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صادق اعضاء اللجنة العلمية على قبول المطبوعة المنجزة من طرف الأستاذة: بن خرباش سعاد

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تحت عنوان : Heat Transfer II

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Course Handout

Heat transfer II

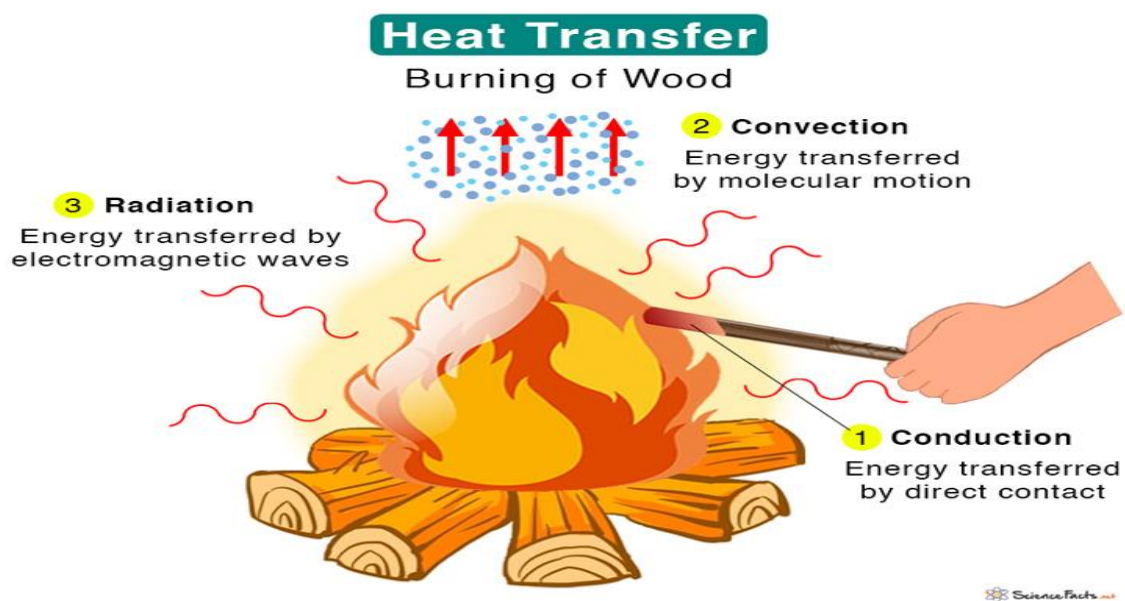
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Preface

These lecture notes, entitled Heat Transfer II, have been prepared for third-year undergraduate students in Energy Engineering during the sixth semester. This course is designed as a continuation of the Heat Transfer I module taught in the fifth semester.

Heat transfer is one of the fundamental disciplines in mechanical, energy, chemical, and process engineering. It plays a crucial role in understanding and designing many industrial systems such as heat exchangers, boilers, cooling systems, thermal insulation devices, energy conversion systems, and numerous other engineering applications. Mastering the principles of heat transfer is therefore essential for engineering students and researchers working in thermal sciences.

The main objective of this course is to provide students with a deeper understanding of heat transfer mechanisms and their engineering applications. It covers advanced topics in thermal sciences, including convective heat transfer, radiative heat transfer, and heat exchangers, with emphasis on both theoretical foundations and practical engineering analysis.

These lecture notes are organized into three main chapters. The first chapter is devoted to convective heat transfer, covering the governing conservation equations, boundary layer concepts, dimensionless analysis, and practical correlations for forced and natural convection. The second chapter introduces radiative heat transfer, including the physical nature of thermal radiation, radiometric quantities, blackbody radiation laws, real surfaces, and radiative exchange between surfaces. The third chapter focuses on heat exchangers, presenting their classifications, thermal analysis methods, overall heat transfer coefficient, logarithmic mean temperature difference method, effectiveness-NTU approach, as well as practical aspects related to boilers and thermal efficiency.

To reinforce the understanding of theoretical concepts and develop problem-solving skills, each chapter is supplemented with application exercises and practical examples, allowing students to apply the fundamental principles studied throughout the course.

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Chapter I

Heat Convection

I. Introduction

Convection is the process by which heat is transferred between a solid surface and a surrounding fluid in motion. When a solid surface is warmer than the surrounding fluid, heat is transferred by convection in several steps:

- Heat circulates by convection from the solid surface to the adjacent particles of the fluid. This transferred energy increases the temperature and internal energy of these fluid particles.
- The particles move to a region of fluid with a lower temperature, where, when they mix, they transfer some of their energy to other fluid particles.
- The energy is stored in the fluid particles and transported by the movement of the masses.

There are two types of convective processes: forced convection and natural convection.

Forced convection: If the fluid's movement is produced by the action of an external agent, such as a hydraulic pump, agitator, or fan. Situations can occur where the two forms of convection act simultaneously, with forced convection being of greater practical interest than natural convection due to industrial applications.

Natural convection occurs when the fluid's movement is caused by natural means, such as buoyancy, which manifests as the ascent of warm fluid and the descent of cold fluid (a difference in density that occurs in the fluid) as a result of a temperature difference.

Convection is also classified as external or internal, depending on whether the fluid is forced to flow on a surface or in a flow.

In simple terms, the main objective of studying the phenomenon of convection is to develop methods for evaluating the convective heat transfer coefficient, h .

I.2 Conservation equation

I.2.1 Conservation equation of mass or continuity equation

This equation expresses the conservation of mass of the fluid during the flow. The continuity equation is written as follows:

$$\frac{\partial \rho}{\partial t} + \text{div}(\rho \vec{V}) = 0 \quad (\text{I.1})$$

For a stationary ($\partial/\partial t = 0$) and incompressible ($\rho = \text{constant}$) flow, the continuity equation reduces to: $\text{div} \vec{V} = 0$

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} + \frac{\partial w}{\partial z} = 0 \quad (I.2)$$

I.2.2 Conservation equation of momentum

For an incompressible and Newtonian fluid in flow, the conservation equation of momentum (Navier-Stokes) is written as follows:

$$\rho \frac{D\vec{V}}{Dt} = -\nabla P + \vec{F} + \mu \nabla^2 \vec{V} \quad (I.3)$$

Or

$$\begin{cases} \rho \frac{Du}{Dt} = F_x - \frac{\partial P}{\partial x} + \mu \left(\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} + \frac{\partial^2 u}{\partial z^2} \right) \\ \rho \frac{Dv}{Dt} = F_y - \frac{\partial P}{\partial y} + \mu \left(\frac{\partial^2 v}{\partial x^2} + \frac{\partial^2 v}{\partial y^2} + \frac{\partial^2 v}{\partial z^2} \right) \\ \rho \frac{Dw}{Dt} = F_z - \frac{\partial P}{\partial z} + \mu \left(\frac{\partial^2 w}{\partial x^2} + \frac{\partial^2 w}{\partial y^2} + \frac{\partial^2 w}{\partial z^2} \right) \end{cases} \quad (I.4)$$

With $\frac{D}{Dt} = \frac{\partial}{\partial t} + u \frac{\partial}{\partial x} + v \frac{\partial}{\partial y} + w \frac{\partial}{\partial z}$ is the particulate derivative.

ρ : is the density

P : Pressure.

μ : Dynamic viscosity.

u, v, w : Components of the velocity vector.

I.2.3 Energy conservation equation

For an incompressible Newtonian fluid, the energy equation is given by:

$$\frac{DT}{Dt} = \frac{\lambda}{\rho C_p} \left(\frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial y^2} + \frac{\partial^2 T}{\partial z^2} \right) \quad (I.5)$$

I.3 Flow regime

- **Laminar flow**: the fluid layers are parallel; between two fluid streams, heat transfer occurs via:
 - conduction if we consider a direction perpendicular to the fluid streams.
 - convection if we consider a direction not perpendicular to the fluid streams.
- **Turbulent flow**: the flow is not unidirectional; in the turbulent zone, heat transfer occurs via convection and conduction in all directions.

I.4 Boundary Layer Concepts

Solving the Navier-Stokes equations for viscous flow presents significant mathematical challenges. Consequently, a generalization of these equations is necessary. In 1905, Ludwig Prandtl noted that the effect of viscosity on most flows was significant only in a very thin region near the solid wall. The rest of the fluid can be considered non-viscous and optimal. This region is called the boundary layer. Fluid particles in contact with the surface have a zero velocity due to viscous forces. These particles slow down neighboring particles in the upper layers over a certain distance "y," until this braking effect becomes negligible. This delay is accompanied by shear stresses " τ " in planes parallel to the fluid's velocity.

Consider a fluid flowing near a solid wall (Figure I.1). If the fluid is Newtonian, the shear stress will be expressed by Newton's law, which can be written as follows:

$$\tau = \mu \left(\frac{\partial u}{\partial y} \right)_{y=0} \quad (\text{I.6})$$

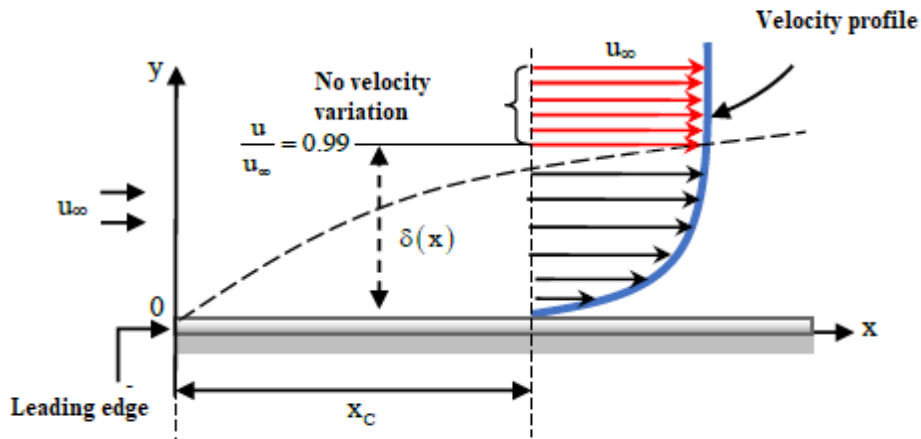


Figure I.1 Hydrodynamic boundary layer on a flat surface

As the distance from the surface increases, the fluid's velocity "u" tends towards the velocity " u_∞ ". The value of y where the velocity ($u = 0.99 u_\infty$) is called the hydrodynamic boundary layer thickness, denoted as δ . The velocity profile in the boundary layer represents the velocity variation as a function of y, through the boundary layer. The boundary layer separates two distinct domains. In the first domain, the velocity gradients and shear stresses are significant, while they are negligible in the second domain. The hydrodynamic boundary layer develops regardless of the flow and is fundamental to convective transfer. The thickness of the boundary layer increases with distance "x". For an external flow, it provides a basis for evaluating the local friction coefficient.

$$C_f = \frac{2\tau}{\rho u_\infty^2} \quad (\text{I.7})$$

When a dye is injected using a capillary placed near the leading edge of the plate, it is observed that the colored stream retains a regular shape until an abscissa x_c , and then bursts in an unordered manner. The average position of x_c is called the transition abscissa: $x_c < x$, the boundary layer is laminar; $x_c > x$, the boundary layer is turbulent.

As explained previously, the hydrodynamic boundary layer develops for any flow.

When dye is injected with a capillary placed near the leading edge of the wing, the thermal boundary layer develops. However, the thermal boundary layer will only develop if the temperature of the fluid and the surface are different. Let's consider the flow depicted in Figure I.2. The fluid arrives at a temperature T_∞ on a horizontal plate.

Particles in contact with the surface of the plate are at the temperature of that surface. These particles exchange energy with neighboring particles, creating temperature gradients in the fluid. The domain in which these gradients develop is called the thermal boundary layer, and its thickness, δ , is defined as the distance for which the ratio $(T_p - T)/(T_p - T_\infty) = 0.99$. As with the hydrodynamic boundary layer, the thickness of the thermal boundary layer increases with distance x . The flow can now be considered as consisting of two regions. In the first, velocity gradients and shear stresses are significant, whereas they are negligible in the second.

The thickness of the hydrodynamic boundary layer where $(u=99\%U_\infty)$:

$$\delta = \frac{5x}{\sqrt{Re_x}} \quad (I.8)$$

Re_x is the local Reynolds Number $\left(Re_x = \frac{\rho u_\infty x}{\mu} \right)$

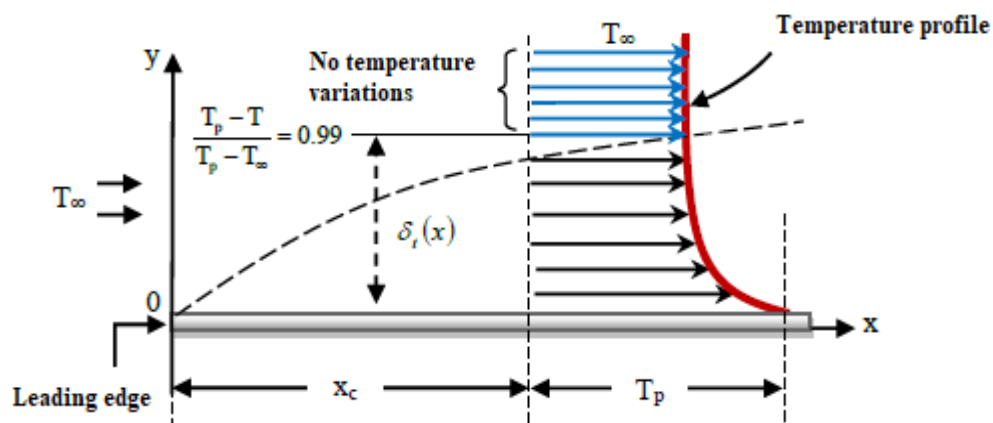


Figure I.2 Thermal boundary layer on a flat surface

- **Significance of the Boundary layer**

- The hydrodynamic boundary layer is characterized by the presence of velocity and shear stress gradients.
- The thermal boundary layer is characterized by the presence of temperature gradients.
- The key parameter for the hydrodynamic layer is the friction coefficient.

The key parameter for the thermal boundary layer is the coefficient of thermal exchange by convection.

The flux density in the fluid at the surface ($y = 0$) is expressed using Fourier's law.

$$\phi = -\lambda \left(\frac{\partial T}{\partial y} \right)_{y=0} \quad (\text{I.9})$$

This heat is then transported by convection in the fluid, so the flux density is also expressed according to Newton's general law:

$$\phi = h(T_p - T_\infty) \quad (\text{I.10})$$

This makes it possible to highlight the relationship between the convective heat transfer coefficient "h" and the temperature gradient in the fluid at the wall.

$$h = \frac{-\lambda \left(\frac{\partial T}{\partial y} \right)_{y=0}}{(T_p - T_\infty)} \quad (\text{I.11})$$

The major problem to be solved before calculating the heat flux is to determine h, which depends on many parameters:

- Characteristics of the fluid,
- Nature of the flow,
- Temperature,
- Shape of the exchange surface, etc.

I.5 Order of magnitude of the heat transfer coefficient h for different configurations:

Configurations	h (W.m ⁻² .K ⁻¹)
Natural convection	
Vertical plate of height 0.3 m in air	4.5
Horizontal cylinder of diameter 5 cm in air	6.5
Horizontal cylinder of diameter 5 cm in water	890
Forced convection	
Airflow at 2 m/s over a square plate of side 2 m	12
Airflow at 35 m/s over a square plate of side 0.75 m	75
Water at 0.5 kg/s in a tube of diameter 2.5 cm	3500
Airflow at 50 m/s perpendicular to a tube of diameter 5 cm	180
Water boiling	
In a container	2500-35000
Flow in a tube	5000-10000

Table I.1 Order of magnitude of the heat transfer coefficient h for different configurations

- **Orders of magnitude of the heat transfer coefficient h ($\text{W}\cdot\text{m}^{-2}\cdot\text{K}^{-1}$)**
- Free convection (air) 5 – 25
- Free convection (water) 100 – 900
- Forced convection (air) 10 – 500
- Forced convection (water) 100 – 15,000
- Forced convection (oil) 50 – 2,000
- Forced convection (molten metals) 6,000 – 120,000
- Forced convection (boiling water) 2,500 – 25,000
- Condensation of water vapor 50,000 – 100,000

I.6 Method for evaluating the convective heat transfer coefficient

The main techniques used in convection to determine the convective heat transfer coefficient " h " are:

- Dimensional analysis combined with experimentation.
- Analytical or numerical solution of the conservation equations of mass, momentum, and energy.
- Analogy between heat and momentum transfer.
- Experimental determination.

In this course, we will limit ourselves to the first technique.

I.6.1 Vaschy–Buckingham theorem or Pi theorem

When the system under study is too complex to allow a complete solution of the fundamental equations, dimensional analysis (Vaschy–Buckingham theorem or π theorem) differs from other methods in that it provides simple access to relationships between the various quantities characterizing the system without introducing mathematical equations to be solved. It allows the combination of a certain number of variables (or dimensionless groupings) which will lead to empirical relationships describing experimental results in an acceptable and widely usable way.

Statement: Given a physical phenomenon involving " p " physical quantities denoted $E_1, E_2, \dots, E_p$ and " q " the number of fundamental units involved in the measurement of the " p " quantities.

The most general form of any complete physical equation is: $f(E_1, E_2, \dots, E_p) = 0$.

Which can be written as: $F(\pi_1, \pi_2, \dots, \pi_{p-q}) = 0$

The $\pi_1, \pi_2, \dots, \pi_{p-q}$ denote independent dimensionless products that can be formed from the p physical quantities considered. The function " f " must be homogeneous with respect to the fundamental dimensions: mass $[M]$, length $[L]$, time $[T]$, and temperature $[\theta]$.

Suppose that h is a function of the following variables:

ν : kinematic viscosity (m^2s^{-1})

C_p : specific heat ($J \cdot kg^{-1} \cdot K^{-1}$)

λ : thermal conductivity ($W \cdot m^{-1} \cdot K^{-1}$)

μ : dynamic viscosity ($kg \cdot m^{-1} \cdot s^{-1}$)

v : velocity ($m \cdot s^{-1}$)

d : characteristic dimension (m)

$\nu = (\mu/\rho)$ kinematic viscosity ($m^2 s^{-1}$)

$$h = h(C_p, \lambda, \mu, d, v); (W \cdot m^{-2} \cdot K^{-1})$$

h is also an implicit function of temperature T , since ρ , C_p , and λ depend on it.

These variables do not all intervene at the same time.

Let us use the dimensional equations for each term:

$$[\delta Q] = [\text{energy}] = [\text{force}] \cdot [\text{displacement}] = M \cdot L^2 \cdot T^{-2}$$

$$[\text{Thermal conductivity } \lambda] = \lambda = \frac{[\delta Q]}{L \cdot T \cdot \theta} = M \cdot L \cdot T^{-3} \theta^{-1}$$

$$[\text{Velocity } v] = v = \frac{L}{T} = L \cdot T^{-1}$$

$$[\text{specific heat } C_p] = C_p = \frac{[\delta Q]}{M \cdot \theta} = L^2 \cdot T^{-2} \theta^{-1}$$

$$[\text{Dynamic viscosity } \mu] = \mu = \frac{M}{L \cdot T} = M L^{-1} \cdot T^{-1}$$

The dimensional equation for h is obtained from Newton's law:

$$[h] = \frac{[\delta Q]}{[(T_p - T_f)] [ds] [dt]} = \frac{M L^2 \cdot T^{-2}}{\theta L^2 T} = M T^{-3} \theta^{-1}$$

Writing $[h]$ in the form:

$$[h] = [C_p]^a \cdot [\lambda]^b [\mu]^c [d]^d [v]^e$$

$$[h] = (L^2 \cdot T^{-2} \theta^{-1})^a \cdot (M \cdot L \cdot T^{-3} \theta^{-1})^b \cdot (M L^{-1} \cdot T^{-1})^c \cdot (L)^d \cdot (L T^{-1})^e \quad (I.12)$$

The fundamental quantities involved in the calculation of h are: mass M , time T , length L , temperature θ . Identifying the exponents in the dimensional equation of h provides a linear system of equations allowing the calculation of a , b , c , d , and e .

$$(L^2 \cdot T^{-2} \theta^{-1})^a \cdot (M \cdot L \cdot T^{-3} \theta^{-1})^b \cdot (M L^{-1} \cdot T^{-1})^c \cdot (L)^d \cdot (L T^{-1})^e = M T^{-3} \theta^{-1} \quad (I.13)$$

Therefore:

- Exponent of M : $b+c=1$
- Exponent of θ : $a+b=1$

- Exponent of L: $2a+b-c+d+e=0$
- Exponent of T: $2a+3b+c+e=3$

Solving dimensional equations reveals dimensionless numbers that are very useful in the study of fluid mechanics, particularly in convective phenomena.

These numbers are:

1. The Reynolds number
2. The Nusselt number
3. The Eckert number
4. The Grashof number
5. The Prandtl number

1. The Reynolds number:

The flow of a fluid may be laminar or turbulent. The transition from one regime to another is characterized by the Reynolds number. $Re = \frac{\text{inertia forces}}{\text{viscous forces}} = \frac{\rho v D}{\mu}$

Where:

D: characteristic dimension [m]

v : characteristic velocity [$\text{m}\cdot\text{s}^{-1}$]

ν : kinematic viscosity [$\text{m}^2\cdot\text{s}^{-1}$]

ρ : fluid density [$\text{kg}\cdot\text{m}^{-3}$]

μ : dynamic viscosity of the fluid [$\text{kg}\cdot\text{m}^{-1}\cdot\text{s}^{-1}$]

If it is a circular pipe, then D is the diameter. In all other cases, D is the hydraulic diameter.

$$D_h = \frac{4S}{P}$$

where S is the area and P is the perimeter.

Experience shows that when the value of Re is lower than a critical value Re_c , the flow in a pipe is always laminar. For internal flow, the value 2300 can be used for Re_c .

2. The Nusselt number

It characterizes the importance of convection relative to conduction: it is the ratio of the amount of heat exchanged by convection $h\cdot S\cdot\Delta T$ to the amount of heat exchanged by conduction $\lambda\cdot S\cdot\Delta T/d$.

$$Nu = \frac{hS\Delta T}{\lambda S \frac{\Delta T}{L}} = \frac{hL}{\lambda}$$

Note: Nu is a direct function of h; knowing it allows determining the value of h.

3. The Grashof number

Characterizes the viscosity force of the fluid

$$Gr = \frac{g\beta d^3 \Delta T}{\nu^2}$$

d: characteristic dimension of the wall [m]

g: acceleration due to gravity [$m \cdot s^{-2}$]

$\Delta T = T_p - T_f$: characteristic temperature difference [$^{\circ}C$]

β : volumetric thermal expansion coefficient of the fluid [$^{\circ}C^{-1}$]

$$\beta = \frac{1}{\rho} \left(\frac{\partial \rho}{\partial T} \right)_p$$

For an ideal gas:

$$\beta = \frac{1}{T}$$

4. The Prandtl number

Characterizes the velocity distribution relative to the temperature distribution.

$$Pr = \frac{\mu C_p}{\lambda}$$

5. The Rayleigh number

$$Ra = Gr Pr$$

It is identical to the Reynolds number in natural convection.

The different calculation steps :

- Calculate Re and compare it to Rec;
- If $Re < Rec$, the flow is said to be laminar; if $Re > Rec$, the flow is said to be turbulent;
- Use one of the given empirical formulas to determine h;
- Calculate δQ using Newton's formula and integrate to obtain Q.

I.7 Forced Convection

In this section of the chapter, we will see how heat is transferred by convection without phase change. The most common experimental correlations in forced convection are of the following type:

$$Nu = f(Re, Pr)$$

For a given fluid temperature, the Prandtl number, which appears in the expression of the Nusselt

number and characterizes the flowing fluid, must be determined. Alternatively, if heat transfer occurs, the surface temperature and the fluid temperature would be different; the fluid properties would simply be taken at the mean temperature T_m or the film temperature.

For a given fluid temperature, the Prandtl number, which appears in the expression of the Nusselt number and characterizes the flowing fluid, must be determined. Alternatively, if heat transfer occurs, the surface temperature and the fluid temperature would be different; the fluid properties would simply be taken at the mean temperature T_m .

$$T_m = \frac{T_f + T_p}{2} \quad (\text{I.14})$$

1.7.1 External forced convection. Heat transfer along a flat plate $Re_c=5 \times 10^5$.

Laminar regime: $Re \leq 5 \times 10^5$

$$Nu_L = 0.66(Re_L)^{1/2}(Pr)^{1/3} \quad (\text{I.15})$$

Turbulent regime: $Re \geq 5 \times 10^5$

$$Nu_L = 0.036(Re_L)^{4/5}(Pr)^{1/3} \quad (\text{I.16})$$

The thermophysical properties of the various fluids are either given directly in the exercises or interpolated from tables found in the literature.

1.7.2 Flow perpendicular to the axis of a cylinder:

The fluid flows perpendicular to the axis of the cylinder. Its upstream velocity is u_∞ and its temperature T_∞ (Figure I.3). A wake forms downstream of the flow, leading to a non-uniform distribution of the heat transfer coefficient on the cylinder periphery.

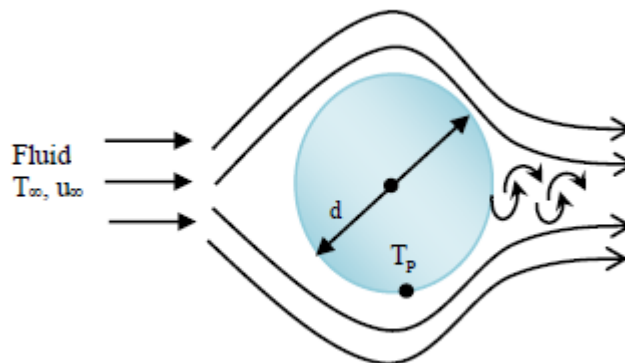


Figure I.3 Flow perpendicular to the axis of a cylinder

The convective heat transfer coefficient is given by:

$$Nu = C Re^m Pr^n \quad (\text{I.17})$$

The constants $n=1/3$, C and m depend on the Reynolds number and the nature of the fluid; their values are given in Table I.2 as well as various noncircular cylinders. Table I.3:

Of the several such relations available in the literature for the average Nusselt number for cross flow over a cylinder, we present the one proposed by Churchill and Bernstein:

$$Nu = 0.3 + \frac{0.62 Re^{1/2} Pr^{1/3}}{\left[1 + (0.4/Pr)^{1/4}\right]^{1/4}} \left[1 + \left(\frac{Re}{282.000}\right)^{5/8}\right]^{4/5}$$

This relation is quite comprehensive in that it correlates available data well for $Re Pr > 0.2$.

Re	n	C	
		Gaz	Liquide
1 – 4	0.330	0.891	$0.989 Pr^{1/3}$
4 – 40	0.385	0.821	$0.911 Pr^{1/3}$
40 – 4000	0.466	0.615	$0.683 Pr^{1/3}$
4000 – 40000	0.618	0.174	$0.193 Pr^{1/3}$
40000 - 250000	0.805	0.0239	$0.0266 Pr^{1/3}$

Table I.2 Value of C and n for the Nusselt number of flow perpendicular to a cylinder

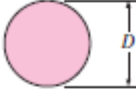
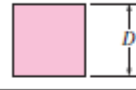
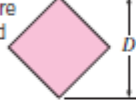
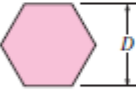
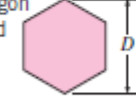
Cross-section of the cylinder	Fluid	Range of Re	Nusselt number
Circle 	Gas or liquid	0.4–4 4–40 40–4000 4000–40,000 40,000–400,000	$Nu = 0.989Re^{0.330} Pr^{1/3}$ $Nu = 0.911Re^{0.385} Pr^{1/3}$ $Nu = 0.683Re^{0.466} Pr^{1/3}$ $Nu = 0.193Re^{0.618} Pr^{1/3}$ $Nu = 0.027Re^{0.805} Pr^{1/3}$
Square 	Gas	5000–100,000	$Nu = 0.102Re^{0.675} Pr^{1/3}$
Square (tilted 45°) 	Gas	5000–100,000	$Nu = 0.246Re^{0.588} Pr^{1/3}$
Hexagon 	Gas	5000–100,000	$Nu = 0.153Re^{0.638} Pr^{1/3}$
Hexagon (tilted 45°) 	Gas	5000–19,500 19,500–100,000	$Nu = 0.160Re^{0.638} Pr^{1/3}$ $Nu = 0.0385Re^{0.782} Pr^{1/3}$
Vertical plate 	Gas	4000–15,000	$Nu = 0.228Re^{0.731} Pr^{1/3}$
Ellipse 	Gas	2500–15,000	$Nu = 0.248Re^{0.612} Pr^{1/3}$

Table I.3 Empirical correlations for the average Nusselt number for forced convection over circular and noncircular cylinders in cross flow

I.7.3 Flow perpendicular to a tube bundle:

Many industrial installations consist of rows of parallel tubes immersed in a flow perpendicular or inclined to their axes. As shown in Figure I.4, the tubes can be arranged in an in-line or staggered configuration. The staggered arrangement leads to higher turbulence and therefore a higher heat transfer coefficient than for an in-line bundle.

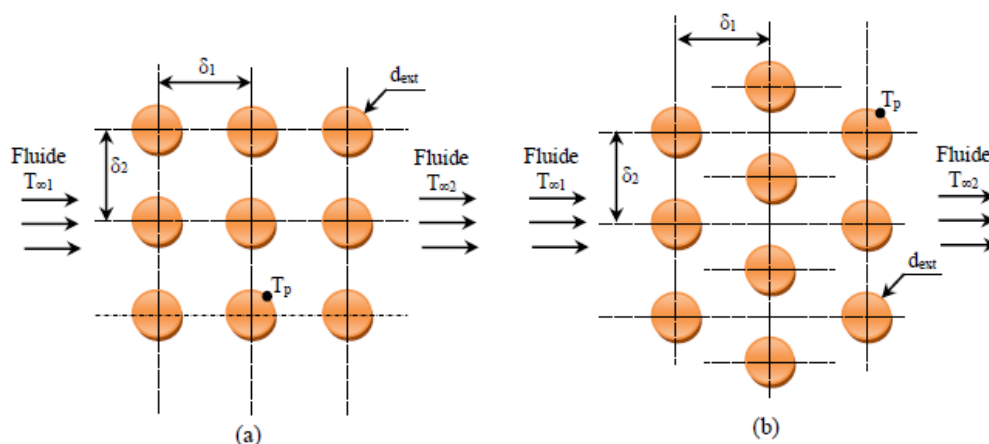


Figure I.4 Flow perpendicular to a tube bundle : (a) Linear arrangement; (b) staggered arrangement

The Nusselt number for a row "i" is calculated using the following expression:

$$Nu = C Re^n Pr_f^{0.33} \left(\frac{Pr_f}{Pr_p} \right)^{0.25} \epsilon_\delta \epsilon_\alpha \epsilon_i \quad (I.18)$$

The constants C, n, ϵ_δ , ϵ_α , ϵ_i are reported in the following table.

Arrangement	C	n	ϵ_i	ϵ_δ	ϵ_α
In-line	0.26	0.65	0.6 for row 1 0.9 for row 2 1 for row 3 and beyond	$\left(\frac{\delta_2}{d_{ext}} \right)^{-0.15}$	10° → 0,42 20° → 0,52 30° → 0,67 40° → 0,78 50° → 0,88
Staggered	0.41	0.6	0.6 for row 1 0.7 for row 2 1 for row 3 and beyond	$\left(\frac{\delta_1}{\delta_2} \right)^{\frac{1}{6}}$ pour $\frac{\delta_1}{\delta_2} < 2$ 1.12 pour $\frac{\delta_1}{\delta_2} \geq 2$	60° → 0,94 70° → 0,98 80° → 1 90° → 1

Table I.4 Constants for the Nusselt number :Flow perpendicular to a tube bundle

The average convective heat transfer coefficient of the bundle is therefore:

$$h = \frac{\sum_{i=1}^n h_i S_i}{\sum_{i=1}^n S_i} \quad (I.19)$$

Note: The fluid properties are determined at the mean temperature given by: $T_m = \frac{T_{\infty 1} + T_{\infty 2}}{2}$

I.7.4 Flow around a sphere:

For a sphere, the wake effects are similar to those encountered in the case of flow around a cylinder; however, the literature recommends the correlation given by Whitakar:

- Whitakar correlation:

$$Nu = 2 + \left(0.4 Re^{0.5} + 0.06 Re^{0.66} \right) Pr^{0.4} \left(\frac{\mu_f}{\mu_p} \right)^{0.25} \quad \text{valable for : } \begin{cases} 0.71 < Pr < 380 \\ 3.5 < Re < 7.6 \times 10^4 \\ 1 < \left(\frac{\mu_f}{\mu_p} \right) < 3.2 \end{cases} \quad (I.20)$$

- Kreith correlation:

$$Nu = 0.48 Re^{0.5} + 2.2 Pr \quad \text{valid for : } 1 < Re < 25 \quad (I.21)$$

- Katsnel'son and Timofeyava correlation: whatever the value of Re

$$Nu = 2 + 0.03 Pr^{0.33} Re^{0.54} + 0.35 Pr^{0.36} Re^{0.58} \quad (I.22)$$

- Froessling correlation:

$$Nu = 2 + 0.60 Pr^{0.33} Re^{0.5} \quad \text{valid for} \quad \begin{bmatrix} 0.6 \leq Pr \leq 400 \\ 3.5 \leq Re \leq 7.10^4 \end{bmatrix} \quad (I.23)$$

- Kramers correlation:

$$Nu = 2 + 1.3 Pr^{0.15} + 0.66 Pr^{0.31} Re^{0.5} \quad \text{valid for} \quad \begin{bmatrix} 0.6 \leq Pr \leq 380 \\ 10 \leq Re \leq 10^5 \end{bmatrix} \quad (I.24)$$

Note: The thermophysical properties of the fluid are evaluated at T_m . The dynamic viscosities of the fluid, μ_f and μ_p , are determined at T_∞ and T_p , respectively.

I.7.5 Internal forced convection

Liquid or gas flow through pipes or ducts is commonly used in heating and cooling applications. The fluid in such applications is forced to flow by a fan or pump through a tube that is sufficiently long to accomplish the desired heat transfer. Most fluids, especially liquids, are transported in circular pipes. This is because pipes with a circular cross section can withstand large pressure differences between the inside and the outside without undergoing any distortion. Noncircular pipes are usually used in applications such as the heating and cooling systems of buildings where the pressure difference is relatively small and the manufacturing and installation costs are lower (Fig. I.5).

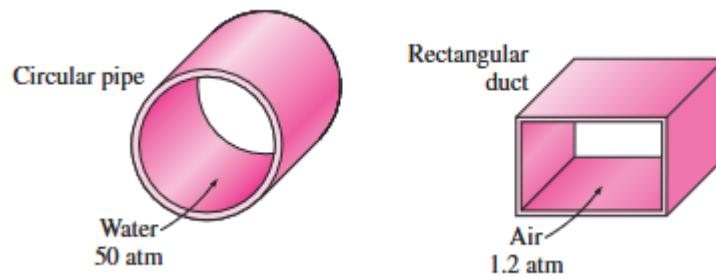


Figure I.5 Circular pipes can withstand large pressure differences between the inside and the outside without undergoing any distortion, but the noncircular pipes cannot

The terms pipe, duct, tube, and conduit are usually used interchangeably for flow sections. In general, flow sections of circular cross section are referred to as pipes (especially when the fluid is a liquid), and the flow sections of non-circular cross section as ducts (especially when the fluid is a gas). Small diameter pipes are usually referred to as tubes.

When a fluid enters a circular tube at a uniform velocity, the layer in contact with the wall stops, and adjacent layers slow due to friction. To maintain constant mass flow, the central fluid accelerates. A velocity boundary layer develops and grows until it fills the entire tube. The region from the inlet to where the boundary layer merges at the centerline is the hydrodynamic entrance region where the

flow is hydrodynamically developing. Beyond this is the hydrodynamically fully developed region, where the velocity profile is parabolic for laminar flow and flatter for turbulent flow due to radial eddy motion. (Fig I.6).

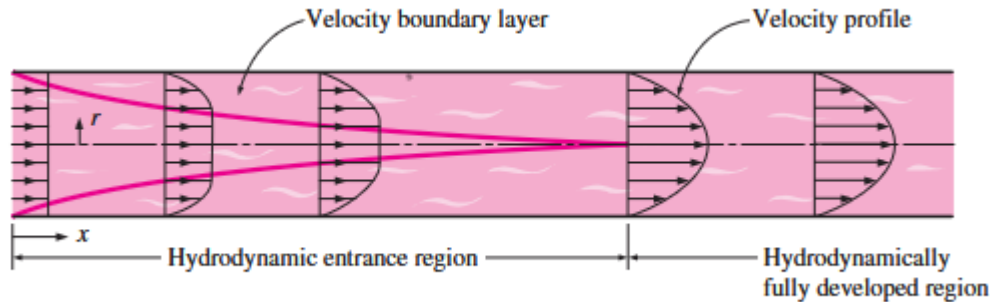


Figure I.6 The development of the velocity boundary layer in a tube.

- **Laminar regime ($Re \leq 2,100$):** Two zones are distinguished: the entrance region and the developed region.

In the developed region, theoretical studies show that the Nusselt number tends toward a limiting value ($Nu = \text{constant}$). It depends only on the thermal boundary conditions imposed on the outer surface of the duct, namely:

Constant temperature (T_p constant): $Nu = 3.66$.

Constant heat flux (Φ constant): $Nu = 4.36$.

In the entrance region: For the entrance region, the literature recommends the following two correlations:

- of Haussen correlation:

$$Nu = 3.66 + \frac{0.0668 Re Pr (D/L)}{1 + 0.04 [Re Pr (D/L)]^{2/3}} \quad (I.25)$$

- Seider and Tate Correlation

$$Nu = 1.86 (Re Pr)^{1/3} \left(\frac{D}{L} \right)^{1/3} \left(\frac{\mu_m}{\mu_p} \right)^{0.14} \quad \text{valid for} \quad \left[\begin{array}{l} 0.48 \leq Pr \leq 16700 \\ 0.0044 < \frac{\mu_m}{\mu_p} < 9.75 \end{array} \right] \quad (I.26)$$

The term $Re Pr (D/L)$ is called The Graetz number

- **Turbulent regime ($Re \geq 2500$)**

For fully developed turbulent flow in smooth tubes, a simple relation for the Nusselt number can be expressed as:

- Colburn correlation: Used for small or moderate temperature differences ($T_p - T_m$)

$$Nu = 0.023 Re^{0.8} Pr^{0.33} \text{ valid for } \left[\begin{array}{l} 0.7 \leq Pr \leq 160, \\ Re > 10000 \\ \frac{L}{D} \geq 10 \end{array} \right] \quad (I.27)$$

The accuracy of this equation can be improved by modifying it .

- Dittus–Boelter correlation

$$Nu = 0.023 Re^{0.8} Pr^n \text{ valid for } \left[\begin{array}{l} 0.7 \leq Pr \leq 160, \\ Re > 10000 \\ \frac{L}{D} \geq 10 \end{array} \right] \quad (I.28)$$

where $n = 0.4$ for **heating** and 0.3 for **cooling** of the fluid flowing through the tube.

- Sieder and Tate correlation: For flows characterized by large variations in thermophysical properties, the Sieder and Tate equation is recommended:

$$Nu = 1.86 (Re Pr)^{1/3} \left(\frac{D}{L} \right)^{1/3} \left(\frac{\mu_m}{\mu_p} \right)^{0.14} \text{ valid for } \left[\begin{array}{l} 0.48 \leq Pr \leq 16700 \\ Re > 10000 \\ \frac{L}{D} \geq 10 \end{array} \right] \quad (I.29)$$

Remarks:

The thermophysical properties of the fluid are evaluated at the mean temperature between the inlet and outlet of the tube, except for the dynamic viscosity μ_p which is determined at T_p .

For tubes with non-circular cross-sections (square, rectangular, triangular, rhomboidal, etc.), the dimensionless numbers are calculated by replacing the internal diameter with the hydraulic diameter D_h (paragraph I.5:

I.8 Natural convection

The flow regime in natural convection is governed by the dimensionless Grashof number and the Rayleigh number, which represents the ratio of the buoyancy force to the viscous force acting on the fluid.

- Grashof number

$$Gr = \frac{g \beta d^3 \Delta T}{\nu^2} \quad (I.31)$$

d : characteristic dimension of the wall [m]

g : acceleration due to gravity [$m \cdot s^{-2}$]

$\Delta T = T_p - T_f$: characteristic temperature difference [$^{\circ}C$]

β : volumetric thermal expansion coefficient of the fluid [$^{\circ}C^{-1}$], $\beta = 1/T_m$

- Rayleigh number

$$Ra = Gr.Pr \quad (I.32)$$

I.8.1 Natural convection over surfaces

The simple empirical correlations for the average Nusselt number Nu in natural convection are of the form: $Nu = \frac{hL}{\lambda} = C(Gr Pr)^n = CRa_L^n$ (I.33)

where Ra_L is the Rayleigh number, which is the product of the Grashof and Prandtl numbers:

$$Ra_L = Gr.Pr = \frac{g\beta L^3 \Delta T}{\nu^2} Pr \quad (I.34)$$

The values of the constants C and n depend on the geometry of the surface and the flow regime, which is characterized by the range of the Rayleigh number. The value of n is usually for laminar flow and for turbulent flow. The value of the constant C is normally less than 1. Simple relations for the average Nusselt number for various geometries are given in Table I.4, together with sketches of the geometries. Also given in this table are the characteristic lengths of the geometries and the ranges of Rayleigh number in which the relation is applicable. All fluid properties are to be evaluated at the film temperature T_f (eq.I.14).

When the average Nusselt number and thus the average convection coefficient is known, the rate of heat transfer by natural convection from a solid surface at a uniform temperature T_s to the surrounding fluid is expressed by Newton's law of cooling as:

$$\dot{Q}_{conv} = hS(T_s - T_\infty) \quad (I.35)$$

where S is the heat transfer surface area and h is the average heat transfer coefficient on the surface.

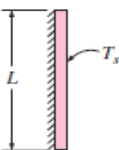
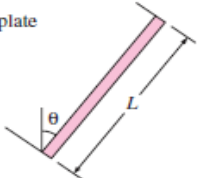
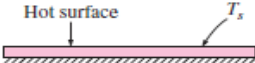
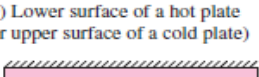
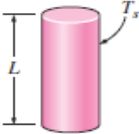
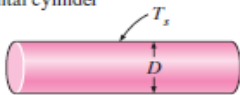
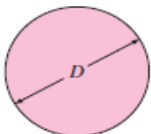
Geometry	Characteristic length L_c	Range of Ra	Nu
Vertical plate 	L	10^4-10^9 10^9-10^{13} Entire range	$Nu = 0.59Ra_L^{1/4}$ $Nu = 0.1Ra_L^{1/3}$ $Nu = \left\{ 0.825 + \frac{0.387Ra_L^{1/6}}{[1 + (0.492/Pr)^{9/16}]^{8/27}} \right\}^2$ (complex but more accurate)
Inclined plate 	L		Use vertical plate equations for the upper surface of a cold plate and the lower surface of a hot plate Replace g by $g \cos \theta$ for $Ra < 10^9$
Horizontal plate (Surface area A and perimeter p) (a) Upper surface of a hot plate (or lower surface of a cold plate)  (b) Lower surface of a hot plate (or upper surface of a cold plate) 	A_s/p	10^4-10^7 10^7-10^{11} 10^5-10^{11}	$Nu = 0.54Ra_L^{1/4}$ $Nu = 0.15Ra_L^{1/3}$ $Nu = 0.27Ra_L^{1/4}$
Vertical cylinder 	L		A vertical cylinder can be treated as a vertical plate when $D \geq \frac{35L}{Gr_L^{1/4}}$
Horizontal cylinder 	D	$Ra_D \leq 10^{12}$	$Nu = \left\{ 0.6 + \frac{0.387Ra_D^{1/6}}{[1 + (0.559/Pr)^{9/16}]^{8/27}} \right\}^2$
Sphere 	D	$Ra_D \leq 10^{11}$ ($Pr \geq 0.7$)	$Nu = 2 + \frac{0.589Ra_D^{1/4}}{[1 + (0.469/Pr)^{9/16}]^{4/9}}$

Table I.5 Empirical correlations for the average Nusselt number for natural convection over surfaces

I.9 Natural convection inside enclosures

Enclosures are frequently encountered in practice, and heat transfer through them is of practical interest. Heat transfer in enclosed spaces is complicated by the fact that the fluid in the enclosure, in general, does not remain stationary. In a vertical enclosure, the fluid adjacent to the hotter surface rises and the fluid adjacent to the cooler one falls, setting off a rotary motion within

the enclosure that enhances heat transfer through the enclosure. Typical flow patterns in vertical and horizontal rectangular enclosures are shown in Figures I.7 and I.8.

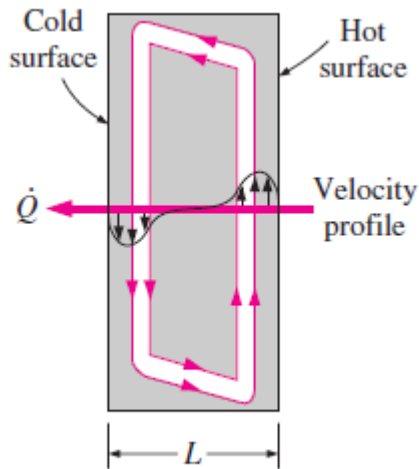


Figure I.7 Convective currents in a vertical rectangular enclosure

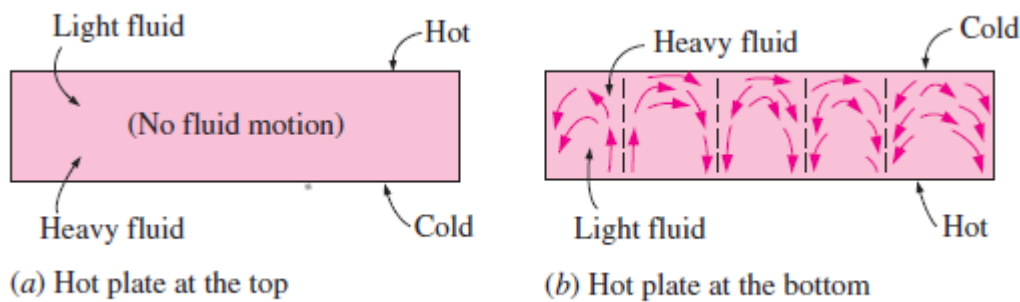


Figure I.8 Convective currents in a horizontal enclosure with
(a) hot plate at the top and (b) hot plate at the bottom.

The characteristics of heat transfer through a horizontal enclosure depend on whether the hotter plate is at the top or at the bottom, as shown in Figure I.8. When the hotter plate is at the top, no convection currents will develop in the enclosure, since the lighter fluid will always be on top of the heavier fluid. Heat transfer in this case will be by pure conduction, and we will have $Nu = 1$. When the hotter plate is at the bottom, the heavier fluid will be on top of the lighter fluid, and there will be a tendency for the lighter fluid to topple the heavier fluid and rise to the top, where it will come in contact with the cooler plate and cool down. Until that happens, however, the heat transfer is still by pure conduction and $Nu = 1$. When $Ra > 1708$, the buoyant force overcomes the fluid resistance and initiates natural convection currents, which are observed to be in the form of hexagonal cells called Bénard cells. For $Ra = 3 \times 10^5$, the cells break down and the fluid motion becomes turbulent.

- **Horizontal Rectangular Enclosures**

For horizontal enclosures that contain air, Jakob (1939) recommends the following simple correlations

$$\begin{aligned}
 \text{Nu} &= 0.195 \text{Ra}_L^{1/4} & 10^4 < \text{Ra} < 4 \times 10^5 \\
 \text{Nu} &= 0.068 \text{Ra}_L^{1/3} & 4 \times 10^5 < \text{Ra} < 10^7
 \end{aligned}
 \tag{I.36}$$

These relations can also be used for other gases with $0.5 < \text{Pr} < 2$. Using water, silicone oil, and mercury in their experiments, Globe and Dropkin (1959) obtained this correlation for horizontal enclosures heated from below,

$$\text{Nu} = 0.069 \text{Ra}_L^{1/3} \text{Pr}^{0.074} \quad 3 \times 10^5 < \text{Ra} < 7 \times 10^9 \tag{I.37}$$

Based on experiments with air, Hollands et al (1976) recommend this correlation for horizontal enclosures,

$$\text{Nu} = 1 + 1.44 \left[1 - \frac{1708}{\text{Ra}_L} \right]^+ + \left[\frac{\text{Ra}_L^{1/3}}{18} - 1 \right]^+ \quad \text{Ra}_L < 10^8 \tag{I.38}$$

The notation $[]^+$ indicates that if the quantity in the bracket is negative, it should be set equal to zero. This relation also correlates data well for liquids with moderate Prandtl numbers for $\text{Ra}_L < 10^5$, and thus it can also be used for water.

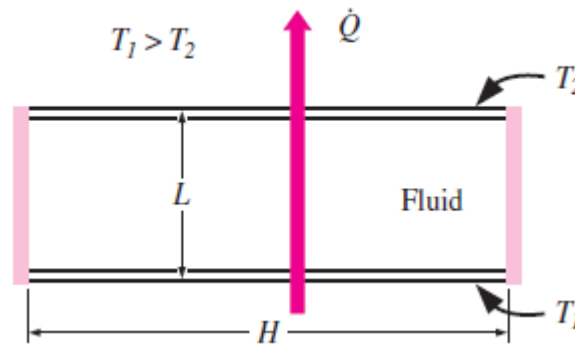


Figure I.9 A horizontal rectangular enclosure with isothermal surfaces

- **Inclined Rectangular Enclosures**

Air spaces between two inclined parallel plates are commonly encountered in flat-plate solar collectors (between the glass cover and the absorber plate) and the double-pane skylights on inclined roofs. Heat transfer through an inclined enclosure depends on the aspect ratio H/L as well as the tilt angle θ from the horizontal (Figure I.10).

For large aspect ratios ($H/L \geq 12$), this equation Hollands et al. (1976), correlates experimental data extremely well for tilt angles up to 70° ,

$$Nu = 1 + 1.44 \left[1 - \frac{1708}{Ra_L \cos \theta} \right]^+ \left(1 - \frac{1708 (\sin 1.80)^{1.6}}{Ra_L \cos \theta} \right) + \left[\frac{(Ra_L \cos \theta)^{1/3}}{18} - 1 \right]^+ \quad (I.39)$$

For $Ra_L < 10^5$, $0 < \theta < 70^\circ$, and $H/L \geq 12$. Again any quantity in $[]^+$ should be set equal to zero if it is negative. This is to ensure that $Nu = 1$ for $Ra_L \cos \theta < 1708$. Note that this relation reduces to (Eq. I.38) for horizontal enclosures for $\theta = 0^\circ$, as expected.

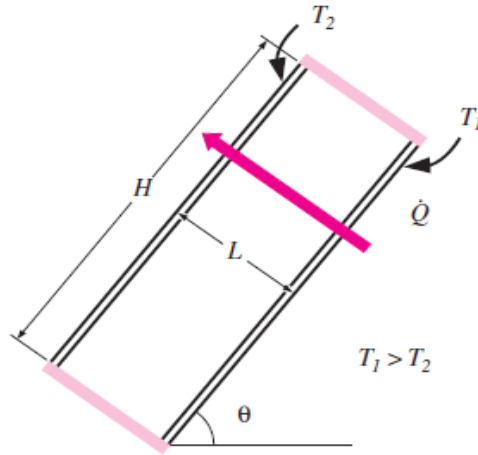


Figure I.10 An inclined rectangular enclosure with isothermal surfaces

- **Vertical rectangular enclosures**

For vertical enclosures (Figure I.11), Catton (1978) recommends these two correlations due to Berkovsky and Plevikov (1977),

$$Nu = 0.18 \left(\frac{Pr}{0.2 + Pr} Ra_L \right)^{0.29} \quad \begin{array}{l} 1 < H/L < 2 \\ \text{any } Pr \text{ and } l \text{ number} \\ Ra_L Pr / (0.2 + Pr) > 10^3 \end{array} \quad (I.40)$$

$$Nu = 0.22 \left(\frac{Pr}{0.2 + Pr} Ra_L \right)^{0.28} \left(\frac{H}{L} \right)^{-1/4} \quad \begin{array}{l} 2 < H/L < 10 \\ \text{any } Pr \text{ and } l \text{ number} \\ Ra_L < 10^{10} \end{array} \quad (I.41)$$

For vertical enclosures with larger aspect ratios, the following correlations can be used MacGregor and Emery (1969).

$$Nu = 0.42 Ra_L^{1/4} Pr^{0.012} \left(\frac{H}{L} \right)^{-0.3} \quad \begin{array}{l} 10 < H/L < 40 \\ 1 < Pr < 2 \times 10^4 \\ 10^4 < Ra_L < 10^7 \end{array} \quad (I.42)$$

$$Nu = 0.46 Ra_L^{1/3} \quad \begin{array}{l} 1 < H/L < 40 \\ 1 < Pr < 20 \\ 10^6 < Ra_L < 10^9 \end{array} \quad (I.43)$$

Again all fluid properties are to be evaluated at the average temperature $(T_1 + T_2)/2$.

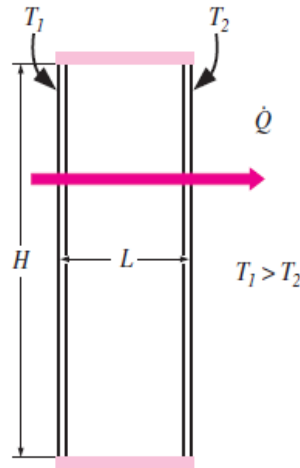


Figure I.11 A vertical rectangular enclosure with isothermal surfaces.

I.10 Exercises

Exercise1:

Engine oil at 60°C flows over the upper surface of a 5-m-long flat plate whose temperature is 20°C with a velocity of 2 m/s (Fig. I). Determine the rate of heat transfer per unit width of the entire plate.

Solution

The properties of engine oil at the film temperature of $T_f = (T_s + T_\infty)/2 = (20 + 60)/2 = 40^\circ\text{C}$

(Table I.6). $\rho = 876 \text{ kg/m}^3$ $\text{Pr} = 2870$ $\lambda = 0.144 \text{ W/m } ^\circ\text{C}$, $\nu = 242 \times 10^{-6} \text{ m}^2/\text{s}$, $L = 5 \text{ m}$

We have $\text{Re}_c = 5 \times 10^5$

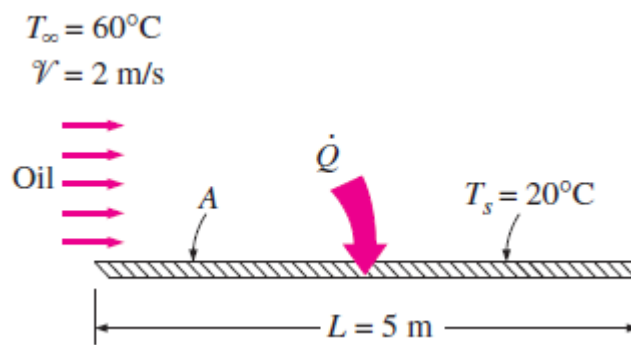


Figure I schematic view

$$\text{Re} = \frac{vL}{\nu} = \frac{2 \times 5}{242 \times 10^{-6}} = 4.13 \times 10^4 < 5 \times 10^5$$

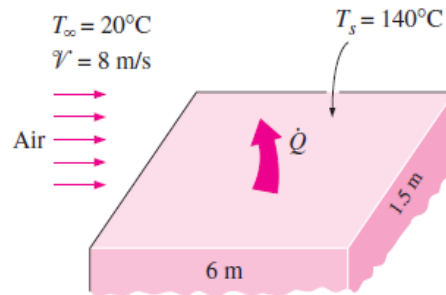
The flow is Laminar, $\text{Nu} = hL/k = 0.664 \text{ Re}_L^{0.5} \text{Pr}^{1/3} = 0.664 \times (4.13 \times 10^4)^{0.5} \times (2870)^{0.33} = 1918$

$$h = \frac{\lambda \text{Nu}}{L} = \frac{0.144 \times 1918}{5} = 55.2 \text{ W/m}^2\text{ } ^\circ\text{C}$$

$$\dot{Q}_{\text{conv}} = hS(T_s - T_\infty) = 55.2 \times 5 \times 1 \times (60 - 20) = 11.04 \text{ W}$$

Exercise 2:

Air at 20°C flows with a velocity of 8 m/s over a 1.5 m x 6 m flat plate whose temperature is 140°C. Determine the rate of heat transfer from the plate if the air flows parallel to the (a) 6-m long side and (b) the 1.5m side.

**Solution**

The top surface of a hot block is to be cooled by forced air. The rate of heat transfer is to be determined for two cases. $Re_c = 5 \times 10^5$

$$T_f = \frac{T_s + T_\infty}{2} = \frac{20 + 140}{2} = 80^\circ\text{C}$$

$$\lambda = 0.02953 \text{ W/m}^\circ\text{C}, Pr = 0.7154, \nu = 2.548 \times 10^{-5} \text{ m}^2/\text{s},$$

a) 6-m long side

$$Re = \frac{vL}{\nu} = \frac{8 \times 6}{2.54 \times 10^{-5}} = 1.884 \times 10^6 > Re_c \text{ The flow is turbulent}$$

$$Nu = \frac{hL}{\lambda} = \frac{0.144 \times 1918}{5} = 55.2 \text{ W/m}^2\text{C}$$

$$Nu_L = 0.036(Re_L)^{4/5}(Pr)^{1/3}$$

$$Re_L^{4/5} = (1,884 \times 10^6)^{0.8} \approx 1,047 \times 10^5$$

$$Pr^{1/3} = (0,7154)^{1/3} \approx 0,8944$$

$$Nu_L = 0,036 \times 1,047 \times 10^5 \times 0,8944 \approx 3371$$

Coefficient h :

$$h = \frac{\lambda Nu}{L} = \frac{3371 \times 0.02953}{6} = 16.59 \text{ W/m}^2\text{C}$$

$$\text{Thermal flux : } \dot{Q}_{\text{conv}} = hS(T_s - T_\infty) = 16.59 \times 9 \times 120 = 17920 \text{ W} = 17.92 \text{ kW}$$

b) 1.5m side,

$$Re = \frac{vL}{\nu} = \frac{8 \times 1.5}{2.54 \times 10^{-5}} = 4.71 \times 10^5 < 5 \times 10^5 \text{ the flow is Laminar}$$

Correlation for laminar flow over a flat plate: $Nu = 0.664 Re_L^{0.5} Pr^{1/3}$

$$Re_L^{1/2} = 4.71 \times 10^5 \approx 686.3$$

$$Pr^{1/3} \approx 0.8944$$

$$Nu_L = 0.664 \times 686.3 \times 0.8944 \approx 408$$

Heat transfer coefficient:

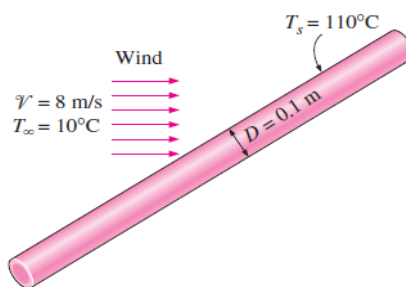
$$h = 408 \times 0.0295315 \approx 8.03 \text{ W/(m}^2 \cdot ^\circ\text{C)}$$

Heat transfer rate:

$$\Phi = 8.03 \times 9 \times 120 \approx 8672 \text{ W} = 8.67 \text{ KW}$$

Exercise 3:

A long 10cm-diameter steam pipe whose external surface temperature is 110°C passes through some open area that is not protected against the winds (Fig). Determine the rate of heat loss from the pipe per unit of its length when the air is at 1 atm pressure and 10°C and the wind is blowing across the pipe at a velocity of 8 m/s.



Solution

The properties of air at the average film temperature of $T_f = \frac{T_s + T_\infty}{2} = \frac{110 + 10}{2} = 60^\circ\text{C}$

$$\lambda = 0.02808 \text{ W/m } ^\circ\text{C}, \quad Pr = 0.7202, \quad \nu = 1.896 \times 10^{-5} \text{ m}^2/\text{s}$$

$$Re = \frac{vL}{\nu} = \frac{8 \times 0.1}{1.896 \times 10^{-5}} = 4.219 \times 10^4$$

The Nusselt number can be determined from

$$Nu = 0.3 + \frac{0.62 Re^{1/2} Pr^{1/3}}{\left[1 + (0.4/Pr)^{2/3}\right]^{1/4}} \left[1 + \left(\frac{Re}{282.000}\right)^{5/8}\right]^{4/5}$$

$$= 0.3 + \frac{0.62(4.219 \times 10^4)^{1/2} (0.7202)^{1/3}}{\left[1 + (0.4/0.7202)^{2/3}\right]^{1/4}} \left[1 + \left(\frac{4.219 \times 10^4}{282.000}\right)^{5/8}\right]^{4/5} = 124$$

$$h = \frac{\lambda Nu}{D} = \frac{0.02808 \times 124}{0.1} = 34.8 \text{ W/m}^2 \cdot ^\circ\text{C}$$

$$S = PL = \pi DL = \pi \times 0.1 \times 1 = 0.314 \text{ m}^2$$

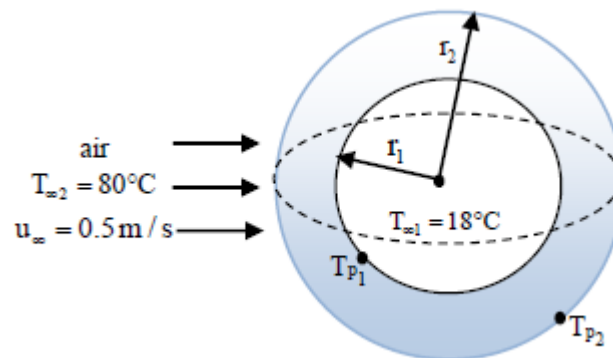
$$\dot{Q}_{\text{conv}} = hS(T_s - T_\infty) = 34.8 \times 0.314 \times (110 - 10) = 1093 \text{ W} = 1.093 \text{ kW}$$

Exercise 4:

Consider a hollow sphere with inner radius $r_1 = 0.3 \text{ m}$ and outer radius $r_2 = 0.5 \text{ m}$, having a thermal conductivity of $25 \text{ W/m} \cdot \text{K}$. The convective heat transfer coefficients of the interior and exterior environments are respectively $h_{\text{int}} = 10 \text{ W/m}^2 \cdot \text{K}$ and $h_{\text{ext}} = ?$. Knowing that the outer surface temperature of the sphere is $T_{p2} = 40^\circ\text{C}$, calculate:

The heat flux exchanged through the sphere.

The temperature of the inner surface of the sphere T_{p1} .



Solution

$$T_f = \frac{T_s + T_\infty}{2} = \frac{80 + 40}{2} = 60^\circ\text{C}$$

The properties of air at the average film temperature are:

$$\lambda = 0.0287 \text{ W/m} \cdot ^\circ\text{C}, Pr = 0.70, \rho = 1.059 \text{ kg/m}^3, C_p = 1008 \text{ J/Kg} \cdot ^\circ\text{C}, \mu = 1.59 \times 10^{-5} \text{ pas}, \mu_f = 2.09 \times 10^{-5} \text{ Pas}, \mu_p = 1.90 \times 10^{-5} \text{ Pas}.$$

$$Re = \frac{\rho v L}{\mu} = \frac{\rho v d_{\text{ext}}}{\mu} = \frac{1.059 \times 0.5 \times 1}{1.59 \times 10^{-5}} = 0.226 \times 10^5$$

The Nusselt number for this configuration is as follows:

$$Nu = 2 + \left(0.4 Re^{0.5} + 0.06 Re^{0.66}\right) Pr^{0.4} \left(\frac{\mu_f}{\mu_p}\right)^{0.25}$$

$$Nu = 2 + \left(0.4(2.66 \times 10^4)^{0.5} + 0.06(2.66 \times 10^4)^{0.66}\right) 0.7^{0.4} \left(\frac{2.05}{1.90}\right)^{0.25} = 104.278$$

The coefficient h: $h = \frac{\lambda Nu}{d_{ext}} = 3 \text{ W/m}^2 \text{ } ^\circ\text{C}$

The heat flux

$$\Phi = \frac{T_{\infty 1} - T_{\infty 2}}{\frac{1}{h_{int} S_{int}} + \frac{1}{4\pi\lambda \left(\frac{1}{r_1} - \frac{1}{r_2}\right) + \frac{1}{h_{ext} S_{ext}}}} = \quad \text{with } S_{ext} = 4\pi r_2^2 \quad \text{and } S_{int} = 4\pi r_1^2$$

$$\text{then } \Phi = \frac{80 - 18}{\frac{1}{10 \times 4\pi \times (0.3)^2} + \frac{0.5 - 0.3}{4\pi \times 25 \times (0.3) \times (0.5)} + \frac{1}{3 \times 4\pi \times (0.5)^2}} = 312.122 \text{ W}$$

Calculation of the inner wall temperature: knowing that the heat flux is conserved, we can write:

$$\Phi = \frac{T_{p1} - T_{\infty 1}}{\frac{1}{h_{int} S_{int}}} = \frac{T_{p1} - T_{\infty 1}}{\frac{1}{10 \times 4\pi \times (0.3)^2}} \Rightarrow T_{p1} = 45.591^\circ\text{C}$$

Exercise 5:

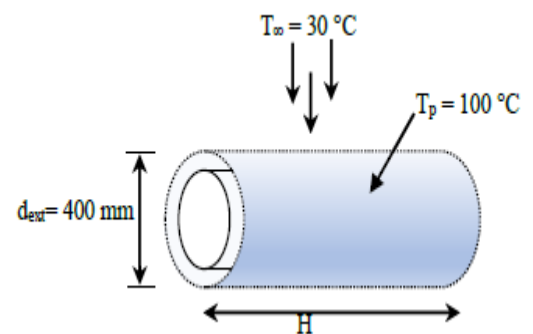
Calculate the heat losses per unit time and per unit surface area of a horizontal heat exchanger whose cylindrical body is cooled by a free air stream at 30 °C. The external diameter of the pipe is 400mm and its wall temperature is 100 °C.

Solution

$$T_f = \frac{T_s + T_{\infty}}{2} = \frac{100 + 30}{2} = 65^\circ\text{C}$$

$$\lambda = 0.029 \text{ W/m } ^\circ\text{C}, Pr = 0.7147,$$

$$\rho = 1.04 \text{ kg/m}^3, C_p = 1008 \text{ J/Kg } ^\circ\text{C}, \mu = 2.01 \times 10^{-5} \text{ Pas},$$



$$\beta = \frac{1}{T_f} = \frac{1}{65 + 273} = 2.95 \times 10^{-3} \text{ K}^{-1}$$

It concerns natural convection on the outside of a horizontal cylinder; the Nusselt number is calculated using two dimensionless numbers, Gr and Pr. The characteristic length in this case is $L=d_{\text{ext}}$ and the Nusselt number is given by: $Nu=C(Ra)^n$

The Grashof number is calculated as follows

$$Gr = \frac{g\beta d^3 \Delta T}{\nu^2} = \frac{g\rho^2 \beta d_{\text{ext}}^3 \Delta T}{\mu^2} = \frac{(1.04)^2 \times 9.81 \times 2.95 \times 10^{-3} \times (100 - 30 \times (400 \times 10^{-3}))}{(2.01 \times 10^{-5})^2} = 0.034 \times 10^{10}$$

The Rayleigh number $Ra=Gr.Pr=2.3 \times 10^2$ for $10^4 < Ra < 10^9$, laminar convection

We have from the table $C=0.53$ and $n=0.25$ then

$$Nu = C(GrPr)^n = 0.53 \times (2.3 \times 10^2)^{0.25} = 65.59$$

$$h = \frac{\lambda Nu}{d_{\text{ext}}} = 4.75 \text{ W/m}^2 \text{ } ^\circ\text{C}, \text{ and the heat flux is } \Phi = hS\Delta T = 333.2 \text{ W}$$

Saturated water properties						
T_m [°C]	ρ [kg/m]	C_p [J/kg·°C]	λ [W/m·°C]	$\mu \times 10^{-4}$ [Pa.s]	$a \times 10^{-7}$ [m ² /s]	Pr
0	1002	4218	0.552	17.90	1.31	13.06
20	1001	4182	0.597	10.10	1.43	7.02
40	995	4178	0.628	6.55	1.51	4.34
60	985	4184	0.651	4.71	1.55	3.02
80	974	4196	0.668	3.55	1.64	2.22
100	960	4216	0.680	2.82	1.68	1.74
120	945	4250	0.685	2.33	1.71	1.45
140	928	4283	0.684	1.99	1.72	1.24
160	910	4342	0.680	1.73	1.73	1.10
180	889	4417	0.675	1.54	1.72	1.00
200	867	4505	0.665	1.39	1.71	0.94
220	842	4610	0.652	1.26	1.68	0.89
240	816	4756	0.635	1.17	1.64	0.88
260	786	4949	0.611	1.08	1.58	0.87
280	753	5208	0.580	1.02	1.48	0.91
300	714	5728	0.540	0.96	1.32	1.02
Saturated air properties						
T_m [°C]	ρ [kg/m]	C_p [J/kg·°C]	λ [W/m·°C]	$\mu \times 10^{-5}$ [Pa.s]	$a \times 10^{-5}$ [m ² /s]	Pr
0	1.292	1006	0.0242	1.72	1.86	0.72
20	1.204	1006	0.0257	1.81	2.12	0.71
40	1.127	1007	0.0272	1.90	2.40	0.70
60	1.059	1008	0.0287	1.99	2.69	0.70
80	0.999	1010	0.0302	2.09	3.00	0.70
100	0.946	1012	0.0318	2.18	3.32	0.69
120	0.898	1014	0.0333	2.27	3.66	0.69
140	0.854	1016	0.0345	2.34	3.98	0.69
160	0.815	1019	0.0359	2.42	4.32	0.69
180	0.779	1022	0.0372	2.50	4.67	0.69
200	0.746	1025	0.0386	2.57	5.05	0.68
220	0.700	1028	0.0399	2.64	5.43	0.68
240	0.688	1032	0.0412	2.72	5.80	0.68
260	0.662	1036	0.0425	2.79	6.20	0.68
280	0.638	1040	0.0437	2.86	6.59	0.68
300	0.616	1045	0.0450	2.93	6.99	0.68

Table I.6 Thermophysical characteristics of water and air

Chapter II

Radiative Heat Transfer

II. Introduction of radiative heat transfer

In radiation heat transfer, thermal energy is transferred via electromagnetic waves propagating in a straight line, requiring no material medium for transmission.

Radiation is fundamentally different from conduction and convection

In thermal radiation, heat is transferred through electromagnetic waves similar to light but with distinct wavelengths. The amount of radiant energy emitted depends on the surface properties and the temperature of the emitting body. When this radiation strikes a receiver, a portion of the energy is absorbed based on the receiver's characteristics and absorptivity, leading to an increase in its temperature. Radiative heat transfer involves only the transport of energy and does not require any material medium; it can take place even in a complete vacuum.

II.1 Thermal Radiation and Electromagnetic Spectrum

Thermal radiation is one of the three mechanisms by which electromagnetic energy can be transferred between bodies at different temperatures. While the electromagnetic radiation spectrum also includes radio waves, microwaves, gamma rays, and certain cosmic rays, thermal radiation is specifically the emission, by all bodies above absolute zero, of electromagnetic waves due to their temperature, or more precisely due to molecular and atomic agitation associated with their internal thermal energy, and at the expense of this energy (which is proportional to temperature; there is thermodynamic equilibrium).

Everybody acts both as a source (it emits thermal radiation into its surroundings) and as a receiver (it receives thermal radiation from bodies that surround it, even those very far away).

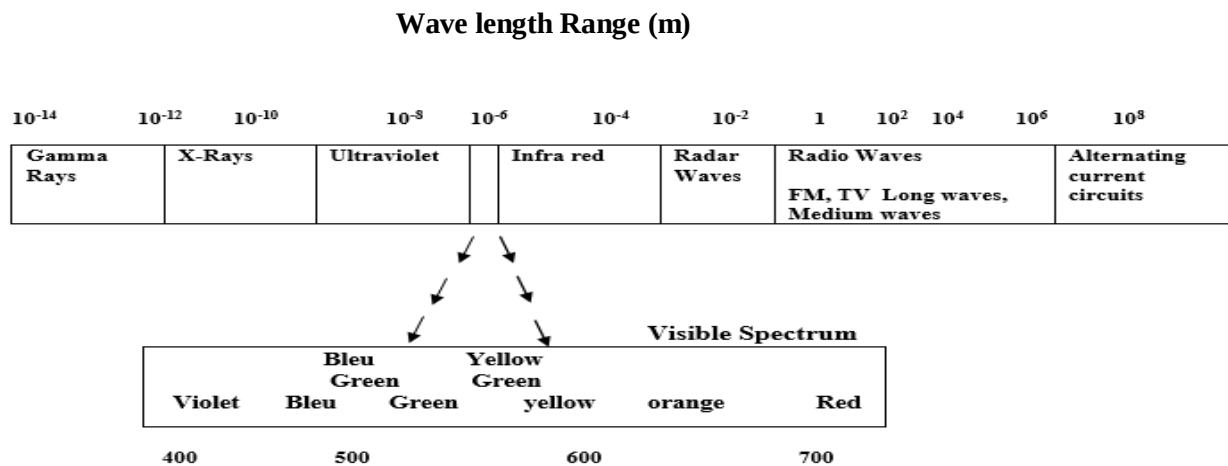


Figure II.1 Electromagnetic spectrum

II.2 The Nature of Thermal Radiation

Thermal radiation transfers heat via electromagnetic waves following Planck's law ($E=h\nu$). It requires no material medium and propagates rectilinearly at the speed of light ($c=3\times 10^8$ m/s). The wavelength range for thermal radiation is typically $0.1\text{ }\mu\text{m}$ to $100\text{ }\mu\text{m}$. Although all three heat transfer modes usually coexist, radiation becomes predominant at high temperatures. All matter emits thermal radiation. Vacuum and simple gases (O_2 , H_2 , N_2) are transparent, while compound gases (CO_2 , H_2O , CO , CH_4) and certain liquids and solids (glass, polymers) are partially transparent. Most solids and liquids are opaque, blocking radiation at their surfaces.

Human perception of electromagnetic radiation occurs in two main ways:

- **Visual perception (by the eye):** Wavelengths between $0.38\text{ }\mu\text{m}$ and $0.78\text{ }\mu\text{m}$ correspond to visible light.
- **Thermal perception (by the skin):** Wavelengths between $0.78\text{ }\mu\text{m}$ and $314\text{ }\mu\text{m}$ correspond to infrared radiation, which is felt as heat.

II.3 Radiometric quantities

A body at temperature T emits thermal radiation (electromagnetic waves) across wave lengths and in all directions. It may also receive radiation from surrounding bodies.

For radiation analysis, several fundamental quantities are defined:

- **Monochromatic:** per unit wavelength interval (λ to $\lambda+d\lambda$)
- **Total:** integrated over the entire spectrum

- **Directional:** specific to a given emission direction
- **Hemispherical:** integrated over all directions in the upper half-space

These categories are often combined (e.g., monochromatic directional radiance) to characterize radiation fully.

II.4 Classification of quantities associated with radiation

Classification of energy quantities depends on the spectral composition and spatial distribution of radiation.

II.4.1 Spectral Composition

Radiation is a function of wavelength. We distinguish :

- **Monochromatic quantity:** a radiative quantity that concerns only a narrow spectral interval $d\lambda$ centered around a wavelength λ ; it is then said to be spectral or monochromatic.
- **Total quantity:** a quantity associated with the entire electromagnetic spectrum. It is obtained by integrating the monochromatic quantity over the entire spectrum.

II.4.2 Spatial Composition

Radiation is a function of the propagation direction. We distinguish:

- **Directional quantity:** a quantity that concerns only a single given direction in space.
- **Hemispherical quantity:** a quantity associated with all directions in space. It is obtained by integrating the directional quantity over the hemisphere.

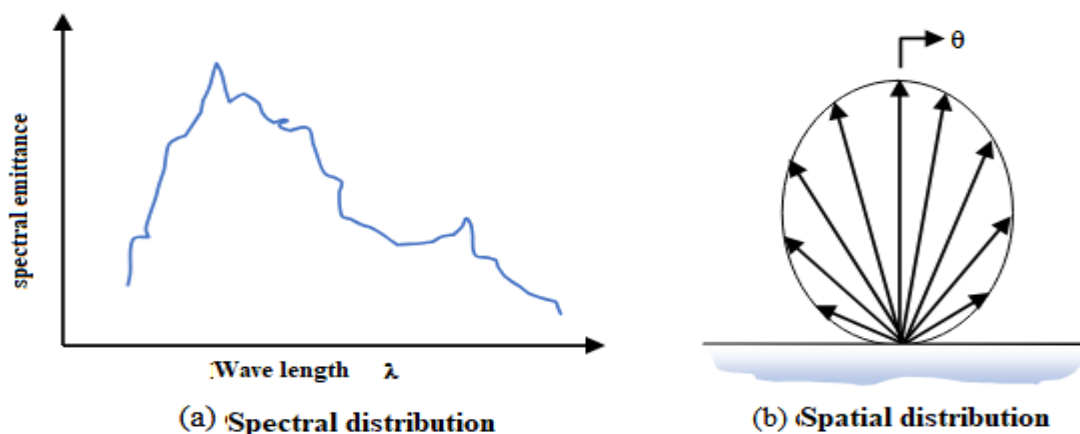


Figure II.2 Emitted radiation

II.5 Quantities Related to Emitting Surfaces

II.5.1 Solid Angle

Definition: In plane geometry, the plane angle $d\theta$ is defined as the ratio of the arc length dl to the radius r :

$$d\theta = \frac{dl}{r} \quad (\text{II.1})$$

By analogy, the solid angle $d\Omega$ extends this concept to three dimensions. It is defined as the area dS_n intercepted on a sphere of unit radius by a cone with its vertex at the sphere's center (Figure II.3). The solid angle is measured in steradians (sr) and is given by:

$$\frac{d\Omega}{1^2} = d\Omega = \frac{dS_n}{r^2} \quad (\text{II.2})$$

The solid angle is obtained by integrating over the entire surface under consideration:

$$\Omega = \iint_{\text{surface}} d\Omega \quad (\text{II.3})$$

The total solid angle is obtained by integrating over the surface of interest:

For example, the solid angle associated with the entire hemisphere can be obtained by integrating

over the limits from 0 to 2π and 0 to $\pi/2$. We obtain: $\Omega = \int_{\text{hemisphere}} d\Omega = \int_0^{2\pi} \int_0^{\pi/2} \sin\theta d\theta d\phi = 2\pi$ sr

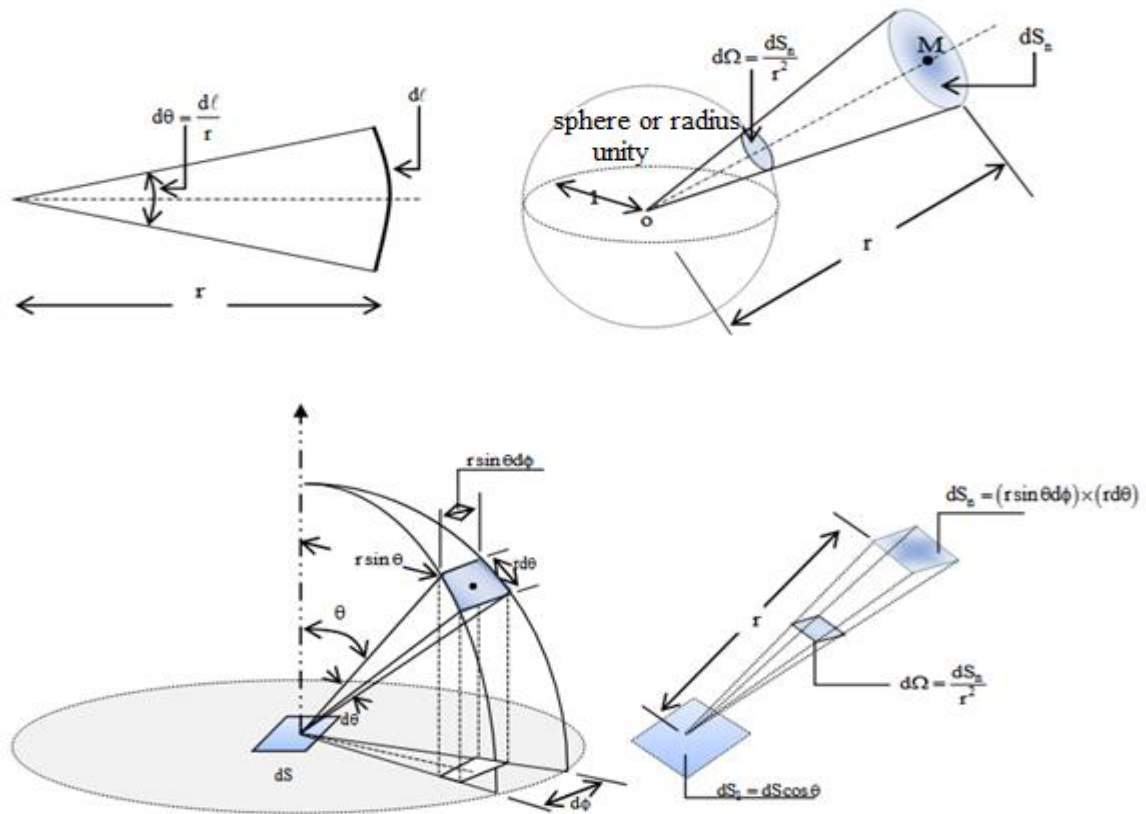


Figure II.3 Solid angle

- **Generalization of the Solid Angle Concept**

Consider an infinitesimal surface element dS' located at point M, inclined at an angle θ with respect to the direction of dS_n (the surface perpendicular to the line of sight). The projected area in the direction perpendicular to $\overrightarrow{OM}$ is :

$$dS_n = dS' \cos \theta = dS' (\vec{u} \cdot \vec{n}) \quad (\text{II.4})$$

Where

- $\vec{u}$ is the unit vector pointing from the observation point to M,
- $\vec{n}$ is the unit vector normal to dS' at M.

The dot product yields:

$$\vec{u} \cdot \vec{n} = \cos \theta \quad (\text{II.5})$$

Thus, the elementary solid angle subtended by dS' is:

$$d\Omega = \frac{dS_n}{r^2} = \frac{ds' \cos \theta}{r^2} = \frac{dS' (\vec{u} \cdot \vec{n})}{r^2} \quad (\text{II.6})$$

Special case: If dS' is already perpendicular to $\overrightarrow{OM}$, then $\theta=0$, $\cos\theta=1$, and the expression simplifies

$$\text{to: } d\Omega = \frac{dS'}{r^2} \quad (\text{II.7})$$

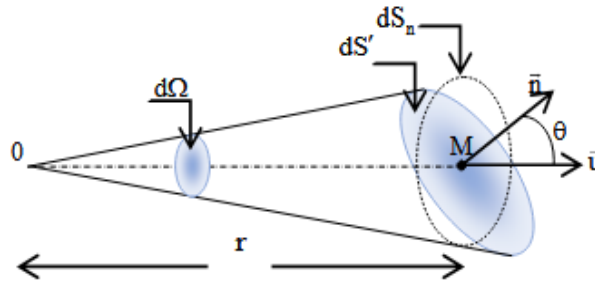


Figure II.4 Solid angle: Generalization

II.5.2 Radiant intensity

The radiant flux emitted by a source spreads out in all directions. To describe how this energy is distributed directionally, we introduce the concept of radiant intensity.

Radiant intensity characterizes the power emitted by a source.

- In a specific direction D
- Per unit solid angle $d\Omega$ around that direction

The radiant intensity I is defined as the radiant flux emitted per unit solid angle in a given direction:

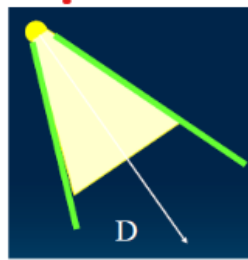


Figure II.5 Radiant Intensity

where:
$$I = \frac{d\Phi}{d\Omega} \text{ (W / sr)} \quad (\text{II.8})$$

- $d\Phi$ is the elementary radiant flux emitted in the elementary solid angle $d\Omega$.
- The direction is specified by angles (θ, ϕ) in spherical coordinates

Unit: steradian (sr) is the unit of solid angle.

Radiant intensity is a valuable quantity for characterizing the spatial distribution of emission from a surface. It describes how the emitted power is distributed in different directions throughout space.

II.5.3 Radiant Flux

This is the total power radiated by a source into all space.

This flux ϕ is emitted at different wavelengths, so it can be decomposed into elementary **monochromatic flux** ϕ_λ , which allows precise characterization of the way the body radiates.

$$\phi = \int \phi_\lambda d\lambda \quad (\text{W}) \quad (\text{II.9})$$

To quantify the flux emitted by a source in a given wavelength range (λ_1, λ_2) , we express:

$$\phi = \int_{\lambda_1}^{\lambda_2} \phi_\lambda d\lambda \quad (\text{II.10})$$

II.5.4 Luminance

In the case of an extended source, luminance L is the flux emitted per unit of apparent area, per solid angle, and per wavelength. It is a directional quantity.

It can be monochromatic or total.

It characterizes an observer's perception of this radiation.

If we consider a surface element dS emitting toward a surface element dS' , we have:

$$\textbf{Monochromatic Luminance: } L_{\lambda,\theta} = \frac{d\phi}{d\Omega dS \cdot \cos\theta d\lambda} \left(W / m^2 sr.m \right) \quad (II.11)$$

$$\textbf{Total Luminance : } L_{\theta} = \frac{d\phi}{d\Omega \cdot dS \cdot \cos\theta} \left(W / m^2 sr \right) \quad (II.12)$$

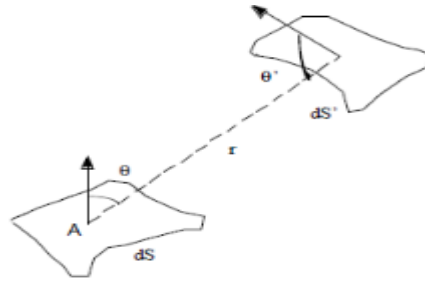


Figure II.6 Luminance between two surfaces

II.5.5 Emittance

For an extended source, **emittance** (also called radiant exitance) represents the total radiant flux emitted per unit surface area into the entire hemispherical space above the surface.

Unlike radiance, which is directional, emittance is a **hemispherical quantity** obtained by integrating the directional emission over all directions in the upper half-space (2π steradians).

- **Total Emittance** $M = \frac{d\Phi}{dS} \left(W / m^2 \right) \quad (II.13)$

- **Monochromatique Emittance** $M = \frac{d\Phi}{dS d\lambda} \left(W / m^2 sr \right) \quad (II.14)$

II.6 Quantities Related to the Reception of Radiation

II.6.1 Illuminance

Illuminance is the total luminous flux incident on a surface per unit area. It quantifies how much light (visible radiation) illuminates a surface.

$$E = \frac{d\phi}{dS} \left(W / m^2 \right) \quad (II.15)$$

where:

$d\Phi$ is the luminous flux (in lumens) received by the surface element dS .

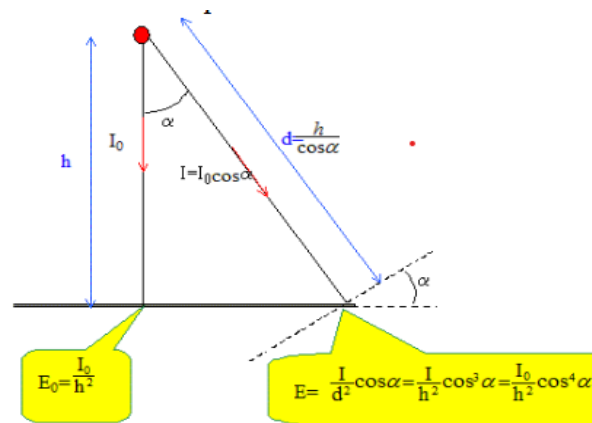


Figure II.7 Projection of Illuminance in Space

I_0 Radiant (or luminous) intensity in the direction normal to the source

I Radiant (or luminous) intensity in the direction toward point P

h Height of the source above the horizontal receiving plane

d Distance from the source to the receiving point P

α Angle between the normal direction and the direction to point P

- Having an intensity I
- Radiating toward a receiving surface dS'

The illuminance on the surface dS' is:
$$E = \frac{I}{d^2} \cos \alpha \quad (\text{II.16})$$

$$\text{with } \cos \alpha = \frac{h}{d}$$

II.7 Laws of Radiation

II.7.1 Lambert's Law (diffuse emission)

A source is said to obey Lambert's law, or to be a Lambertian (diffuse) emitter, when its radiance L is independent of the direction of emission. This means that the surface appears equally bright from all viewing angles. Most opaque emitting surfaces, such as matte paints, ceramics, and many natural materials, approximately satisfy this property. We then obtain a simple relationship between radiance and emittance.

$$M = \pi \cdot L \left(\text{W} / \text{m}^2 \right) \quad (\text{II.17})$$

II.7.2 Black Body

A blackbody is an ideal body (which therefore does not exist in nature, serving as a reference for physicists) possessing the following properties:

- It **absorbs** all incident radiation regardless of the wavelength and direction of this radiation.
- At an equivalent surface temperature, the radiation of a blackbody is greater than that of any other surface.
- The radiation of a blackbody is **isotropic** (characterizing the invariance of the physical properties of a medium with respect to direction).

By convention, all quantities related to the blackbody will have the exponent °.(e.g., M° , L° ...)

Examples

Quantity	Black Body notation	Meaning
Emissive Power	M° or E°	Blackbody emissive power: $E^\circ = \sigma T^4$
Luminance	L°	Blackbody Luminance: given by Planck's law
Spectral Luminance	L°_λ	Blackbody Spectral Luminance
Luminous intensity	I°	Blackbody Luminous intensity



Figure II.8 Theoretical model of the Blackbody

II.7.2.1 Properties of the Blackbodies

1. Universality of emission

All blackbodies, regardless of their composition or structure, radiate identically when at the same temperature. Their emission depends solely on temperature and wavelength.

2. Maximum emission

A blackbody radiates more energy than any non-blackbody (real surface) at the same temperature. The emission of a real surface is given by the blackbody emission multiplied by the emissivity ($\epsilon \leq 1$).

II.7.2.2 Quantities related to black bodies

1. Planck's Law

Planck's law describes the **spectral radiance** (or spectral emissive power) of a blackbody. Based on the quantum theory, Max Planck established the fundamental relationship that links the monochromatic emission M_λ^0 of a blackbody to its temperature and wavelength.

$$M_{\lambda,T}^0 = \frac{C_1}{\lambda^5} \frac{1}{\left(e^{\frac{C_2}{\lambda T}} - 1 \right)} \quad \left(\text{W} / \text{m}^2 \cdot \mu\text{m} \right) \text{ or } \left(\text{W} / \text{m}^3 \right) \quad (\text{II.18})$$

C_1 and C_2 are two physical constants whose values, in SI units, are:

- h is Planck's constant
- c is the speed of light
- k is Boltzmann's constant

T	λ	$C_1 = 2\pi h c^2$	$C_2 = h \cdot c/k$	M_λ^0
K	<i>m</i>	$3.741 \times 10^{-16} \text{ W/m}^2$	0.01439 m.K	W/m^3
	<i>μm</i>	$3.741 \times 10^{-8} \text{ W}/\mu\text{m}^4/\text{m}^2$	$1.439 \times 10^4 \mu\text{m.K}$	$\text{W} / \text{m}^2 \cdot \mu\text{m}$

Table II.1 Plancks constants

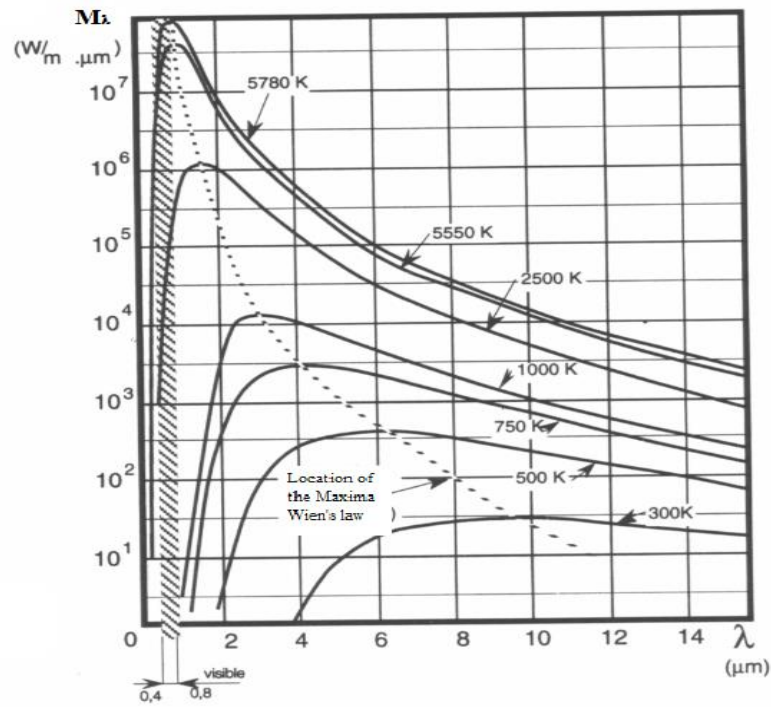


Figure II.9 Theoretical model of the Blackbody

Planck's law is the fundamental law governing the emission of radiation. The figure below shows the general shape of this function for different temperatures.

These curves all pass through the origin, tangent to the x-axis. The x-axis is their asymptote as wavelength tends to infinity. Their peak value increases with temperature, while the corresponding peak wavelength decreases as temperature rises.

For any given wavelength, emittance increases with temperature.

- **Approximation of Planck's law:** Planck's expression admits simplifying approximations, namely: For short wavelengths $\lambda \leq 5 \mu\text{m}$ (visible and near-infrared), the term

$\frac{C_2}{e^{\lambda T}}$ is large compared to 1, and Planck's formula becomes:

$$M_{\lambda,}^0 = \frac{C_1}{\lambda^5} e^{-\frac{C_2}{\lambda T}} \quad (\text{II.19})$$

(valid for $\lambda T \ll 14000 \mu\text{m} \cdot \text{K}$)

For long wavelengths in the far infrared, the expansion of $e^{\frac{C_2}{\lambda T}}$

allows us to write:

$$M_{\lambda,}^0 = \frac{C_1 T}{C_2 \lambda^4} \quad (\text{II.20})$$

2. Wien's laws

- **Wien's first law**

This law expresses the wavelength λ_m at which the monochromatic emittance is maximum.

It suffices to set the derivative of the emittance to zero:

$$\frac{dM_{\lambda,T}^0}{d\lambda} = 0 \quad (\text{II.21})$$

This yields a very simple relation: $\lambda_m T = 2880 \mu\text{m} \cdot \text{K}$.

This relation is very useful because it allows us to determine the wavelength λ_m at which a black body at temperature T emits the maximum radiation.

$$T=300\text{K} \quad \lambda_m=9.6 \mu\text{m}$$

$$T=500 \text{ K} \quad \lambda_m=5.8 \mu\text{m}$$

$$T=750 \text{ K} \quad \lambda_m=3.8 \mu\text{m}$$

$$T=1000 \text{ K} \quad \lambda_m=2.9 \mu\text{m}$$

$$T=2000 \text{ K} \quad \lambda_m=1.44 \mu\text{m}$$

$$T=3000 \text{ K} \quad \lambda_m=0.96 \mu\text{m}$$

$$T=5790 \text{ K} \quad \lambda_m=0.5 \mu\text{m}$$

We observe that a body at room temperature ($\sim 300 \text{ K}$) primarily emits far-infrared radiation (long-wave infrared). This radiation is present all around us, but we cannot perceive it because our eyes are only sensitive to radiation with wavelengths between 0.36 and $0.76 \mu\text{m}$. In contrast, the Sun, whose surface temperature is on the order of 5500°C , emits its maximum radiation at a wavelength of $0.5 \mu\text{m}$ (yellow) in the visible spectrum. This is precisely the wavelength at which our eyes have maximum luminous efficacy.

- **Wien's second law**

It expresses the value of the maximum monochromatic emittance. To do this, simply substitute λ_m into Planck's law.

We then obtain
$$\left(M_{m\lambda,T}^0\right)_{\max} = BT^5 \quad (\text{II.22})$$

$$\text{With } T \text{ in Kelvin, } B = 1.2376 \times 10^{-11} \text{ W} \cdot \text{m}^{-2} \mu\text{m}^{-1} \text{ K}^{-5}$$

This relation shows the importance of temperature with respect to emittance.

For example, the ratio of the maximum emittance of the Sun ($\sim 6000 \text{ K}$) to that of a body at room temperature ($\sim 300 \text{ K}$) is 3×10^6 .

Note:

Any object at a temperature other than absolute zero emits radiation.

3. Stefan-Boltzmann Law

This law expresses the total emissive power of the blackbody. It suffices to sum all the monochromatic emissive powers for all wavelengths.

$$M^0 = \int_0^{\infty} M_{\lambda,T}^0 d\lambda \quad (\text{II.23})$$

After integration, we obtain

$$M^0 = \sigma_0 T^4 \quad (\text{II.21})$$

T in Kelvin

σ_0 is called the Stefan-Boltzmann constant $\sigma_0 = 5.68 \times 10^{-8} \text{ W m}^{-2} \text{ K}^{-4}$.

For numerical applications, since temperatures are generally above 100K, it is advisable to use the relation

$$M^0 = 5.68 \left(\frac{T}{100} \right)^4 \quad (\text{II.22})$$

II.8 Real bodies

II.8.1 The Gray Body. Definition of Emissivity

Since the blackbody is the "standard" of radiation, the emission of a real body is evaluated in relation to that of a blackbody placed under the same temperature conditions, by defining global, monochromatic, and directional coefficients called emissivities.

Thus, the total or monochromatic emissive powers of a real body are written as:

$$\varepsilon = \frac{M}{M^0}, \quad M = \varepsilon M^0 \quad (\text{II.23})$$

$$\text{And} \quad \varepsilon_{\lambda} = \frac{M_{\lambda}}{M_{\lambda}^0} \quad (\text{II.24})$$

This emissivity may also depend on direction. It is then called directional emissivity $\varepsilon_{\theta\phi}$. If for the real body, this emissivity is constant regardless of the wavelength, this type of emission is then called gray or diffuse. This approach simplifies problem-solving.

When it is not possible to consider the body as gray across the entire spectrum, this approximation may be narrowed to large bands of the spectrum under consideration. In that case, it becomes necessary to use the fraction of total emissive power to quantify the emission in the various emission "bands."

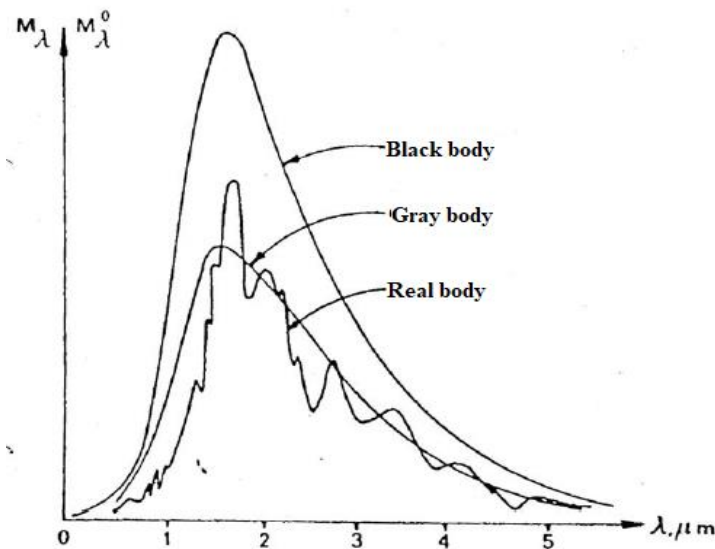


Figure II.10 Luminance of real bodies as a function of wavelength.

II.8.2 Thermal Radiated Power (Emittance) of a Real Body (Kirchhoff's Law)

This law establishes a relationship between the absorption coefficient α and the emission coefficient ε of a given surface.

$$\text{Thus: } \varepsilon_\lambda = \frac{M_\lambda(T)}{M_\lambda^0(T)} \quad (\text{II.25})$$

$$\text{Hence } M_\lambda(T) = \varepsilon_\lambda M_\lambda^0(T) = \varepsilon_\lambda \sigma T^4 \quad (\text{II.26})$$

Emissivity is always less than 1.

$$M(T) = \varepsilon_\lambda \sigma T^4 = \varepsilon S \sigma T^4 \quad [\text{W} / \text{m}^2] \quad (\text{II.27})$$

II.9 Gray body

A gray body is a body whose absorptive power is independent of the wavelength of the radiation it receives. Generally, solid bodies are considered gray bodies.

- Emissivity of a gray body “ ϵ ”

Material	Temperature	Emissivity
Aluminum, bare after rolling	170°C	0.04
Aluminum, black anodized	50°C	0.98
Concrete	20°C	0.93
Ice, smooth	0°C	0.97
Polished iron	20°C	0.24
Glass	90°C	0.94
Copper, slightly tarnished	20°C	0.04
Copper, oxidized	130°C	0.76
Steel, hardened	200°C	0.52
Steel, oxidized	200°C	0.79
Varnish, matte black	80°C	0.97

Table II.2 Some values of gray body emissivity

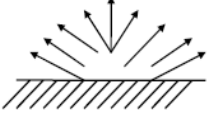
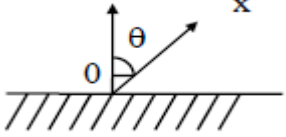
Classification		According to spatial distribution of emitted radiation	Hemispherical
According to spectral composition of emitted radiation	Nature of radiation considered	Hemispherical 	Directed along a normal direction Ox 
	Monochromatic	Monochromatic (hemispherical) emissivity $\varepsilon_{\lambda} = \frac{M_{\lambda}}{M_{\lambda}^0}$	Directional monochromatic emissivity $\varepsilon_{ox,\lambda} = \frac{L_{ox,\lambda}}{L_{\lambda}^0} = \frac{L_{ox,\lambda}}{M_{\lambda}^0 / \pi}$
	Total	Total (hemispherical) emissivity $\varepsilon_{\lambda} = \frac{M}{M^0} = \frac{\int_0^{\infty} \varepsilon_{\lambda} M_{\lambda}^0 d\lambda}{\sigma T^4}$	Total directional emissivity $\varepsilon_{ox} = \frac{\int_0^{\infty} \varepsilon_{ox,\lambda} L_{\lambda}^0 d\lambda}{\int_0^{\infty} L_{\lambda}^0 d\lambda}$ $= \frac{\int_0^{\infty} \varepsilon_{ox,\lambda} M_{\lambda}^0 d\lambda}{\sigma T^4}$
The emissive powers of the real body and the blackbody are taken at the same temperature T			
Special (usual) cases Gray body: emissivity independent of wavelength $\varepsilon_{\lambda} \rightarrow \varepsilon$ and $\varepsilon_{ox,\lambda} \rightarrow \varepsilon_{ox}$ Diffuse (or isotropic) emitting body: emissivity independent of direction $\varepsilon_{ox} \rightarrow \varepsilon$ and $\varepsilon_{ox,\lambda} \rightarrow \varepsilon_{\lambda}$ Gray and diffuse body: emissivity independent of wavelength and direction: <i>single</i> parameter ε.			

Table II.3 Different definitions of emissivity

II.10 Radiative exchanges separated by a transparent medium

Assumptions:

- The surfaces considered are assumed to be homogeneous, opaque, isothermal, and gray.
- The irradiances are assumed to be homogeneous and reflections diffuse.

Definition:

Transparent medium = Non-absorbing (and non-emitting) medium. It therefore does not participate in radiative exchanges.

II.10.1 Radiative Exchanges between Two Black Surfaces

a. Form Factors

This is the fraction of the radiant power leaving surface S_i and intercepted directly by surface S_j .

F_{ij} = the fraction of radiation leaving surface i that directly strikes surface j .

Consequently, the view factor F_{12} represents the fraction of radiation leaving surface 1 that directly strikes surface 2, and F_{21} represents the fraction of radiation leaving surface 2 that directly strikes surface 1.

Note that radiation striking a surface does not need to be absorbed by that surface. Furthermore, radiation that strikes a surface after being reflected by other surfaces is not taken into account in the evaluation of view factors.

Assuming that S_1 and S_2 are two black surfaces maintained at uniform temperatures T_1 and T_2

The total hemispherical flux emitted by S_1 is: $\phi_1 = M_1^0 S_1$ of which only a fraction Φ_{12} reaches S_2 .

With : M_1^0 total emissive power of the blackbody.

In general: $\phi_{12} = F_{12} \phi_1 = \phi_{12} = F_{12} \phi_1 = M_1^0 S_1 F_{12}$ (II.28)

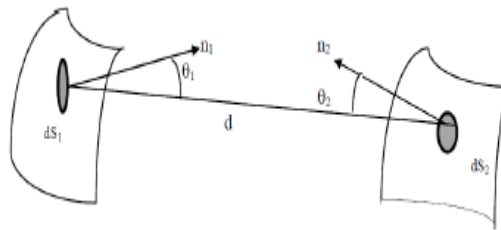


Figure II.11 Radiative exchange between two surfaces

$$F_{12} = \frac{1}{S_1} \iint \frac{dS_i \cos \beta_i dS_j \cos \beta_j}{\pi d_{ij}^2} \quad (II.29)$$

or $F_{ij} = \frac{\phi_{i \rightarrow j}}{\phi_i}$ with $0 \leq F_{i \rightarrow j} \leq 1$

b. Relationships between form factors

- **Reciprocity relation**

The symmetry of the expressions leads to the reciprocity of view factors.

$$\phi_{12} = M_1^0 S_1 F_{12} = M_2^0 S_2 F_{21} \quad (\text{II.30})$$

$$\phi_{21} = M_2^0 S_2 F_{21} = M_1^0 S_1 F_{12} \quad (\text{II.31})$$

$$S_1 F_{12} = S_2 F_{21} \quad (\text{II.32})$$

- **Summation relation**

In an enclosure, the flux ϕ_i emitted by a surface S_i is likely to be received by the n surfaces that constitute the enclosure (surface i included).

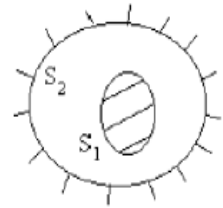
$$\phi_i = \phi_{i \rightarrow 1} + \phi_{i \rightarrow 2} + \dots + \phi_{i \rightarrow i} + \phi_{i \rightarrow n} \quad (\text{II.33})$$

c. Evaluation of form factors

- **Two parallel surfaces of large dimensions relative to their distance**

surfaces with “total influence”

$$F_{12} = 1 \Rightarrow F_{21} = \frac{S_1}{S_2} \quad F_{22} = 1 - \left(\frac{R_1}{R_2} \right)^2$$



- **Case of two concentric spheres:**

$$F_{21} = \frac{S_1}{S_2} = \left(\frac{R_1}{R_2} \right)^2 \Rightarrow F_{22} = F_{12} - F_{21}$$



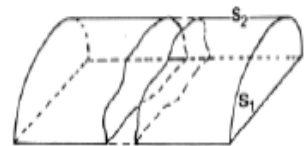
- **Hemisphere, relative to its base plane S_1 :**

In this case as well, all the flux emitted by S_1 reaches S_2

$$F_{12} = 1/2$$

- **Very long half-cylinder, relative to its base plane S_1 :**

$$F_{12} = 2/\pi$$



- **Two perpendicular joined plates**

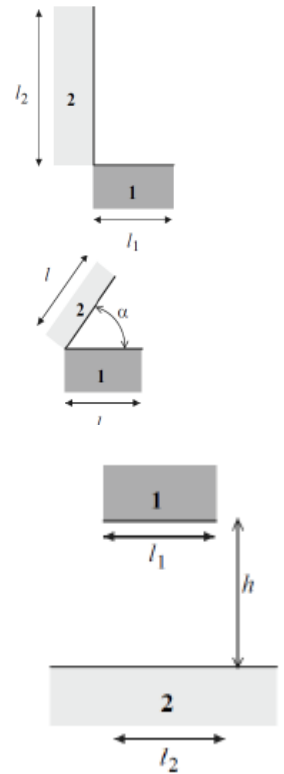
$$F_{12} = \frac{1 + \gamma - \sqrt{1 + \gamma^2}}{2} \quad \text{with} \quad \gamma = \frac{I_1}{I_2}$$

- **Two plates of equal length, inclined and joined**

$$F_{12} = 1 - \sin \frac{\alpha}{2}$$

- **Two parallel plates**

$$F_{12} = \frac{\sqrt{(L_1 + L_2)^2 + 4} - \sqrt{(L_2 - L_1)^2 + 4}}{2L_1} \quad \text{with} \quad L_1 = \frac{I_1}{h} \quad \text{and} \quad L_2 = \frac{I_2}{h}$$



II.10.2 Net power exchanged

The net flux exchanged by radiation between two black surfaces S_1 and S_2 , denoted $\phi_{12} \text{ net}$, is obtained by performing an energy balance on S_1 :

$$\phi_{12} \text{ net} = \phi_{12} - \phi_{21} = M_1^0 S_1 F_{12} - M_2^0 S_2 F_{21} \quad (\text{II.34})$$

$$\phi_{12} \text{ net} = S_1 F_{12} (M_1^0 - M_2^0) = S_2 F_{21} (M_2^0 - M_1^0) \quad (\text{II.35})$$

$$\phi_{12} \text{ net} = S_1 F_{12} \sigma (T_1^4 - T_2^4) = S_2 F_{21} (T_2^4 - T_1^4) \quad (\text{II.36})$$

Note:

Depending on the sign of ϕ_{net} , it is possible to know whether the surface loses energy by radiation ($\phi_{\text{net}} > 0$), whether it gains energy ($\phi_{\text{net}} < 0$), or whether losses are compensated by gains ($\phi_{\text{net}} = 0$); in the latter case, the surface is adiabatic with respect to the exterior.

II.10.3 Radiative exchanges in a black enclosure

Consider an enclosure consisting of n black surfaces S_j with $j=1, \dots, n$, and let us perform an energy balance of radiative exchanges on one of the surfaces, S_i :

$$\phi_{12} \text{ net} = \phi_i - \sum_{j=1}^n \phi_{ij} = S_i M_i^0 - \sum_{j=1}^n S_j F_{ji} M_j^0 \quad (\text{II.37})$$

II.10.4 Electrical analogy for radiative exchanges

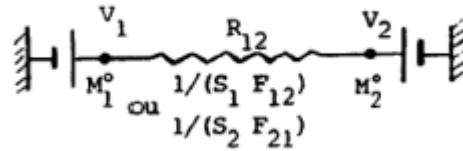


Figure II.12 Electrical diagrams

$$\phi_{12\text{net}} = S_1 F_{12} (M_1^0 - M_2^0), \text{ By electrical analogy } I_{12} = \frac{1}{R_{12}} (V_1 - V_2)$$

II.11 Radiative exchanges between real bodies through a transparent medium

1. Definition of radiosity

Radiative exchanges between gray surfaces are more complex because surfaces reflect radiation, and multiple reflections then appear within an enclosure. A new concept is then used: radiosity J .

The radiosity of a surface is the sum of the emitted flux and the reflected flux per unit area.

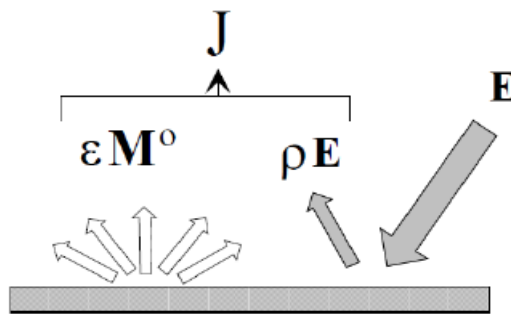


Figure II.13 Radiosity

$$\text{With } J = \varepsilon M^0 + \rho E \quad (\text{II.38})$$

J : Radiosity of the gray surface

εM^0 : Emittance of the gray surface

E : illuminance of the surface

The net flux from the surface is then:

$$\phi_{\text{net}} = \varepsilon M^0 S + \alpha E S \quad (\text{II.39})$$

Recall:

When a radiative flux ϕ reaches a surface, it is decomposed into:

$$\phi = \phi_{\tau} + \phi_r + \phi_a \quad (\text{II.40})$$

The transmissivity, reflectivity, and absorptivity coefficients:

$$\tau = \frac{\phi_\tau}{\phi}; \rho = \frac{\phi_r}{\phi}; \alpha = \frac{\phi_a}{\phi} \quad (\text{II.41})$$

With $\tau + \rho + \alpha = 1$ (II.42)

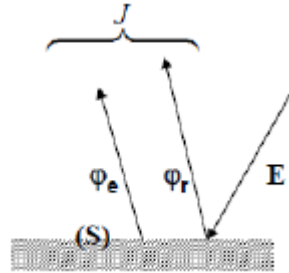


Figure II.14 Radiative flux

For a gray body: $\alpha = \epsilon$, for an opaque surface: $\tau = 0$ therefore: $\rho = 1 - \epsilon$

The special cases related to this expression are:

Grey bodies: when the surface is opaque, $\tau = 0$, expression $\tau + \rho + \alpha = 1$ becomes: $\rho + \alpha = 1$. This relationship is very important because it allows us to determine both absorptivity and reflectivity by measuring either of these properties. This means that a body with good reflectivity has poor absorptivity, and vice versa.

White bodies or mirrors: a perfectly reflecting surface is characterized by total diffuse reflection of the incident flux, $\alpha = 0$ and $\tau = 0$, i.e.: $\rho = 1$

Matte bodies: a matte surface is represented by $\rho = 0$, therefore: $\alpha + \tau = 1$

Transparent or diathermic bodies: a perfectly transparent surface allows the entire incident flux to pass through; in this case, $\alpha = 0$ and $\rho = 0$, so: $\tau = 1$

Black bodies: a black surface is characterized by total absorption of the incident flux; $\rho = 0$ and $\tau = 0$, so: $\alpha = 1$.

Note: These limiting cases do not occur in nature, but they are nevertheless very useful (especially the concept of a blackbody) for the study and calculation of heat transfer by radiation

Assumptions:

There is no total absorption by the surfaces receiving radiation.

Let J be the radiosity, the sum of the surface's emittance and the reflected flux per unit area:

With $\phi_e = \epsilon M^0 = \epsilon \sigma T^4$ and $\phi_r = \rho E$ Then $J = \epsilon M^0 + \rho E = \epsilon M^0 + (1 - \epsilon)E$

With E being the illuminance of surface S , which equals:

$$E = \frac{J - \varepsilon M^0}{1 - \varepsilon} \quad \left[\text{W} / \text{m}^2 \right] \quad (\text{II.43})$$

Now consider surface S_j chosen from n isothermal and homogeneous surfaces that bound a

volume: $\frac{\phi_{\text{net}}}{S} = J - E$

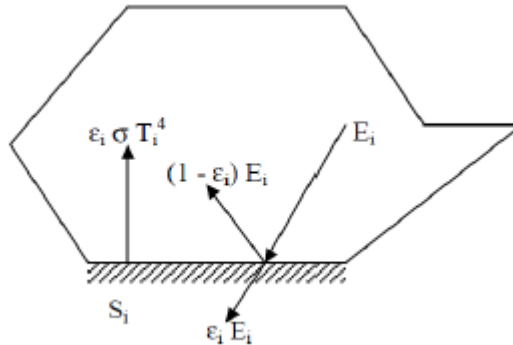


Figure II.15. Illumination of multiple surfaces

The net flux lost by surface SS is equal to the difference between the flux emitted and absorbed by this surface:

$$\phi_{\text{net}} = S_i (j_j - E_i) \quad \text{alternative expression}$$

$$\phi_{\text{net}} = \frac{\varepsilon S}{1 - \varepsilon} (M^0 - J) \quad (\text{II.44})$$

The net flux density lost by a surface is equal to the difference between its radiosity (flux density leaving the surface by emission and reflection) and its irradiance (flux density arriving at the surface).

2. Interpretation in terms of electrical analogy

The net flux radiated by a gray surface that is diffuse in emission and reflection is:

$$\phi_{\text{net}} = \frac{(M^0 - J)}{\frac{1 - \varepsilon}{\varepsilon S}} \quad (\text{II.45})$$

It can be interpreted in terms of electrical analogy as follows:

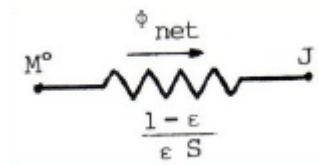


Figure II.16 Surface flow diagrams

The resistance $\frac{1-\varepsilon}{\varepsilon S}$ is subjected to the potential difference M^0 - J_1 is subjected to the potential difference and is traversed by the current ϕ_{net} .

- **Case of two real surfaces**

This leads to the diagram of mutual radiative exchanges between two gray, diffuse surfaces.

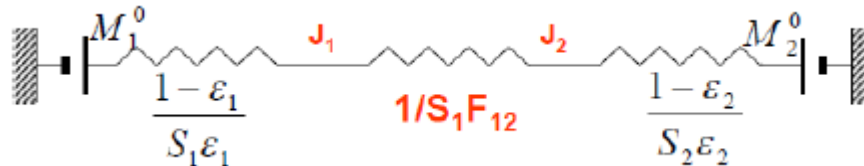


Figure II.17 Flow diagrams for two surfaces

- **Case of three real surfaces**

For three surfaces, one of which is adiabatic, we obtain:

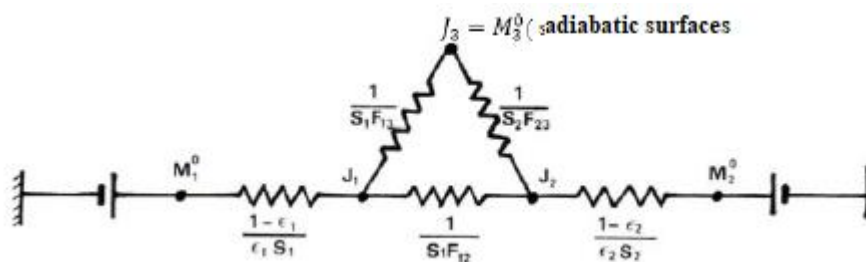


Figure II.18 Flow diagrams for three surfaces

$\lambda T (\mu\text{m}) \text{K}$	$F_{0-\lambda T}$	$\lambda T (\mu\text{m}) \text{K}$	$F_{0-\lambda T}$
500	0.13×10^{-7}	5000	0.63371
600	0.112×10^{-6}	5200	0.65792
700	0.22×10^{-5}	5400	0.68031
800	0.184×10^{-4}	5600	0.701
900	0.912×10^{-4}	5800	0.72009
1000	0.321×10^{-3}	6000	0.73777
1100	0.926×10^{-3}	6200	0.75408
1200	0.00216	6400	0.76917
1300	0.00436	6600	0.78315
1400	0.00783	6800	0.79607
1500	0.01285	7000	0.80806
1600	0.01977	7500	0.83435
1700	0.0286	8000	0.85624
1800	0.03941	8500	0.87455
1900	0.05215	9000	0.88997
2000	0.06672	9500	0.90303
2200	0.10093	10 000	0.91414
2400	0.14027	11 000	0.93183
2600	0.18312	12 000	0.94504
2800	0.22788	13 000	0.95508
3000	0.27322	14 000	0.96284
3200	0.31807	15 000	0.96892
3400	0.3617	16 000	0.97375
3600	0.40357	18 000	0.9808
3800	0.44334	20 000	0.98554
4000	0.48085	30 000	0.99528
4200	0.51596	40 000	0.99791
4400	0.54875	50 000	0.99889
4600	0.57923	.	1.00000
4800	0.60751		

TableII.4 Values of the $F_{0-\lambda T}$ function

II.12 Exercises

Exercise 1:

The Sun emits maximum radiation at a wavelength of $\lambda_{\max} = 0.5 \mu\text{m}$. Estimate its surface temperature.

A ceramic tile in an oven emits maximum radiation at $\lambda_{\max} = 3.8 \mu\text{m}$. What is its temperature?

Solution:

According to Wien's law: $\lambda_{\max} \cdot T = 2898 \mu\text{m} \cdot \text{K}$ (standard value).

For the Sun: $T = 2898 / 0.5 \approx 5796 \text{ K}$.

For ceramics: $T = 2898 / 3.8 \approx 762.6 \text{ K}$ (approximately 490°C).

Exercise 2:

A sphere with a diameter of 10 cm, treated as a blackbody, is maintained at 800 K.

Calculate the total power radiated by this sphere.

Stefan-Boltzmann constant: $\sigma = 5.67 \times 10^{-8} \text{ W} \cdot \text{m}^{-2} \cdot \text{K}^{-4}$.

Solution:

The total power is $\Phi = \sigma \cdot T^4 \cdot A$, where $A = 4\pi R^2 = 4\pi(0.05)^2 = 0.031416 \text{ m}^2$.

$$\Phi = 5.67 \times 10^{-8} \times (800)^4 \times 0.031416.$$

$$800^4 = 4096 \times 10^8 = 4.096 \times 10^{11} \text{ (let's check: } 800^2 = 640000, 800^4 = 640000^2 = 4.096 \times 10^{11}\text{)}.$$

$$\sigma T^4 = 5.67 \times 10^{-8} \times 4.096 \times 10^{11} = 5.67 \times 4.096 \times 10^3 = 23.22 \times 10^3 = 23220 \text{ W/m}^2.$$

$$\Phi = 23220 \times 0.031416 \approx 729.4 \text{ W}.$$

Exercise 3:

A metal surface oxidized at 500 K has a total emissivity of $\varepsilon = 0.75$.

Calculate its actual total emittance. Compare this with that of a blackbody at the same temperature.

Solution:

$$\text{Blackbody emissivity: } M^0 = \sigma T^4 = 5.67 \times 10^{-8} \times (500)^4 = 5.67 \times 10^{-8} \times 625 \times 10^8 = 5.67 \times 625 = 3543.75 \text{ W/m}^2.$$

$$\text{Actual emissivity: } M = \varepsilon M^0 = 0.75 \times 3543.75 = 2657.8 \text{ W/m}^2.$$

Exercise 4:

Two parallel infinite planes, treated as black bodies, are maintained at temperatures $T_1 = 1000 \text{ K}$ and $T_2 = 600 \text{ K}$.

Calculate the net heat flux per unit area.

Solution:

For two infinite planes, $F_{12} = 1$. The net flux per unit area:

$$\phi_{12} = \sigma (T_1^4 - T_2^4) = 5.67 \times 10^{-8} \times (1000^4 - 600^4).$$

$$1000^4 = 1 \times 10^{12}; 600^4 = 1296 \times 10^8 = 1.296 \times 10^{11}.$$

$$\text{Difference} = 10^{12} - 1.296 \times 10^{11} = 8.704 \times 10^{11}.$$

$$\phi_{12} = 5.67 \times 10^{-8} \times 8.704 \times 10^{11} = 5.67 \times 8.704 \times 10^3 = 49.36 \times 10^3 = 49,360 \text{ W/m}^2.$$

Exercise 5:

Consider two concentric spheres maintained at uniform temperatures $T_1 = 900 \text{ K}$ and $T_2 = 600 \text{ K}$, with emissivities $\varepsilon_1 = 0.3$ and $\varepsilon_2 = 0.8$, respectively.

Given that the inner and outer diameters are 20 cm and 25 cm, respectively, determine the radiative flux exchanged.

Solution:

The radiative flux transferred from sphere 1 to sphere 2 can be easily determined as

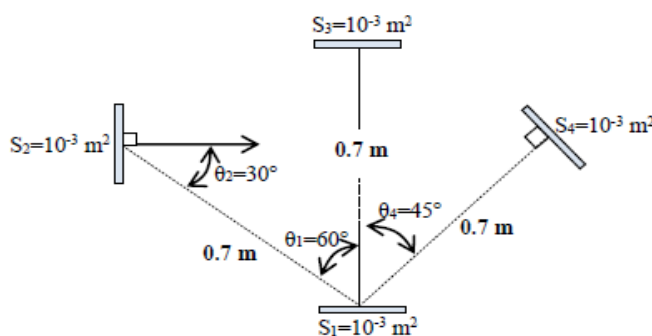
$$\text{follows : } \Phi_{12} = \frac{S_1 \sigma (T_1^4 - T_2^4)}{\frac{1}{\varepsilon_1} + \frac{1 - \varepsilon_2}{\varepsilon_2} \left(\frac{r_1}{r_2} \right)^2} = \frac{4\pi (20 \times 10^{-2})^2 \times 5.67 \times 10^{-8} [900^4 - 600^4]}{\frac{1}{0.3} + \frac{1 - 0.8}{0.8} \left(\frac{20}{25} \right)} = 4293.298 \text{ W}$$

Exercise 6 :

A small diffuse surface with area $S_1 = 10^{-3} \text{ m}^2$ emits light isotropically (see figure below). The total luminance in the direction normal to the surface is measured as:

$L_n = 4500 \text{ W} \cdot \text{m}^{-2} \cdot \text{sr}^{-1}$. The radiant flux is intercepted by four other surfaces, each with the same area "A". The surfaces are oriented as shown in the figure below ($r = 0.7 \text{ m}$).

1. What is the total luminance in the 4 emission directions associated with the 4 receivers?
2. What are the values of the solid angles under which the 4 receiving surfaces are viewed from the emitter?
3. What are the values of the fluxes intercepted by the 4 surfaces?.



Solution:

Since the radiation is isotropic, its intensity does not depend on the direction of emission. The luminance is therefore: $L = L_n = 4500 \text{ W/m}^2 \cdot \text{sr}$

The solid angle between surface i (2, 3, and 4) and surface 1 is given by the relation

$$d\Omega_{i \rightarrow 1} = \frac{dA \cos \theta_i}{r^2}$$
 In this relation, “ θ_i ” denotes the angle between surface “i” and the direction of the incident radiation coming from “1”. Therefore

$$\Omega_{2 \rightarrow 1} = \frac{A \cos \theta_2}{r_0^2} = \frac{10^{-3} \times \cos 30}{0.7^2} = 1.767 \times 10^{-3} \text{ sr}$$

$$\Omega_{3 \rightarrow 1} = \frac{A \cos \theta_3}{r_0^2} = \frac{10^{-3} \times \cos 0}{0.7^2} = 2.041 \times 10^{-3} \text{ sr}$$

$$\Omega_{4 \rightarrow 1} = \frac{A \cos \theta_4}{r_0^2} = \frac{10^{-3} \times \cos 45}{0.7^2} = 1.443 \times 10^{-3} \text{ sr}$$

To calculate the flux intercepted by surface “i” (2, 3, and 4) as it comes from 1, we use the relation: $\Phi_{1 \rightarrow j} = L A \cos \theta_1 \Phi_{j \rightarrow 1}$. In this equation, “ θ_i ” denotes the angle between the direction of the radiation emanating from 1 and incident on surfaces i and surface 1. We then find:

$$\Phi_{1 \rightarrow 2} = L A \cos \theta_2 \Phi_{2 \rightarrow 1} = 4500 \times 10^{-3} \cos 60 \times 1.767 \times 10^{-3} = 3.98 \times 10^{-3} \text{ W}$$

$$\Phi_{1 \rightarrow 3} = L A \cos \theta_3 \Phi_{3 \rightarrow 1} = 4500 \times 10^{-3} \cos 0 \times 2.041 \times 10^{-3} = 9.19 \times 10^{-3} \text{ W}$$

$$\Phi_{1 \rightarrow 4} = L A \cos \theta_4 \Phi_{4 \rightarrow 1} = 4500 \times 10^{-3} \cos 45 \times 1.443 \times 10^{-3} = 4.59 \times 10^{-3} \text{ W}$$

Exercise 7 :

The Sun is a sphere with a radius of 688,000 km, located 150 million kilometers from Earth. Its surface, considered a blackbody, has a temperature of 5,750 K.

Earth has a radius of 6,380 km and an average surface temperature of 27°C

1. Calculate the total emittance of the Sun (specify the unit)
2. Calculate the total power radiated by the Sun (specify the unit)
3. Calculate its radiant power, assuming that the intensity is constant in all directions (specify the unit)
4. Calculate the solid angle under which the Sun is visible from Earth

Solution:

1. The total emittance of the Sun, $M(T) = \sigma T^4 = 6.198 \times 10^7 \text{ W/m}^2$
2. Total power, $\phi = S \times M(T) = \sigma T^4 \times 4\pi R^2 = 3.686 \times 10^{22} \text{ W}$
3. Luminance: according to Alembert's law, $M(T) = \pi \cdot L$, hence $L = 19.728 \times 10^6 \text{ W/sr}\cdot\text{m}^2$
4. The solid angle $\Omega = S \cdot \cos\theta \cdot d^2$ with $\theta = 0$ and $\cos\theta = 1$; $d = 150 \times 10^9 \text{ m}$, so $\Omega = 6.609 \times 10^5 \text{ W}$

Exercise 8:

A black surface radiating into a vacuum is maintained at a temperature of 1000 °C. At a wavelength of 6 μm, what is the emitted luminance, the spectral directional emittance of the blackbody at an angle of 60° relative to the surface normal, and the hemispherical spectral emittance?

Solution :

$$L_{6\mu\text{m}}^0 = \frac{\frac{C_1 \lambda^{-5}}{\pi}}{e^{\frac{C_2}{\lambda T}} - 1} = \frac{1.194 \times 10^8 (6)^5}{e^{\left(\frac{1.439 \times 10^4}{6 \times 1273}\right)} - 1} = 2745.9 \text{ W / m}^2 \text{sr}\cdot\mu\text{m}$$

Spectral directional emittance:

$$M_{\lambda}^0(6, 60^\circ) = L_{\lambda}^0 \cos \theta = 2745.9 \cos 60 = 1372.95 \text{ W / m}^2 \mu\text{m}$$

Hemispheric spectral emittance

$$M_{\lambda}^0 = \pi L_{\lambda}^0 = 9622.126 \text{ W / m}^2 \mu\text{m}$$

Exercise 9:

The Sun is considered to radiate like a blackbody at 5800 K.

1. Determine the fraction of the Sun's radiated energy that falls within the following ranges:
 - Visible spectrum ($0.4 \leq \lambda \leq 0.76 \mu\text{m}$)
 - Ultraviolet ($0.01 \leq \lambda \leq 0.4 \mu\text{m}$)
 - Solar $0 \leq \lambda \leq 3 \mu\text{m}$
 - Infrared $0.76 \leq \lambda \leq 100 \mu\text{m}$
2. At what wavelength is the maximum energy emitted? What is the maximum value of the hemispheric spectral irradiance?

Solution:

1. Calculation of the fraction of radiated energy in the different wavelength ranges:

- **Visible:** Table II.4 $\lambda_1 T = 0.4 \times 5800 = 2320 \Rightarrow F_{0 \rightarrow 2320} = 0.124534$
 $\lambda_2 T = 0.76 \times 5800 = 4408 \Rightarrow F_{0 \rightarrow 4408} = 0.549969$

Thus, in the visible range, the fraction of radiant energy is:

$$F_{0 \rightarrow \lambda_2 T} - F_{0 \rightarrow \lambda_1 T} = 0.549969 - 0.124534 = 0.425 = 42.5\%$$

- **Ultra-violet :** Table II.4 $\lambda_1 T = 0.01 \times 5800 = 58 \Rightarrow F_{0 \rightarrow 58} = 0$
 $\lambda_2 T = 0.4 \times 5800 = 2320 \Rightarrow F_{0 \rightarrow 2320} = 0.124534$

Thus, in the visible range, the fraction of radiant energy is:

$$F_{0 \rightarrow \lambda_2 T} - F_{0 \rightarrow \lambda_1 T} = 0.124534 - 0 = 0.124534 = 12.4534\%$$

- **Solar :** Table II.4 $\lambda_1 T = 0 \times 5800 = 0 \Rightarrow F_{0 \rightarrow 0} = 0$
 $\lambda_2 T = 3 \times 5800 = 17400 \Rightarrow F_{0 \rightarrow 17400} = 0.978685$

Thus, in the visible range, the fraction of radiant energy is:

$$F_{0 \rightarrow \lambda_2 T} - F_{0 \rightarrow \lambda_1 T} = 0.978685 - 0 = 0.978685 = 97.8685\%$$

- **Infrared :** Table II.4 $\lambda_1 T = 0.76 \times 5800 = 4408 \Rightarrow F_{0 \rightarrow 4408} = 0.549969$
 $\lambda_2 T = 100 \times 5800 = 580000 \Rightarrow F_{0 \rightarrow 580000} = 1$

Thus, in the visible range, the fraction of radiant energy is:

$$F_{0 \rightarrow \lambda_2 T} - F_{0 \rightarrow \lambda_1 T} = 1 - 0.549969 = 0.45 = 45\%$$

2. The wavelength corresponding to the maximum emissivity is calculated using Wien's first law:

$$\lambda_{max} T = 2898 \Rightarrow \lambda_{max} = \frac{2898}{T} = \frac{2898}{5800} = 0.5 \mu\text{m}$$

The hemispheric spectral emittance:

$$M_{\lambda,}^0 = BT^5 = 1.287 \times 10^{-5} T^5 = 1.287 \times 10^{-5} (5800)^5 = 8.447 \times 10^{13} \text{ W / m}^3$$

Chapter III**Heat Exchangers****III. Introduction**

Heat exchangers are devices that facilitate the exchange of heat between two fluids that are at different temperatures while keeping them from mixing with each other. Heat exchangers are commonly used in practice in a wide range of applications, from heating and air-conditioning systems in a household, to chemical processing and power production in large plants.

Heat exchangers differ from mixing chambers in that they do not allow the two fluids involved to mix. In a car radiator, for example, heat is transferred from the hot water flowing through the radiator tubes to the air flowing through the closely spaced thin plates outside attached to the tubes.

Heat transfer in a heat exchanger usually involves convection in each fluid and conduction through the wall separating the two fluids. In the analysis of heat exchangers, it is convenient to work with an overall heat transfer coefficient U that accounts for the contribution of all these effects on heat transfer.

The rate of heat transfers between the two fluids at a location in a heat exchanger depends on the magnitude of the temperature difference at that location, which varies along the heat exchanger. In the analysis of heat exchangers, it is usually convenient to work with the logarithmic mean temperature difference LMTD, which is an equivalent mean temperature difference between the two fluids for the entire heat exchanger.

III.1 Types of Heat Exchangers

Different heat transfer applications require different types of hardware and different configurations of heat transfer equipment. The attempt to match the heat transfer hardware to the heat transfer requirements within the specified constraints has resulted in numerous types of innovative heat exchanger designs.

III.1.1 Double-pipe heat exchanger

The simplest type of heat exchanger consists of two concentric pipes of different diameters, as shown in Figure III.1. One fluid in a double-pipe heat exchanger flows through the smaller pipe while the other fluid flows through the annular space between the two pipes. Two types of flow arrangement are possible in a double-pipe heat exchanger: in parallel flow, both the hot and cold fluids enter the heat exchanger at the same end and move in the same direction. In counter flow, on the other hand, the hot and cold fluids enter the heat exchanger at opposite ends and flow in opposite directions.

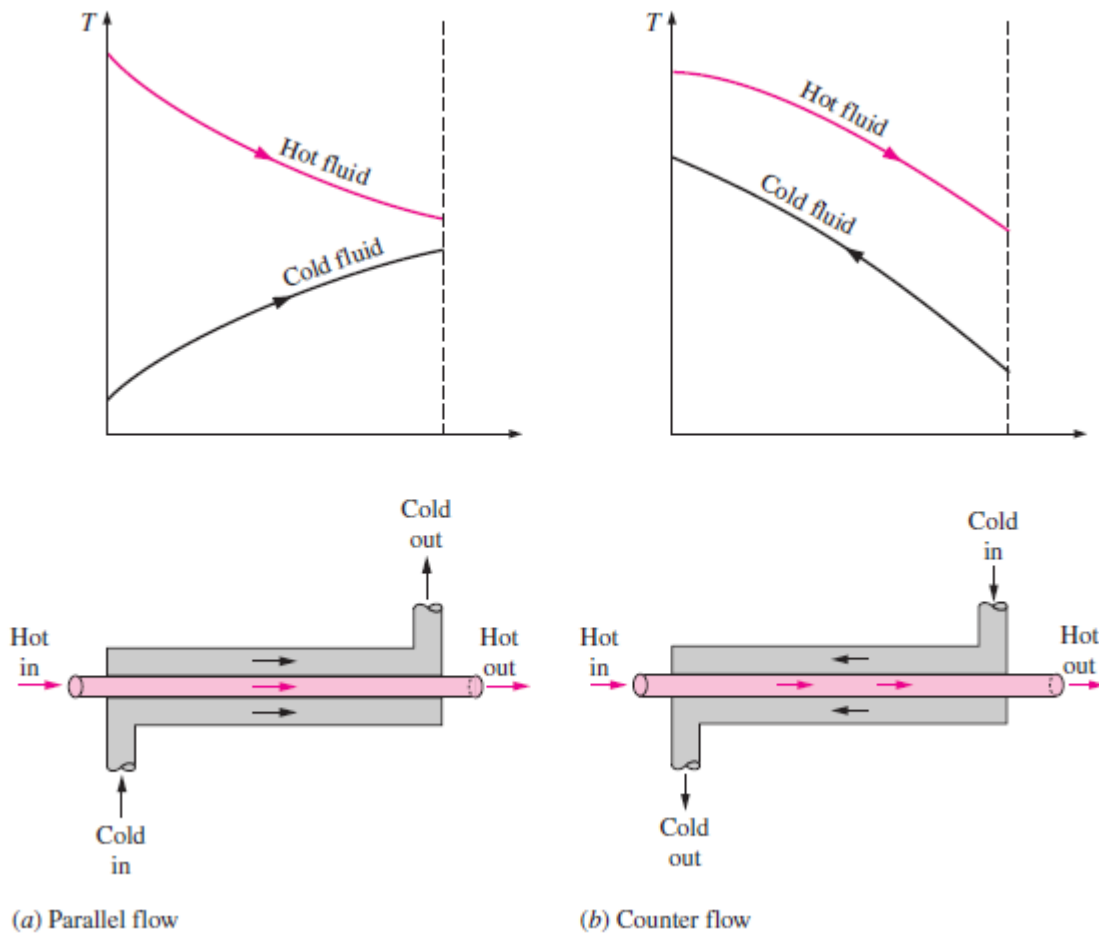


Figure III.1 Different flow regimes and associated temperature profiles in a double-pipe heat exchanger

III.1.2 Compact Heat Exchanger

The compact heat exchanger is specifically designed to realize a large heat transfer surface area per unit volume. The ratio of the heat transfer surface area of a heat exchanger to its volume is called the area density β . A heat exchanger with $\beta > 700 \text{ m}^2/\text{m}^3$ is classified as being compact. Compact heat exchangers enable us to achieve high heat transfer rates between two fluids in a small volume, and they are commonly used in applications with strict limitations on the weight and volume of heat exchangers Figure III.2.

The large surface area is achieved by attaching closely spaced fins to the separating walls. They are commonly used in gas-to-gas or gas-to-liquid applications to compensate for the low heat transfer coefficient of gases by increasing surface area (e.g., car radiator fins on the air side). Fluids typically flow perpendicular to each other (**cross-flow**), which can be unmixed (fins prevent transverse movement) or mixed (fluid free to move transversely). In a car radiator, both fluids are unmixed. Fluid mixing significantly affects heat transfer performance, as shown in Figure III.3.

transverse direction.

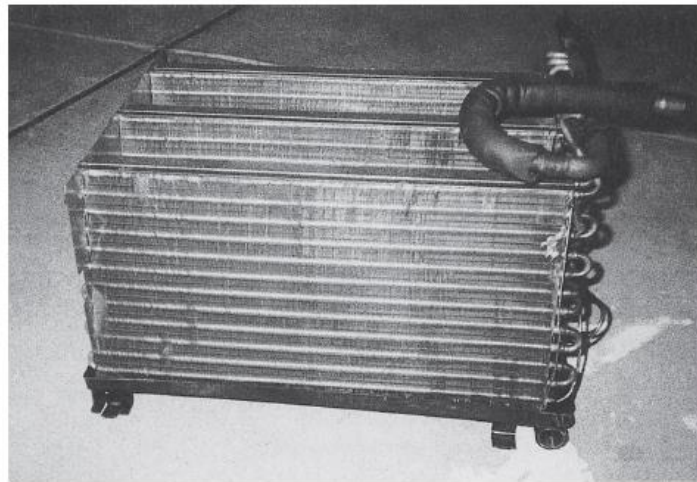


Figure III.2 A gas-to-liquid compact heat exchanger for a residential air conditioning system.

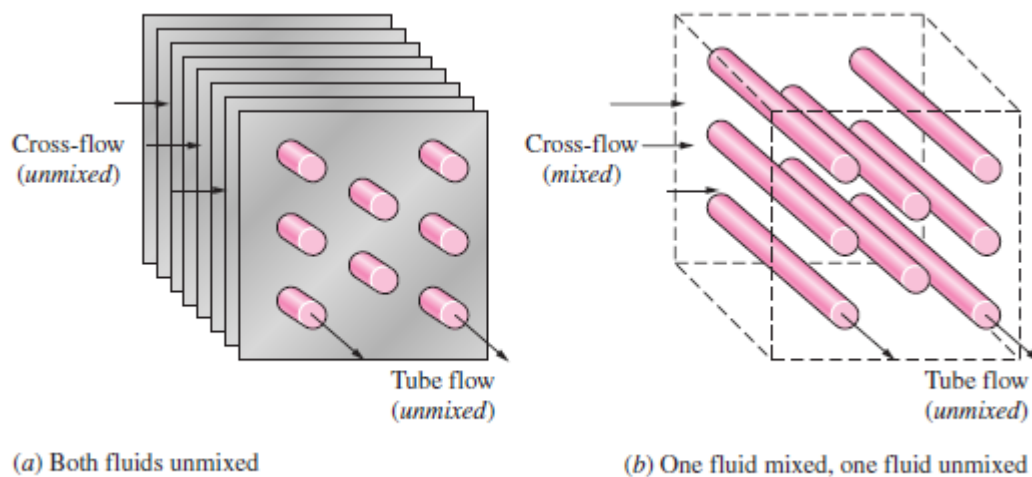


Figure III.3 Different flow configurations in cross-flow heat exchangers.

III.2.3 Shell-and-Tube

The shell-and-tube heat exchanger is the most widely used industrial design, consisting of numerous parallel tubes housed within a shell. Heat transfer occurs between two fluids: one flowing through the tubes and the other through the shell, with baffles used to improve efficiency and spacing. While highly effective, their size and weight make them unsuitable for automotive or aircraft use. These systems are categorized by the number of "passes" the fluids make through the shell and tubes to optimize heat exchange. Heat exchangers in which all the tubes make one U-turn in the shell, for example, are called **one-shell-pass** and **two-tube-passes** heat exchangers. Likewise, a heat exchanger that involves two passes in the shell and four passes in the tubes is called a **two-shell-passes** and **four-tube-passes** heat exchanger. Figure III.5.

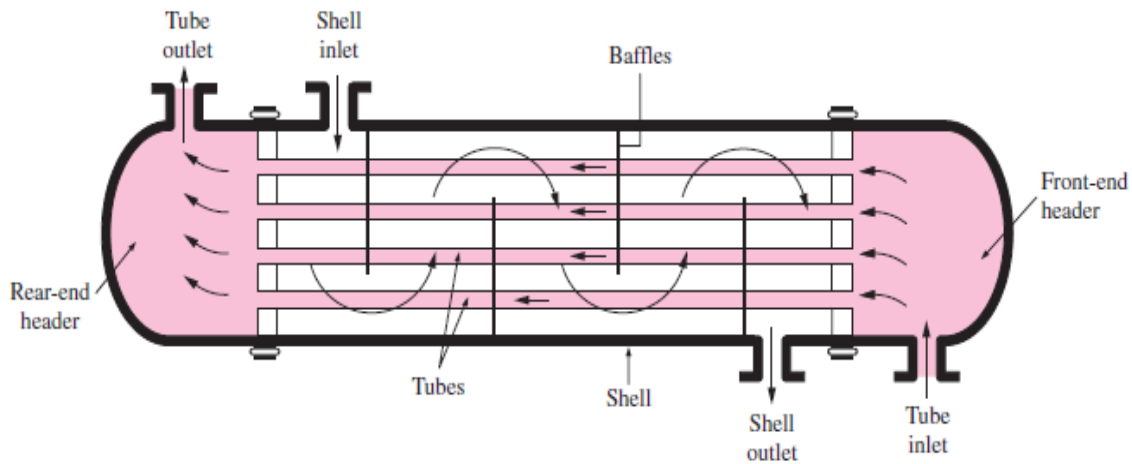


Figure III.4 The schematic of a shell-and-tube heat exchanger (one-shell pass and one-tube pass).

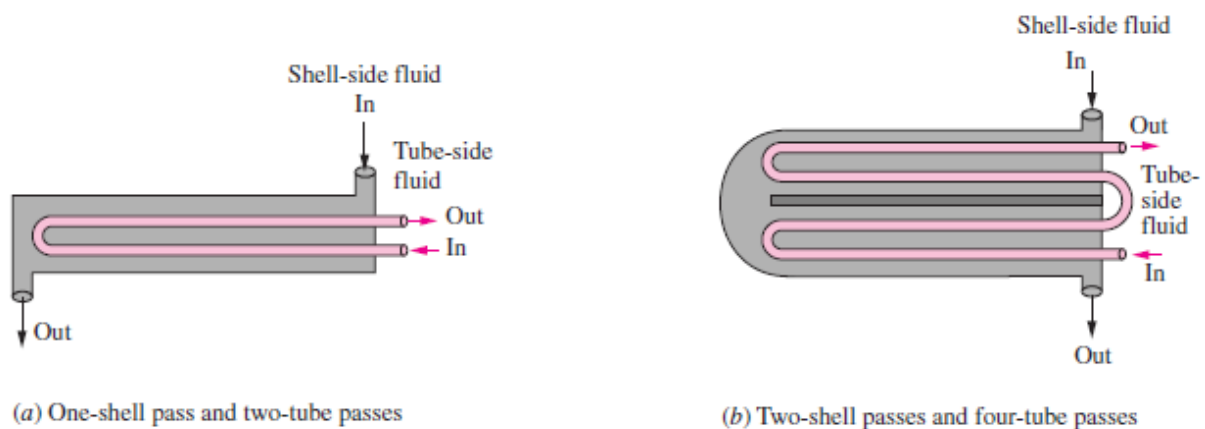


Figure III.5 Multi pass flow arrangements in shell and- tube heat exchangers

III.2.4 Plate and Frame Heat Exchanger

The plate and frame (plate) heat exchanger consists of a series of corrugated plates forming alternating flow passages for hot and cold fluids. Each cold stream is surrounded by two hot streams, enabling very effective heat transfer. The capacity can be easily increased by adding more plates. It is ideal for liquid-to-liquid applications where both fluids are at similar pressures. Figure III.6.

Heat exchangers are often given specific names to reflect the specific application for which they are used. For example, a condenser is a heat exchanger in which one of the fluids is cooled and condenses as it flows through the heat exchanger. A boiler is another heat exchanger in which one of the fluids absorbs heat and vaporizes. A space radiator is a heat exchanger that transfers heat from the hot fluid to the surrounding space by radiation.

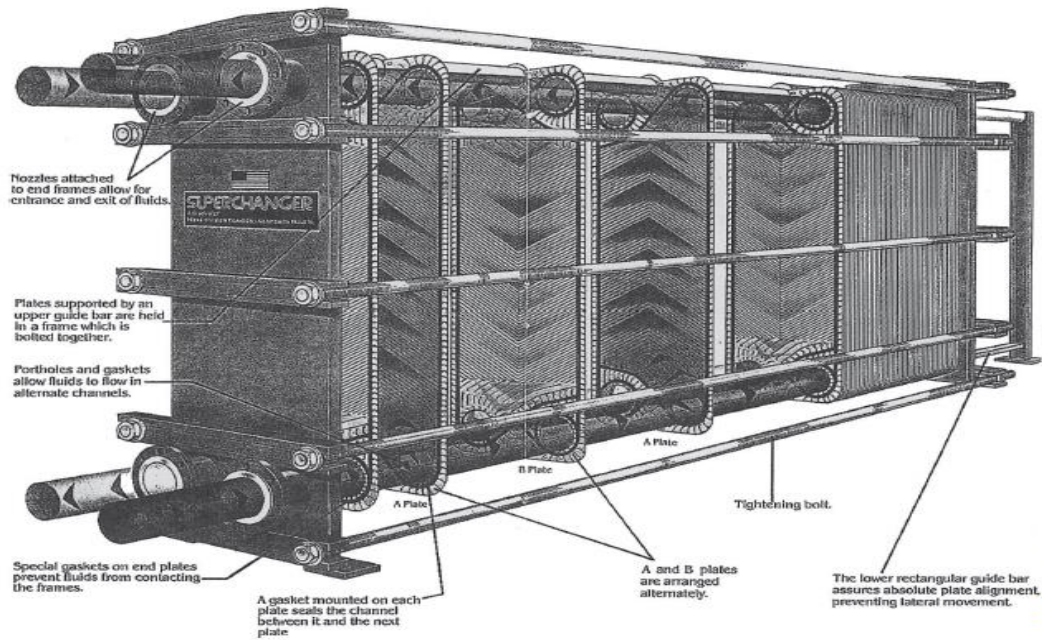


Figure III.6 A plate-and-frame liquid-to-liquid heat exchanger

III.3 The overall heat-transfer Coefficient

A heat exchanger typically involves two flowing fluids separated by a solid wall. Heat is first transferred from the hot fluid to the wall by convection, through the wall by conduction, and from the wall to the cold fluid again by convection. Any radiation effects are usually included in the convection heat transfer coefficients. The thermal resistance network associated with this heat transfer process involves two convection and one conduction resistances, as shown in Figure III.7. Here the subscripts i and o represent the inner and outer surfaces of the inner tube. For a double-pipe heat exchanger, we have $A_i = \pi D_i L$ and $A_o = \pi D_o L$, and the thermal resistance of the tube wall in this case is:

$$R_{\text{wall}} = \frac{\ln\left(\frac{D_o}{D_i}\right)}{2\pi kL} \quad (\text{III.1})$$

where k is the thermal conductivity of the wall material and L is the length of the tube. Then the total thermal resistance becomes

$$R = R_{\text{total}} = R_i + R_{\text{wall}} + R_o = \frac{1}{h_i A_i} + \frac{\ln\left(\frac{D_o}{D_i}\right)}{2\pi kL} + \frac{1}{h_o A_o} \quad (\text{III.2})$$

The A_i is the area of the inner surface of the wall that separates the two fluids, and A_o is the area of the outer surface of the wall. In other words, A_i and A_o are surface areas of the separating

wall wetted by the inner and the outer fluids, respectively. When one fluid flows inside a circular tube and the other outside of it, we have $A_i = \pi D_i L$ and $A_o = \pi D_o L$.

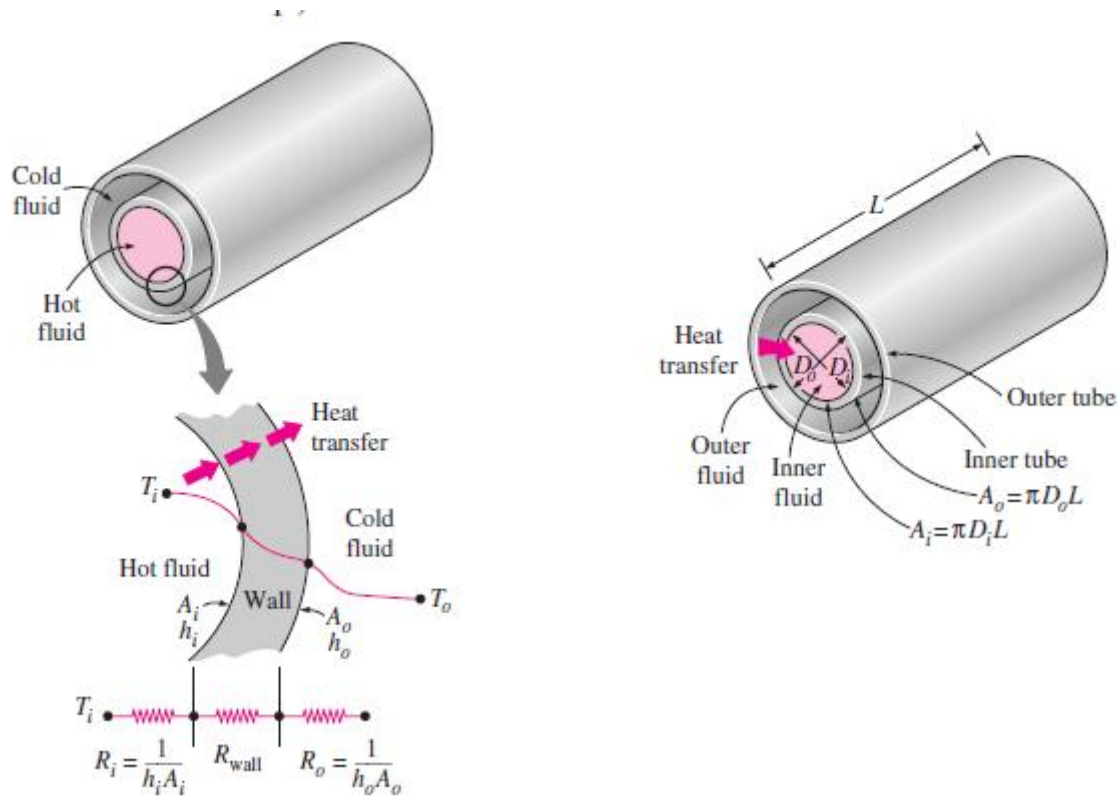


Figure III.7 Thermal resistance network associated with heat transfer in a double-pipe heat exchanger.

In the analysis of heat exchangers, it is convenient to combine all the thermal resistances in the path of heat flow from the hot fluid to the cold one into a single resistance R , and to express the rate of heat transfer between the two fluids as:

$$q = \frac{\Delta T}{R} = UA\Delta T = U_i A_i \Delta T = U_o A_o \Delta T \quad (\text{III.3})$$

where U is the overall heat transfer coefficient, whose unit is $\text{W/m}^2 \cdot ^\circ\text{C}$, which is identical to the unit of the ordinary convection coefficient h . Canceling ΔT Eq. III.3 reduces to:

$$\frac{1}{UA_s} = \frac{1}{U_i A_i} = \frac{1}{U_o A_o} = R = \frac{1}{h_i A_i} + R_{\text{wall}} + \frac{1}{h_o A_o} \quad (\text{III.4})$$

Every heat exchanger has two heat transfer surface areas A_i and A_o , which, in general, are not equal to each other and we have two overall heat transfer coefficients U_i and U_o for a heat exchanger.

The overall heat-transfer coefficient of heat transfer through the plane wall of Figure III.8 expressed as:

$$q = \frac{T_A - T_B}{\frac{1}{h_1 A} + \frac{\Delta x}{kA} + \frac{1}{h_2 A}} \quad (\text{III.5})$$

Where T_A and T_B are the fluid temperatures on each side of the wall. From Eq III.3 and III.5. The overall heat-transfer coefficient U is defined by the relation:

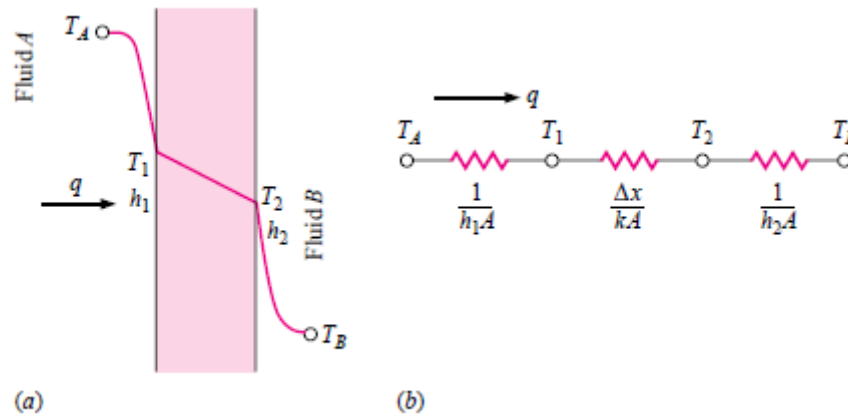


Figure III.8 Overall heat transfer through a plane wall.

From the standpoint of heat-exchanger design, the plane wall is of infrequent application; a more important case for consideration would be that of a double-pipe heat exchanger, as shown in Figure III.9. In this application one fluid flows on the inside of the smaller tube while the other fluid flows in the annular space between the two tubes. The convection coefficients are calculated by the methods described in chapter I, and the overall heat transfer is obtained from the thermal network of Figure III.9 (b) as:

$$q = \frac{T_A - T_B}{\frac{1}{h_i A_i} + \frac{\ln\left(\frac{r_o}{r_i}\right)}{2\pi k L} + \frac{1}{h_o A_o}} \quad (\text{III.6})$$

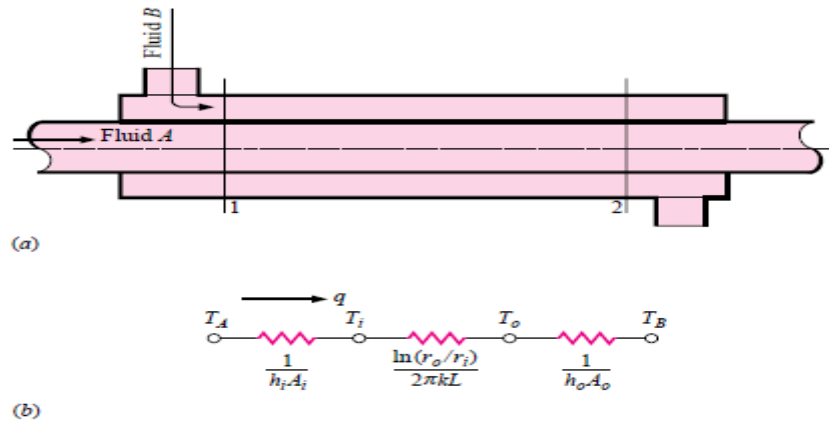


Figure III.9 Double-pipe heat exchange: (a) schematic; (b) thermal-resistance network for overall heat transfer

The overall heat-transfer coefficient may be based on either the inside or outside area of the tube at the discretion of the designer. Accordingly,

$$U_i = \frac{1}{\frac{1}{h_i} + \frac{A_i \ln\left(\frac{r_o}{r_i}\right)}{2\pi kL} + \frac{1}{h_o} \frac{A_i}{A_o}} \quad (\text{III.7})$$

$$U_o = \frac{1}{\frac{A_o}{A_i} \frac{1}{h_i} + \frac{A_o \ln\left(\frac{r_o}{r_i}\right)}{2\pi kL} + \frac{1}{h_o}} \quad (\text{III.8})$$

Although final heat-exchanger designs will be made on the basis of careful calculations of U , it is helpful to have a tabulation of values of the overall heat-transfer coefficient for various situations that may be encountered in practice. An abbreviated list of values of U is given in Table III.1. We should remark that the value of U is governed in many cases by only one of the convection heat-transfer coefficients. In most practical problems the conduction resistance is small compared with the convection resistances.

Physical situation	U
	$\text{W/m}^2 \cdot ^\circ\text{C}$
Brick exterior wall, plaster interior, uninsulated	2.55
Frame exterior wall, plaster interior: uninsulated	1.42
with rock-wool insulation	0.4
Plate-glass window	6.2
Double plate-glass window	2.3
Steam condenser	1100–5600
Feedwater heater	1100–8500
Freon-12 condenser with water coolant	280–850
Water-to-water heat exchanger	850–1700
Finned-tube heat exchanger, water in tubes, air across tubes	25–55
Water-to-oil heat exchanger	110–350
Steam to light fuel oil	170–340
Steam to heavy fuel oil	56–170
Steam to kerosene or gasoline	280–1140
Finned-tube heat exchanger, steam in tubes, air over tubes	28–280
Ammonia condenser, water in tubes	850–1400
Alcohol condenser, water in tubes	255–680
Gas-to-gas heat exchanger	10–40

Table III.1 Approximate values of overall heat-transfer coefficients

III.4 The Log Mean Temperature Difference

Heat exchangers are commonly used in practice, and an engineer often finds himself or herself in a position to select a heat exchanger that will achieve a specified temperature change in a fluid stream of known mass flow rate, or to predict the outlet temperatures of the hot and cold fluid streams in a specified heat exchanger.

The method used in the analysis of heat exchangers is the log mean temperature difference (or LMTD) method.

Consider the double-pipe heat exchanger shown in Figure III.9(a). The fluids may flow in either parallel flow or counterflow, and the temperature profiles for these two cases are indicated in Figure (III.10). We propose to calculate the heat transfer in this double-pipe arrangement

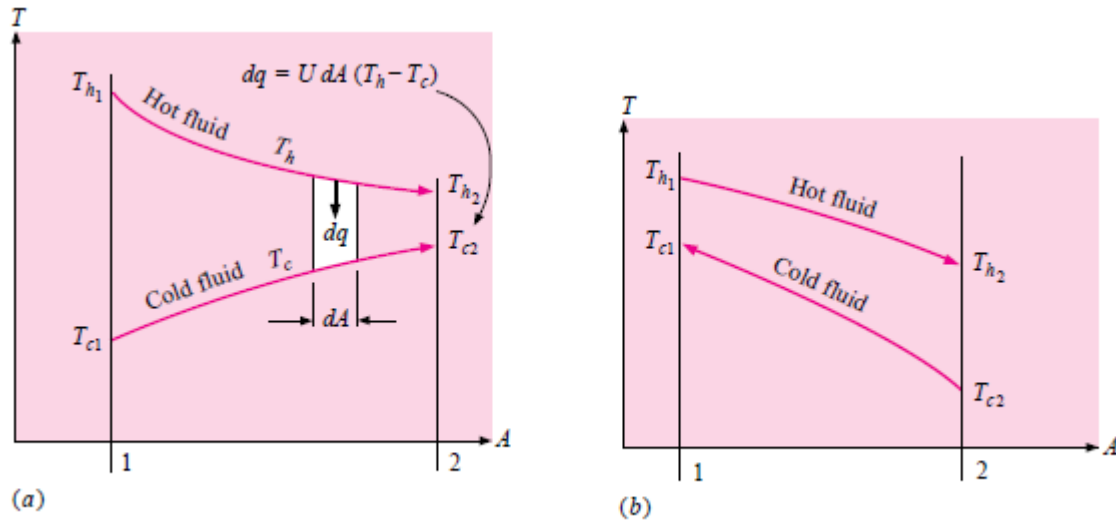


Figure III.10 Temperature profiles for (a) parallel flow and (b) counterflow in double-pipe heat exchanger.

From Eq III.3, we have $q = UA\Delta T_m$

where

U = overall heat-transfer coefficient

A = surface area for heat transfer consistent with definition of U

ΔT_m = suitable mean temperature difference across heat exchanger

Figure III.10, shows that the temperature difference between the hot and cold fluids varies between inlet and outlet. the heat transferred through an element of area dA may be written:

$$dq = -\dot{m}_h c_h dT_h = -\dot{m}_c c_c dT_c \quad (\text{III.9})$$

Where the subscripts h and c designate the hot and cold fluids, respectively. The heat transfer could also be expressed

$$dq = U(T_h - T_c)dA \quad (\text{III.10})$$

From Equation (III.9)

$$dT_h = \frac{-dq}{\dot{m}_h c_h}$$

$$dT_c = \frac{dq}{\dot{m}_c c_c}$$

where $\dot{m}$ represents the mass-flow rate and c is the specific heat of the fluid. Thus

$$dT_h - dT_c = d(T_h - T_c) = -dq \left(\frac{1}{\dot{m}_h c_h} + \frac{1}{\dot{m}_c c_c} \right) \quad (\text{III.11})$$

Solving for dq from Equation (III.10) and substituting into Equation (III.11) gives

$$\frac{d(T_h - T_c)}{T_h - T_c} = -U \left(\frac{1}{\dot{m}_h c_h} + \frac{1}{\dot{m}_c c_c} \right) dA \quad (\text{III.12})$$

This differential equation may now be integrated between conditions 1 and 2 as indicated in Figure III.10. The result is

$$\ln \frac{(T_{h2} - T_{c2})}{(T_{h1} - T_{c1})} = -UA \left(\frac{1}{\dot{m}_h c_h} + \frac{1}{\dot{m}_c c_c} \right) \quad (\text{III.13})$$

Returning to Equation (III.9), the products $\dot{m}_c c_c$ and $\dot{m}_h c_h$ may be expressed in terms of the total heat transfer q and the overall temperature differences of the hot and cold fluids. Thus

$$\dot{m}_h c_h = \frac{q}{T_{h1} - T_{h2}}$$

$$\dot{m}_c c_c = \frac{q}{T_{c2} - T_{c1}}$$

Substituting these relations into Equation (III.13) gives

$$q = UA \frac{(T_{h2} - T_{c2}) - (T_{h1} - T_{c1})}{\ln \left[(T_{h2} - T_{c2}) / (T_{h1} - T_{c1}) \right]} \quad (\text{III.14})$$

Comparing Equation (III.14) with Equation (III.3), we find that the mean temperature difference is the grouping of terms in the brackets. Thus

$$\Delta T_m = \frac{(T_{h2} - T_{c2}) - (T_{h1} - T_{c1})}{\ln \left[(T_{h2} - T_{c2}) / (T_{h1} - T_{c1}) \right]} \quad (\text{III.15})$$

This temperature difference is called the log mean temperature difference (LMTD). Stated verbally, it is the temperature difference at one end of the heat exchanger less the temperature difference at the other end of the exchanger divided by the natural logarithm of the ratio of these two temperature differences. It is left as an exercise for the reader to show that this relation may also be used to calculate the LMTDs for counterflow conditions. The above derivation for LMTD involves two important assumptions: (1) the fluid specific heats do not vary with temperature, and (2) the convection heat-transfer coefficients are constant throughout the heat exchanger. The second assumption is usually the more serious one because of entrance effects, fluid viscosity, and thermal-conductivity changes, etc. If a heat exchanger other than the double-pipe type is used, the heat transfer is calculated by using a correction factor applied to the LMTD for a counterflow double-pipe arrangement with the same hot and cold fluid temperatures. The heat-transfer equation then takes the form:

$$q = UAF\Delta T_m \quad (\text{III.16})$$

In condensation or boiling (evaporation), the fluid normally remains at essentially constant temperature and the relations are simplified. For this condition, P or R becomes zero and we obtain $F = 1.0$ for boiling or condensation

Values of the correction factor F are plotted in Figures III.11 to III.14 for several different types of heat exchangers.

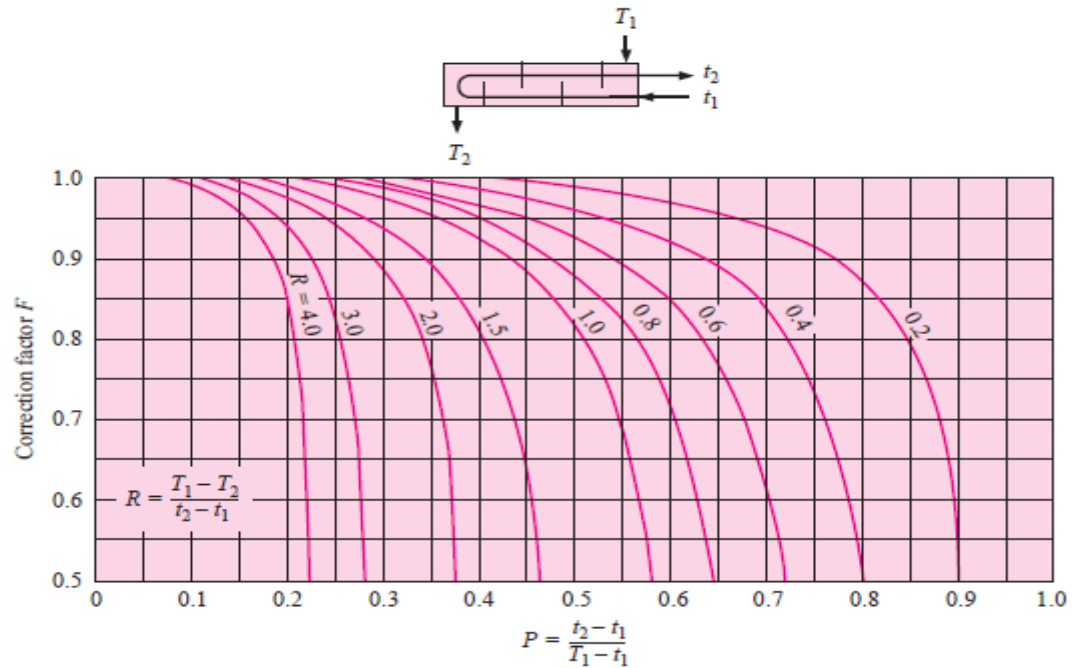


Figure III.11 Correction-factor plot for exchanger with one shell pass and two, four, or any multiple of tube passes.

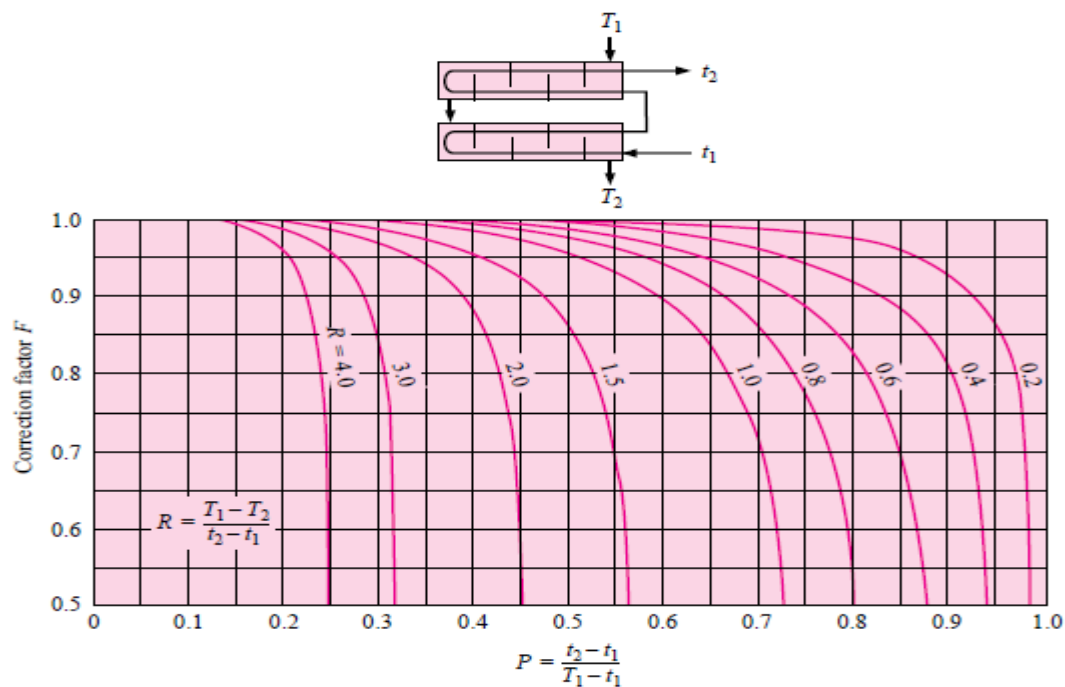


Figure III.12 Correction-factor plot for exchanger with two shell passes and four, eight, or any multiple of tube passes.

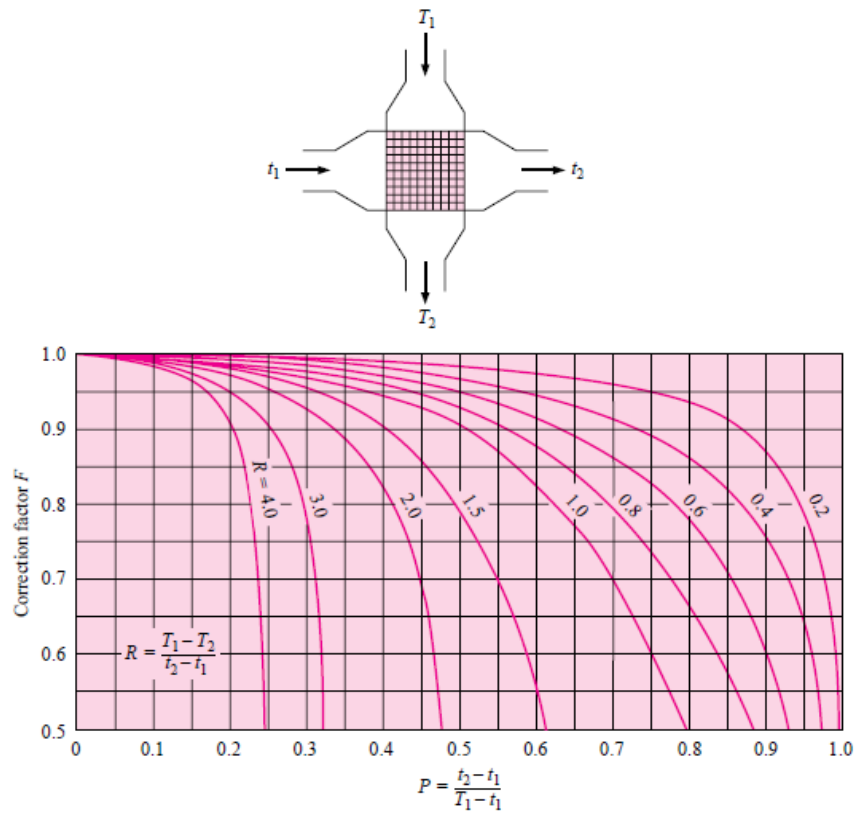


Figure III.13 Correction-factor plot for single-pass cross-flow exchanger, both fluids unmixed.

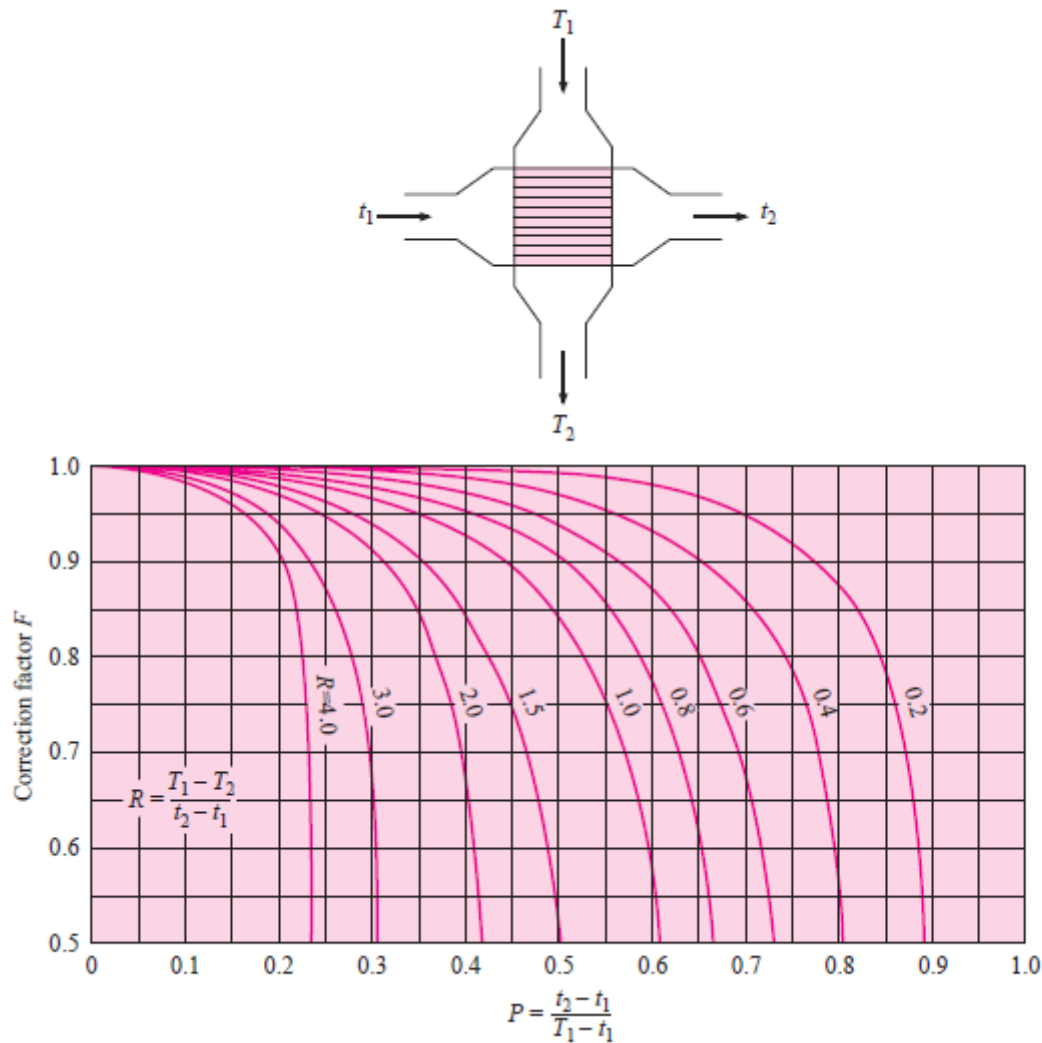


Figure III.14 Correction-factor plot for single-pass cross-flow exchanger, one fluid mixed, the other unmixed.

The LMTD method is very suitable for determining the size of a heat exchanger to realize prescribed outlet temperatures when the mass flow rates and the inlet and outlet temperatures of the hot and cold fluids are specified.

With the LMTD method, the task is to select a heat exchanger that will meet the prescribed heat transfer requirements. The procedure to be followed by the selection process is:

1. Select the type of heat exchanger suitable for the application.
2. Determine any unknown inlet or outlet temperature and the heat transfer rate using an energy balance.
3. Calculate the log mean temperature difference ΔT_m and the correction factor F , if necessary.
4. Obtain (select or calculate) the value of the overall heat transfer coefficient U .
5. Calculate the heat transfer surface area A_s .

A second kind of problem encountered in heat exchanger analysis is the determination of the heat transfer rate and the outlet temperatures of the hot and cold fluids for prescribed fluid mass flow rates and inlet temperatures when the type and size of the heat exchanger are specified. The heat

transfer surface area A of the heat exchanger in this case is known, but the outlet temperatures are not. Here the task is to determine the heat transfer performance of a specified heat exchanger or to determine if a heat exchanger available in storage will do the job.

III.5 Effectiveness-NTU Method

The effectiveness-NTU method, is a greatly simplified heat exchanger analysis. This method is based on a dimensionless parameter called the heat transfer effectiveness ε , defined as:

$$\varepsilon = \frac{\dot{Q}}{\dot{Q}_{\max}} = \frac{\text{actual heat transfer rate}}{\text{maximum possible heat transfer rate}} \quad (\text{III.17})$$

The actual heat transfer rate in a heat exchanger can be determined from an energy balance on the hot or cold fluids and can be expressed as:

$$\dot{Q} = C_c (T_{c,\text{out}} - T_{c,\text{in}}) = C_h (T_{h,\text{out}} - T_{h,\text{in}}) \quad (\text{III.18})$$

where $C_c = \dot{m}_c C_{p_c}$ and $C_h = \dot{m}_h C_{p_h}$ are the heat capacity rates of the cold and the hot fluids, respectively. To determine the maximum possible heat transfer rate in a heat exchanger, we first recognize that the maximum temperature difference in a heat exchanger is the difference between the inlet temperatures of the hot and cold fluids. That is, To determine the maximum possible heat transfer rate in a heat exchanger, we first recognize that the maximum temperature difference in a heat exchanger is the difference between the inlet temperatures of the hot and cold fluids. That is,

$$\Delta T_{\max} = T_{h,\text{in}} - T_{c,\text{in}} \quad (\text{III.19})$$

The heat transfer in a heat exchanger will reach its maximum value when (1) the cold fluid is heated to the inlet temperature of the hot fluid or (2) the hot fluid is cooled to the inlet temperature of the cold fluid. These two limiting conditions will not be reached simultaneously unless the heat rates of the hot and cold fluids are identical (i.e., $C_c = C_h$). When $C_c \neq C_h$, which is usually the case, the fluid with the smaller heat capacity rate will experience a larger temperature change, and thus it will be the first to experience the maximum temperature, at which point the heat transfer will come to a halt. Therefore, the maximum possible heat transfer rate in a heat exchanger is

$$\dot{Q} = C_{\min} (T_{h,\text{in}} - T_{c,\text{in}}) \quad (\text{III.20})$$

where $C_{\min}$ is the smaller of $C_h = \dot{m}_h C_{p_h}$ and $C_c = \dot{m}_c C_{p_c}$

Then, once the effectiveness of the heat exchanger is known, the actual heat transfer rate $\dot{Q}$ can be determined from

$$\dot{Q} = \varepsilon \dot{Q}_{\max} = \varepsilon C_{\min} (T_{h,\text{in}} - T_{c,\text{in}}) \quad (\text{III.21})$$

Therefore, the effectiveness of a heat exchanger enables us to determine the heat transfer rate without knowing the outlet temperatures of the fluids. The effectiveness of a heat exchanger depends on the geometry of the heat exchanger as well as the flow arrangement. Therefore, different types of heat exchangers have different effectiveness relations. Below we illustrate the development of the effectiveness ε relation for the double-pipe parallel-flow heat exchanger.

Equation III.13 developed for a parallel-flow heat exchanger can be rearranged as:

$$\ln \frac{(T_{h,out} - T_{c,out})}{T_{h,in} - T_{c,in}} = -\frac{UA_s}{C_c} \left(1 + \frac{C_c}{C_h}\right) \quad (\text{III.22})$$

Also, solving Eq. III.18 for $T_{h,out}$ gives

$$T_{h,out} = T_{h,in} - \frac{C_c}{C_h} (T_{c,out} - T_{c,in}) \quad (\text{III.23})$$

Substituting this relation into Eq. III.22 after adding and subtracting $T_{c,in}$ gives

$$\ln \frac{T_{h,in} - T_{c,in} + T_{c,in} - T_{c,out} - \frac{C_c}{C_h} (T_{c,out} - T_{c,in})}{T_{h,in} - T_{c,in}} = -\frac{UA_s}{C_c} \left(1 + \frac{C_c}{C_h}\right)$$

Which simplifies to

$$\ln \left[1 - \left(1 + \frac{C_c}{C_h}\right) \frac{T_{c,out} - T_{c,in}}{T_{h,in} - T_{c,in}} \right] = -\frac{UA_s}{C_c} \left(1 + \frac{C_c}{C_h}\right) \quad (\text{III.24})$$

We now manipulate the definition of effectiveness to obtain

$$\varepsilon = \frac{\dot{Q}}{Q_{\max}} = \frac{C_c (T_{c,out} - T_{c,in})}{C_{\min} (T_{h,in} - T_{c,in})} \rightarrow \frac{T_{c,out} - T_{c,in}}{T_{h,in} - T_{c,in}} = \varepsilon \frac{C_{\min}}{C_c} \quad (\text{III.25})$$

Substituting this result into Eq. III.24 and solving for ε gives the following relation for the effectiveness of a parallel-flow heat exchanger:

$$\varepsilon_{\text{parallelflow}} = \frac{1 - \exp \left[-\frac{UA_s}{C_c} \left(1 + \frac{C_c}{C_h}\right) \right]}{\left(1 + \frac{C_c}{C_h}\right) \frac{C_{\min}}{C_c}} \quad (\text{III.26})$$

Taking either C_c or C_h to be $C_{\min}$ (both approaches give the same result), the relation above can be expressed more conveniently as

$$\varepsilon_{\text{parallelflow}} = \frac{1 - \exp \left[-\frac{UA_s}{C_{\min}} \left(1 + \frac{C_{\min}}{C_{\max}} \right) \right]}{\left(1 + \frac{C_{\min}}{C_{\max}} \right)} \quad (\text{III.27})$$

Again $C_{\min}$ is the smaller heat capacity ratio and $C_{\max}$ is the larger one, and it makes no difference whether $C_{\min}$ belongs to the hot or cold fluid.

Effectiveness relations of the heat exchangers typically involve the dimensionless group $UA_s/C_{\min}$. This quantity is called **the number of transfer units NTU** and is expressed as

$$\text{NTU} = \frac{UA_s}{C_{\min}} = \frac{UA_s}{(\dot{m}C_p)_{\min}} \quad (\text{III.29})$$

For specified values of U and $C_{\min}$, the value of NTU is a measure of the heat transfer surface area A_s . Thus, the larger the NTU, the larger the heat exchanger.

In heat exchanger analysis, it is also convenient to define another dimensionless quantity called the capacity ratio c as

$$c = \frac{C_{\min}}{C_{\max}} \quad (\text{III.30})$$

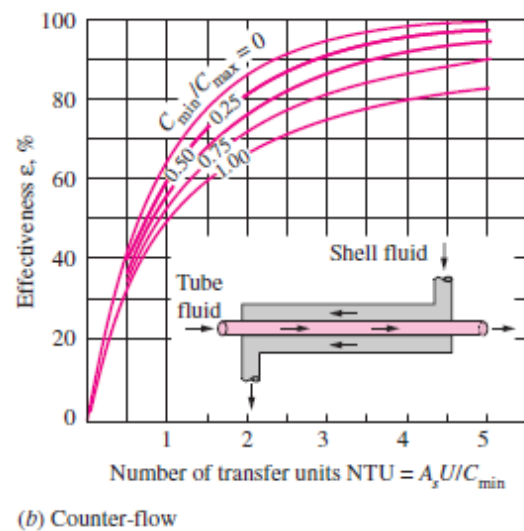
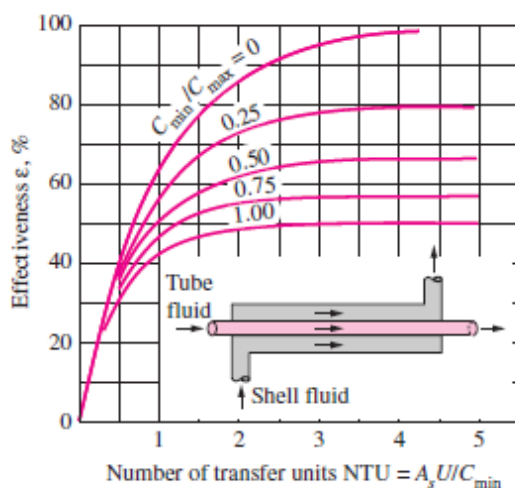
It can be shown that the effectiveness of a heat exchanger is a function of the number of transfer units NTU and the capacity ratio c . That is,

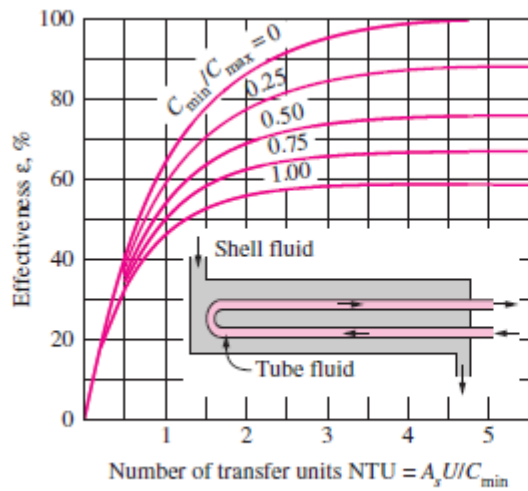
$$\varepsilon = \text{function} (UA_s / C_{\min}, C_{\min} / C_{\max}) = \text{function} (\text{NTU}, c)$$

Effectiveness relations have been developed for a large number of heat exchangers, and the results are given in Table III.2. The effectivenesses of some common types of heat exchangers are also plotted in Figure III.15.

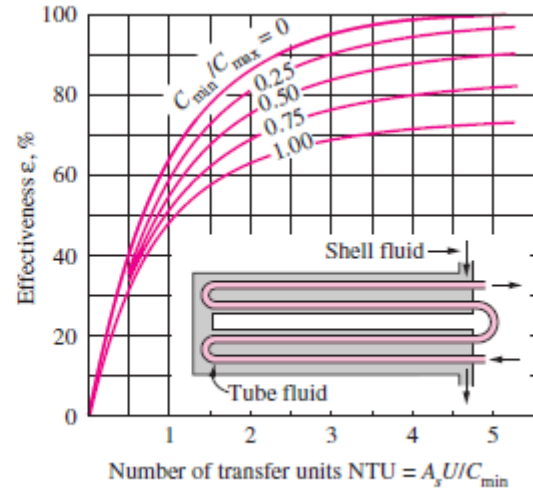
Heat exchanger type	Effectiveness relation
1 <i>Double pipe:</i>	
Parallel-flow	$\varepsilon = \frac{1 - \exp[-NTU(1 + c)]}{1 + c}$
Counter-flow	$\varepsilon = \frac{1 - \exp[-NTU(1 - c)]}{1 - c \exp[-NTU(1 - c)]}$
2 <i>Shell and tube:</i>	
One-shell pass	
2, 4, . . . tube passes	$\varepsilon = 2 \left\{ 1 + c + \sqrt{1 + c^2} \frac{1 + \exp[-NTU\sqrt{1 + c^2}]}{1 - \exp[-NTU\sqrt{1 + c^2}]} \right\}^{-1}$
3 <i>Cross-flow (single-pass)</i>	
Both fluids unmixed	$\varepsilon = 1 - \exp \left\{ \frac{NTU^{0.22}}{c} [\exp(-c NTU^{0.78}) - 1] \right\}$
$C_{\max}$ mixed, $C_{\min}$ unmixed	$\varepsilon = \frac{1}{c} (1 - \exp[1 - c[1 - \exp(-NTU)]])$
$C_{\min}$ mixed, $C_{\max}$ unmixed	$\varepsilon = 1 - \exp \left\{ -\frac{1}{c} [1 - \exp(-c NTU)] \right\}$
4 <i>All heat exchangers with $c = 0$</i>	$\varepsilon = 1 - \exp(-NTU)$

Table III.2 Effectiveness relations for heat exchangers: $NTU = UA_s / C_{\min}$ and $c = C_{\min} / C_{\max}$ [20]

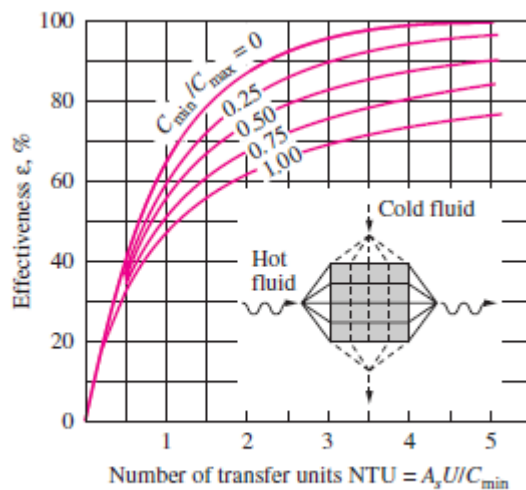




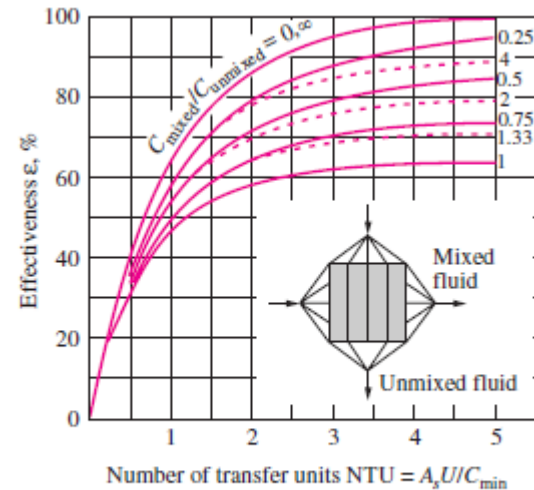
(c) One-shell pass and 2, 4, 6, ... tube passes



(d) Two-shell passes and 4, 8, 12, ... tube passes



(e) Cross-flow with both fluids unmixed



(f) Cross-flow with one fluid mixed and the other unmixed

Figure III.15 Effectiveness for heat exchangers [20]

III.6 Boilers

III.6.1 Introduction

A boiler is a closed pressure vessel in which water is heated to produce hot water or steam by transferring heat released from fuel combustion.

Boilers are widely used in:

- Power plants
- Industrial processes
- Heating systems
- Refrigeration and air conditioning plants

Main functions:

- Generate steam or hot water
- Transfer thermal energy efficiently
- Operate safely under pressure

III.6.2 Classification of Boilers

III.6.2.1 According to Flow Arrangement

A. Fire-Tube Boilers

In a fire-tube boiler, hot combustion gases pass through tubes surrounded by water. Boilers are engineered to produce steam by heating water with a burner, leveraging water's remarkable heat transfer properties for numerous applications, including space heating, process hot water, and electricity generation. In a traditional fire tube boiler, the tubes are submerged in a water-filled tank. A furnace heats the tube walls, which in turn heat the water, converting it into hot water and eventually steam. Despite water's excellent heat transfer capabilities, conventional boilers consume a significant amount of energy due to the lengthy process of heating the tubes through water's heat transfer.

Conventional boilers for industrial use require substantial space, often necessitating a dedicated boiler room due to their size and maintenance requirements. This spatial requirement restricts their application primarily to large industrial, manufacturing, and institutional facilities, and makes conventional boilers less appealing for modern manufacturing processes that demand compact, modular, and energy-efficient heating solutions.

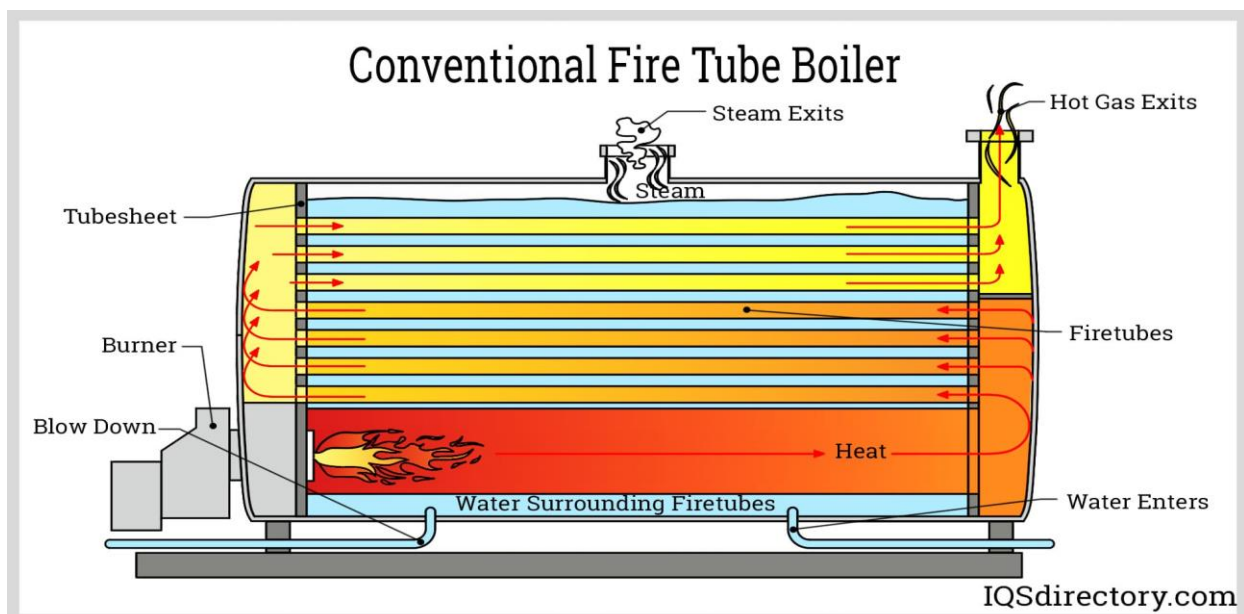


Figure III.16 Conventional Fire Tube boiler

Principle

Heat transfer occurs:

- Convection from gases to tube walls
- Conduction through tube walls
- Convection from wall to water

Characteristics

- Low to medium pressure
- Small steam capacity
- Simple construction

Advantages

- Low cost
- Easy maintenance
- Simple operation

Disadvantages

- Lower efficiency
- Limited pressure capability
- Slow steam generation

Applications

- Small industries
- Heating plants
- Locomotives

Examples:

- Cochran boiler
- Lancashire boiler
- Scotch marine boiler

B. Water-Tube Boilers

A water tube boiler is designed with a series of tubes that carry water, which is heated by hot gases originating from a furnace or combustion chamber located below these tubes. When fuel is burned in the furnace, it produces gases that transfer heat to the metal tubes, subsequently warming the water inside. This heated water ascends to a steam drum, where saturated steam is released. The fuels used in a water tube boiler include natural gas, propane, coal, diesel fuel, or heating oil, which are combusted in the combustion chamber or furnace to generate hot gases, heating both the tubes and the water within them. Traditional boilers operated in the opposite manner, heating tubes immersed in water via a furnace utilizing coal or wood. As water temperature increased, it resulted

in a time-intensive heating process. This inefficiency led to the innovation of the water tube boiler design, where water flows through the tubes for more effective heating.

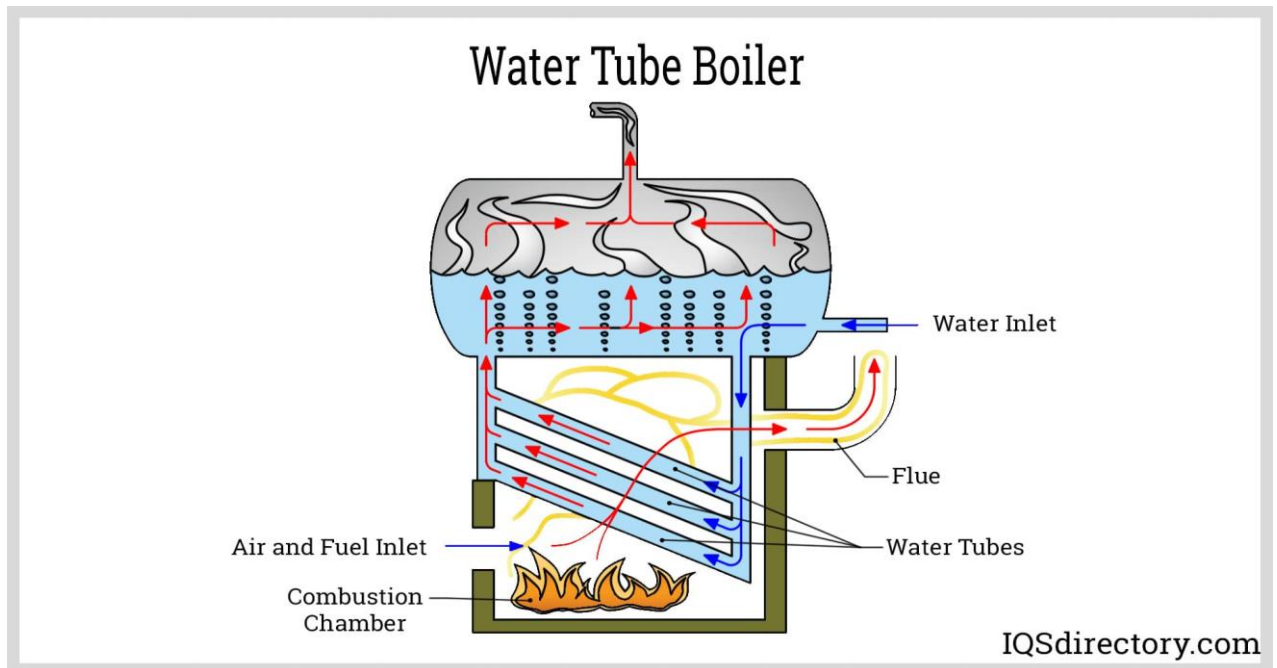


Figure III.17 Water Tube boiler

Features

- High pressure operation
- High evaporation rate
- Better heat transfer

Examples:

- Babcock and Wilcox boiler
- Benson boiler
- LaMont boiler

Advantages

- High efficiency
- Rapid steam production
- Suitable for power stations

Disadvantages

- Expensive
- Complex construction

III.6.3 Classification According to Pressure**Low-pressure boilers**

Up to 15 bar

Medium-pressure boilers

15–80 bar

High-pressure boilers

Above 80 bar

III.6.4 Classification According to Fuel**Solid fuel boilers**

Coal

Biomass

Wood

Liquid fuel boilers

Fuel oil

Diesel

Gas-fired boilers

Natural gas

LPG

Electric boilers**III.6.5 Main Boiler Components****III.6.5.1 Essential Parts**

- Furnace
- Burner
- Combustion chamber
- Water drum
- Steam drum
- Tubes

Economizer

Recovers waste heat from flue gases:

$$Q = \dot{m}C_p(T_{\text{out}} - T_{\text{in}})$$

Improves efficiency.

Superheater

Raises steam temperature beyond saturation.

Air Preheater

Preheats combustion air using flue gas heat.

III.6.6 Boiler Heat Losses

Total energy balance:

$$Q_{\text{fuel}} = Q_{\text{useful}} + Q_{\text{losses}}$$

III.6.6.1 Flue Gas Losses

Largest boiler loss.

$$Q_{\text{fgl}} = \dot{m}_g C_p (T_g - T_a)$$

Where:

- m_g : mass of flue gases
- C_p : specific heat
- T_g : flue gas temperature
- T_a : ambient temperature

Typically

5-20%

III.6.6.2 Incomplete Combustion Losses

Due to:

- Carbon monoxide
- Unburned carbon
- Soot

Chemical energy remains unused.

III.6.6.3 Radiation and Convection Losses

Heat lost through external surfaces:

$$Q = hs(T_s - T_{\infty})$$

Usually:

1–5 %

III.6.6.4 Blowdown Losses

To remove dissolved solids.

Loss:

$$Q_{\text{bd}} = \dot{m}_{\text{bd}}(h_{\text{bd}} - h_{\text{fw}})$$

III.6.6.5 Moisture Losses

Energy consumed in evaporating:

- Moisture in fuel
- Water from hydrogen combustion

III.6.7 Boiler Efficiency

III.6.7.1 Direct Method

$$\text{Boiler efficiency: } \eta_b = \frac{\text{Useful heat output}}{\text{Fuel heat input}} \times 100$$

$$\text{For steam boiler: } \eta_b = \frac{\dot{m}_s (h_s - h_w)}{\dot{m}_f \text{CV}} \times 100$$

$\dot{m}_s$: steam flow rate

$\dot{m}_f$: Fuel flow rate

CV : calorific value

III.6.7.2 Indirect Method (Heat Loss Method)

$$\eta_b = 100 - \sum \text{Losses}$$

$$\eta_b = 100 - (L_1 + L_2 + L_3 \dots)$$

Where:

- L_1 : flue losses
- L_2 : moistures losses
- L_3 : radiation losses

III.7 Exercises

Exercise1

Hot oil is to be cooled in a double-tube counter-flow heat exchanger. The copper inner tubes have a diameter of 2 cm and negligible thickness. The inner diameter of the outer tube (the shell) is 3 cm. Water flows through the tube at a rate of 0.5 kg/s, and the oil through the shell at a rate of 0.8 kg/s. Taking the average temperatures of the water and the oil to be 45°C and 80°C, respectively, determine the overall heat transfer coefficient of this heat exchanger.

The properties of water at 45°C are $\rho = 990 \text{ kg/m}^3$ $\text{Pr} = 3.91$, $k = 0.637 \text{ W/m} \cdot ^\circ\text{C}$ $\nu = 0.602 \times 10^{-6} \text{ m}^2/\text{s}$

The properties of oil at 80°C are $\rho = 852 \text{ kg/m}^3$ $\text{Pr} = 490$ $k = 0.138 \text{ W/m} \cdot ^\circ\text{C}$ $\nu = 37.5 \times 10^{-6} \text{ m}^2/\text{s}$

Solution

The overall heat transfer coefficient U can be determined from

$$\frac{1}{U} \approx \frac{1}{h_i} + \frac{1}{h_o}$$

Where h_i and h_o are the convection heat transfer coefficients inside and outside the tube, respectively, which are to be determined using the forced convection relations.

The hydraulic diameter for a circular tube is the diameter of the tube itself, $D_h = D = 0.02$ m. The mean velocity of water in the tube and the Reynolds number are

$$\dot{m} = \rho V A_s \Rightarrow V = \frac{\dot{m}}{\rho \left(\pi \frac{D^2}{4} \right)} = \frac{0.5}{990 \times \left(\frac{\pi \times 0.02^2}{4} \right)} = 1.62 \text{ m/s}$$

$$Re = \frac{\rho V D_h}{\nu} = \frac{990 \times 1.62 \times 0.02}{0.602 \times 10^{-6}} = 53490 \text{ The flow of water is turbulent}$$

The Nusselt number can be determined from

$$Nu = \frac{h D_h}{k} = 0.023 Re^{0.8} \cdot Pr^{0.33} = 0.023 (53490)^{0.8} (3.91)^{0.33} = 240.6$$

$$h = \frac{Nu k}{D_h} = \frac{240.6 \times 0.637}{0.02} = 7663 \text{ W/m}^2\text{°C}$$

Now we repeat the analysis above for oil. The properties of oil

The hydraulic diameter for the annular space is $D_h = D_o - D_i = 0.03 - 0.02 = 0.01$ m

The mean velocity and the Reynolds number in this case are

$$\dot{m} = \rho V A_s \Rightarrow V = \frac{\dot{m}}{\rho \left(\pi \frac{D_o^2 - D_i^2}{4} \right)} = \frac{0.8}{852 \times \left(\frac{\pi \times (0.03^2 - 0.02^2)}{4} \right)} = 2.39 \text{ m/s}$$

$$Re = \frac{\rho V D_h}{\nu} = \frac{852 \times 2.39 \times 0.02}{37.5 \times 10^{-6}} = 637 < 4000 \Rightarrow \text{the flow is laminar}$$

Assuming fully developed flow, the Nusselt number on the tube side of the annular space Nu_i corresponding to $D_i/D_o = 0.02/0.03 = 0.667$ can be determined from Table III.3 by interpolation to be $Nu = 5.45$

D_i/D_o	Nu_i	Nu_o
0.00	—	3.66
0.05	17.46	4.06
0.10	11.56	4.11
0.25	7.37	4.23
0.50	5.74	4.43
1.00	4.86	4.86

Table III.3 Nusselt number for fully developed laminar flow in a circular annulus with one surface insulated and the other isothermal

$$h = \frac{Nu_k}{D_h} = \frac{5.45 \times 0.138}{0.01} = 75.2 \text{ W/m}^2 \cdot ^\circ\text{C}$$

Then the overall heat transfer coefficient for this heat exchanger becomes

$$U = \frac{1}{\frac{1}{h_i} + \frac{1}{h_o}} = \frac{1}{\frac{1}{7663} + \frac{1}{75.2}} = 74.5 \text{ W/m}^2 \cdot ^\circ\text{C}$$

Exercise2:

Steam in the condenser of a power plant is to be condensed at a temperature of 30°C with cooling water from a nearby lake, which enters the tubes of the condenser at 14°C and leaves at 22°C. The surface area of the tubes is 45 m², and the overall heat transfer coefficient is 2100 W/m² · °C.

Determine the mass flow rate of the cooling water needed and the rate of condensation of the steam in the condenser. The heat of vaporization of water at 30°C is $h_{fg} = 2431 \text{ kJ/kg}$ and the specific heat of cold water at the average temperature of 18°C is $C_p = 4184 \text{ J/kg} \cdot ^\circ\text{C}$

Solution

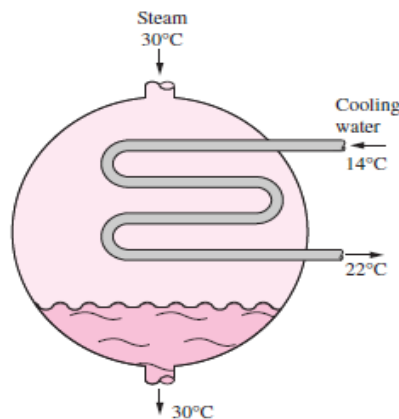


Figure 1 Schematic of the condenser

The condenser can be treated as a counter-flow heat exchanger since the temperature of one of the fluids (the steam) remains constant. The temperature difference between the steam and the cooling water at the two ends of the condenser is

$$\Delta T_1 = T_{h,in} - T_{c,out} = 30 - 22 = 8^\circ\text{C}$$

$$\Delta T_2 = T_{h,out} - T_{c,in} = 30 - 14 = 16^\circ\text{C}$$

The proper average temperature difference between the two fluids is the logarithmic mean temperature difference, which is determined from

$$\Delta T_{lm} = \frac{\Delta T_1 - \Delta T_2}{\ln(\Delta T_1/\Delta T_2)} = \frac{8 - 16}{\ln(8/16)} = 11.5^\circ\text{C}$$

This is a little less than the arithmetic mean temperature difference of $(8 + 16) / 12^\circ\text{C}$. Then the heat transfer rate in the condenser is determined from

$$q = UA\Delta T_m = 2100 \times 45 \times 11.5 = 1.087 \times 10^6 = 1087 \text{ kW}$$

The mass flow rate of the cooling water and the rate of the condensation of the steam are determined from

$$q = \dot{m} C_p (T_{\text{out}} - T_{\text{in}})_{\text{cooling water}} = (\dot{m} h_{\text{fg}})_{\text{steam}}$$

$$\dot{m}_{\text{cooling water}} = \frac{q}{C_p (T_{\text{out}} - T_{\text{in}})} = \frac{1087}{4.184 \times (22 - 14)} = 32.5 \text{ kg/s}$$

$$\text{And } \dot{m}_{\text{steam}} = \frac{q}{h_{\text{fg}}} = \frac{1087}{2431} = 0.45 \text{ kg/s}$$

Therefore, we need to circulate about 72 kg of cooling water for each 1 kg of steam condensing to remove the heat released during the condensation process.

Exercise 3

Water at the rate of 68 kg/min is heated from 35 to 75°C by an oil having a specific heat of 1.9 kJ/kg °C. The fluids are used in a counterflow double-pipe heat exchanger, and the oil enters the exchanger at 110°C and leaves at 75°C. The overall heat-transfer coefficient is 320 W/m² °C.

Calculate the heat-exchanger area.

Solution

The total heat transfer is determined from the energy absorbed by the water:

$$q = \dot{m}_w c_w \Delta T_w = (68/60) \times 4180 \times (75 - 35) = 189.49 \text{ kW}$$

$$\Delta T_m = \frac{(T_{h2} - T_{c2}) - (T_{h1} - T_{c1})}{\ln[(T_{h2} - T_{c2}) / (T_{h1} - T_{c1})]} \Rightarrow \Delta T_m = \frac{(110 - 75) - (75 - 35)}{\ln[(110 - 75) / (75 - 35)]} = 37.44^\circ\text{C}$$

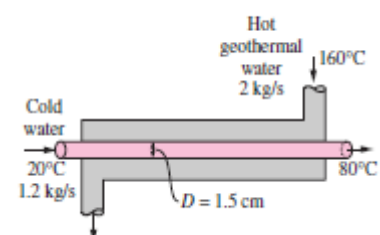
$$\text{Since } q = UA\Delta T_m \Rightarrow A = \frac{q}{U\Delta T_m} = \frac{189.49 \times 1000}{320 \times 37.44} = 18.82 \text{ m}^2$$

Exercise 4

A counter-flow double-pipe heat exchanger is to heat water from 20°C to 80°C at a rate of 1.2 kg/s. The heating is to be accomplished by geothermal water available at 160°C at a mass flow rate of 2 kg/s. The inner tube is thin-walled and has a diameter of 1.5 cm. If the overall heat transfer coefficient of the heat exchanger is 640 W/m² °C, determine the length of the heat exchanger required to achieve the desired heating.

The specific heats of water $C_p = 4.18 \text{ kJ/kg}$

The specific heats of geothermal fluid $C_p = 4.31 \text{ kJ/kg}$



Solution

The rate of heat transfer in the heat exchanger can be determined from:

$$q = \dot{m}C_p(T_{\text{out}} - T_{\text{in}})_{\text{water}} = 1.2 \times 4.18 \times (80 - 20) = 301 \text{ kW} \quad \textbf{Figure 2} \text{ Schematic of heat exchanger}$$

the outlet temperature of the geothermal water is determined to be

$$q = \dot{m}C_p(T_{\text{out}} - T_{\text{in}})_{\text{geothermal}} \Rightarrow T_{\text{out}} = T_{\text{in}} - \frac{q}{\dot{m}C_p} = 160 - \frac{301 \times 1000}{2 \times 4.31 \times 1000} = 125^\circ\text{C}$$

Knowing the inlet and outlet temperatures of both fluids, the logarithmic mean temperature difference for this counter-flow heat exchanger becomes

$$\begin{aligned} \Delta T_1 &= T_{\text{h,in}} - T_{\text{c,out}} = 160 - 80 = 80^\circ\text{C} \\ \Delta T_2 &= T_{\text{h,out}} - T_{\text{c,in}} = 125 - 20 = 105^\circ\text{C} \\ \Delta T_m &= \frac{\Delta T_1 - \Delta T_2}{\ln[\Delta T_1/\Delta T_2]} \Rightarrow \Delta T_m = \frac{80 - 105}{\ln[80/105]} = 92^\circ\text{C} \end{aligned}$$

The surface area of the heat exchanger is determined to be

$$q = UA\Delta T_m \Rightarrow A = \frac{q}{U\Delta T_m} = \frac{301 \times 1000}{6400 \times 92} = 5.11 \text{ m}^2$$

$$\text{we have } A = \pi DL \Rightarrow L = \frac{A}{\pi D} = \frac{5.11}{3.14 \times 1.5 \times 10^{-2}} = 108 \text{ m}$$

Exercise 5

Instead of the double-pipe heat exchanger of Exercise 1, it is desired to use a shell-and-tube exchanger with the water making one shell pass and the oil making two tube passes. Calculate the area required for this exchanger, assuming that the overall heat-transfer coefficient remains at $320 \text{ W/m}^2 \text{ }^\circ\text{C}$

Solution

We determine a correction factor from Figure III.11, to be used with the LMTD calculated on the basis of a counterflow exchanger. The parameters according to this figure.

$$T_1 = 35^\circ\text{C}, T_2 = 75^\circ\text{C}, t_1 = 110^\circ\text{C}, t_2 = 75^\circ\text{C}$$

$$P = \frac{t_2 - t_1}{T_1 - T_2} = \frac{75 - 110}{35 - 110} = 0.467$$

$$R = \frac{T_1 - T_2}{t_2 - t_1} = \frac{35 - 75}{75 - 110} = 1.143$$

So the correction factor is $F = 0.81$

$$\text{The heat transfer is } q = UAF\Delta T_m \Rightarrow A = \frac{q}{UF\Delta T_m} = \frac{1.895 \times 10^5}{320 \times 0.81 \times 37.44} = 19.53 \text{ m}^2$$

Exercise 6

A 2-shell passes and 4-tube passes heat exchanger is used to heat glycerin from 20°C to 50°C by hot water, which enters the thin-walled 2cm-diameter tubes at 80°C and leaves at 40°C (Fig. 2). The total length of the tubes in the heat exchanger is 60 m. The convection heat transfer coefficient is 25 W/m² °C on the glycerin (shell) side and 160 W/m² °C on the water (tube) side. Determine the rate of heat transfer in the heat exchanger (a) before any fouling occurs and (b) after fouling with a fouling factor of 0.0006 m² °C/ W occurs on the outer surfaces of the tubes.

Solution

The heat transfer surface area is

$$\text{we have } A = \pi DL = 3.14 \times 0.02 \times 60 = 3.77 \text{ m}^2$$

The rate of heat transfer in this heat exchanger can be determined from:

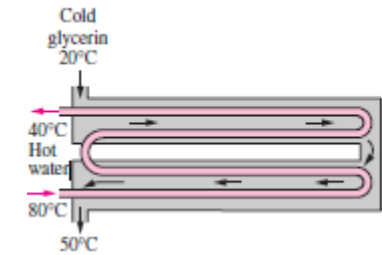


Figure 3. Schematic of heat exchanger

$$q = UAF\Delta T_m$$

The correction factor F and ΔT_m are determined in the form

$$\Delta T_1 = T_{h,in} - T_{c,out} = 80 - 50 = 30^\circ\text{C}$$

$$\Delta T_2 = T_{h,out} - T_{c,in} = 40 - 20 = 20^\circ\text{C} \quad \Delta T_{m,CF} = \frac{\Delta T_1 - \Delta T_2}{\ln[\Delta T_1/\Delta T_2]} \Rightarrow \Delta T_m = \frac{30 - 20}{\ln[30/20]} = 24.7^\circ\text{C}$$

$$\left. \begin{aligned} P &= \frac{t_2 - t_1}{T_1 - T_2} = \frac{40 - 80}{20 - 80} = 0.67 \\ R &= \frac{T_1 - T_2}{t_2 - t_1} = \frac{20 - 50}{40 - 80} = 0.75 \end{aligned} \right\} F = 0.91$$

a) In the case of no fouling, the overall heat transfer coefficient U is determined

$$\text{from } U = \frac{1}{\frac{1}{h_i} + \frac{1}{h_o}} = \frac{1}{\frac{1}{160} + \frac{1}{252}} = 21.6 \text{ W/m}^2\text{°C}$$

$$\text{The rate of heat transfer is } q = UAF\Delta T_m = 21.6 \times 3.77 \times 24.7 = 1830 \text{ W}$$

b) When there is fouling on one of the surfaces, the overall heat transfer coefficient

$$U \text{ is } U = \frac{1}{\frac{1}{h_i} + \frac{1}{h_o} + R} = \frac{1}{\frac{1}{160} + \frac{1}{252} + 0.0006} = 21.3 \text{ W/m}^2\text{°C}$$

$$\text{The rate of heat transfer in this case becomes } q = UAF\Delta T_m = 21.3 \times 3.77 \times 24.7 = 1805 \text{ W}$$

Exercise7

Hot oil is to be cooled by water in a 1-shell-pass and 8-tube-passes heat exchanger. The tubes are thin-walled and are made of copper with an internal diameter of 1.4 cm. The length of each tube pass in the heat exchanger is 5 m, and the overall heat transfer coefficient is $310 \text{ W/m}^2 \cdot ^\circ\text{C}$. Water flows through the tubes at a rate of 0.2 kg/s , and the oil through the shell at a rate of 0.3 kg/s .

The water and the oil enter at temperatures of 20°C and 150°C , respectively. Determine the rate of heat transfer in the heat exchanger and the outlet temperatures of the water and the oil.

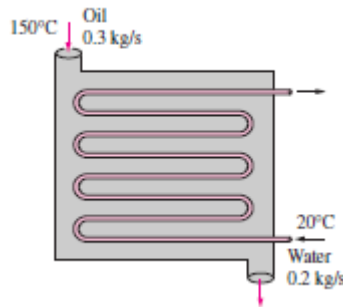


Figure 3. Schematic of heat exchanger

Solution

$$C_h = \dot{m}_h C_{p_c} = 0.3 \times 2.13 = 0.639 \text{ kW}/^\circ\text{C}$$

$$C_{\min} = C_h = 0.639 \text{ kW}/^\circ\text{C}$$

$$C_c = \dot{m}_c C_{p_c} = 0.3 \times 4.18 = 0.836 \text{ kW}/^\circ\text{C}$$

$$c = \frac{C_{\min}}{C_{\max}} = \frac{0.639}{0.836} = 0.764$$

$$\text{And } \dot{Q}_{\max} = C_{\min} (T_{h,\text{in}} - T_{c,\text{in}}) = 0.639 \times (150 - 20) = 83.1 \text{ kW}$$

$$\text{The heat transfer surface area is } A = n(\pi DL) = 3.14 \times 0.014 \times 5 = 1.76 \text{ m}^2$$

$$\text{Then the NTU of this heat exchanger becomes } NTU = \frac{UA}{C_{\min}} = \frac{310 \times 1.76}{639} = 0.853$$

The effectiveness of this heat exchanger corresponding to $c = 0.764$ and $NTU = 0.853$ is

$$E = 0.47 \text{ from figure } \dot{Q} = \varepsilon \dot{Q}_{\max} = 0.47 \times 83.1 = 39.1 \text{ kW}$$

Finally, the outlet temperatures of the cold and the hot fluid streams are determined to be

$$\dot{Q} = C_c (T_{c,\text{out}} - T_{c,\text{in}}) \rightarrow T_{c,\text{out}} = T_{c,\text{in}} + \frac{\dot{Q}}{C_c} = 20 + \frac{39.1}{0.836} = 66.8^\circ\text{C}$$

$$\dot{Q} = C_h (T_{h,\text{in}} - T_{h,\text{out}}) \rightarrow T_{h,\text{out}} = T_{h,\text{in}} - \frac{\dot{Q}}{C_h} = 150 - \frac{39.1}{0.639} = 88.8^\circ\text{C}$$

Therefore, the temperature of the cooling water will rise from 20°C to 66.8°C as it cools the hot oil from 150°C to 88.8°C in this heat exchanger.

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Abstract

These lecture notes, entitled Heat Transfer II, have been developed for third-year undergraduate students in Energy Engineering as part of the sixth-semester curriculum. Serving as a continuation of the Heat Transfer I module, this course aims to deepen students' understanding of advanced heat transfer phenomena and their applications in engineering systems. The material covers three major topics: convective heat transfer, including conservation equations, boundary layer theory, dimensionless analysis, and correlations for forced and natural convection; radiative heat transfer, addressing the fundamentals of thermal radiation, blackbody laws, radiative properties of real surfaces, and radiation exchange between surfaces; and heat exchangers, focusing on their classification, thermal analysis methods, overall heat transfer coefficient, logarithmic mean temperature difference (LMTD) method, effectiveness–NTU approach, and practical considerations related to boilers and thermal efficiency. To enhance learning outcomes and develop analytical skills, each chapter is complemented by illustrative examples and application exercises. These lecture notes provide students with the theoretical knowledge and practical tools required for the analysis and design of thermal systems encountered in energy and mechanical engineering applications.

Keywords: Heat transfer; convection; thermal radiation; heat exchangers; thermal systems; energy engineering.