionally executed by human operators can be effectively delegated to articulated mechanical systems, commonly referred to as manipulators. While these systems may not exhibit the same level of dexterity as human counterparts, their capabilities are adequately proximate to enable the execution of complex movements akin to those executed by a human arm.

1.4.2Mobile robot

A defining characteristic inherent to mobile robots resides in the inclusion of a mobile base facilitating unimpeded traversal within their operational milieu. Such robots possess the capacity for autonomous locomotion, enabling independent movement within diverse environments, including industrial complexes, laboratories, planetary surfaces, among others devoid of requisite intervention from external human operators.

Figure1.2: mobile robot[6].

1.5 Industrial robot

An industrial robot is a programmable mechanical device designed to perform tasks with a high degree of precision and repeatability within an industrial setting. Typically equipped with various sensors, actuators, and manipulators, industrial robots are capable of executing a wide range of manufacturing operations, including assembly, welding, painting, and material handling. These robots are characterized by their ability to operate autonomously or under human supervision, often enhancing productivity, efficiency, and safety in industrial processes. The figure below illustrates two examples of industrial robots[6]:

a-Samsung robot b- Epson robot

Figure 1.3: SCARA type industrial robots[6].

1.6Components of a manipulating robot

The components of a manipulating robot typically encompass both hardware and software elements:

1.6.1 The primary:physical component responsible for manipulation tasks. It consists of joints and links that enable movement and dexterity. Depending on the application, manipulator arms can vary in size, shape, and degrees of freedom.

1.6.2End Effector:This is the tool or device attached to the end of the manipulator arm that interacts with the environment. End effectors can include grippers, suction cups, or specialized tools tailored to specific tasks.

1.6.3 Sensors:These perceptive tools regulate the robot's relationship with its surroundings by reading variables related to its movement, allowing for correct control[2]. Common sensors include:

- **Position Sensors:** Like encoders or potentiometers, to measure joint angles and positions.
- **Force/Torque Sensors**: To measure forces and torques exerted on the end effector.
- **Vision Systems**.
- **Tactile Sensors**.
- **Proximity Sensors**.

1.6.4 Actuators: Actuators are critical components for turning hydraulic, electric, or pneumatic energy into mechanical energy. This conversion is accomplished using rotary movements (motors) or linear translation movements (double-acting simple cylinders)[7].

1.6.5 Controller:The controller is the brain of the robot, responsible for processing sensor inputs, generating control signals for the actuators, and executing high-level commands. This can involve both hardware (like microcontrollers or PLCs) and software (such as control algorithms and motion planning).

1.6.6 Power Supply: Robots require a reliable power source to operate their motors, sensors, and controllers.

1.6.7 Communication Interface: This allows the robot to communicate with external devices or systems, enabling remote control, data exchange, or integration into larger automated systems.

1.6.8 Safety Features: In academic settings, safety is paramount. Safety features can include emergency stop buttons, collision detection systems, and protective barriers to ensure the well-being of operators and bystanders.

1.6.9 Software Frameworks: Academic research often involves developing algorithms and software frameworks for various robot tasks such as motion planning, manipulation, perception, and learning [7].

1.6.10 Human-Machine Interface (HMI): An HMI connects a person to a machine, system, or device, typically in an industrial process [8].

1.7 Robot movement

1.7.1 Degrees of freedom

In mechanics, degrees of freedom (DOF) refers to the number of independent variables that describe a mechanical system's possible positions or motions in space.An articulated robotic arm is made up of links that connect a set number of rotary joints in series to an end effector. The number of joints represents the robotic arm's degrees of freedom (DOF). The joints are often activated by servomotors, which generate the torque required to rotate the associated links[9]. Degree of freedom for planar linkages joined with

common joints can be calculated through Grubler'sequation**:**

M= degree of freedom= 3(n-1)-2jp-jh

Where n = total number of links in the mechanism

jp = total number of primary joints (pins or sliding joints)

jh = total number of higher-order joints (cam or gear joints)[10].

1.7.2 Robots with two degrees of freedom

Robots having two degrees of freedom are robotic devices that can move along two separate axes or directions. The two-degree-of-freedom robot manipulator is a mechanical system that supports a single platform, or end-effector, via several computer-controller serial chains. It gradually becomes less stiff as the number of components increases. They can be the first to act in compared to a serial manipulator. 2-DOF robot manipulators are typically limited in their workspace, as they cannot reach around obstructions. The calculations required to execute the desired manipulation (inverse kinematics) are typically more challenging and can result in many solutions[10].Robots with two degrees of freedom (DOF) have a variety of applications, particularly in industrial settings where tasks require high-speed and high-precision movements like sacra robots. They are often used in applications where manipulation tasks are required, such as pick-and-place operations.

1.8Advantages and disadvantages of the robot

1.8.1The advantages

- Profitability: configuring robots to operate in a continuous, repetitive cycle boosts production rates, leading to increased output. This heightened productivity aids in cost recuperation and generates additional profits.
- Quality improvement: manual involvement in repetitive tasks can impact concentration levels, increasing the likelihood of errors and quality defects. Robotic automation mitigates these risks by ensuring precise production and verification according to set standards. This results in products manufactured with enhanced precision and consistent quality, paving the way for new business opportunities for companies.

- Dexterity and precision : robots cqn be used to To improve precision and dexterity [11].

1.8.2Disadvantages

- Potential job loss: the introduction of robots raises concerns about potential job displacement for workers. With robots capable of delivering faster, more consistent, and precise outputs, there's a possibility that certain tasks may no longer require human intervention in the future.

- Hiring qualified personnel: introducing automation through robots necessitates skilled personnel proficient in advanced programming, operations, and maintenance. These experts play a crucial role in deploying and managing robotic systems effectively, ensuring optimal performance and addressing any technical issues that may arise.
- Lack of Creativity:Robots are programmed to execute specified jobs and lack the creativity and adaptability of human workers. This may hinder their ability to adjust to changing circumstances and innovate[13][12].

1.9 Modeling of a manipulator arm

- Controlling the behavior of articulated mechanical systems, such as robots, requires a multi-level modeling approach tailored to specific objectives, task constraints, and desired performance criteria. The modeling levels of robotic systems can be summarized as follows:
- Direct geometric models describe the position of the terminal organ (like a hand or foot) based on the given joint variables (like angles or distances). In simpler terms, it tells us where the end point of a limb is located based on the movements or positions of the joints while the Inverse geometric models, on the other hand, work the other way around. They determine the joint variables required to achieve a specific position of the terminal organ. So, if you want to reach a certain point with your hand, the inverse geometric model calculates the angles and movements needed in your joints to get there.
- Direct cinematic models express the velocities of the terminal organ (such as a hand or foot) in relation to the joint variables. In simpler terms, they describe how fast the end point of a limb is moving based on the velocities of the joints. inverse cinematic models, on the contrary, determine the joint velocities required to achieve a specific velocity of the terminal organ. So, if you want the hand to move at a certain speed, the inverse cinematic model calculates the velocities needed in the joints to achieve that speed.
- Dynamic models The dynamic model is the relationship between the couples (and/or forces) applied to the actuators and the joint positions, speeds and accelerations. It's describe how systems change over time [14].

1.10Lagrange formalism

The dynamic model is obtained by the Lagrange method with r and θ as coordinates

$$L = \frac{m}{2}\left(\dot{r}^2 + r^2\dot{\theta}^2\right) - mg(r\,sin\,\theta) \qquad (1.1)$$

The Lagrange equations are:

$$\begin{cases} \frac{\mathrm{d}}{\mathrm{dt}}\frac{\partial \mathrm{L}}{\partial \dot{\mathrm{r}}} - \frac{\partial \mathrm{L}}{\partial \mathrm{r}} = \mathrm{F} \\ \frac{\mathrm{d}}{\mathrm{dt}}\frac{\partial \mathrm{L}}{\partial \dot{\theta}} - \frac{\partial \mathrm{L}}{\partial \theta} = \mathrm{T} \end{cases} \qquad (1.2)$$

Which gives:

$$\begin{cases} \ddot{r} - r\dot{\theta}^2 + g\sin\theta = \frac{F}{m} \\ (1 + \sin r^2)\ddot{\theta} + 2r\dot{r}\dot{\theta} + gr\cos\theta = \frac{T}{m} \end{cases} \qquad (1.3)$$

1.11 Dynamic modeling of a manipulator arm

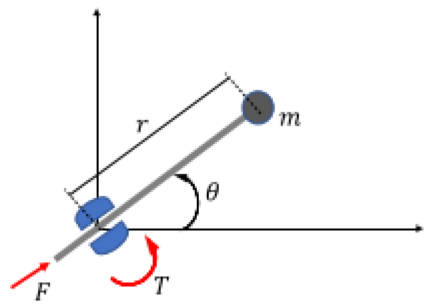


Figure1.4: Schematic diagram of robot of 2DOF.

- θ : **the angular.**
- r : **radial positions**.
- m : **weight.**
- T : **a couple.**
- F : **theforce.**

we have:

$$\begin{cases} \ddot{r} - r\dot{\theta}^2 + g\sin\theta = \frac{F}{m} \\ r^2\ddot{\theta} + 2r\dot{r}\dot{\theta} + gr\cos\theta = \frac{T}{m} \end{cases} \tag{1.4}$$

$$\begin{cases} \ddot{r} = r\dot{\theta}^2 - g\sin\theta + \frac{F}{m} \\ \ddot{\theta} = r^2\left(-2r\dot{r}\dot{\theta} - gr\cos\theta + \frac{T}{m}\right) \end{cases} \tag{1.5}$$

In pose:

$$\begin{cases} y_1 = x_1 = r \\ y_2 = x_3 = \theta \end{cases} \tag{1.6}$$

Therefore:

$$\begin{cases} x_1 = r \\ x_2 = \dot{x}_1 = \dot{r} \\ \dot{x}_2 = \ddot{r} = r\dot{\theta}^2 - g\sin\theta + \frac{F}{m} \end{cases} \tag{1.7}$$

$$\begin{cases} x_3 = \theta \\ x_4 = \dot{x}_3 = \dot{\theta} \\ \dot{x}_4 = \ddot{\theta} = \frac{1}{1+\sin{x_1}^2}\left(-2r\dot{r}\dot{\theta} - gr\cos\theta + \frac{T}{m}\right) \end{cases} \tag{1.8}$$

The state model is built by defining the inputs$U_1 = F \ \ and \ \ U_2 = T$, Then:

$$\begin{cases} \dot{x}_1 = \dot{r} \\ \dot{x}_2 = \ddot{r} = r\dot{\theta}^2 - g\sin\theta + \frac{U_1}{m} \\ \dot{x}_3 = \dot{\theta} \\ \dot{x}_4 = \ddot{\theta} = \frac{1}{1+\sin{x_1}^2}\left(-2r\dot{r}\dot{\theta} - gr\cos\theta + \frac{U_2}{m}\right) \end{cases} \tag{1.9}$$

$$\begin{cases} \dot{x}_1 = x_2 \\ \dot{x}_2 = x_1{x_4}^2 - g\sin x_3 + \frac{U_1}{m} \\ \dot{x}_3 = x_4 \\ \dot{x}_4 = \frac{1}{1+\sin{x_1}^2}\left(-2x_1x_2x_4 - gx_1\cos x_3 + \frac{U_2}{m}\right) \end{cases} \tag{1.10}$$

1.12Manipulator Arm Open Loop Simulation

Parameters of the manipulator arm:

m = 0.5 kg ;g=9.8m/s^2

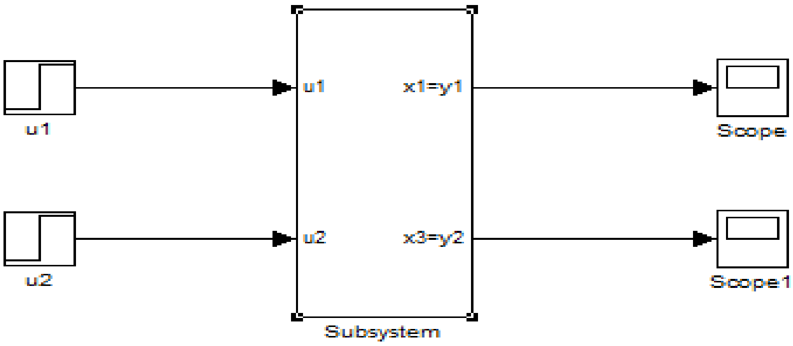


Figure 1.5: Block diagram of manipulator arm.

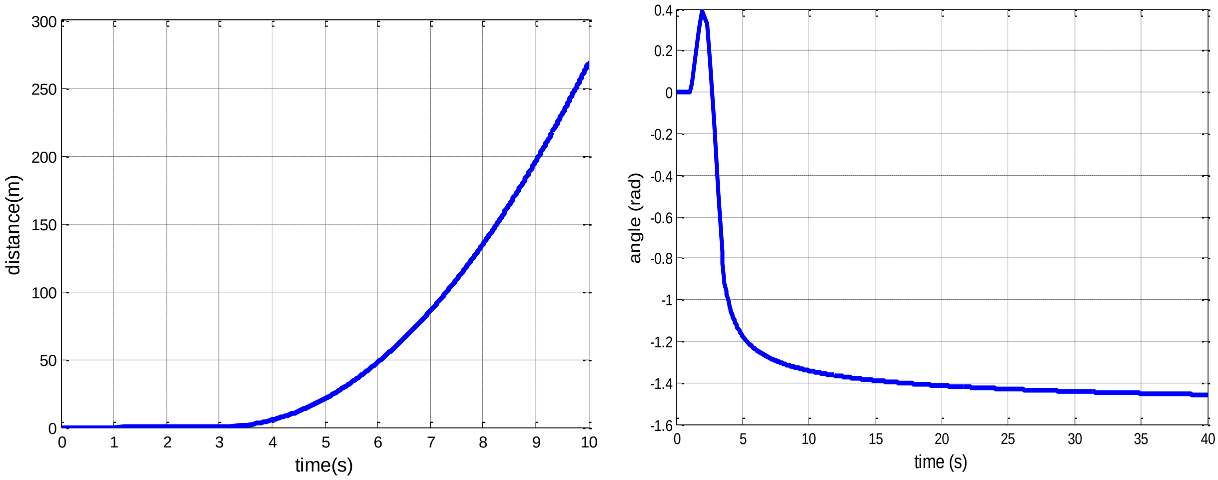


Figure 1.6: the response of the manipulating arm.

1.13PID controller

A PID controller is a type of feedback controller and it is mainly a closed loop feedback system. generic that uses three terms to calculate the output of a system: proportional, integral and derivative. The proportional term is proportional to the error, which is the difference between the setpoint (the desired value) and the return (the actual value).

The full term is proportional to the sum of errors over time, which helps eliminate errors in a steady state. The derived term is proportional to the rate of change of the error, which helps to reduce overshoot and oscillations. The output of the PID controller is the sum of these three terms, multiplied by their respective gains. The differential equation of the PID controller is[15]:

$$u(t) = k_p\, e(t) + k_i \int_0^{tf} e(t)\, dt + k_d \frac{de(t)}{dt} \qquad (1.11)$$

Andk_p , k_i et $k_d$are proportional, integral and derivative gains respectively.

The superposition of these three actions constitutes the mechanism for adjusting process performance, as shown in the figure.:

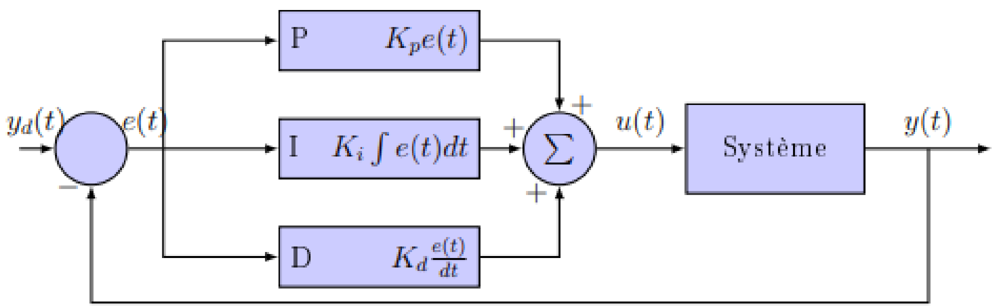


Figure 1.7: PID regulator block diagram [6].

1.13.1 Application of PID control of a 2DOF manipulator arm

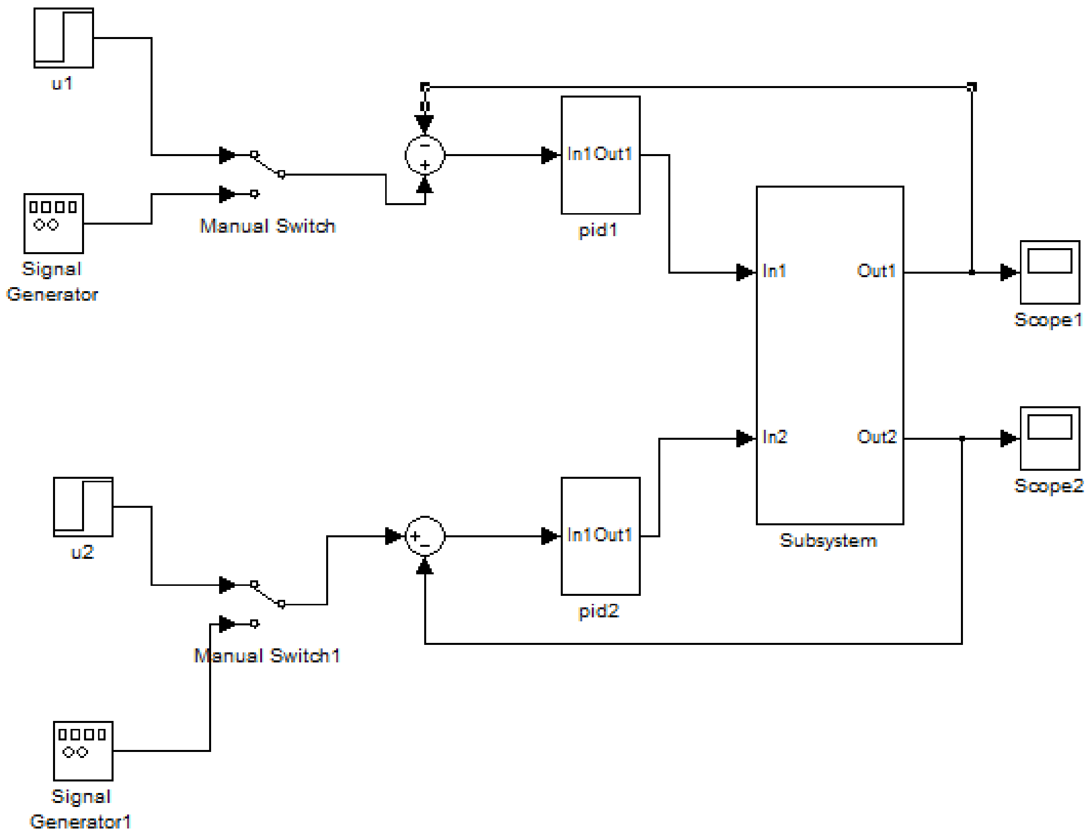


Figure 1.8: Block diagram of the PID control of a 2DOF manipulator arm.

1.13.2 simulation results

Gains:$k_{p1} = 14$; $k_{i1} = 5$;$k_{d1} = 28$

$k_{p2} = 35$; $k_{i2} = 35$; $k_{d2} = 20$

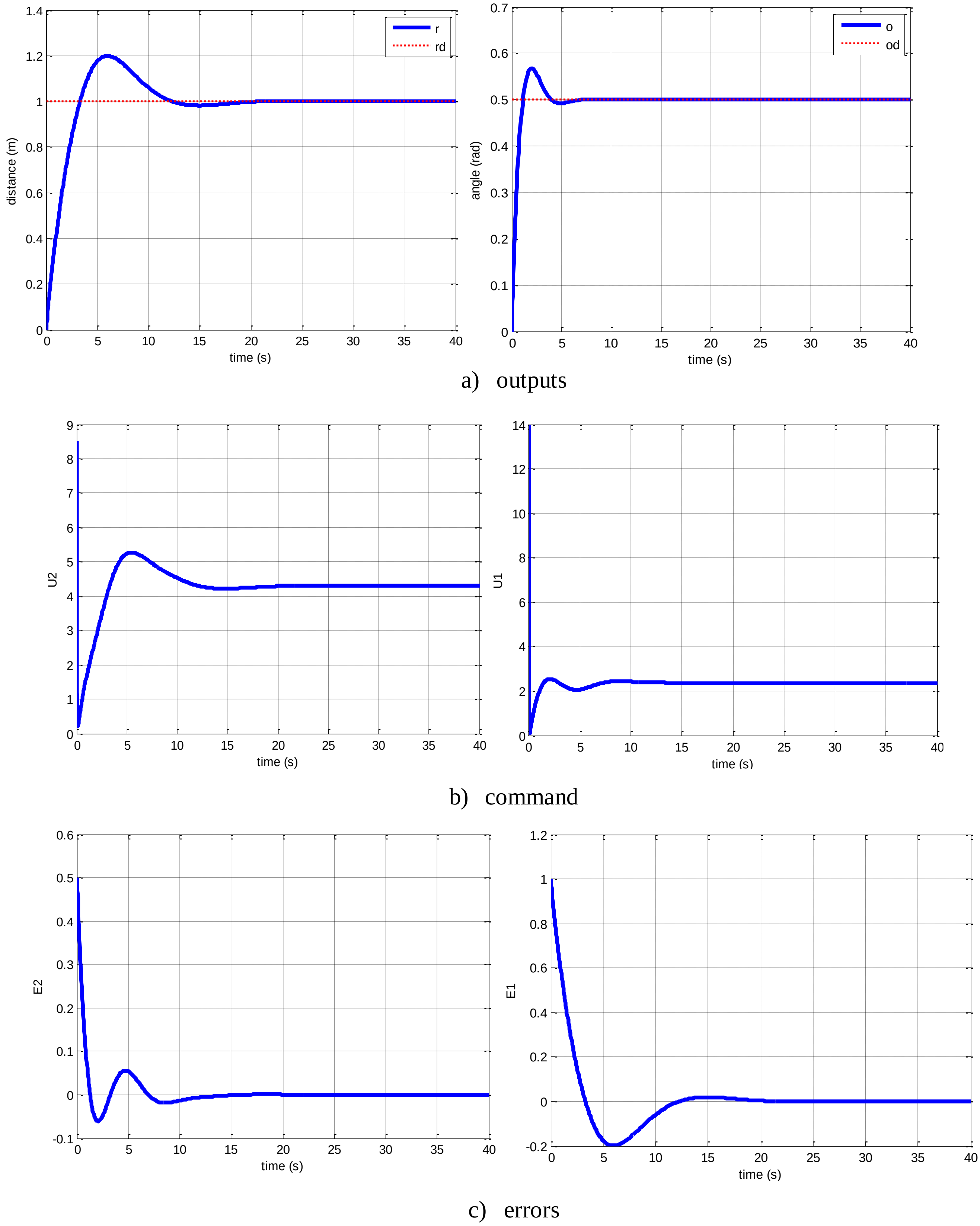
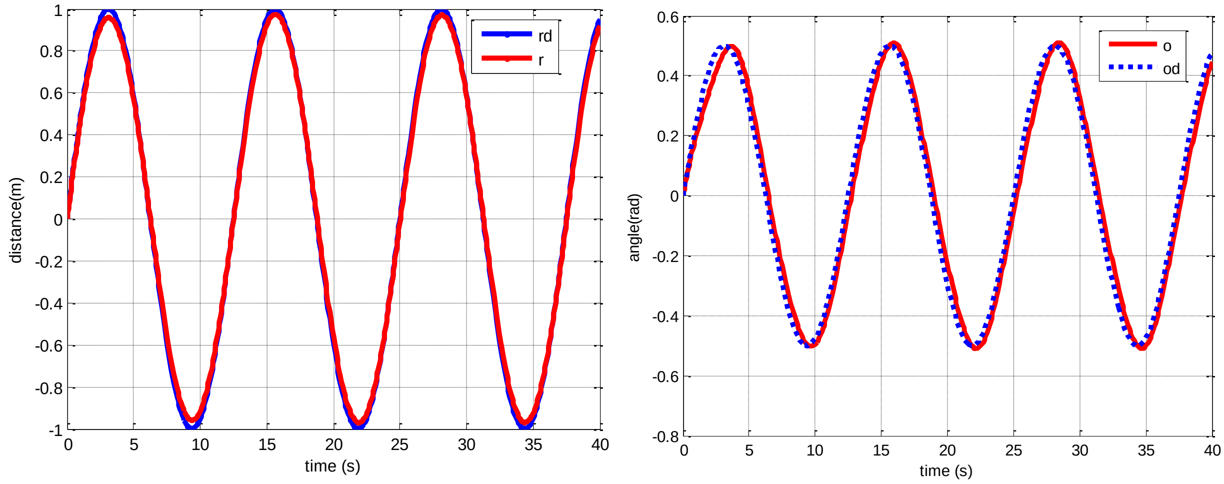


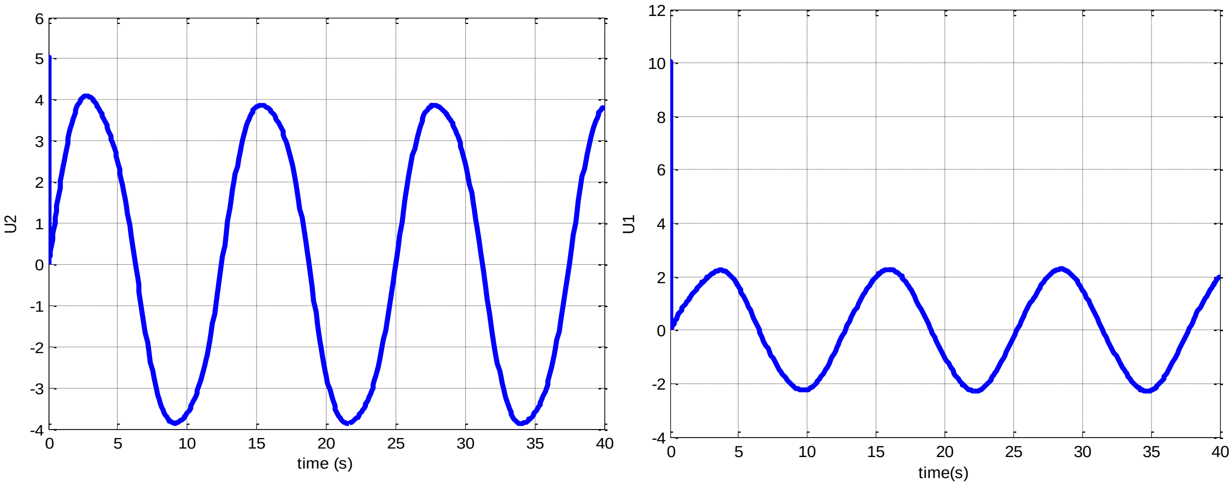
Figure 1.11: Answers in BF for entry level.

The position, control and error results for a sinusoidal reference (amplitude 0.5 and frequency 0.08rad/s) shown in Figure (2.12).

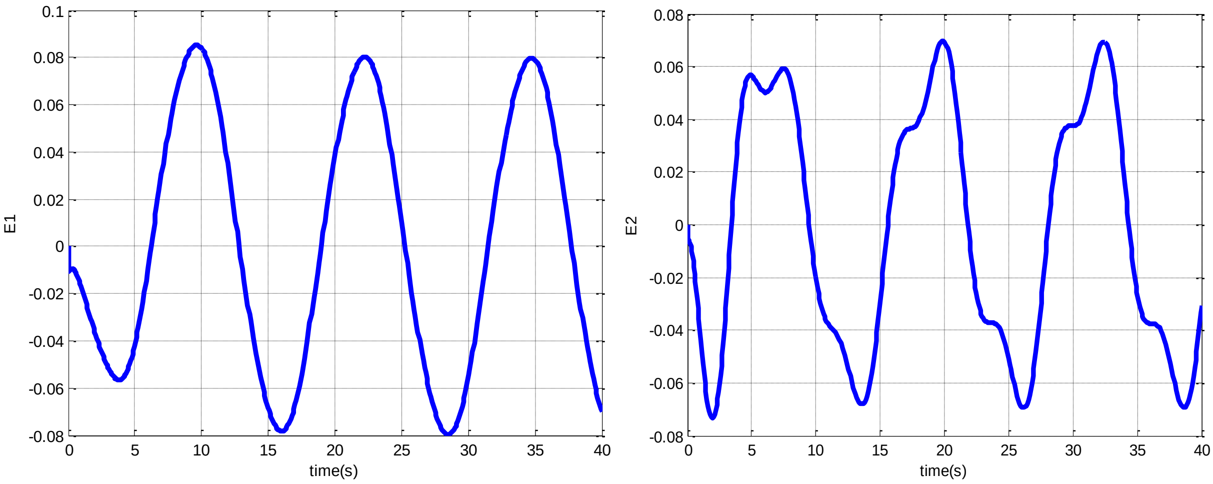
For the synthesis parameters we took ($k_{p2} = 11$; $k_{i2} = 37$; $k_{d2} = 20$):



a) outputs



b) command



c) errors

Figure 2.14: Simulation results of the PID control, for a sinusoidal input.

- From these figures we can see, that the errors of regulation and pursuit and the value of overrun are small (Fig. 2.14).
- We can also see that the control signals are relatively smooth.

1.14 Conclusion

In conclusion, this chapter has provided a comprehensive overview of the critical aspects of robotics, from basic definitions and types to advanced control techniques and simulations. The 2 degrees of freedom manipulator arm is a non-linear robotic model with two independent movements: inputs and outputs. In order to demonstrate how the system responds to changes in the output position angle, an open-loop simulation test is provided. As a result, our system became unstable, and the PID regulator was used to provide control by fine-tuning the Proportional-Integral-Derivative parameters so we could significantly enhance the accuracy and stability of the manipulating arm. In the next chapter, the controlling manipulator arm using the sliding mode will be introduced.

2.1 Introduction

When developing a control issue, inconsistencies between the actual system and the mathematical model used for control design are common. This disparity might be due to differences in system dynamics parameters or a model's approximation of complex system behavior. This has resulted in a strong interest in creating effective control approaches to address this issue [17]. Sliding mode control has mostly demonstrated its utility in theoretical research, and its primary application fields include robotics and electric motors. The importance of such a control stems from its robustness to model perturbations and uncertainty. However, these performances come at the cost of some drawbacks, such as chattering or reluctance induced by the discontinuous element of this control, which might harm the actuators[18][19].

2.2 General and principles

Various path modes in the phase plan.The Sliding Mode Control (SMC) look at was first created in the 1950s and popularized by Utkin's major study [20]. It is a control technique that has gained popularity in current control theory due to its simplicity and resilience in the face of parametric fluctuations [21]. With the expanding development of power electronics, this control method has proved particularly effective in the realm of electrical equipment[22].

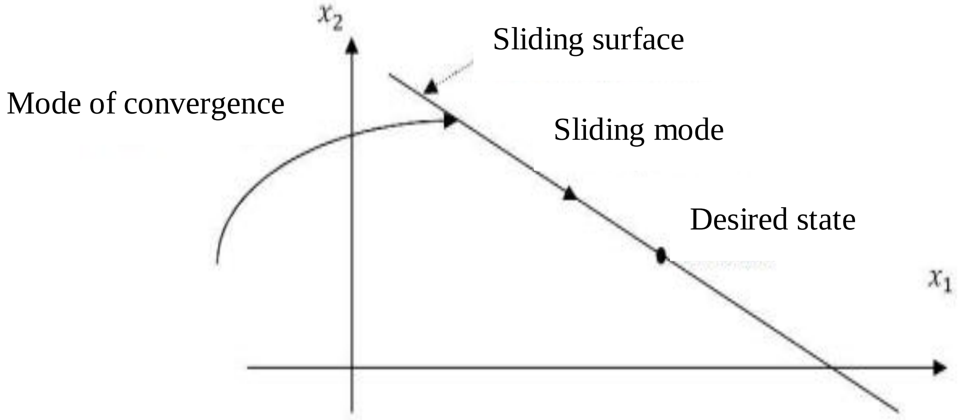


Figure 2.1:Different modes of convergence for the state trajectory.

2.3 Variable structure system

- Using the proper switching logic, the sliding mode control approach returns the system's state trajectory to the sliding surface before switching it to the balance point.This trajectory is made up of three different components.
- The convergence mode (CM) is the mode in which the variable to be modified travels from any starting point in the phase plane to the switching surface S(x)=0.This mode is defined by a control rule and a convergence condition.
- Slip mode (SM): This is the mode in which the state variable has reached the slip surface and is moving toward the origin of the phase plane. The sliding surface S(x)=0 determines the dynamics of this mode.
- Steady state mode (SSM): This mode is included to look into the system's behavior near its equilibrium point. It describes the quality and performance of the control (Gao and Hung, 1993)[23].

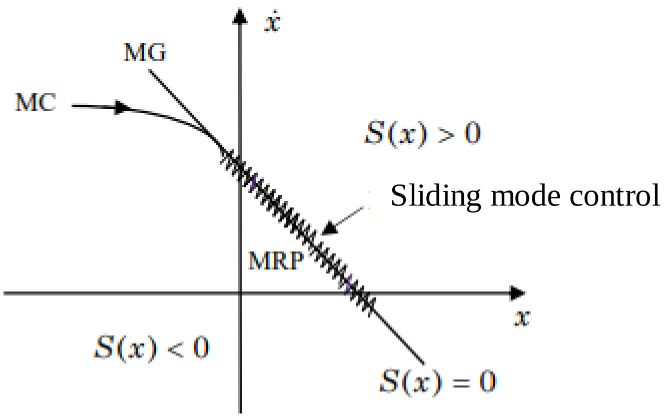


Figure 2.2:Different modes for the trajectory in the phase plane[6].

2.4 Sliding mode

Sliding mode control is a nonlinear control look at distinguished by the control's discontinuity as it travels over a switching surface known as the sliding surface. The discontinuous control law is utilized to keep the system on this surface. Thus, the phenomena of sliding is represented as follows: [24][25].

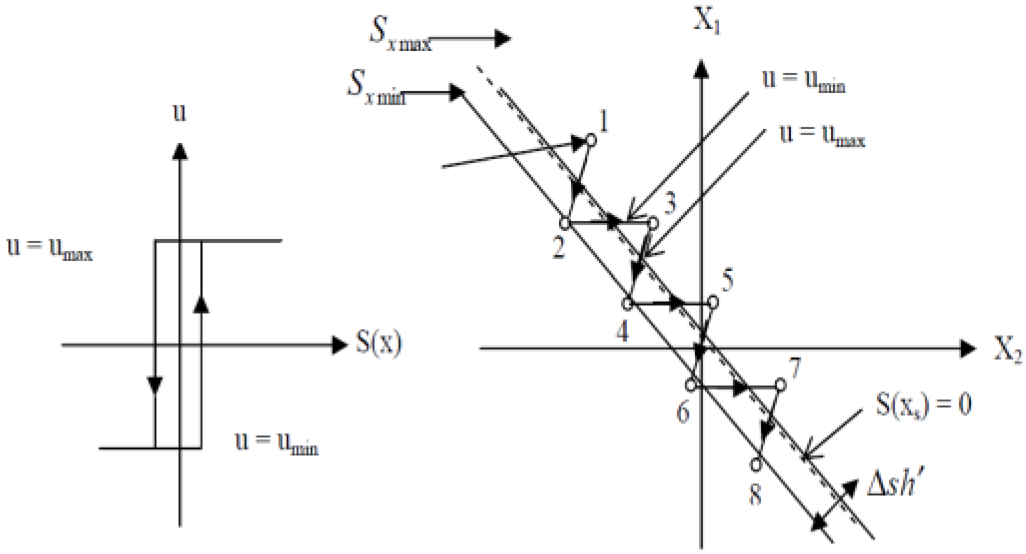


Figure 2.3: Sliding mode.

We initially recognize a hysteresis in the switching law S(x) = 0 (mixed line). As a result, the switches take place on two lines offset parallel to *Sh'*. A trajectory with *u=Umax* intersects the lower tipping point at point 1. If the trajectory is pointed towards the hysteresis zone with u=Umin, it will come into contact with point 2, the upper tipping threshold at which a switch occurs when u = Umax. If the trajectory is shifted inward again, it will reach the lower tipping threshold at point 3, and so on. [26].

2.5Sliding mode control design

The sliding mode control technology is characterized by the use of an intermittent surveillance Law to force the system to stay on that surface. So we observe the phenomenon of chattering:

•Choice of the surface.

• Establishment of living conditions.

• Determination of the control law.

2.5.1 Choice of the sliding surface

The choice of the sliding surface concerns the number and shape of the necessary functions. These two factors depend on the application and the objective.
For a system defined by equation (2.1), the surface vector s has the same dimension as the control vector *u:*

$$\dot{x} = f(x,t)x + g(x,t)u \quad (2.1)$$

The sliding surface is a scalar function such that the variable to be adjusted slides on this surface and tends towards the origin of the phase plane

$$s(x) = \left(\frac{\partial}{dt} + \lambda\right)^{r-1} e(x) \quad (2.2)$$

With:

- e(x) :is the difference between the variable to be adjusted and its reference
- λ : s a positive constant.
- r: is a relative degree, it presents the number of times it is necessary to derive the surface to make appear the command.

The surface expression is a differential equation whose only solution is *e(x)=0.*
The purpose of the control is to keep the surface to zero [27].

2.5.2Condition of existence and convergence

For the sliding surface to exist, the convergence mode must be ensured, there are two conditions:

a. The direct function of switching

This is the first condition of convergence (attractiveness). It is proposed and studied by EMILYANOV and UTKIN [28]. [29]
It is a question of giving the surface a dynamic converging towards zero. It is formulated by:

$$\begin{cases} \dot{s}(x) > 0 \; si \; s(x) < 0 \\ \dot{s}(x) < 0si \;\; s(x) > 0 \end{cases} \quad (2.3)$$

This condition may be given otherwise by:

$$\dot{s}(x).\, s(x) < 0 \quad (2.4)$$

b. The Function of Lyapunov

The determination of the sliding domain may be simplified to a study of the system's stability in sliding mode. This work makes use of the Lyapunov function[6]. The Lyapunov function is a positive scalar function (v(x)>0) for system state variables, and the control rule must reduce it (v(x)<0). The fundamental concept is to select a scalar function s(x) to assure the attraction of the variable to be controlled to its reference value, and to create a U command so that the square of the surface corresponds to a Lyapunov function that is specified by

$$V(x) = \frac{1}{2} s^2(x) (2.5)$$

By deriving the latter, one obtains:

$$\dot{v}(x) = \dot{s}(x) s(x) (2.6)$$

For Lyapunov's candidate position to decline, it is enough to ensure that:$\dot{v}(x) < 0$

If the v(x) function drops, ensure that its derivative is negative. This can only be accomplished if the condition (2.5) is confirmed. According to equation (2.6), the distance measured by S(x) to the surface decreases continually, pushing the system's trajectory to move towards the surface from both sides. [6].

2.5.3Order Summary

A sliding regime requires discontinuous control. The sliding surface should be appealing from both sides. As a result, if this discontinuous command is required, it will not block the inclusion of a continuous component. The amplitude of the discontinuous section is lowered to the greatest extent feasible by the continuous part. When there is a problem, the discontinuous component mostly serves to verify the conditions of attraction. To get the system to follow the desired trajectory, simply make S = 0 appealing. Thus, an u_n command is appended to the equivalent command u_{eq} in the following manner:

$$u = u_{eq} + u_n \qquad (2.7)$$

The necessary condition for the system states to follow the trajectory defined by the sliding surfaces is $S = 0 \Rightarrow \dot{S} = 0$which brings us back to defining the equivalent U_{eq}command.

$$u_n = -K\, sign(s(x,t)) \qquad (2.8)$$

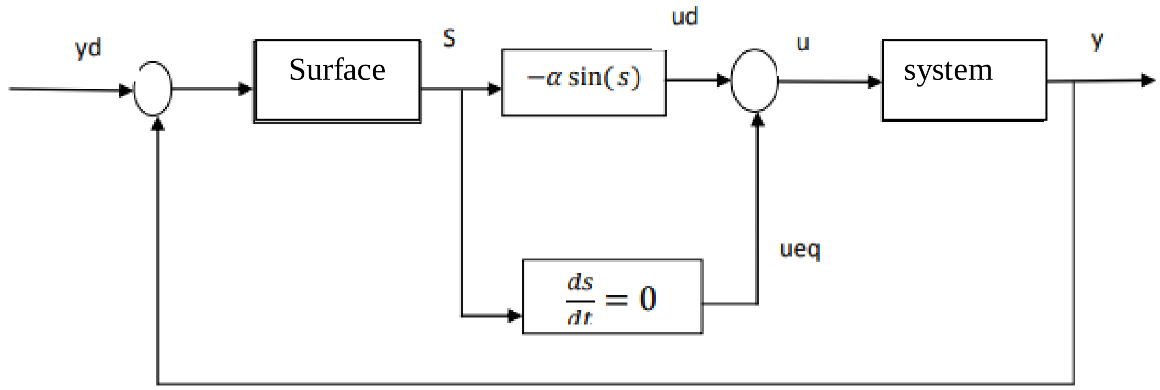


Figure2.4: The structure of a controller by sliding mode.

2.6The phenomenon of chattering

In practical implementations of sliding mode control, engineers may encounter the unwanted phenomena of oscillations with limited frequency and amplitude, known as 'chattering'. Chattering was the primary impediment to the application of sliding mode control theory during its early stages of development. Chattering is a negative phenomenon because it causes poor control precision, excessive wear of moving mechanical parts, and significant heat losses in power circuits. There are two factors that might cause chattering.

- The ideal model failed to account for quick dynamics, leading to chattering. These 'unmodeled' dynamics with tiny time constants are typically ignored in models of servomechanisms, sensors, and data processors.
- Utilizing digital controllers with finite sample rates leads to 'discretization noise'. The perfect sliding mode implies an unlimited switching frequency. Because the control is constant inside a sample period, switching frequency cannot exceed that of sampling, which also causes chattering.[30].

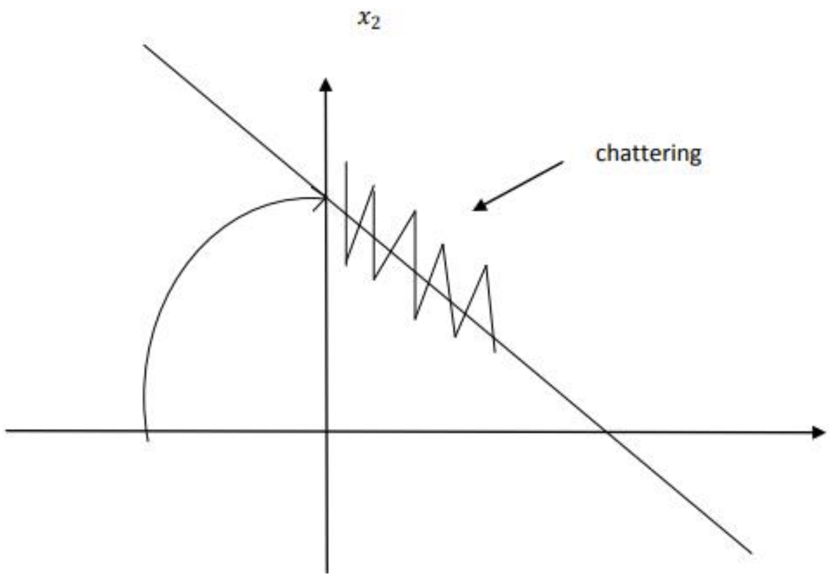


Figure 2.5:the phenomenon of chattering[29].

2.7Solutions to mitigate the phenomenon of chattering

Many methods have been proposed to decrease or eliminate the phenomena of reluctance, such as the boundary layer, which consists of substituting the "sign" function of the control law with a continuous approximation in a neighborhood of the sliding surface[31].

We propose to describe skid reduction techniques; the two most used functions are:

2.7.1The saturation function

This involves replacing the *sign*(σ) function with the slope line 1/8 inside a width band 28 on either side of the sliding surface, while retaining the discontinuity outside this band. Its expression is presented as:

$$sat(\sigma,\delta) = \begin{cases} \frac{\sigma}{\delta} & si \quad |\sigma| \leq \delta \\ sign(\sigma) & si \quad |\sigma| > \delta \end{cases} \tag{2.9}$$

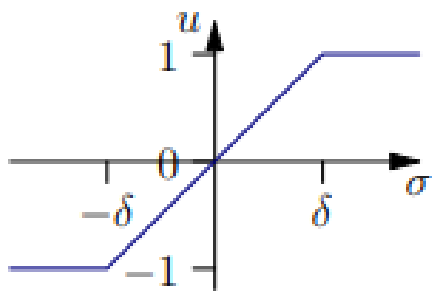


Figure 2.6: Function saturation.

2.7.2 The sign + saturation function

The previous saturation function can be combined with the sign function.

$$sat(\sigma,\delta) = \begin{cases} \alpha\frac{\sigma}{\delta} + bsign(\sigma) & si \quad |\sigma| \leq \delta \\ (a+b)sign(\sigma) & si \quad |\sigma| > \delta \end{cases} \tag{2.10}$$

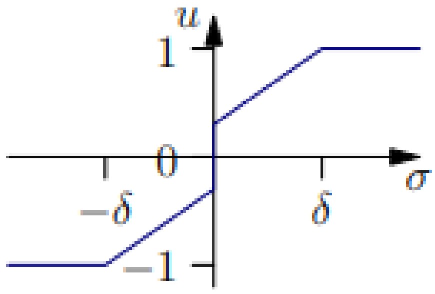


Figure 2.7 : The sign + saturation function.

There are other less used approximations, as they are more expensive in computation time

2.7.3The law of power interpolation

$$v(\sigma,\delta)=\begin{cases} sign(\sigma) & \text{si } |\sigma|>\delta \\ (\delta/|\sigma|)^{(q-1)} & \text{si } 0<|\sigma|\leq\delta \\ 0 & \text{si } s=0 \end{cases} \quad (2.11)$$

With **:** $q \in [0,1]$

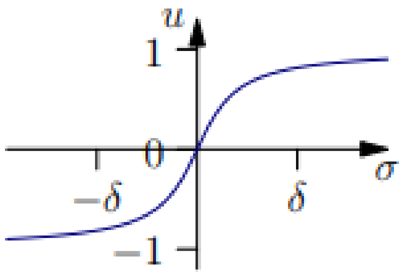


Figure 2.8: The law of power interpolation

2.7.4 The pseudo-sign function

$$v(\sigma,\delta)=\frac{\sigma}{\sigma+|\sigma|} \qquad (2.12)$$

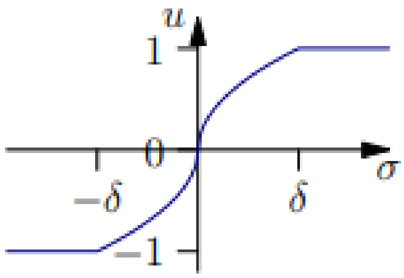


Figure 2.9 The pseudo-sign function.

2.7.5 The arctangent function

$$v(\sigma,\delta)=\frac{2}{\pi}arc\ \tan\frac{\sigma}{\delta} \qquad (2.13)$$

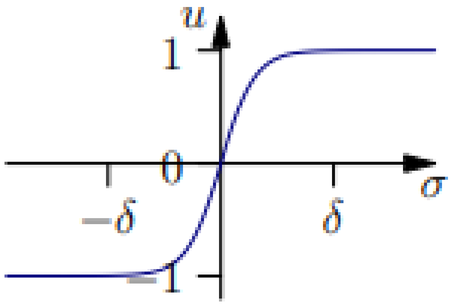


Figure2.10: The arctangent function.

2.7.6 The hyperbolic tangent function

$$v(\sigma,\delta)=tanh(\frac{\sigma}{\delta}) \qquad (2.14)$$

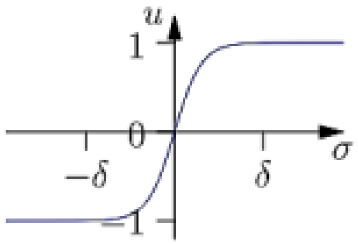


Figure2.11: The hyperbolic tangent function

2.8Application of the control by sliding mode to the manipulator arm

2.8.1 principle of operation

we have the dynamic model:

$$\begin{cases} \ddot{r} - r\dot{\theta}^2 + g\sin\theta = \frac{F}{m} \\ r^2\ddot{\theta} + 2r\dot{r}\dot{\theta} + gr\cos\theta = \frac{T}{m} \end{cases} \quad (2.15)$$

we pose:

$$\begin{cases} y_1 = x_1 = r \\ y_2 = x_3 = \theta \end{cases} \quad (2.16)$$

we calculate the relative degree:

$$\begin{cases} y_1 = x_1 = r \\ y_1^{(1)} = \dot{x}_1 = \dot{r} \\ y_1^{(2)} = \dot{x}_2 = \ddot{r} = x_1 {x_4}^2 - g\sin x_3 + \frac{U_1}{m} \end{cases} \quad (2.17)$$

and it's where:$r_1 = 2$

$$\begin{cases} y_2 = x_3 = \theta \\ y_2^{(1)} = x_4 = \dot{x}_3 = \dot{\theta} \\ y_2^{(2)} = \dot{x}_4 = \ddot{\theta} = \frac{1}{1+\sin {x_1}^2}\left(-2x_1x_2x_4 - gx_1\cos x_3 + \frac{U_2}{m}\right) \end{cases} \quad (2.18)$$

and $r_2 = 2$

$$\begin{cases} e_1 = x_1 - x_{1d} \\ e_2 = x_3 - x_{3d} \end{cases} \quad (2.19)$$

$$s_1 = (\frac{\mathrm{d}}{\mathrm{dt}} + \lambda_1)^{r_1 - 1} e_1 \quad (2.20)$$

$$s_1 = \left(\frac{\mathrm{d}}{\mathrm{dt}} + \lambda_1\right)^{2-1} (x_1 - x_{1d}) \quad (2.21)$$

$$s_1 = \left(\frac{\mathrm{d}}{\mathrm{dt}} + \lambda_1\right)^{1} (x_1 - x_{1d}) \quad (2.22)$$

$$s_2 = (\frac{\mathrm{d}}{\mathrm{dt}} + \lambda_2)^{r_2 - 1} e_2 \quad (2.23)$$

$$s_2 = (\frac{\mathrm{d}}{\mathrm{dt}} + \lambda_2)^{2-1} (x_3 - x_{3d}) \quad (2.24)$$

$$s_2 = (\frac{\mathrm{d}}{\mathrm{dt}} + \lambda_2)^{1} (x_3 - x_{3d}) \quad (2.25)$$

then:

$$\begin{cases} s_1 = (\dot{x}_1 - \dot{x}_{1d}) + \lambda_1 (x_1 - x_{1d}) \\ s_2 = (\dot{x}_3 - \dot{x}_{3d}) + \lambda_2 (x_3 - x_{3d}) \end{cases} \quad (2.26)$$

$$\begin{cases} \dot{s}_1 = (\ddot{x}_1 - \ddot{x}_{1d}) + \lambda_1 (\dot{x}_1 - \dot{x}_{1d}) \\ \dot{s}_2 = (\ddot{x}_3 - \ddot{x}_{3d}) + \lambda_2 (\dot{x}_3 - \dot{x}_{3d}) \end{cases} \quad (2.27)$$

$$\begin{cases} \dot{s}_1 = (\dot{x}_2 - \ddot{x}_{1d}) + \lambda_1(\dot{x}_1 - \dot{x}_{1d}) \\ \dot{s}_2 = (\dot{x}_4 - \ddot{x}_{3d}) + \lambda_2(\dot{x}_3 - \dot{x}_{3d}) \end{cases} \tag{2.28}$$

$$\begin{cases} \dot{s}_1 = \left(x_1 {x_4}^2 - g\sin x_3 + \frac{U_1}{m}\right) + \lambda_1(x_2 - \dot{x}_{1d}) \\ \dot{s}_2 = \left(\frac{1}{1+\sin {x_1}^2}\left(-2x_1x_2x_4 - g x_1\cos x_3 + \frac{U_2}{m}\right) - \ddot{x}_{3d}\right) + \lambda_2(x_4 - \dot{x}_{3d}) \end{cases} \tag{2.29}$$

And we have sliding condition:

$$\begin{cases} \dot{s}_1 = -k_1 sign(s) \\ \dot{s}_2 = -k_2 sign(s) \end{cases} \tag{2.30}$$

The control law is as follows:

$$\begin{cases} U1 = m * [-x_1x_4^2 + g\sin x_3 + \ddot{x}_1 d - \lambda_1(-x_2 + \dot{x}_1 d) - k_1 sign(s)] \\ U2 = m * [(g x_1 \cos x_3 + 2x_1x_2x_4) + (1 + \sin {x_1}^2) * \left(\ddot{x}_3 d - k_2 sign(s) + \lambda_2(\dot{x}_3 d - x_4)\right)] \end{cases} \tag{2.31}$$

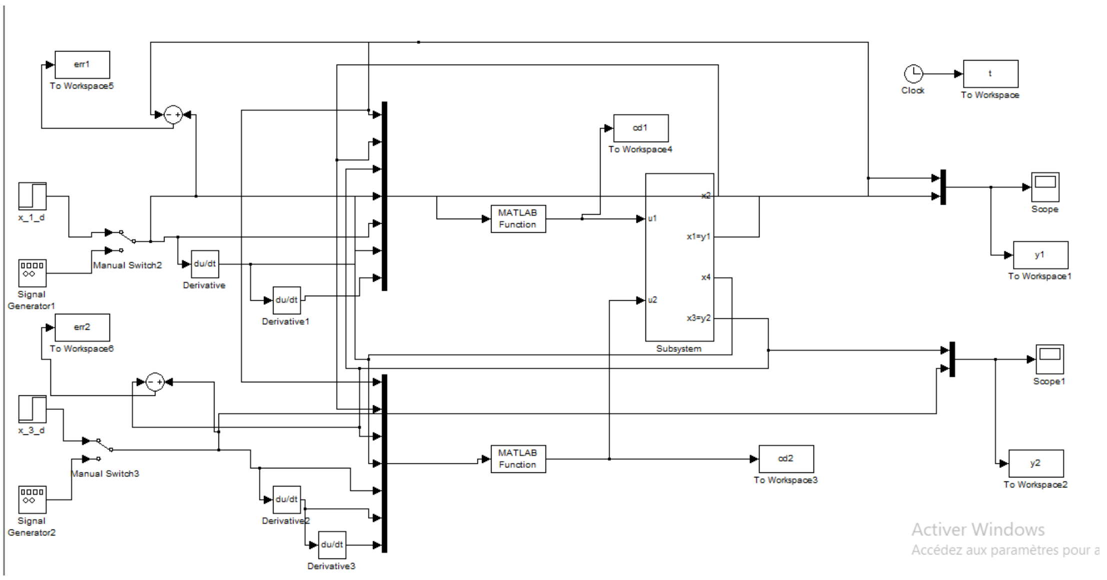


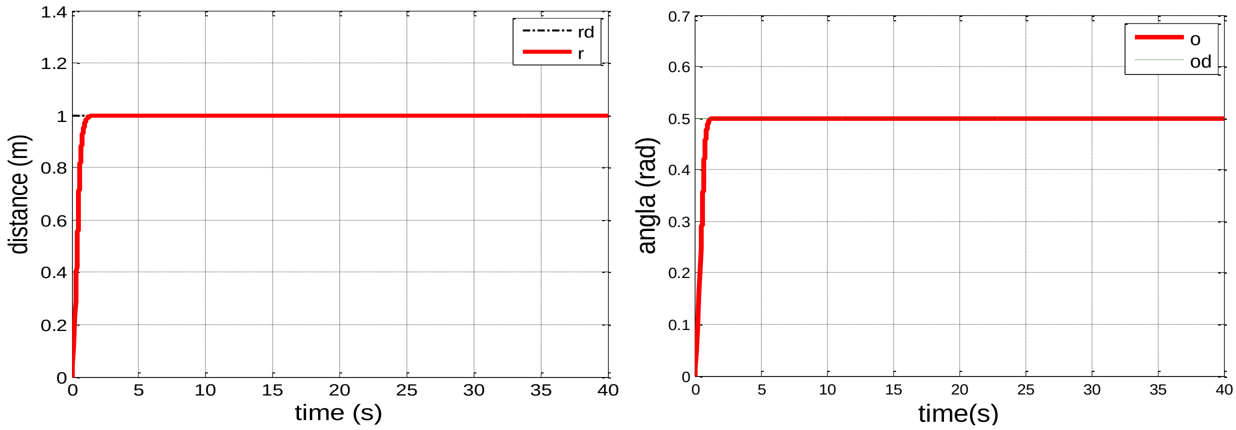
Figure 2.12:block diagram of the control by sliding mode technique.

2.8.3 Simulation results

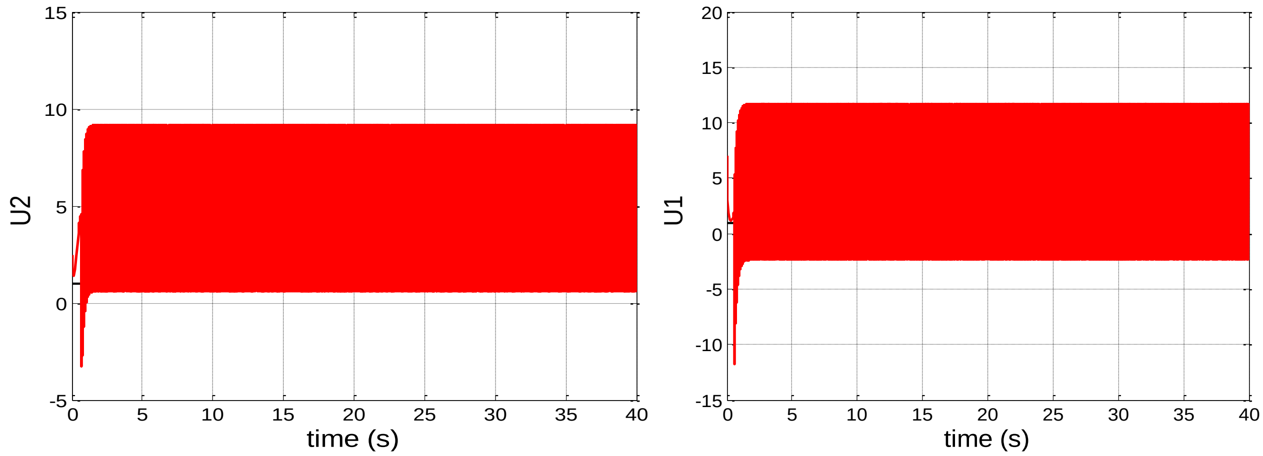
For the synthesis parameters we took:

$$k_1 = 7\,; \qquad \lambda_1 = 5$$
$$k_2 = 5; \qquad \lambda_2 = 7$$

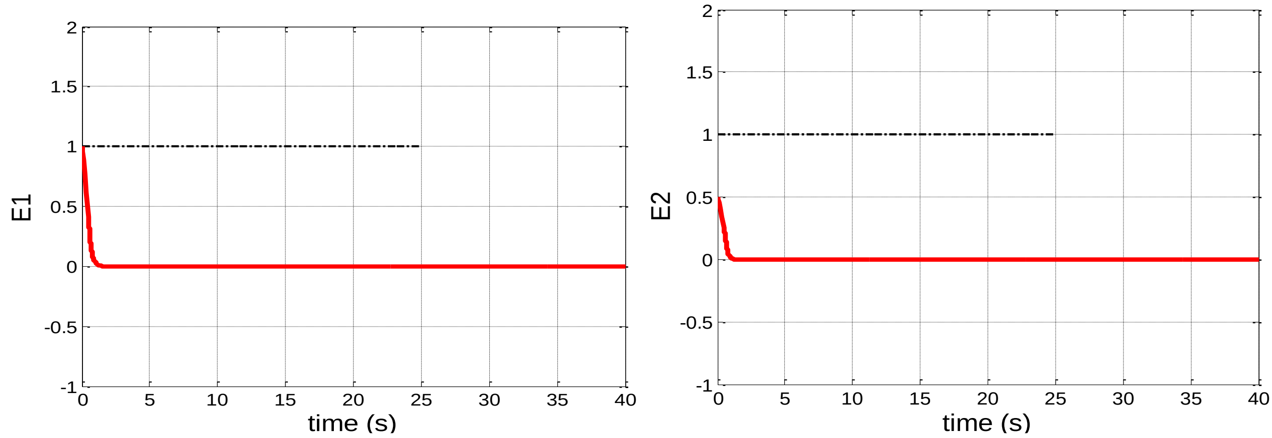
The results of the position, control and error in figure 2.13:



a) outputs



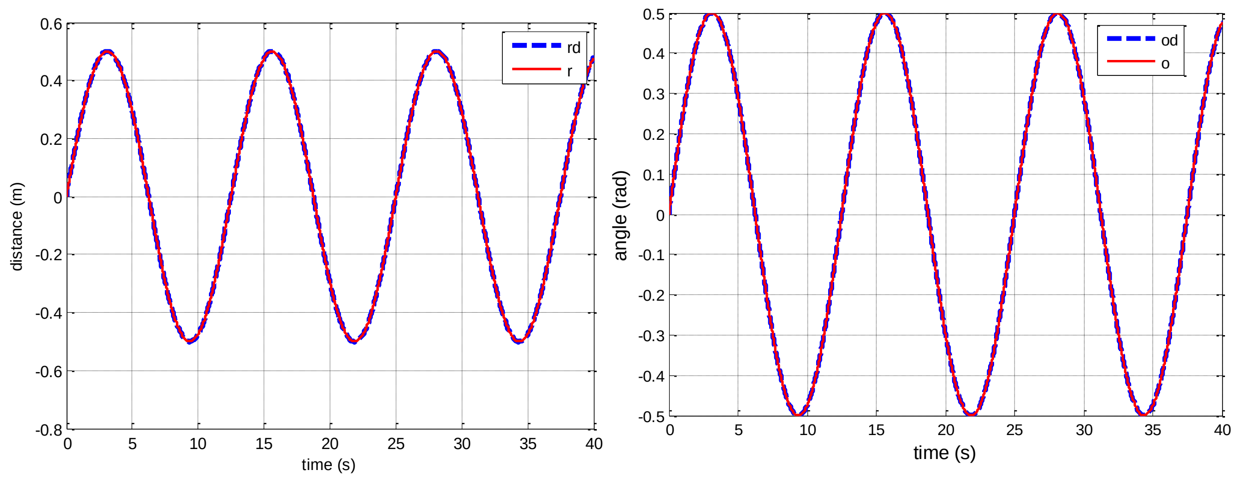
b) command



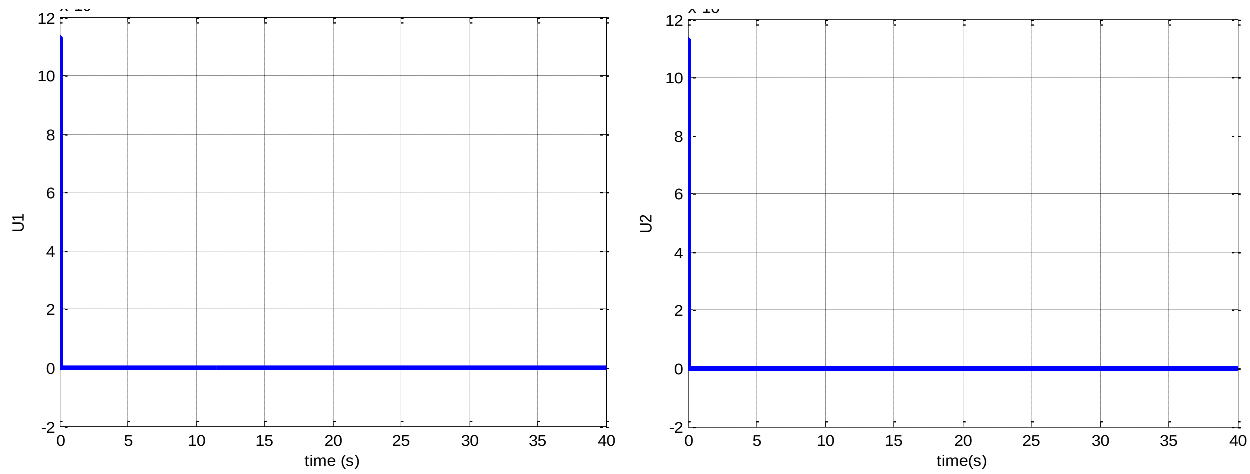
c) errors

Figure 2.13: Simulation results of the control by sliding mode, for step reference.

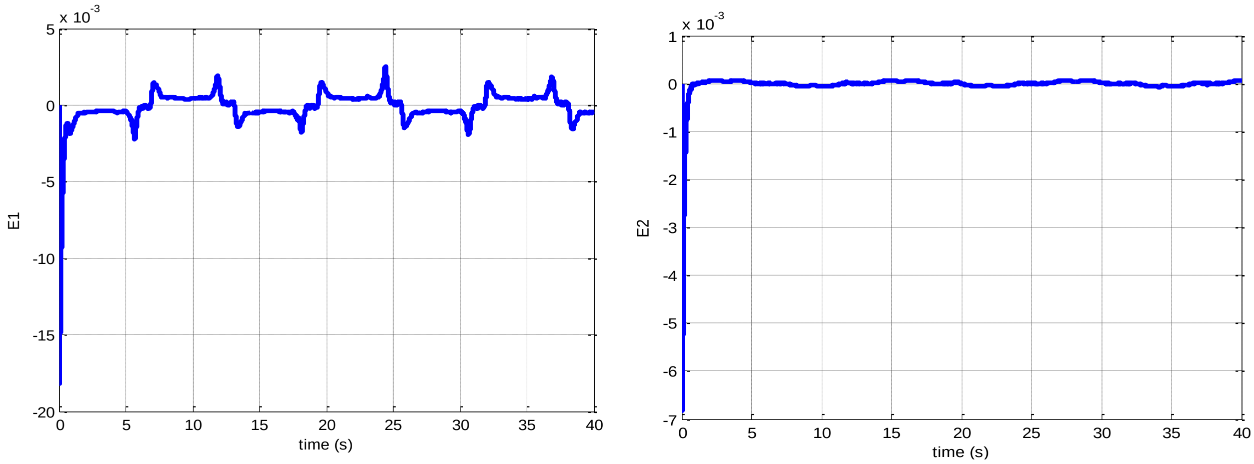
Theposition, control and error results for a sinusoidal reference (amplitude 0.5 and frequency 0.08rad/s) shown in Figure 2.14:



a) outputs.



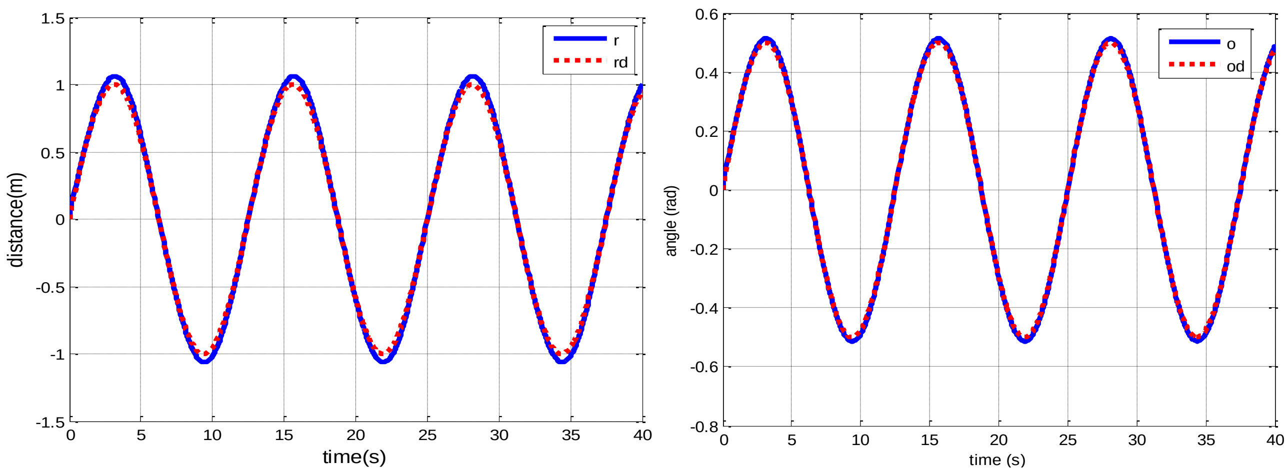
b) command.



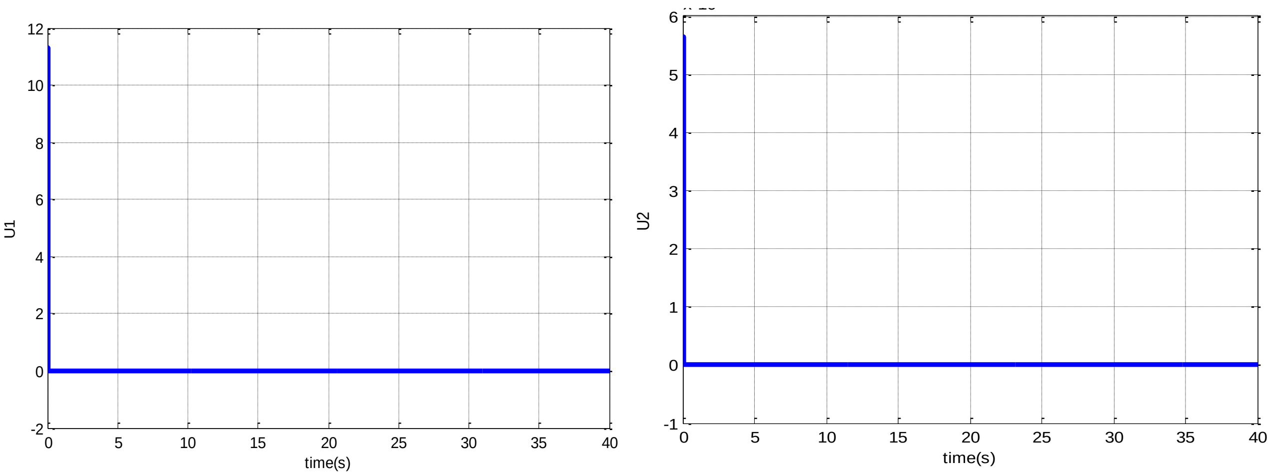
c) errors

Figure 2.14: Simulation results of the control by sliding mode, for a sinusoidal input.

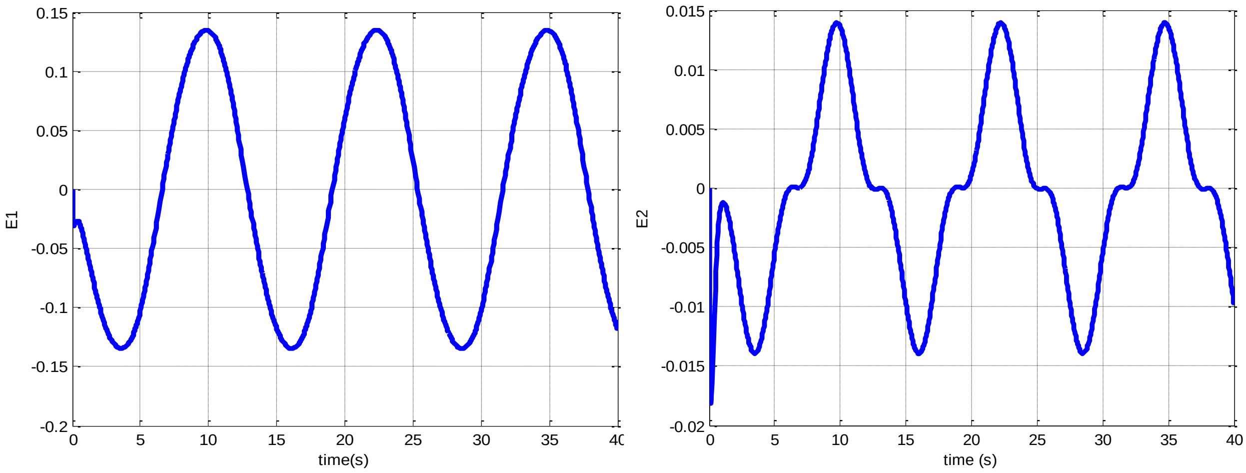
The hyperbolic tangent function (*tanh*) was used to reduce chatter. The results are shown in the figure(2.15):



a) outputs.



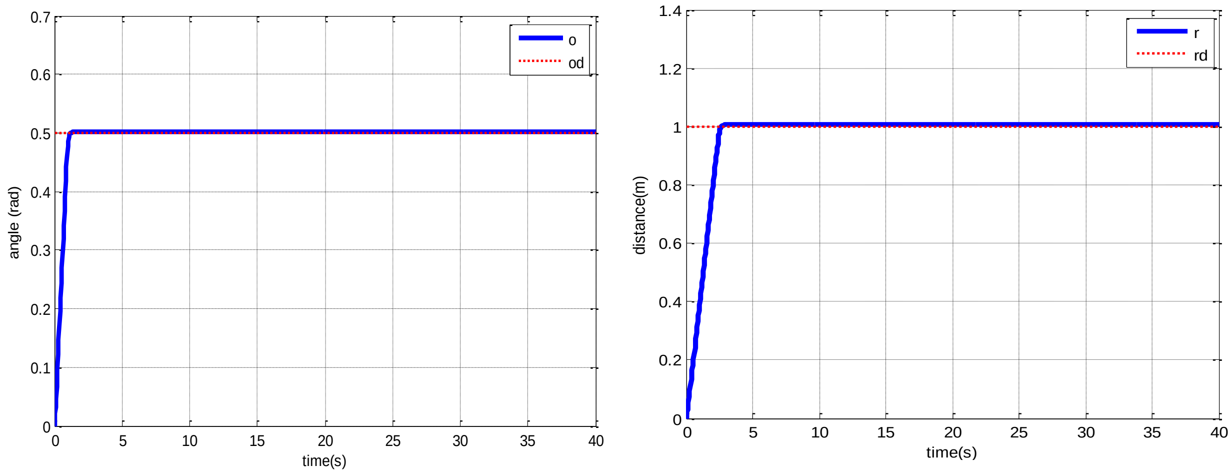
b) command.



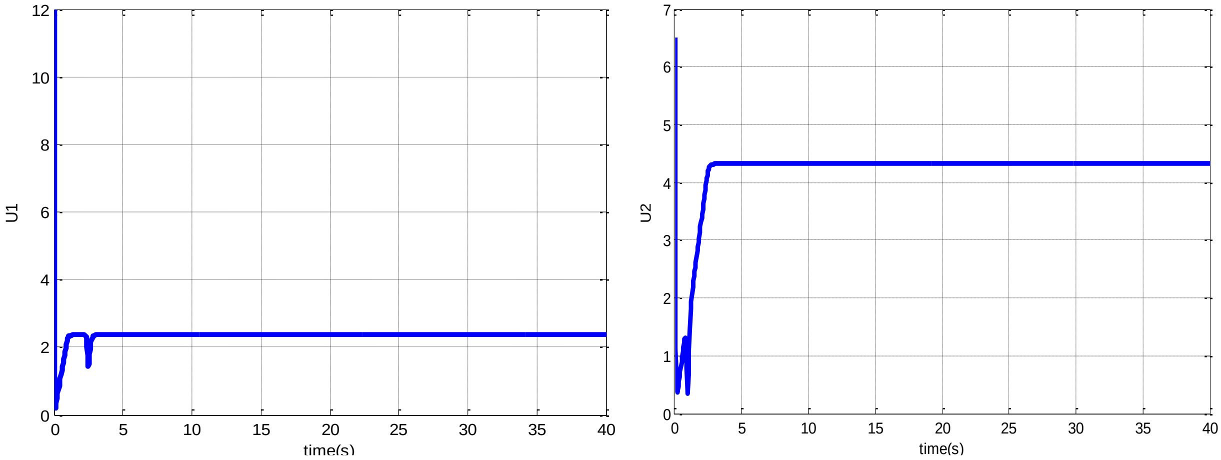
c) errors.

Figure 2.15: Simulation results of the control by sliding mode, for a sinusoidal input.

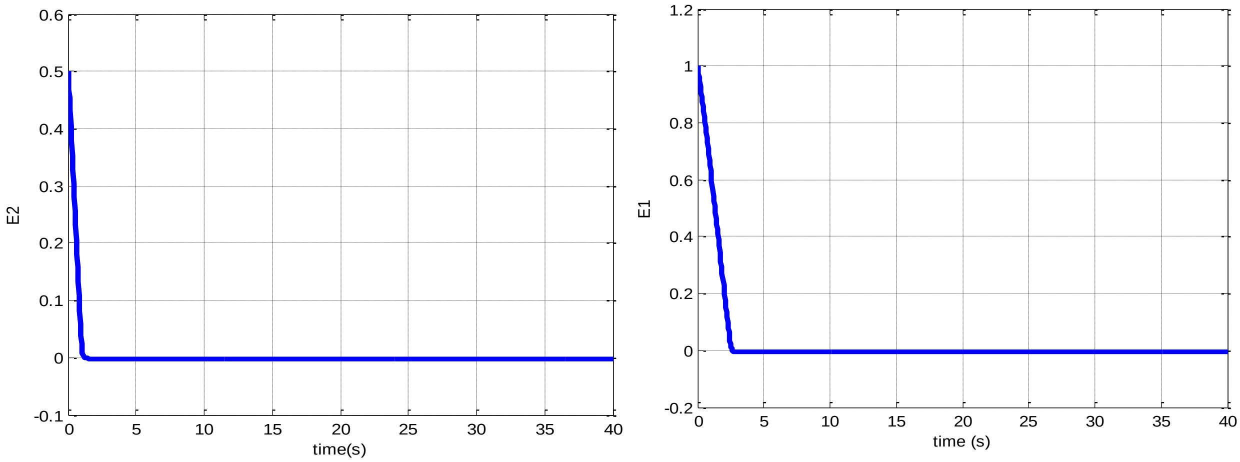
For the synthesis parameters we took: $k_1 = 12$; $\lambda_1 = 35$; $k_2 = 13$; $\lambda_2 = 25$:



a) outputs



b) command



c) errors

Figure 2.16: Simulation results of the control by sliding mode, for step reference.

- We note that there is a good continuation of the desired trajectories and that the control signals and the estimated parameters are bounded. Figures 2.13-2.16 show that the SMC reduces the effect of defects on control performance by rapidly varying the values of the controller parameters to bring the system outputs to their desired values quickly.

2.9Conclusion

Applying the sliding mode on the arm manipulator will make its performance more accurate and stable, increasing its efficiency and effectiveness in interacting with the specific environment and tasks. This is what we touched upon in this file. We allocated the first part of the basic concepts to command sliding mode (General and principles, Different modes for the trajectory in the phase plane, Order Summary, the phenomenon of chattering and their solutions).The simulation results demonstrate the benefits of the command by showing its consistency with the resilience it provides against external disturbances...However, we noted that the system's performance was affected by the robust oscillations of the control unit. This phenomenon called chatting has several solutions, including replacing the sign function with a hyperbolic tangent(*tanh*), which reduces or reduces oscillations and improves the performance of control, stability, accuracy, etc. In the next chapter, we propose using the command feedback linearization.

3.1 Introduction

This chapter describes a controller for MIMO systems that may ensure specified performance, known as feedback linearization.

Feedback linearization is a powerful control approach for nonlinear control systems that simplifies controller design and analysis. This method makes it easier to apply well-established linear control techniques to systems that are intrinsically nonlinear by converting a nonlinear system into an analogous linear one via variable changes and feedback. The method is especially useful for dealing with complex dynamics, which are common in the robotics, aerospace, and chemical process industries.

The basic principle underlying feedback linearization is to cancel out the nonlinearities in system dynamics. This is accomplished by creating a control rule that effectively converts the nonlinear system into a linear one. The transformation consists of two critical steps: selecting an appropriate coordinate transformation and applying a feedback control law that cancels the nonlinear effects.

3.2History of the Feedback linearization control

Feedback linearization control is a more advanced control method that use linear control techniques to control nonlinear systems. Researchers in automatic control created this technology during the 1980s and 1990s. The latter has expanded the framework of systems theory with fundamental physical notions. Also, the work of Yula and al created the first relationship between circuit theory and I/O stability theory [32].

Throughout the late 20th and early 21st centuries, feedback linearization became increasingly relevant in various high-tech applications. It has been particularly influential in fields such as robotics, where precise control of nonlinear dynamics is crucial. For example, in robotic manipulators and mobile robots, feedback linearization techniques are employed to achieve desired motion trajectories and stability.

3.3 The theory of feedback linearization

Feedback linearization is a nonlinear control design approach that has sparked significant research interest in recent years. The approach's basic idea is to algebraically reduce nonlinear system dynamics into (fully or partially) linear ones, allowing linear control techniques to be employed. This is fundamentally different from conventional linearization (Jacobian linearization) in that feedback linearization is accomplished by exact state transformations and feedback rather than linear approximations to the dynamics [32].

The robust feedback linearization technique is concerned with the local stability of a nonlinear process. It formalizes the intuition that a nonlinear process should behave similarly to its linearized approximation for short-range motions. Because all physical processes are intrinsically nonlinear, the robust feedback linearization technique serves as the primary basis for employing linear control strategies in practice, demonstrating that stable design via linear control ensures the stability of the original physical process locally[33].

3.4 Applications of Feedback Linearization

Feedback linearization is widely utilized in various engineering fields, allowing precise control of nonlinear systems. Here are some key areas where this technique is applied:

3.4.1 Robotics and Automation

- Robotic Manipulators
- Mobile Robots

3.4.2 Aerospace

- Aircraft
- Spacecraft

3.4.3 Electrical Systems

- Electric Motors
- Power Systems

3.4.4Automotive

- Autonomous Vehicles
- Advanced Driver Assistance Systems (ADAS)

3.4.5 Medical Applications

- Medical Devices
- Drug Delivery Systems

3.4.6Marine Systems

- Submarines
- Ships

3.5 mathematical tools

In this section, we present some mathematical tools necessary to assimilate the input-output linearization technique.

- **Gradient**

A scalar function h(x) is defined with respect to the state vector $x \in R^n$,the gradient of h(x) is given by:

$$\nabla h = \frac{\partial h}{\partial x} = \begin{bmatrix} \frac{\partial h}{\partial x} & .... & \frac{\partial h}{\partial x_n} \end{bmatrix} \quad (3.1)$$

$\nabla h$is a vector line of elements $\nabla h_i = \frac{\partial h}{\partial x_i}$

- **Derived from Lie**

We consider a scalar function$h: R^n \to R$and two vector fields$f, g: R^n \to R^n$. The derivative of Lie as a new scalar function, rated $L_f h$, giving the derivative of h(x) in the direction of f(x), such that[34]:

$$L_f h(x) = [\Delta f h] = \begin{bmatrix} \frac{\partial h}{\partial x_1} & .... & \frac{\partial h}{\partial x_n} \end{bmatrix} \begin{bmatrix} f_1 \\ \vdots \\ \vdots \\ f_2 \end{bmatrix} \quad (3.2)$$

Derivatives of any order of Lie are given by:

$$L_f^0 h = h(x) \quad (3.3)$$

$$L_f^i h = L_f\left(L_f^{i-1} h\right); i = 1,2,3, \cdots \quad (3.4)$$

The Lie derivative of the function $L_f h$, following the vector field g is given by:

$$L_g L_f h = \nabla\left(L_f h\right) g = \sum_{i=1}^{n} \frac{\partial\left(L_f h(x)\right)}{\partial x_i} g_i(x) \quad (3.5)$$

The relative degree (graded r) of a SISO system can be intuitively defined as the minimum number of times required to differentiate the expression from the output (y) versus time to see that the input occurs clearly (u):

$$L_g L_f^i h(x) = 0 \qquad 0 \le i \le -1 \quad (3.6)$$

$$L_g L_f^{r-1} h(x) \neq 0 \quad (3.7)$$

$$\begin{cases} y(x) = h(x) \\ y^{(1)} = h_1(x) \\ y^{(2)} = h_2(x) \\ \vdots \\ y^{(r-1)}(x) = h_{r-1}(x) \\ y^{(r)}(x) = a(x) + b(x)u \end{cases} \quad (3.8)$$

In order to determine the nonlinear control law, we calculate the relative degree of the output i.e. the number of times the output must be derived in order to reveal the input u. The temporal derivation of the output gives:

$$y^{\cdot} = \frac{dh(x)}{dx}\frac{dx}{\partial t} \quad (3.9)$$

$$= \frac{dh(x)}{dx}\left(f(x) + g(x)u\right) \quad (3.10)$$

$$= L_f h(x) + L_g h(x) u \quad (3.11)$$

if $L_g h(x) \neq 0 \forall x \in R^n$,it is easily shown that the control:

$$u = \frac{1}{L_g h(x)}\left(-L_f h(x) + v\right) \quad (3.12)$$

Leads to linear system representing a simple integrator:

$$y^{\cdot} = v \quad (3.13)$$

If $L_g h(x) = 0$the bypass is continued to obtain:

$$y^{(i)} = L_f^i h(x) + L_g L_f^{i-1} h(x) u i = 1,2, \ldots (3.14)$$

With $L_g L_f^{i-1} h(x) \neq 0$.. The method thus consists in determining the degree of derivation *r* from which the multiplier factor of the command $u \left(L_g L_f^{i-1} h(x) \right)$is not zero, *r* is the relative degree of h. It is shown that for:

$$y^{(r)} = L_f^r h(x) + L_g L_f^{r-1} h(x) u \qquad (3.15)$$

The control:

$$u = \frac{1}{L_g L_f^{r-1} h(x)} \left(-L_f^r h(x) + v \right) \qquad (3.16)$$

Leads to linear system equivalent to an integrator chain:

$$y^{(r)} = v \qquad (3.17)$$

If the relative degree is greater than 1, we have:$L_g h(x) = 0$.So :$\dot{y} = L_f h(x)$

And we show that:

$$y^{(i)} = L_f^i h(x) \qquad 0 \leq i \leq r (3.18)$$

$$y^{(r)} = L_f^r h(x) + L_g L_f^{r-1} (x) (3.19)$$

The gain *K* is such that the matrix of the parameters $A_s = A - BK$ is a stable matrix.

$$v = -kz = -k_1 z_1 - k_2 z_2 \cdots - k_r z_r \quad (3.20)$$

The relative degree (r_i) related to the *yi* output represents the number of times that this output must be derived to explicitly show at least one of the u_i inputs$u_i(1 \leq i \leq m)$ We always have for a controlling system. The system is not controlled if the input does not show up after n output derivations. The system exhibits precise linearization for *r* = *n*. The system exhibits partial linearization when *r* < *n*. The Linearity subsystem has an order of *r*.

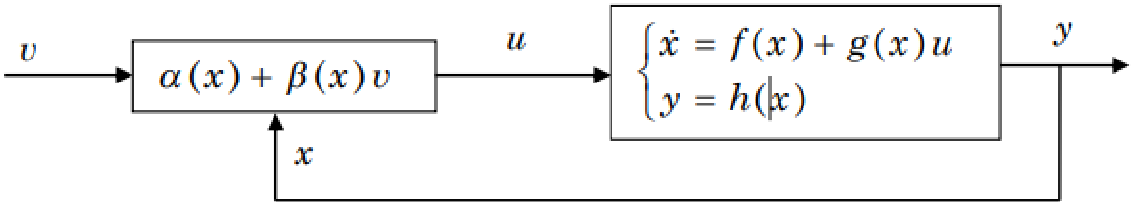


Figure 3.1: Linearization loop of a system.

- **Diffeomorphism**

By changing shape variables, a nonlinear system can be diffeomorphically transformed into another nonlinear system. We have to find the usual form first in order to locate the command. Either a point x = x0 in a nonlinear system of relative degree r = n.
The vector construction gives the basic change [34].

$$z = \Phi(x) = \begin{pmatrix} \Phi_1(x) \\ \Phi_2(x) \\ \vdots \\ \Phi_n(x) \end{pmatrix} = \begin{pmatrix} h(x) \\ L_f h(x) \\ \vdots \\ {L_f}^{n-1} h(x) \end{pmatrix} \quad (3.21)$$

we have $\dot{z}_n = b(z) + a(z)u = v$ since $\dot{z}_r = \dot{z}_n$, in this:

$$u = \frac{v - b(z)}{a(z)} = \frac{v - b(\Phi(x))}{a(\Phi(x))} \quad (3.22)$$

the closed loop system (the exact linearization *r*=*n*)

$$\begin{cases} \dot{z}_1 = z_2 \\ \dot{z}_2 = z_3 \\ \vdots \\ \dot{z}_{n-1} = z_n \\ \dot{z}_n = v \end{cases} \Rightarrow \dot{z} \begin{pmatrix} 0 & 1 & 0 & \dots & 0 \\ 0 & 0 & 1 & \dots & 0 \\ \vdots & & & & \vdots \\ 0 & \cdots & & & 1 \\ 0 & \cdots & & & 0 \end{pmatrix} \begin{pmatrix} z_1 \\ z_2 \\ \vdots \\ \\ z_n \end{pmatrix} + \begin{pmatrix} 0 \\ 0 \\ \vdots \\ \\ 1 \end{pmatrix} v \quad (3.23)$$

It is a linear and controllable form, we have $b(z) = L_f^n h(x)$ and $a(z) = L_g L_f^{n-1} h(x)$ therefore :

$$u = \frac{1}{L_g L_f^{n-1} h(x)} (-L_f^n h(x) + v) \tag{3.24}$$

From (3.20) in the form of:

$$v = k_0 h(x) + k_1 L_f h(x) + \cdots + k_{n-1} L_f^{n-1} h(x) \Rightarrow v = \sum_{i=0}^{n-1} k_i L_f^i h(x) \tag{3.25}$$

So the command (3.23) becomes:

$$u = \frac{1}{L_g L_f^{n-1} h(x)} (-L_f^n h(x) + \sum_{i=0}^{n-1} k_i L_f^i h(x)) \tag{3.26}$$

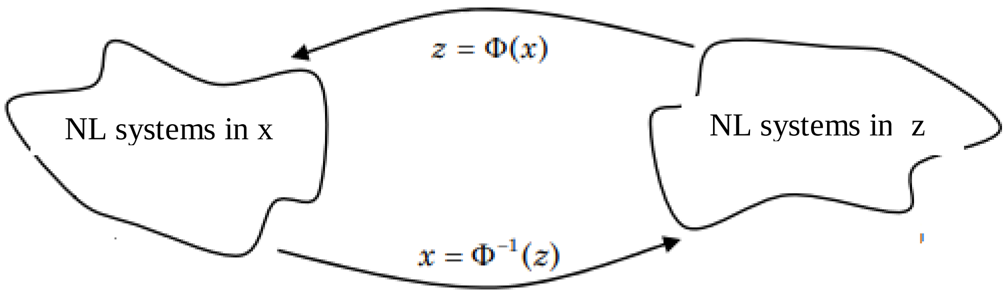


Figure 3.2: Order summary with exact linearization

3.6 Principle of input/output linearization control

Input-output linearization is the process of linearizing the dynamic relationship between system control input and output using a nonlinear control law. The latter is known as linearization. Another linear control law is then applied to stabilize the resulting linear system. Figure 3.1 shows this strategy clearly. On the other hand, feedback linearization produces a valid linear models for the full state space. A linear adjuster that stabilizes the system is therefore, in principle, more effective. Because of its simplicity, a single input and output system is given first [35].

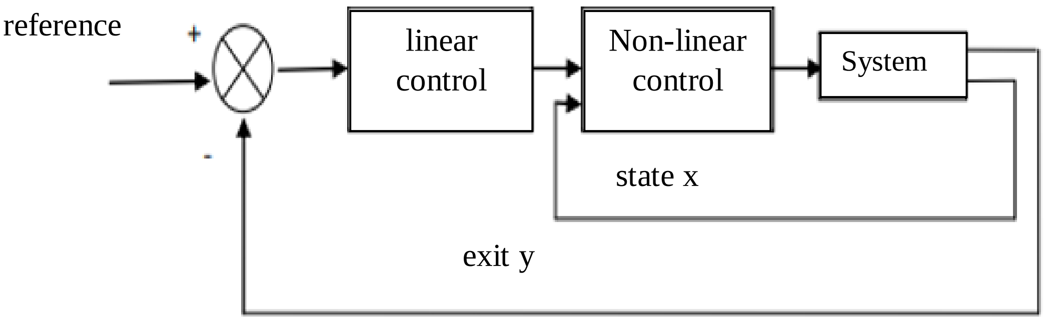


Figure. 3.3 Input/output linearization block diagram.

3.7 Idea of feedback linearization

The notion of feedback linearization, which involves canceling nonlinearities and imposing desirable linear dynamics, is easily applicable to a class of nonlinear systems characterized by the companion form, also known as the controllability canonical form.A system is said to be in companion form if its dynamics is represented by[32] :

$$x^n = f(x) + g(x)u \tag{3.27}$$

3.8Input-output decoupling

We now extend the input-output linearization approach to multiple-input, multiple-output (MIMO) processes. This extension is often called input- output decoupling because the input-output response is both linearized and decoupled. More precisely, the input-output decoupling problem is to find (if possible) a diffeomorphism and a state feedback control law such that: (1) the map between the transformed inputs v and controlled outputs y is linear; and (2) the i-th output yi is decoupled from all inputs vj for ij. In this section, we will only consider static state feedback control laws [34].

3.9System with multiple inputs and multiple outputs

Above all, it is considered that the non-linear system of p inputs and p outputs:

$$\dot{x} = f(x) + \sum_{i=1}^{p} g_i(x).u_i \quad \text{and} \begin{cases} y_1 = h_1(x) \\ \quad\vdots \\ y_m = h_m(x) \end{cases} \tag{3.28}$$

$x = [x_1\ x_2\ ...\ x_n]^T \epsilon\, \mathcal{R}^n$ is the vector of states.

$u = [u_1\ u_2\ ...\ u_n]^T$the control vector

$f(x)$and$g_i(x)$: are vector fields

y : are analytical defined in the vicinity of x0.

3.9.1 Concept of vector relative degree

The system is said to be of vector relative degree$\{r_1, r_2, r_3, ..., r_4\}$ in pointx_0 if, The square matrix defined by (3.28) is not singular in the vicinity of x_0 , the relative degree r_i of the i^{rd} output h_i is the number of times that the output y_i must be derived to show at least one input.

$$A(x) = \begin{pmatrix} L_{g1}L_f^{r_i-1}h_1(x) & \cdots & L_{gm}L_f^{r_i-1}h_1(x) \\ \vdots & \ddots & \vdots \\ L_{g1}L_f^{r_m-1}h_m(x) & \cdots & L_{g1}L_f^{r_m-1}h_m(x) \end{pmatrix} \qquad (3.29)$$

Where A(x) is called the system decoupling matrix. If we assume that is not singular, the linearizing control law has the form:

$$u = A(x).(-\xi(x) + v) \qquad (3.30)$$

And:

$$\xi(x) = \begin{bmatrix} L_f^{ri}h_i(x) \\ \cdots \\ L_f^{ri}h_p(x) \end{bmatrix} \qquad (3.31)$$

3.9.2 Diffeomorphism

$\Phi^1(x), \Phi^2(x), ..., \Phi^m(x)$such as$\Phi^i(x) = [\Phi_1^i(x) ... \Phi_{r_i}^i(x)]$

If $r < n$the new coordinates : $\begin{cases} z_1^i = \Phi_1^i = h_i(x) \\ z_2^i = \Phi_2^i = L_f h_i(x) \\ \vdots \\ z_{r_i}^i = \Phi_{r_i}^i = L_f^{r_i-1}h_i(x) \end{cases}$ (3.32)

The missing (n - r)functions $\Phi_{r+1}(x)$, ... , $\Phi_{r+1}(x)$are chosen so as to have

$L_{gj}\Phi_i(x)$ for $r+1 \le i \le n , 1 \le j \le m$

3.10 Application of the control by Feedback Linearization to themanipulator arm

3.10.1 principle of operation

we have:

$$\begin{cases} \ddot{r} - r\dot{\theta}^2 + g\sin\theta = \frac{F}{m} \\ r^2\ddot{\theta} + 2r\dot{r}\dot{\theta} + gr\cos\theta = \frac{T}{m} \end{cases} \tag{3.33}$$

In pose:

$$\begin{cases} y_1 = x_1 = r \\ y_2 = x_3 = \theta \end{cases} \tag{3.34}$$

We calculate

- ❖ the relative degree:

$$\begin{cases} y_1 = x_1 = r \\ y_1^{(1)} = \dot{x}_1 = \dot{r} \\ y_1^{(2)} = \dot{x}_2 = \ddot{r} = x_1{x_4}^2 - g\sin x_3 + \frac{U_1}{m} \end{cases} \tag{3.35}$$

and it's where:$r_1 = 2$

$$\begin{cases} y_2 = x_3 = \theta \\ y_2^{(1)} = x_4 = \dot{x}_3 = \dot{\theta} \\ y_2^{(2)} = \dot{x}_4 = \ddot{\theta} = \frac{1}{1+\sin{x_1}^2}\left(-2x_1x_2x_4 - gx_1\cos x_3 + \frac{U_2}{m}\right) \end{cases} \tag{3.36}$$

And $r_2 = 2$, So:

$$\begin{cases} r_{total} = r_1 + r_2 \\ r_{total} = 4 = n \end{cases} \quad (3.37)$$

n: order system

- Diffeomorphism:

$$\begin{cases} z_1^1 = \phi_1(x) = h_1(x) = x_1 \\ z_2^1 = \phi_2(x) = L_f h_1(x) = x_2 \\ z_1^2 = \phi_3(x) = h_2(x) = x_3 \\ z_2^2 = \phi_4(x) = L_f h_2(x) = x_4 \end{cases} \quad (3.38)$$

We derivate it and we get:

$$\begin{cases} \dot{z}_1^1 = \dot{x}_1 = x_2 \\ \dot{z}_2^1 = \dot{x}_2 = \ddot{r} = x_1 {x_4}^2 - g \sin x_3 + \frac{U_1}{m} = v_1 \\ \dot{z}_1^2 = \dot{x}_3 = x_4 \\ \dot{z}_2^2 = \dot{x}_4 = \ddot{\theta} = \frac{1}{1+\sin {x_1}^2}\left(-2x_1x_2x_4 - g x_1 \cos x_3 + \frac{U_2}{m}\right) = v_2 \end{cases} \quad (3.39)$$

We have:

$$\begin{cases} v_{1=}-k_{11}z_1 - k_{12}z_2 + k_{\omega_1}\omega_1 \\ v_{2=}-k_{21}z_3 - k_{22}z_4 + k_{\omega_2}\omega_2 \end{cases} \quad (3.40)$$

The control law is as follows:

$$\begin{cases} U_1 = m * (v_1 - x_1 x_4^2 + g \sin x_3) \\ U_2 = m * ((1 + \sin {x_1}^2) v_1 + 2x_1x_2x_4 + g x_1 \cos x_3) \end{cases} \quad (3.41)$$

The relative degree (r_i) related to the yi output represents the number of times that this output must be derived to explicitly show at least one of the u_i inputs$u_i (1 \leq i \leq m)$

3.10.2Presentation of simulation results

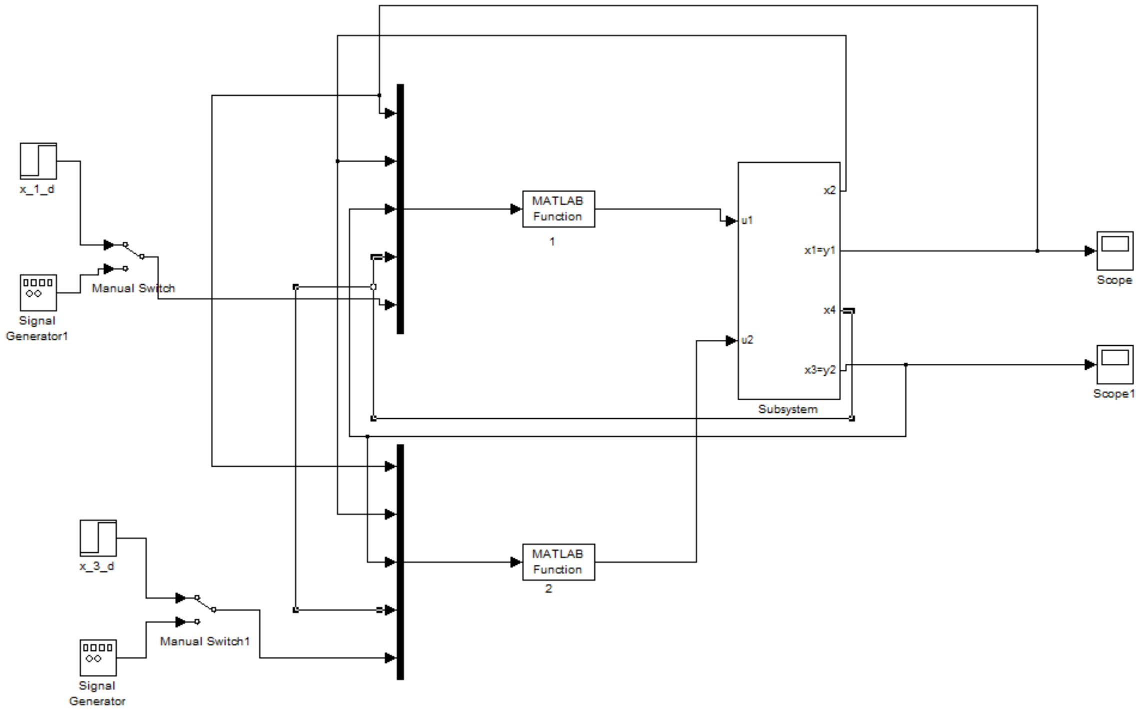


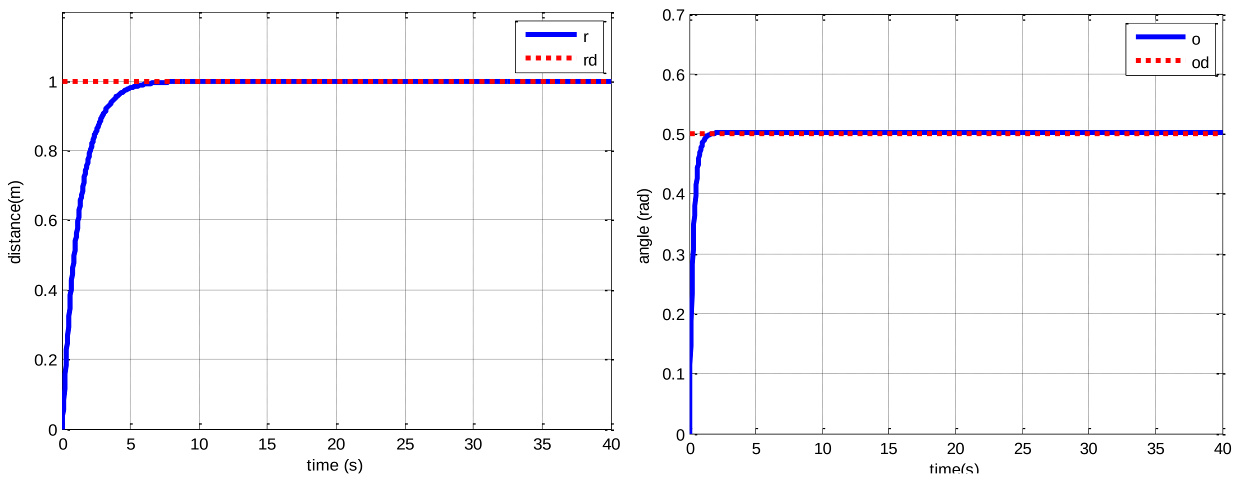
Figure 3.4: block diagram of Feedback Linearizationcontrol.

3.10.3 Simulation results

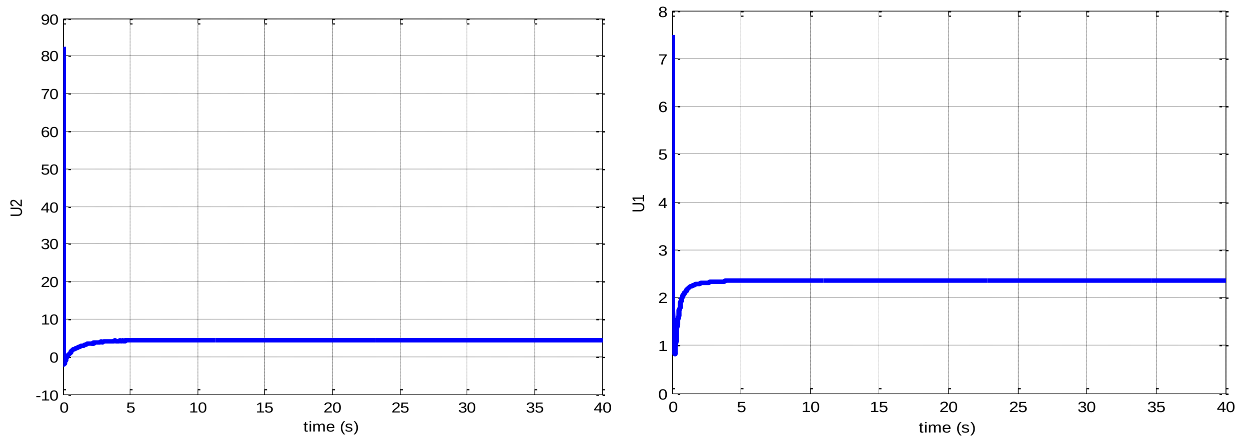
We have:

- $k_{11} = -15\,;\ k_{12} = -20$
- $k_{21} = -330\,;\ k_{22} = -100$
- m = 0.5 m ; g=9.8m/s^2

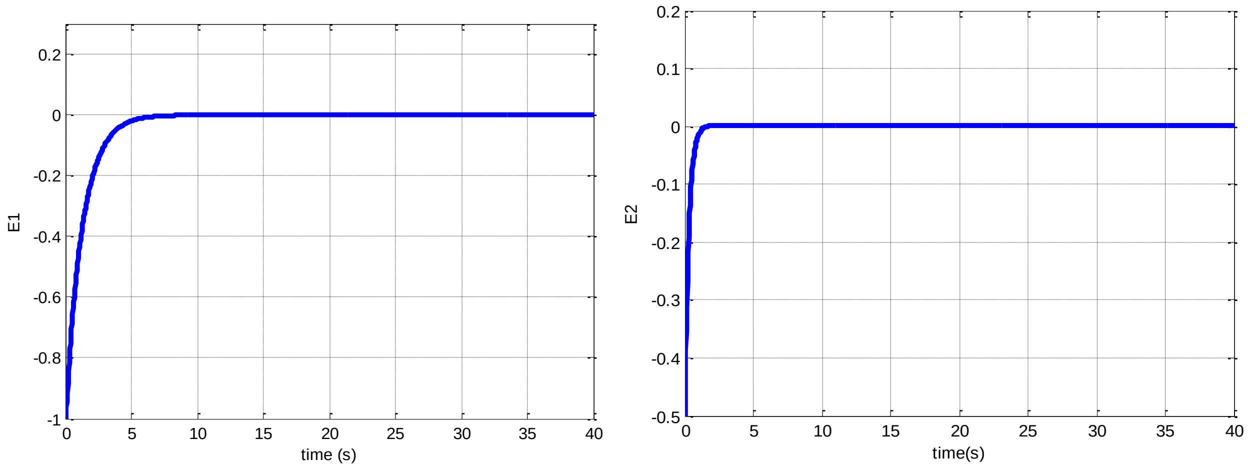
The results of the position, control and error in figure 3.5:



a) outputs



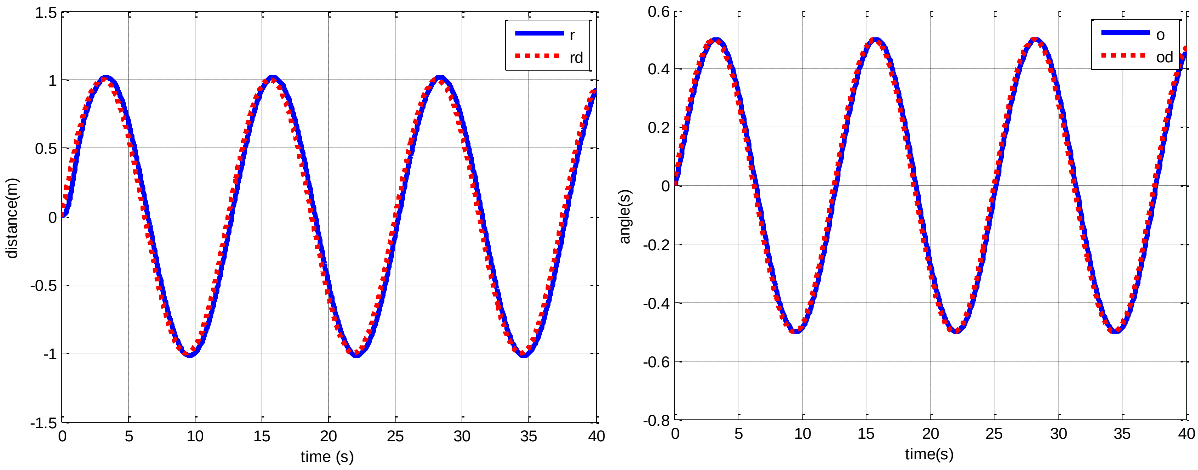
b) command



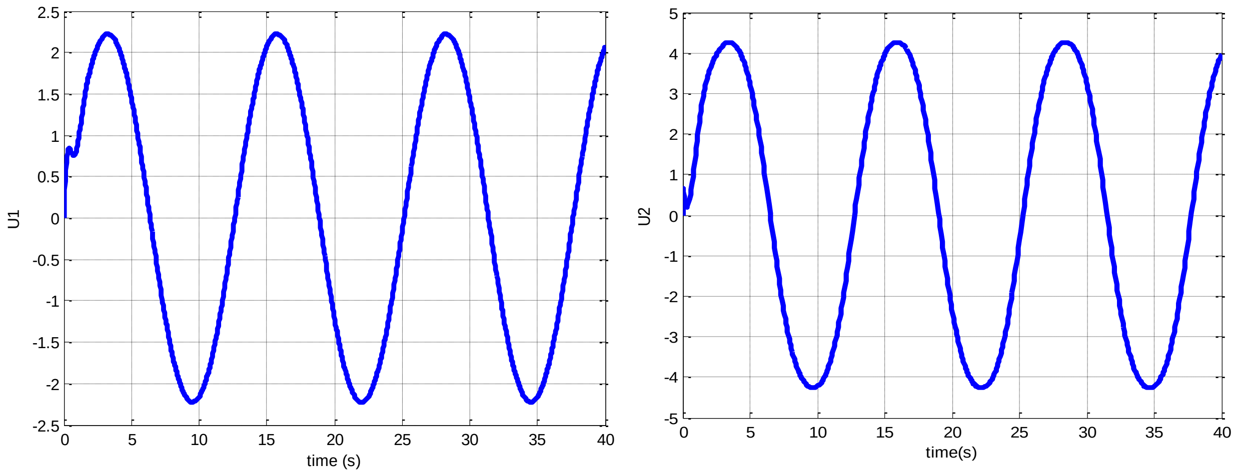
c) errors

Figure 3.5: Simulation results of the control by Feedback Linearization, for a step reference.

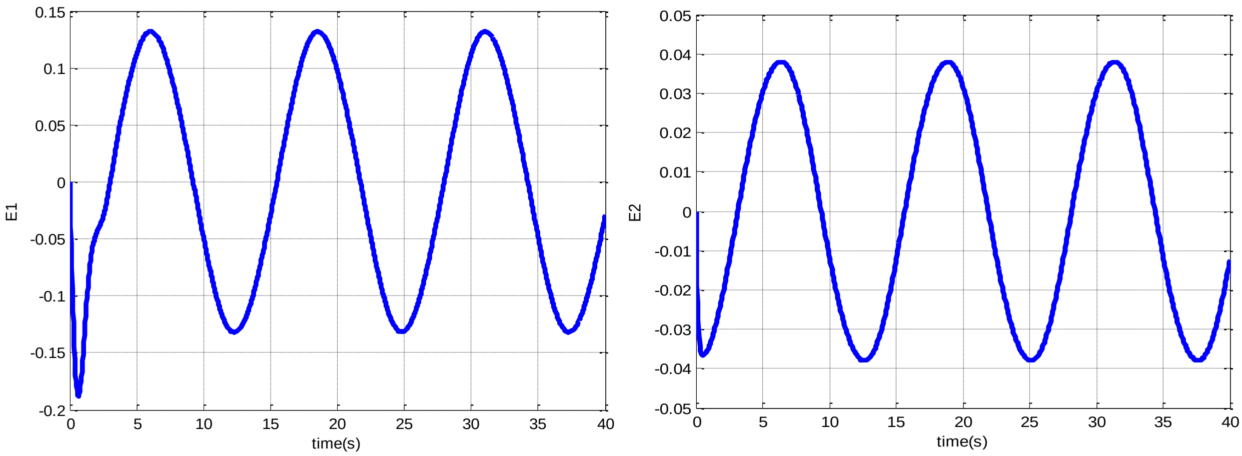
The position, control and error results for a sinusoidal reference are shown in figure 3.3:
$k_{11} = -10$; $k_{12} = -2.5$; $k_{21} = -330$; $k_{22} = -50$



a)outputs



b)command



a) errors

Figure 3.6: Simulationresults of the control by Feedback Linearization, for asinusoidal reference.

- From the simulation results, we note that satisfactory results were obtained in the response time and stability of systems and the convergence accuracy towards their *yd*.

3.7 Comparative Study

To examine the different control laws, developed for our system, we opted for a comparative study between these different commands. For this, we define two criteria, the first is according to the command applied, we can consider it as an energetic criterion, the second is according to the static error of speed:

Energy criterion J_1 and precision criterion J_2 are defined by:

$$J_1 = \frac{1}{2}\sum_{k=1}^{P}(u^T u)(3.42)$$

$$J_2 = \frac{1}{2}\sum_{k=1}^{P}(e^T e)(3.43)$$

Where u_i is the control at the system input and e_i the static setting error:

3.7.1 The 1st subsystem

Criterion	Controls developed for 2 DOF manipulator arm (outputs *r*)		
	PID	Silding mode	Feedback linearization
$j_1 = \frac{1}{2}\sum_{k=1}^{p}(u^T u)$	$6.5218*10^3$	$8.639*10^3$	$1.6588*10^3$
$j_2 = \frac{1}{2}\sum_{k=1}^{p}(e^T e)$	296.5000	498.5000	261.5000

Table 3.1: Comparative study between the commands developed for the first system.

3.7.2 the 2nd subsystem

Criterion	Controls developed for 2 DOF manipulator arm (outputs angle)		
	PID	Silding mode	Feedback linearization
$j_1 = \frac{1}{2}\sum_{k=1}^{p}(u^T u)$	1.9639*10^3	3.4904*10^3	1.1389*10^3
$j_2 = \frac{1}{2}\sum_{k=1}^{p}(e^T e)$	196.5000	498.5000	65.3750

Table 3.2: Comparative study between the commands developed for the second system.

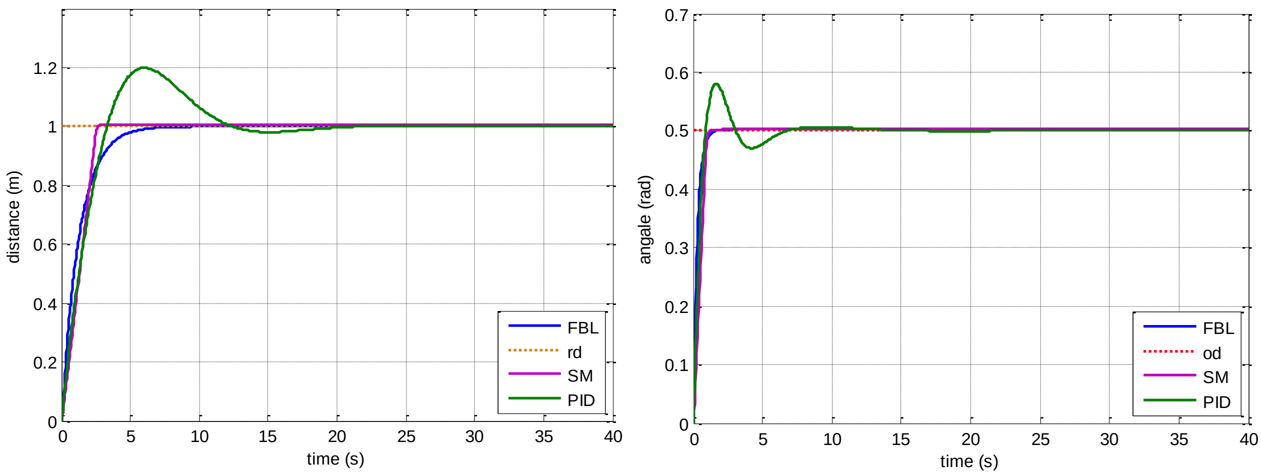


Figure 3.7: Comparative study between the commands developed.

➢ The objective of this section is to examine the different control laws summarized in this document under the same conditions. To highlight the performance of each control law, in terms of the static error and the value of the command needed to perform such an order, we adopted two criteria J_1 and J_2. This comparison is based on the temporal evolution of the command and the resulting error.

For the manipulator, we find that for the feedback linearization control, criterion J_1 and J_2. take the lowest value.We can deduce that for the feedback linearization, the system is well modelled and for this the two criteria take the smallest value.

From the previous analysis, we can say that the feedback linearization control is the most efficient from the point of view of minimizing the two criteria compared to the conventional PID regulation. In general, nonlinear controls ensure good performance even in the presence of external disturbances and modeling errors.

3.8 Conclusion

Finally, this chapter gave a complete overview of the essential elements of feedback systems, which included basic definitions and kinds to advanced control approaches and simulations. Feedback is a crucial part of control systems, allowing for constant adjustment of a system's performance based on its output. This chapter investigated several feedback mechanisms and their effects in stabilizing and destabilizing systems. The next chapter will present a comparative examination of several control systems, including sliding mode control, classic PID control, and feedback linearization.The results of the comparative study between the different controls developed for the manipulator arm confirm that the two nonlinear controls are more efficient. Moreover, if the criterion of precision and energy is privileged, therefore the control by feedback linearization is the most efficient.

General Conclusion

In the present work, we investigated three different methods of control: PID control, sliding mode control and feedback linearization, with the goal of comparing how they performed in regulating a twodegreeof freedom manipulator arm.

The first chapter covered the fundamental ideas of robotics, including multiple models (geometric, kinematic, and dynamic), and we examined manipulator arms, with a special emphasis on the 2DOF manipulator arm utilized in our application, which was operated using PID.

In Chapter 2, we examined the robustness and performance of nonlinear sliding mode control for nonlinear systems. The simulation results showed that sliding mode control is highly resistant against external disturbances.

The third chapter introduced the fundamental principle of feedback linearization and offered simulation results for operating a 2DOF manipulator arm. In this chapter, we analyzed the three control methods used on the system, and while the results were satisfactory, we discovered that feedback linearization produced the best outcomes after comparing their energies.

In the future, we plan to explore with these control methods on a 2DOF manipulator arm. Based on our findings, sliding mode control and feedback the linearization both have advantages, however sliding mode control may be preferable due to its robustness to disturbances.

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Résumé

L'objectif de ce travail est l'application de commandes non linéaires d'un bras manipulateur avec deux degrés de liberté basés sur la commande non linéaire avec mode glissant et linéarisation E/S, et découvrir qui ces approches préservent la performance du système et assurent la stabilité et le suivi de trajectoire. Notre étude se concentre sur trois commandes, la première est le contrôleur PID en raison de leur simplicité de la structure. La deuxième technique est le contrôle de structure variable et la dernière est basée sur la technique de linéarisation E/S. puis nous avons fait une étude comparative entre les trois commandes. Les lois de commande développées, ont été validées par des simulations avec MATLAB/SIMULINK appliquées sur le modèle du robot à 2 degrés de liberté.

Mots clés : robot, bras manipulateur à 2DDL, régulateur PID, commande par mode glissant, la linéarisation par rétroaction.

Abstract

The objective of this work is the application of non-linear controls of a manipulator arm with two degrees of freedom based on the non-linear control with sliding mode and feedback linearization, and discover who these approaches preserve system performance and ensure stability and trajectory tracking. Our study is focused on three commands, the first is the PID controller because of their simplicity of the structure. The second technique is variable structure control and the last one is based on the feedback linearization technique. then we made a comparative study between the three commands. The control laws developed, were validated by simulations with MATLAB/ SIMULINK applied on the model of the robot with 2 degrees of freedom.

Key words:robot, manipulator arm with 2DOF, PID controller, sliding mode, feedback linearization.

ملخص

الهدف من هذا العمل هو تطبيق ضوابط غير خطية لذراع المناورة بدرجتين من الحرية بناءً على التحكم غير الخطي مع نمط الانزلاق والتغذية الراجعة الخطية، واكتشاف من تحافظ بهذا النهج على أداء النظام وتضمن الاستقرار وتتبع المسار. تركز دراستنا على ثلاث أوامر، الأولى هي وحدة تحكم PID بسبب بساطة الهيكل. التقنية الثانية هي التحكم في البنية المتغيرة والأخيرة تعتمد على تقنية التغذية المرتدة الخطية. ثم قمنا بدراسة مقارنة بين الأوامر الثلاثة. تم التحقق من صحة قوانين التحكم من خلال عمليات المحاكاة مع تطبيق MATLAB/SIMULINK على نموذج الروبوت بدرجات 2 من الحرية.

الكلمات الرئيسية: الروبوت، ذراع المتلاعب بـ 2 درجات حرية، وحدة تحكم PID، الوضع المنزلق، التغذية المرتدة الخطية.