الجمهـــورية الجزائــــرية الديمقــــراطية الشعـــــبية

People's Democratic Republic of Algeria

وزارة التعـــليم العـــالي والبحــث العلــمي

Ministry of Higher Education and Scientific Research

جـــامعة محمد بوضـــياف – بالمســـيلة

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Faculty of Technology

كـلية التكنــولــوجيــا

Vice Deanship of Post-Graduation, Scientific Research and External Relations

نيابة العمادة لما بعد التدرج والبحث العلمي والعلاقات الخارجية

رقم: 313 ن.ع.ب.ع /ك.ت /2025

المسيلة في: 16 أكتوبر 2025

شهادة ادارية

المصادقة على تقرير خبير للموافقة على مطبوعة جامعية

بعد الإطلاع على تقارير لجنة الخبراء للموافقة على المطبوعة البيداغوجية للأستاذ : : **بورزق يوسف إسلام– أستاذ محاضرأ**،بالقاعدة المشتركة بكلية التكنولوجيا بجامعة محمد بوضياف بالمسيلة والتي كانت كلها ايجابية ، تمّ تقرير التّالي:

1– المصادقة على تقارير لجنة الخبراء للموافقة المطبوعة البيداغوجية والمعنونة بـــ:

Electricity and magnetism (Physics 2)
Common Base ST– Cycle License

2– حيث تمّ تشكيل هذه اللجنة بناء على اجتماع المجلس العلمي للكلية المنعقد بتاريخ 2025/02/18 المكونة من السادة الآتية أسماؤهم:

- **بكري باديس،** أستاذ محاضر "أ"، جامعة محمد بوضياف– المسيلة
- **ميسوم راجعي،** أستاذ محاضر "أ"،جامعة يحي فارس– المدية
- **خريفي جلال،** أستاذ محاضر "أ"، جامعة محمد بوضياف– المسيلة

وتمت الموافقة بالاجماع على هذه المطبوعة.

رئيس المجلس العلمي للكلية

جامعة محمد بوضياف المسيلة – المجلس العلمي – كلية التكنولوجيا

محمد علي جريوي

People's Democratic Republic of Algeria
Ministry of Higher Education and Scientific Research
University of Msila
Faculty of Technology
Common Base

Module

"*Electricity and Magnetism*"

Courses and Exercises Corrected

Presented by: Dr. Yousf Islem BOUREZG

This course is provided for students at 1st year level in,

*** All engineering specialties**

*** ST/SM-LMD**

2024/2025

OBJECTIVES

This handout on electricity and magnetism is intended for students in the Science and Technology field at the first License of, Engineering, and LMD levels. It can also be used by students in the Materials Science field.

The objectives of this course are to acquire basic concepts on electric charges, the electric field, electric potential, conductors, and electrical circuits.

The course manuscript is structured around five chapters:

- Electric Charge and Coulomb's Law,
- Electric Field and Potential,
- Electric Dipole and Gauss's Law,
- Conductors and Capacitors,
- Electrical Current and Resistors,

At the end of each chapter, a series of corrected and uncorrected exercises are provided for students.

Finally, we conclude this course with a general bibliography. We hope that this course will serve as a reference for students and help them in their graduate studies.

Dr. Yousf Islem BOUREZG

Summary

I.1/ Electric charges and their properties

Electric charge (symbol q, sometimes Q) can be defined as a fundamental property of subatomic particles that gives rise to the phenomenon of experiencing force in the presence of electric and magnetic fields. These fields exert influence on charged particles, resulting in observable effects.

The SI unit of charge is called the Coulomb [C].

I.2/ Types of electric charge

Electric charge comes in two main types: *positive and negative charges*. Positive charges are associated with protons, which are subatomic particles residing in the nucleus of an atom. They are represented by the symbol ‘’+’’. On the other hand, negative charges are linked to electrons, which orbit the atomic nucleus and are denoted by the symbol ‘’-‘’.

There are two kinds of charges in nature, charges of opposite sign attract one another and charges of the same sign repel one another.

Several other properties of charge have been discovered:

* **Charge is quantized**: This means that electric charge comes in discrete amounts, and there is a smallest possible amount of charge that an object can have. In the SI system, this smallest amount is $e \equiv 1.602 \times 10^{-19}$ C.
* **The magnitude of the charge is independent of the type**: The smallest possible positive charge is $+1.602 \times 10^{-19}$ C, and the smallest possible negative charge is -1.602×10^{-19} C; these values are exactly equal.
* **Charge is conserved**: Charge can neither be created nor destroyed; it can only be transferred from place to place, from one object to another.

I.3/ The structure of matter

The atom is made up according to the Bohr model of:

- Z (e-) rotating around the nucleus.
- Z (p) and N (n) constituting the nucleus.

Some intrinsic properties of the constituents of the atom are as following:

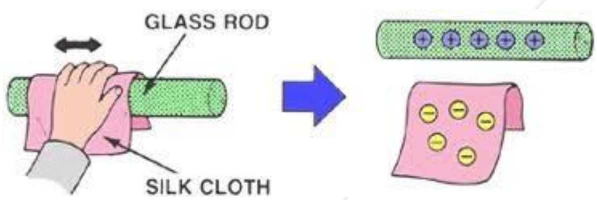
Particle	Symbol	Mass (Kg)	Charge (C)
Proton	P	$1.67261.10^{-27}$	$+1.6021917.10^{-19}$
Neutron	n	$1.67492.10^{-27}$	0
Electron	e	$9.1095.10^{-31}$	$-1.6021917.10^{-19}$

I.4/ Types of charging

There are three types of charging: by rubbing, by contact between charged and uncharged objects and by induction.

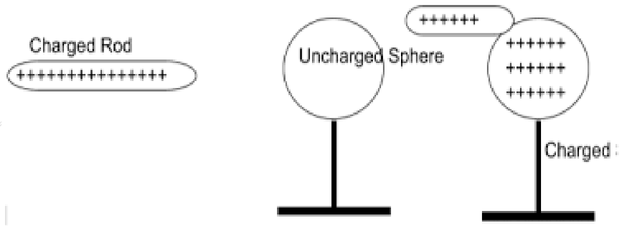
A/ Rubbing/friction method

Consider two objects which rub each other, and then due to friction, one object's electrons get transferred to the other one. The one which loses electrons becomes positively charged and the other, negative. Thus charging by friction is only due to the transfer of electrons.



B/ Contact method

When charged rod touches an uncharged sphere placed on an insulated stand, charges will transfer from the rod to the sphere, and hence, the uncharged object gets charged.



C/ Induction method

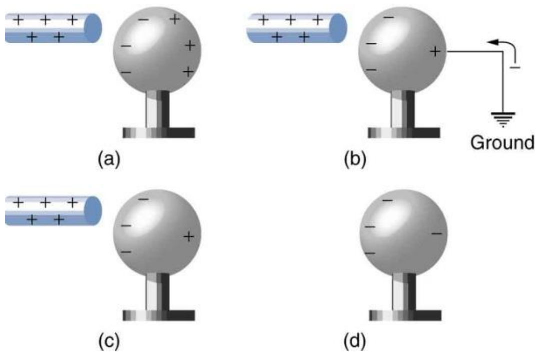
It is also possible to charge a conductor in a way that does not involve contact.

(a) Introducing a positively charged rod near a neutral metal sphere causes a rearrangement of charges within the sphere. Negative charges are attracted towards the rod, while positive charges are repelled to the far side.

(b) Then, the metal sphere is grounded, allowing electrons to flow from the Earth into the sphere to neutralize the positive charges on the far side.

(c) Upon disconnecting the grounding path, the sphere retains a net negative charge, showcasing the principle of induction where charge is induced without direct contact.

(d) With the removal of the positively charged rod, the induced negative charges redistribute themselves uniformly across the surface of the sphere, stabilizing the induced charge.



I.5/ Electric Force -Coulomb's Law

The force of attraction or repulsion between two charged bodies is directly proportional to the product of the quantity of charge on them, and inversely proportional to the square of the distance between them.

I.5.1/ Forces between two charges

If two charges have opposite sign, they have attractive force between them and if charges are same, they will repel each other. The electrostatic force is given by:

$$\vec{F} = k\frac{q_1 . q_2}{r^2}.\vec{u}$$

Noted that the electrostatic force is a vector quantity as it has magnitude:

$$F = k\frac{|q_1 . q_2|}{r^2}$$

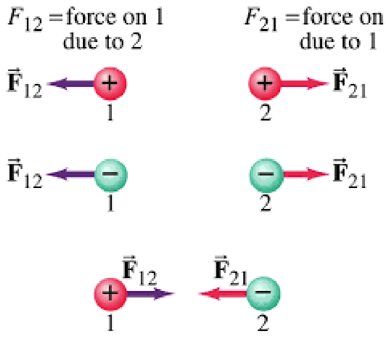
Where:

k is a constant called the Coulomb constant. $k = \frac{1}{4\pi\varepsilon_0} = 8.9876. 10^9\ N.m^2/C^2$

ε_0 is the permittivity of free space and has a value $8.8542. 10^{-12}\ C^2/N.m^2$

$\vec{u}$ is the unit vector pointing from the source charge to the test charge, $\vec{u} = \frac{\vec{r}}{r}$.

Note that, $\overrightarrow{u_{1/2}} = -\overrightarrow{u_{2/1}}$ and $\overrightarrow{F_{1/2}} = -\overrightarrow{F_{2/1}}$.



Example

What must be the distance between point charge q1 = 26,0 µC and point charge q2 = -47,0 µC for the electrostatic force between them to have a magnitude of 5,70 N?

Solution

We are given the charges and the magnitude of the (attractive) force between them. We can use Coulomb's law to solve for r, the distance between the charges:

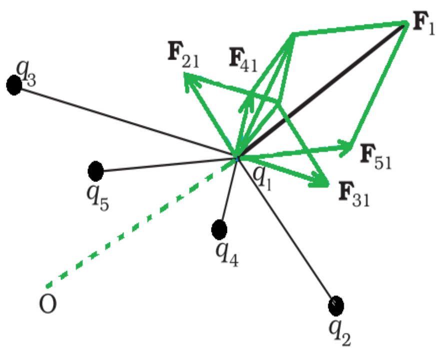
$$F = k\frac{|q_1.q_2|}{r^2} \Longrightarrow r^2 = k\frac{|q_1.q_2|}{F} = 1.39\,m$$

I.5.2/ Forces between multiple charges (Vector superposition of electric forces)

The total force $\overrightarrow{F_1}$ on the charge q_1, due to all other charges, is then given by the vector sum of the forces $\overrightarrow{F_{21}}, \overrightarrow{F_{31}}, \ldots, \overrightarrow{F_{n1}}$: i.e.,

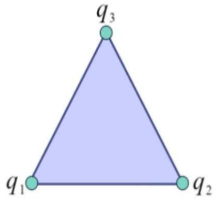
$$\overrightarrow{F_1} = \overrightarrow{F_{21}} + \overrightarrow{F_{31}} + \ldots + \overrightarrow{F_{n1}} = k.\left[\frac{q_1.q_2}{{r_{21}}^2}.\overrightarrow{u_{21}} + \frac{q_1.q_3}{{r_{31}}^2}.\overrightarrow{u_{31}} + \cdots + \frac{q_1.q_n}{{r_{n1}}^2}.\overrightarrow{u_{n1}}\right]$$

$$= k.q_1\sum_{i=2}^{n}\frac{q_i}{{r_{i1}}^2}.\overrightarrow{u_{i1}}$$



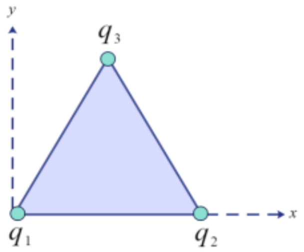
Example

Three charges q_1 =1μC , q_2 = – 2 μC and q_3 = 3 μC are placed on the vertices of an equilateral triangle of side 1.0 m. Find the net electric force acting on charge q_1.



Solution

The coordinates of q_1, q_2 , and q_3 in this coordinate system are (0, 0, 0), (1 m, 0, 0) and (0.5 m,0.87 m, 0), respectively.



$$\overrightarrow{F_1} = \overrightarrow{F_{21}} + \overrightarrow{F_{31}}$$

$$\overrightarrow{F_{21}} = k.\frac{q_1.q_2}{BA^3}.\overrightarrow{BA} = k.\frac{q_1.q_2}{1^3}.(-1\vec{\imath}) = \frac{(9.10^9)(1.10^{-6})(-2.10^{-6})}{1^3}.(-1\vec{\imath})$$

$$= (1.8 \times 10^{-2}\vec{\imath})N$$

$$\overrightarrow{F_{31}} = k.\frac{q_1.q_3}{CA^3}.\overrightarrow{CA} = k.\frac{q_1.q_2}{1^3}.[(0-0.5)\vec{\imath} + (0-0.87)\vec{\jmath}]$$

$$\overrightarrow{F_{31}} = k.\frac{(9.10^9)(1.10^{-6})(3.10^{-6})}{1^3}.[(-0.5)\vec{\imath} + (-0.87)\vec{\jmath}]$$

$$= [(-1.35)\vec{\imath} + (-2.349)\vec{\jmath}].\times 10^{-2}\ N$$

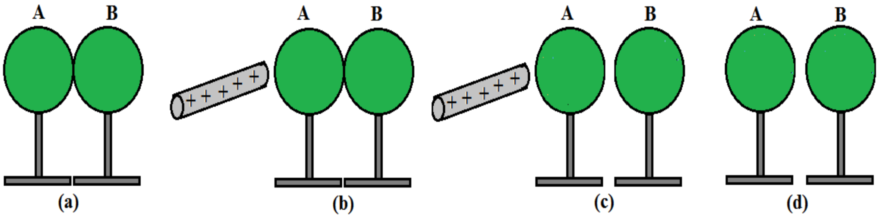
$$\overrightarrow{F_1} = \overrightarrow{F_{21}} + \overrightarrow{F_{31}} = [(-0.45)\vec{\imath} + (-2.349)\vec{\jmath}].\times 10^{-2}N.$$

I.6/ Exercises

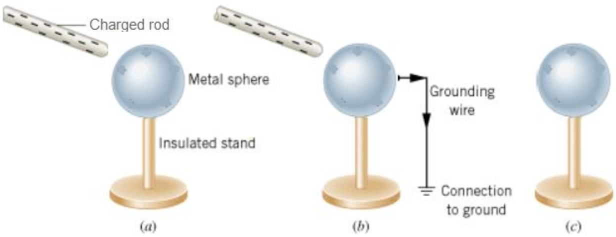
Exercises with solution

Exercise 01

1/ Schematically, draw the distribution of charge induced on neutral metallic spheres (A) and (B) due to a charged stick in the following cases:

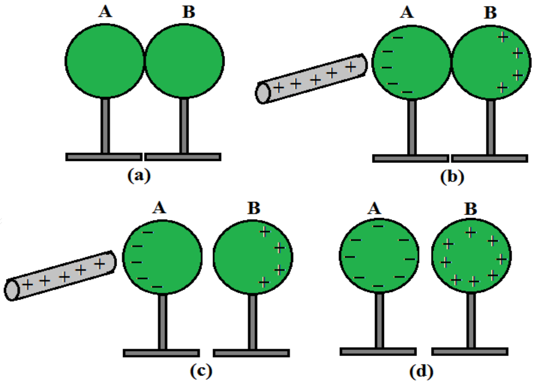


2/ Schematically, draw the distribution of charge induced on neutral metallic sphere as follow:

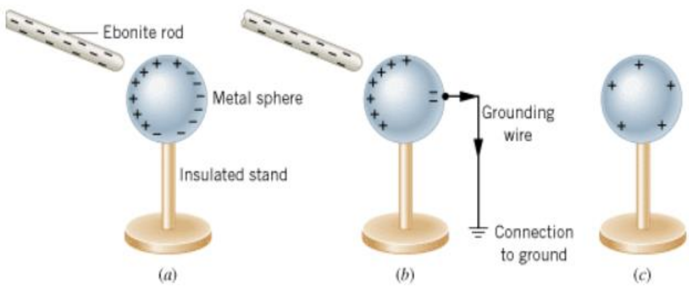


Solution of exercise 01

1/



2/



Exercise 02

The electron and proton of a hydrogen atom are separated by a distance of approximately 5.3×10^{-11} m.

1) Find the magnitude of the electric force between the two particles.
2) Find the magnitude of the gravitational force between the two particles.
3) Comment on the obtained results in questions (1) and (2).

Solution of exercise 02

1/ We use the coulomb's law:

$$F_e = k\frac{|q_1.q_2|}{r^2} \Rightarrow F_e = k\frac{|(e).(-e)|}{r^2} \Rightarrow F_e = 9\times10^9\frac{(1.6\times10^{-19})^2}{(5.3\times10^{-11})^2} = 8.2\times10^{-8}\,N$$

2/ We use the law of Newton of the universal gravitation:

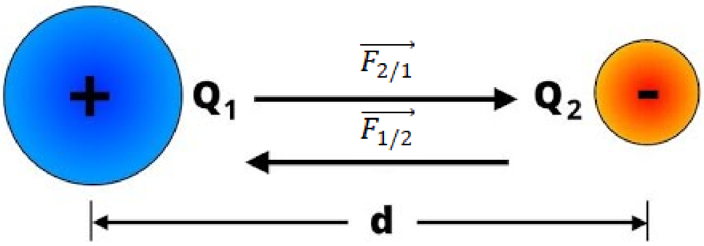
$$F_g = G\frac{m_1.m_2}{r^2} \Rightarrow F_g = 6.67\times10^{-11}\frac{(9.1\times10^{-31}).(1.67\times10^{-27})}{(5.3\times10^{-11})^2} = 3.6\times10^{-47}\,N$$

3/ From the obtained results, the gravitational force F_g between atomic particles, electron and proton, is negligible when compared it with the electric force F_e ($F_g \ll F_e$).

Exercise 03

Two spheres located at distance of d = 5 cm attract one another with a force of F = 3 mN. If one of them has three times more charges than the other, find the magnitude of each charge?

Solution of exercise 03



We put, $q_1 = 3q_2$ (or, $q_2 = 3q_1$):

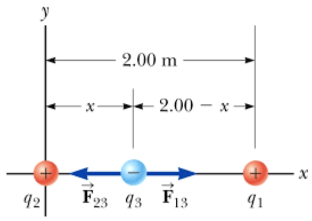
$$F_e = k\frac{|q_1.q_2|}{d^2} \Rightarrow F_e = k\frac{|q_1.3q_1|}{d^2} \Rightarrow F_e.d^2 = 3.k.{q_1}^2$$

$$\Rightarrow q_1 = 1.64\times10^{-8}\,C = 0.0164\,\mu C.$$

We have, $q_2 = 3q_1 \Rightarrow q_2 = 0.0328\,\mu C.$

Exercise 04

Three point charges lie along the x axis as shown in Figure opposite. The positive charge q_1 = 15.0 μC is at x = 2.00 m, the positive charge q_2 = 6.00 μC is at the origin, and the net force acting on negative charge q_3 is zero. What is the x coordinate of q_3?



Solution of exercise 04

$$\overrightarrow{F_3} = \overrightarrow{F_{13}} + \overrightarrow{F_{23}}$$

$$\overrightarrow{F_{13}} = k.\frac{q_1.q_3}{AC^3}.\overrightarrow{AC} = k.\frac{q_1.q_3}{(2-x)^3}.(-(2-x)\vec{\imath}) = -9\times10^9\,\frac{(15\times10^{-6}).(-q_3)}{(2-x)^2}.\vec{\imath}$$

$$\overrightarrow{F_{23}} = k.\frac{q_2.q_3}{BC^3}.\overrightarrow{BC} = k.\frac{q_2.q_3}{(x)^3}.(x\vec{\imath}) = 9\times10^9\,\frac{(6\times10^{-6}).(-q_3)}{(x)^2}.\vec{\imath}$$

The total force exerting on q_3 is zero, $\overrightarrow{F_{13}} + \overrightarrow{F_{23}} = \vec{0}$

$$\Rightarrow -9\times10^9\,\frac{(15\times10^{-6}).(-q_3)}{(2-x)^2}.\vec{\imath} + 9\times10^9\,\frac{(6\times10^{-6}).(-q_3)}{(x)^2}.\vec{\imath} = \vec{0}$$

$$\Rightarrow x = 0.77\ m$$

Exercises without solution

Exercise 01

Three equal point charges are located at the corners of an equilateral triangle. What is the strength of the electrostatic force exerted on a charge placed in the center of the triangle?

Exercise 02

Four equal point charges q are placed at (2; 2; 0), (2; 0; 2), (0; 2; 2) and at the origin.

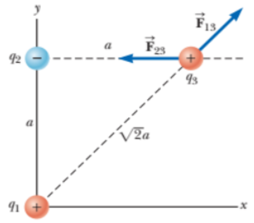
1/ Show that electrostatic force exerted on any charge, placed along the diagonal (x; x; x), is diagonal?

2/ What is the value of x for which the amplitude of this force is zero?

Exercise 03

Consider three point charges located at the corners of a right triangle as shown in below, where $q_1 = q_3$= 5.00 μC , q_2 = -2.00 μC , and a = 0.1 m.

- Find the resultant force F exerted on q_3.



II.1/ Electric field ($\vec{E}$)

An electric field $\vec{E}$ is a physical field that surrounds electrically charged particles. Let us consider a point charge (q) placed in vacuum, at the origin O. The electric field produced by the charge (q) at a point P is given as:

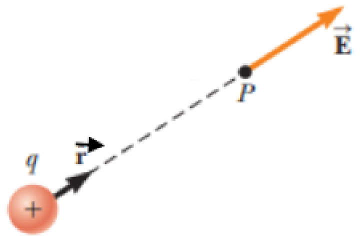
$$\vec{E} = \frac{1}{4\pi\varepsilon_0}.\frac{q}{r^2}.\vec{u} = k.\frac{q}{r^2}.\vec{u} = k.\frac{q}{r^2}.\frac{\vec{r}}{r}$$

$$\vec{E} = k.\frac{q}{r^3}.\vec{r}$$

k is a constant called the Coulomb constant. $k = \frac{1}{4\pi\varepsilon_0} = 8.9876.10^9\ N.m^2/C^2$

ε_0 is the permittivity of free space and has a value $8.8542.10^{-12}\ C^2/N.m^2$

$\vec{u}$ is the unit vector pointing from the source charge to the point P, $\vec{u} = \frac{\vec{r}}{r}$.



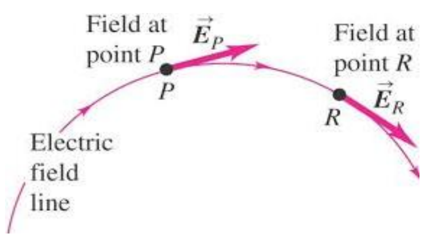
If we place a charge q_0 at point P, hence, according to Coulomb's law, we obtain the force $\vec{F}$ exerted by a charge q on a charge q_0, as:

$$\vec{F} = k.\frac{q.q_0}{r^2}.\vec{u} = q.\vec{E}$$

From this equation, the SI unit of electric field is [N/C**].**

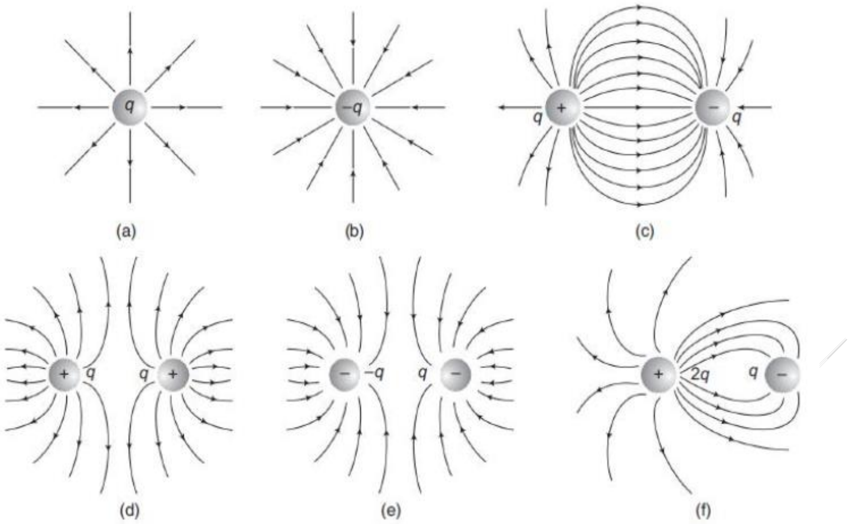
II.1.1/ Electric field lines

An electric field line is an imaginary line or curve drawn through a region of space so that its tangent at any point is in the direction of the electric field vector at that point.



We distinguish according to the sign of charge two cases for the lines of the electric field:

- For a positive point charge, the field lines are directed radially outward.
- For a negative point charge, the field lines are directed radially inward.



II.1.2/ The electric field created by several point charges

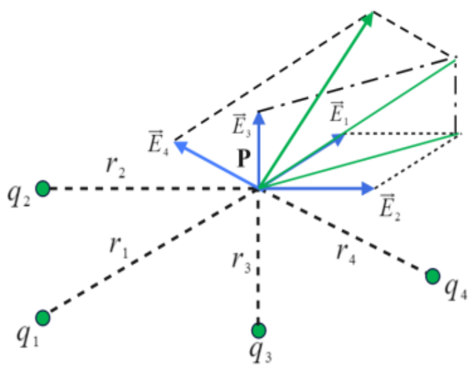
Consider a system of charges q_1, q_2, ..., q_n with position vectors r_1, r_2, ..., r_n relative to the point P. By the superposition principle, the electric field $\vec{E}$ due to the system of charges is :

$$\vec{E}(r) = \overrightarrow{E_1}(r_1) + \overrightarrow{E_2}(r_2) + \cdots \overrightarrow{E_n}(r_n)$$

$$\vec{E}(r) = k.\left[\frac{q_1}{{r_1}^2}.\overrightarrow{u_1} + \frac{q_1}{{r_2}^2}.\overrightarrow{u_2} + \cdots + \frac{q_n}{{r_n}^2}.\overrightarrow{u_n}\right]$$

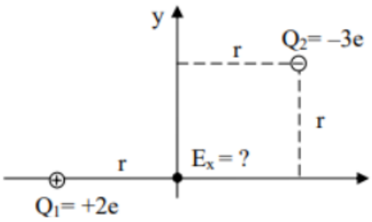
$$\vec{E}(r) = k\sum_{i=1}^{n}\frac{q_i}{r_i^2}.\overrightarrow{u_i}$$

$\vec{E}$ is a vector quantity that varies from one point to another point in space and is determined from the positions of the source charges.



Example

Consider two charges $Q_1 = +2e$ and $Q_2 = -3e$, are placed as shown. Calculate the electric field vector at the origin.



The electric field is the sum, $\vec{E} = \overrightarrow{E_1} + \overrightarrow{E_2}$, where:

$$\overrightarrow{E_1} = k.\frac{Q_1}{AO^3}.\overrightarrow{AO} = k.\frac{2.e}{r^3}.r.\vec{i} = k.\frac{2.e}{r^2}.\vec{i}$$

$$\overrightarrow{E_2} = k.\frac{Q_2}{BO^3}.\overrightarrow{BO} = k.\frac{-3.e}{(\sqrt{2}r)^3}.(-r\vec{i} - r\vec{j}) = k.\frac{3.e}{2\sqrt{2}r^2}.(\vec{i} + \vec{j})$$

$$\vec{E} = k.\frac{2.e}{r^2}.\left[\left(2 + \frac{3}{2\sqrt{2}}\right)\vec{i} + \frac{3}{2\sqrt{2}}\vec{j}\right]$$

II.2 Electric Potential (V)

Electric potential is a physical quantity defined as the amount of work/potential energy needed per unit of electric charge to move the charge from a point (a) to a point (b). This quantity is called the electric potential V:

$$\Delta V = V_b - V_a = \frac{\Delta U}{q} = -\frac{W_{a\to b}}{q} = -\int_a^b \vec{E}.\overrightarrow{dr}$$

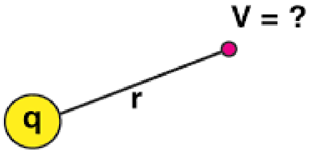
The potential difference dV between two points with a distance $d\ell$ apart can be expressed as:

$$dV = -\vec{E}.\overrightarrow{dr} \Rightarrow V = -\int_a^b k.\frac{q}{r^2}.dr$$

So, for one charge (q), the electric potential (V) is:

$$\mathrm{V = k.\frac{q}{r} + C.}$$

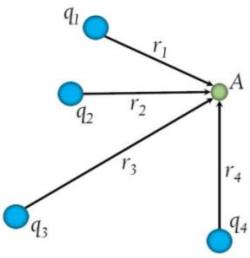
C=0 if (V∞)=0.



So, for several charges, the electric potential (V) is:

$$V = V_1 + V_2 + \cdots + V_n$$

$$V = k.\frac{q_1}{r_1} + k.\frac{q_2}{r_2} + \cdots + k.\frac{q_n}{r_n} + C.$$



Example

Find the electric field vector from the electric potential $V(r) = \vec{a}.\vec{r}$, where:

$$\vec{r} = x\vec{\imath} + y\vec{\jmath} + z\vec{k}$$

$$\vec{a} = a_x\vec{\imath} + a_y\vec{\jmath} + a_z\vec{k}$$

Solution

$$V(r) = \vec{a}.\vec{r} = x.a_x + y.a_y + z.a_z$$

$$\vec{E} = -\overrightarrow{grad}(V) = -\left[\frac{\partial V}{\partial x}\vec{\imath} + \frac{\partial V}{\partial y}\vec{\jmath} + \frac{\partial V}{\partial z}\vec{k}\right] = -a_x\vec{\imath} - a_y\vec{\jmath} - a_z\vec{k} = -\vec{a}$$

II.2.1 Relation between $\vec{E}$ and V

If the electric field has only one component E_x , we can write:

$$dV = -E_x.dx, \text{ so, } E_x = -\frac{dV}{dx}$$

In general, the electric potential is a function with three coordinates, in the Cartesian system. The electric field components E_x, E_y and E_z obtained from V(x, y, z) are:

$$\begin{cases} E_x = -\dfrac{\partial V}{\partial x} \\ E_y = -\dfrac{\partial V}{\partial y} \\ E_z = -\dfrac{\partial V}{\partial z} \end{cases}$$

$$\Rightarrow \vec{E} = -\overrightarrow{grad}(V) = -\left[\frac{\partial V}{\partial x}\vec{\imath} + \frac{\partial V}{\partial y}\vec{\jmath} + \frac{\partial V}{\partial z}\vec{k}\right]$$

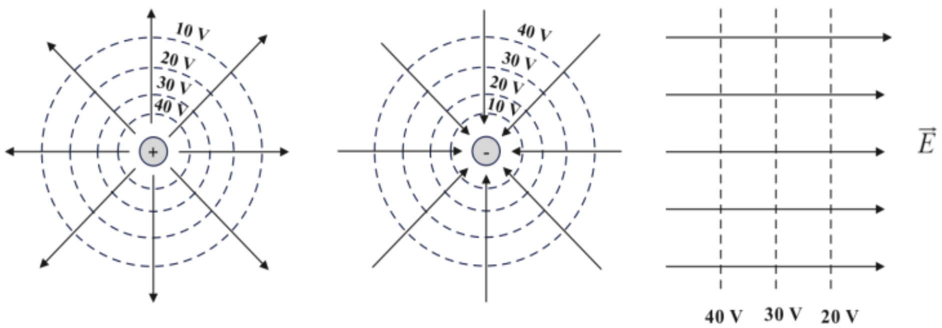
II.2.2/ Equipotential surface

An equipotential surface is a three dimensional surface on which the electric potential V is the same at every point on it. An equipotential surface has the following characteristics:

- The potential difference in any two points on the equipotential is zero.
- Field lines always flow from higher potential to lower potential.
- The potential is constant along an equipotential surface, so:

$$dV = 0 \Rightarrow -\vec{E}.\overrightarrow{dr} = 0 \Rightarrow \vec{E} \perp \overrightarrow{dr}$$

Thus, electric field lines and equipotential surfaces are always mutually perpendicular. Some equipotential surfaces (see dashed lines) are shown in figure opposite.

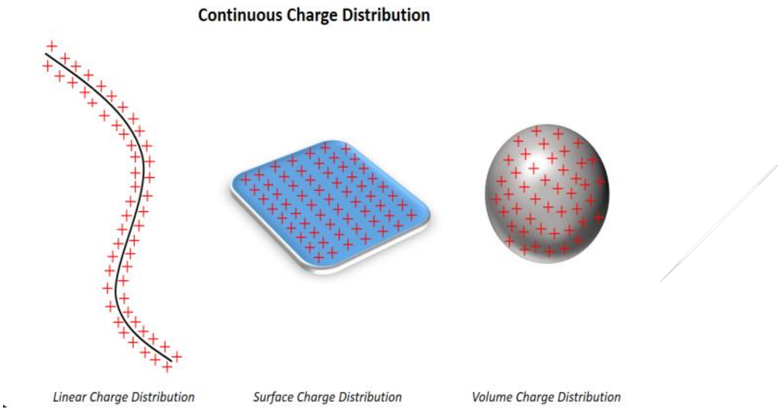


II.3 Continuous Charge Distribution

When the charge is distributed uniformly over a conductor, it is known as a continuous charge distribution.

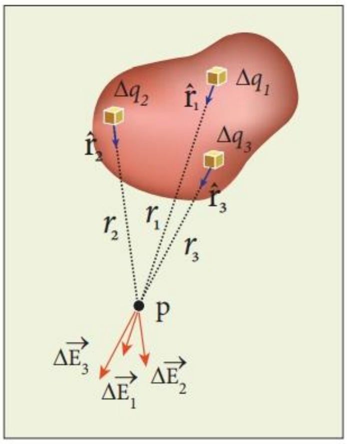
There are 3 types of continues charge distribution system:

- Linear Charge Distribution,
- Surface Charge Distribution,
- Volume Charge Distribution.



- **How to calculate the electric field:** To evaluating the electric field created by a continuous charge distribution, we have utilized the following procedure:

- Divide the charge distribution into small elements, each of which contains a small charge Δq as shown in Figure below.
- Use Equation $\vec{E}(r) = k\sum_{i=1}^{n}\frac{q_i}{r_i^2}.\vec{u_i}$ to calculate the electric field due to one of these elements at a point P.
- The total electric field at P due to all elements in the charge distribution is approximately: $\vec{E}(r) = k\sum_{i=1}^{n}\frac{\Delta q_i}{r_i^2}.\vec{u_i}$.
- In the limit of $\Delta q_i \to 0$, we get, $\vec{E}(r) = k\int\frac{dq}{r^2}.\vec{u}$.

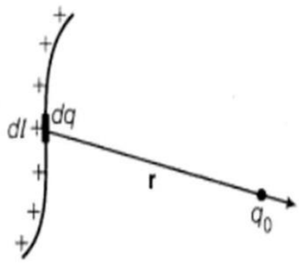


II.3.1/ Linear charge distribution

If a charge q is uniformly distributed along a line of length ℓ, the linear charge density λ is defined by:

$$\lambda = \frac{dq}{d\ell}$$

The unit of the linear load density is [C/m].

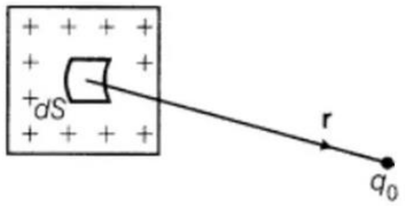


II.3.2/ Surface charge distribution

When the charge is uniformly distributed over the surface of a conductor, it is called Surface Charge Density or Surface Charge Distribution. It is denoted by the symbol σ (sigma) and is defined by:

$$\sigma = \frac{dq}{ds}$$

Where dq is a small element of charge over a small ds surface.

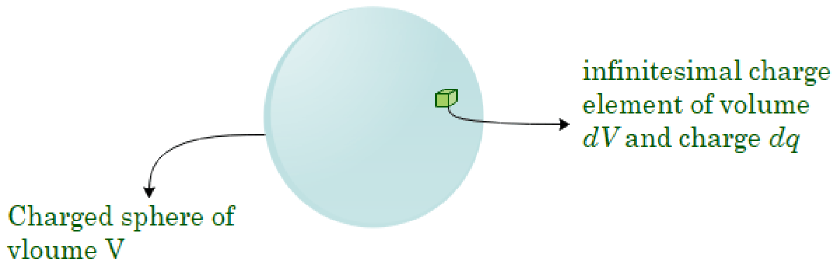


II.3.3/ Volume charge distribution

If the charge is uniformly distributed throughout a volume V, the volume charge density ρ is defined by:

$$\sigma = \frac{dq}{dV}$$

Where dq is a small charge element located in a small volume dV.



II.4 Electric Potential energy

When a charged particle moves in an electric field, the field exerts an electric force $\vec{F}$ that can do work on the particle that moves from point a to point b. The work $W_{a\to b}$ done by the force $\vec{F}$ is given by:

$$W_{a\to b} = \int_a^b \vec{F}.\overrightarrow{dr} = \mathrm{F}.dr.cos\theta$$

Where, dr is an infinitesimal displacement along the particle's path and θ is the angle between $\vec{F}$ and $\overrightarrow{dr}$ at each point along the path.

If the force $\vec{F}$ is conservative, the work done by $\vec{F}$ can always be expressed in terms of a potential energy U.

$$W_{a\to b} = U_a - U_b = -(U_b - U_a) = -\Delta U$$

The electric potential energy of a system of charges is given by:

$$U = k.\sum_{i<j} \frac{q_i . q_j}{r_{ij}}$$

The electric potential energy of a pair of point charges q_1 and q_2 can be found as follows:

$$U = k.\frac{q_1 . q_2}{r_{12}}$$

The electric potential energy of four point charges q_1 q_2 q_3, and q_4 would be given by:

$$U = k.\left[\frac{q_4 . q_3}{r_{43}} + \frac{q_4 . q_2}{r_{42}} + \frac{q_4 . q_1}{r_{41}} + \frac{q_3 . q_2}{r_{32}} + \frac{q_3 . q_1}{r_{31}} + \frac{q_2 . q_1}{r_{21}}\right]$$

Note: Total number of pairs formed by n point charges are, n(n-1)/2.

Example 1

An uniform electric field $\overrightarrow{E_0}$ is directed along positive *y*-direction. Find the change in electric potential energy of a positive test charge q_0 when it is displaced in this field from $y_i = a$ to $y_f = 2a$ along the y-axis.

Solution

The electrostatic force $\overrightarrow{F_e}$ on the test charge along the positive *y*-direction is:

$$F_e = q_0 . E_0$$

And we know, $\Delta U = -W_{i \to f} = -q_0 . E_0(2a - a) = -q_0 . E_0 . a$

It is clear that the work of the electric force F_e is positive and the potential energy is decreasing along the path.

Example 2

The electric potential at point A is 20 V and at B is – 40 V. Find the work done by the electrostatic force for moving an electron slowly from B to A.

Solution

We have , $V_A = 20V \; and \; V_B = -40V$,

Work done by electric force is : $W_{B \to A} = -\Delta U = -q.\Delta V = -q.(V_a - V_b) = q.(V_b - V_a)$

The test charge is an electron, $e = -1.6 \times 10^{-19}$C, so:

$$W_{B\to A} = e.(V_b - V_a) = -1.6 \times 10^{-19}(-40 - 20) = 9.6 \times 10^{-18}\ \text{J}$$

II.5/ Exercises

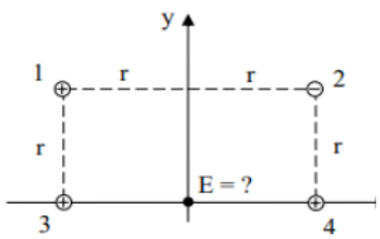
Exercises with solution

Exercise 01

Four point charges, labeled 1 through 4, all with the same magnitude q, are placed around the origin as shown. Charge 2 is negative; the rest are positive.

1/ What is the direction of the $\vec{E}$ field at the origin?

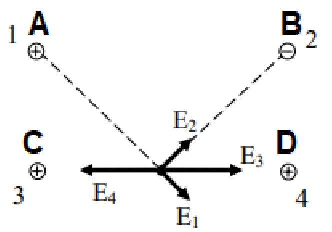
2/ Calculate the electric potential V at the origin.



Solution Exercise 01

Let us label the charges as follow: the charge 1 by the letter A, 2 by B, 3 by C and 4 by D.

The coordinates of these charges are: A(-r, r), B(r, r), C(-r, 0), D(r, 0).



1/ The electric field exerted at the origin is the sum:

$\vec{E} = \overrightarrow{E_1} + \overrightarrow{E_2} + \overrightarrow{E_3} + \overrightarrow{E_4}$, where:

$$\overrightarrow{E_1} = \overrightarrow{E_{AO}} = k.\frac{q_1}{AO^3}.\overrightarrow{AO} = k.\frac{q}{(r\sqrt{2})^3}.\left(r\vec{\imath} - r\vec{\jmath}\right) = k.\frac{q}{(r)^2}.\left(\frac{1}{2\sqrt{2}}\vec{\imath} - \frac{1}{2\sqrt{2}}\vec{\jmath}\right)$$

$$\overrightarrow{E_2} = \overrightarrow{E_{BO}} = k.\frac{q_2}{BO^3}.\overrightarrow{BO} = k.\frac{(-q)}{(r\sqrt{2})^3}.\left(-r\vec{\imath} - r\vec{\jmath}\right) = k.\frac{q}{(r)^2}.\left(\frac{1}{2\sqrt{2}}\vec{\imath} + \frac{1}{2\sqrt{2}}\vec{\jmath}\right)$$

$$\overrightarrow{E_3} = \overrightarrow{E_{CO}} = k.\frac{q_3}{CO^3}.\overrightarrow{CO} = k.\frac{q}{(r)^3}.\left(r\vec{\imath}\right) = k.\frac{q}{(r)^2}.\left(\vec{\imath}\right)$$

$$\overrightarrow{E_4} = \overrightarrow{E_{DO}} = k.\frac{q_4}{DO^3}.\overrightarrow{DO} = k.\frac{q}{(r)^3}.\left(-r\vec{\imath}\right) = k.\frac{q}{(r)^2}.\left(\overrightarrow{-\imath}\right)$$

We get, $\vec{E} = k.\frac{q}{(r)^2}.\left(\frac{1}{\sqrt{2}}\vec{\imath}\right)$

2/ The electric potential V at the origin is the sum:

$$V = V_1 + V_2 + V_3 + V_4$$

$$\mathrm{V} = \mathrm{k}.\frac{\mathrm{q}_1}{\mathrm{r}_1} + \mathrm{k}.\frac{\mathrm{q}_2}{\mathrm{r}_2} + \mathrm{k}.\frac{\mathrm{q}_3}{\mathrm{r}_3} + \mathrm{k}.\frac{\mathrm{q}_4}{\mathrm{r}_4} + \mathrm{C}.$$

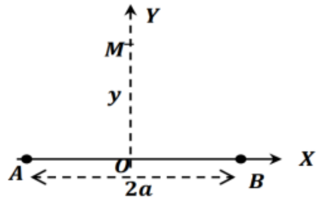
$$\mathrm{V} = \mathrm{k}.\frac{2.\mathrm{q}}{\mathrm{r}} + \mathrm{C}.$$

Exercise 02

Two identical charges $Q/2$ placed at points A and B separated by a distance $2a$, as shown.

1/ Calculate the electric field $\vec{E}$ created at point M.

2/ Calculate the electric potential V at point M.



Solution of Exercise 02

1/ The electric field exerted at the origin is the sum:

$\vec{E} = \overrightarrow{E_1} + \overrightarrow{E_2}$, where:

$q_1 = q_2 = \frac{Q}{2}$ and $AM = BM = \sqrt{a^2 + y^2}$

So, $\vec{E} = \overrightarrow{E_1} + \overrightarrow{E_2} = k.\frac{q_1}{AM^3}.\overrightarrow{AM} + k.\frac{q_2}{BM^3}.\overrightarrow{BM}$

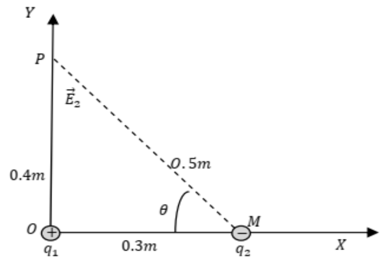
$$\Rightarrow \vec{E} = k.\frac{Q/2}{(a^2+y^2)^{3/2}}.\left[\left(a\vec{i}+y\vec{j}\right)+\left(-a\vec{i}+y\vec{j}\right)\right]$$

$$\Rightarrow \vec{E} = k.\frac{Q/2}{(a^2+y^2)^{3/2}}.y\vec{j}$$

Exercise 03

In orthonormal base OXY, a charge q_1 = 7 μC was placed at the origin (0,0) and charge q_2 = -5 μC at the point M (0.3, 0), (see the figure).

1- Find the electric field $\vec{E}$ at the point P (0, 0.4).
2- Find the electric potential V at the point P (0, 0.4).
3- Calculate the electric potential energy of point charges q_1 and q_2.



Solution of Exercise 03

1/ $\overrightarrow{E_P} = \overrightarrow{E_{OP}} + \overrightarrow{E_{MP}} = \overrightarrow{E_1} + \overrightarrow{E_2}$

$$\Rightarrow \overrightarrow{E_P} = k.\frac{q_1}{OP^3}.\overrightarrow{OP} + k.\frac{q_2}{MP^3}.\overrightarrow{MP}$$

$$\Rightarrow \overrightarrow{E_P} = \frac{(9\times10^9)(7\times10^{-6})}{0.4^3}.(0.4\vec{\imath}) + \frac{(9\times10^9)(-5\times10^{-6})}{0.5^3}.(-0.3\vec{\imath}+0.4\vec{\jmath})$$

$$\Rightarrow \overrightarrow{E_P} = 1.08\times10^5\vec{\imath} + 2.46\times10^5\vec{\jmath}$$

2/ $V_P = V_{OP} + V_{MP} = V_1 + V_2$

$$\Rightarrow V_P = k.\frac{q_1}{OP} + k.\frac{q_2}{MP} = \frac{(9\times10^9)(7\times10^{-6})}{0.4} + \frac{(9\times10^9)(-5\times10^{-6})}{0.5} = 67\,500V$$

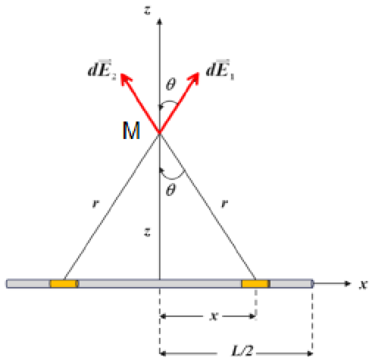
3/ The relation of electric potential energy U is as following:

$$U = k.\sum_{i<j}\frac{q_i.q_j}{r_{ij}}$$

So, for two charges, $U = k.\frac{q_1.q_2}{r_{1/2}} = \frac{(9\times10^9)(7\times10^{-6})(-5\times10^{-6})}{0.3} = -1.05\,J.$

Example 04

Find the electric field a distance z above the midpoint of a straight line segment of length L that carries a uniform line charge density λ.



Solution of Exercise 04

The electric field for a line charge is given by the general expression:

$$\vec{E}(r) = k\int \frac{dq}{r^2}.\vec{u} = k\int \frac{\lambda.\mathrm{dl}}{r^2}.\vec{u}$$

The total field $\vec{E}(P)$ is the vector sum of the fields from each of the two charge elements (call them $\overrightarrow{E_1}$ and $\overrightarrow{E_2}$, for now):

$$\vec{E}(M) = \overrightarrow{E_1} + \overrightarrow{E_2}, so, \overrightarrow{dE}(M) = \overrightarrow{dE_1} + \overrightarrow{dE_2} = \overrightarrow{dE_{1x}} + \overrightarrow{dE_{1z}} + \overrightarrow{dE_{2x}} + \overrightarrow{dE_{2z}}$$

Because the two charge elements are identical and have the same distance away from the point P where we want to calculate the field, $E1x = E2x$, so those components cancel. This leaves:

$$\overrightarrow{dE}(M) = \overrightarrow{dE_{1z}} + \overrightarrow{dE_{2z}} = E_1 cos\theta.\vec{k} + E_2 cos\theta.\vec{k}$$

These components are also equal, so we have:

$$\vec{E}(M) = k\int \frac{\lambda.\mathrm{dl}}{r^2} cos\theta.\vec{k} + k\int \frac{\lambda.\mathrm{dl}}{r^2} cos\theta.\vec{k}$$

$$\vec{E}(M) = k\int_0^{L/2} \frac{2\lambda.\mathrm{dl}}{r^2} cos\theta.\vec{k}$$

We have, $= (z^2 + x^2)^{1/2}$, and $cos\theta = \frac{z}{r} = \frac{z}{(z^2 + x^2)^{1/2}}$, so we get:

$$\vec{E}(M) = k\int_0^{L/2} \frac{2.\lambda.z}{(z^2\ + x^2\ )^{3/2}}\,dx.\vec{k} = \vec{E}(M) = 2.\lambda.z.k\int_0^{L/2} \frac{dx}{(z^2\ + x^2\ )^{3/2}}.\vec{k}$$

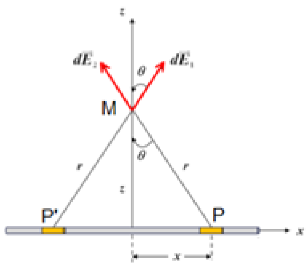
$$\vec{E}(M) = 2.\lambda.z.k\left[\frac{dx}{z^2\,(z^2\ + x^2\ )^{\frac{1}{2}}}\right]_0^{L/2}.\vec{k}$$

$$\vec{E}(M) = k.\frac{\lambda.L}{z.(z^2\ + \frac{L^2}{4}\ )^{\frac{1}{2}}}.\vec{k}$$

Example 05

Find the electric field a distance z above the midpoint of an infinite line of charge that carries a uniform line charge density **λ**.

Solution of Exercise 05



The electric field for a line charge is given by the general expression:

$$\vec{E}(r) = k\int \frac{dq}{r^2}.\vec{u} = k\int \frac{\lambda.\mathrm{dl}}{r^2}.\vec{u}$$

$$\vec{E}(M) = \vec{E_1} + \vec{E_2} = \ k\int \frac{\lambda.\mathrm{dl}}{r^2}.\vec{u_1} + k\int \frac{\lambda.\mathrm{dl}}{r^2}.\vec{u_2} = k\int \frac{\lambda.\mathrm{dl}}{r^2}.(\vec{u_1} + \vec{u_2})$$

$$\vec{E}(M) = k.\lambda\int \frac{\mathrm{dl}}{PM^3}.(\overrightarrow{PM} + \overrightarrow{P'M})$$

Where: $\vec{u_1} = \frac{\overrightarrow{PM}}{PM}$, $\vec{u_2} = \frac{\overrightarrow{P'M}}{P'M}$ and the coordinates of different points are, P (x,0), P’ (-x,0), M (0,z), so:

$$\overrightarrow{PM} = -x\vec{\imath} + z\vec{k}\ and\ \overrightarrow{P'M} = x\vec{\imath} + z\vec{k}$$

By substitution in the relation of electric field above, we get:

$$\vec{E}(M) = k.\lambda \int \frac{\mathrm{dl}}{(z^2 + x^2)^{\frac{3}{2}}}.(2z\vec{k}) = k.2.z.\lambda \int \frac{\mathrm{dx}}{(z^2 + x^2)^{\frac{3}{2}}}.(\vec{k})$$

$$\vec{E}(M) = 2.\lambda.\mathrm{z}.k \int_{-\infty}^{+\infty} \frac{dx}{(z^2 + x^2)^{3/2}}.\vec{k}$$

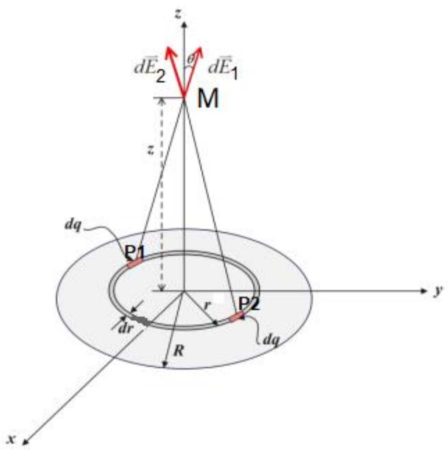
$$\vec{E}(M) = 2.\lambda.\mathrm{z}.k \left[\frac{dx}{z^2 (z^2 + x^2)^{\frac{1}{2}}}\right]_{-\infty}^{+\infty}.\vec{k}$$

$$\vec{E}(M) = \frac{2.k.\lambda}{z}.\vec{k}$$

Example 06

Find the electric field of a circular thin disk of radius R and uniform surface charge density σ at a distance z above the center of the disk.

Solution of Exercise 06



We have, $\vec{E}(r) = k \int \frac{dq}{r^2}.\vec{u}, so, \overrightarrow{dE}(r) = k\frac{\sigma.ds}{r^2}.\vec{u}$

$$\vec{E}(M) = \overrightarrow{E_1} + \overrightarrow{E_2} = k \int \frac{\sigma.ds_1}{r^2}.\overrightarrow{u_1} + k \int \frac{\sigma.ds_2}{r^2}.\overrightarrow{u_2} = k \int \frac{\sigma.ds_1}{r^2}.(\overrightarrow{u_1} + \overrightarrow{u_2})$$

because, $ds = 2.\pi.r.dr\ and\ ds_1 = ds_2 = \frac{ds}{2} = \pi.r.dr.$

Where: $\overrightarrow{u_1} = \frac{\overrightarrow{PM}}{PM}$, $\overrightarrow{u_2} = \frac{\overrightarrow{P'M}}{P'M}$ and the coordinates of different points are, P_1 (0,-r,0), P_2 (0, r,0), M (0,0,z), so:

$$\overrightarrow{P_1M} = r\vec{j} + z\vec{k} \; and \; \overrightarrow{P_2M} = -r\vec{j} + z\vec{k}$$

By substitution in the relation of electric field above, we get:

$$\vec{E}(M) = k.\lambda \int \frac{\sigma.ds_1}{PM^3}.(\overrightarrow{P_1M} + \overrightarrow{P_2M}) = k.\sigma \int \frac{\pi.r.dr}{(r^2 + z^2)^{3/2}}.(2.z.\vec{k})$$

$$\vec{E}(M) = \frac{\sigma.z}{2\varepsilon_0} \int_0^R \frac{r.dr}{(z^2 + r^2)^{3/2}}.\vec{k}$$

We put, $z^2 + r^2 = T^2$, $so, (z^2 + r^2)^{3/2} = T^3$ and $2rdr = 2TdT$

$$\vec{E}(M) = \frac{\sigma.z}{2\varepsilon_0} \int \frac{T.dT}{T^3}.\vec{k} = \frac{\sigma.z}{2\varepsilon_0} \int \frac{dT}{T^2}.\vec{k}$$

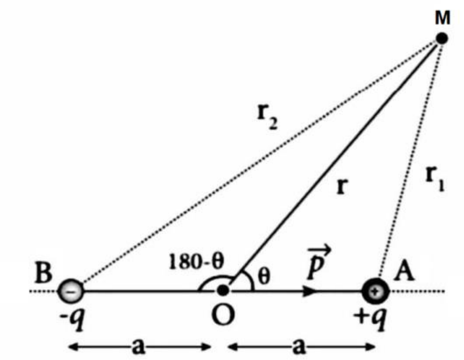
$$\vec{E}(M) = \frac{\sigma.z}{2\varepsilon_0}.\left[-\frac{1}{T}\right].\vec{k} = \frac{\sigma.z}{2\varepsilon_0}.\left[\frac{1}{(r^2 + z^2)^{1/2}}\right]_R^0.\vec{k} = \frac{\sigma.z}{2\varepsilon_0}\left[\frac{1}{|z|} - \frac{1}{(R^2 + z^2)^{\frac{1}{2}}}\right]$$

It is clear that the vector field depends with the sign of z:

$$\vec{E}(M) = \begin{cases} \dfrac{\sigma}{2\varepsilon_0}.\left[1 - \dfrac{1}{(R^2 + z^2)^{\frac{1}{2}}}\right].\vec{k} & , \quad if\ z > 0 \\ \dfrac{-\sigma}{2\varepsilon_0}.\left[1 + \dfrac{1}{(R^2 + z^2)^{\frac{1}{2}}}\right].\vec{k} & , \quad if\ z < 0 \end{cases}$$

III.1/ Electric Dipole

An electric dipole is a system of two equal electric charges that have opposite sign –q and +q, separated by a small distance d as comparing with r, as shown.



We make, d = 2a.

III.1.1/ Calculating of Potential V at point M

$$V_M = V_1 + V_2$$

We have, $V_1 = \frac{kq}{r_1}$ and $V_2 = -\frac{kq}{r_2}$, $\Rightarrow V_M = k.q\left(\frac{1}{r_1} - \frac{1}{r_2}\right)$

Let's search the expressions of $1/r_1$ and $1/r_2$,

- We have, $\overrightarrow{OM} = \overrightarrow{OA} + \overrightarrow{AM} \Rightarrow \overrightarrow{AM} = \overrightarrow{OM} - \overrightarrow{OA} \Rightarrow AM^2 = OM^2 + OA^2 - 2.\overrightarrow{OM}.\overrightarrow{OA}$

$\Rightarrow r_1^2 = r^2 + a^2 - 2.r.a.cos\theta \Rightarrow r_1^2 = r^2\left(1 + \frac{a^2}{r^2} - \frac{2.a.cos\theta}{r}\right)$,

We have, $r \gg a, and\ (1 \pm \varepsilon)^n = 1 \pm n.\varepsilon$,

So, $r_1^2 = r^2\left(1 - \frac{2.a.cos\theta}{r}\right) \Rightarrow r_1 = r\left(1 - \frac{2.a.cos\theta}{r}\right)^{1/2} \Rightarrow \frac{1}{r_1} = \frac{1}{r}\left(1 - \frac{2.a.cos\theta}{r}\right)^{-1/2}$

$$\Rightarrow \frac{1}{r_1} = \frac{1}{r}.\left(1 + \frac{a.cos\theta}{r}\right)$$

- We have, $\overrightarrow{BM} = \overrightarrow{BO} + \overrightarrow{OM} \Rightarrow BM^2 = BO^2 + OM^2 + 2.\overrightarrow{BO}.\overrightarrow{OM}$

$\Rightarrow r_2^2 = a^2 + r^2 + 2.r.a.cos\theta \Rightarrow r_2^2 = r^2\left(1 + \frac{a^2}{r^2} + \frac{2.a.cos\theta}{r}\right)$,

We have, $r \gg a, and\ (1 \pm \varepsilon)^n = 1 \pm n.\varepsilon$,

So, $r_2^2 = r^2\left(1 + \frac{2.a.cos\theta}{r}\right) \Rightarrow r_2 = r\left(1 + \frac{2.a.cos\theta}{r}\right)^{1/2} \Rightarrow \frac{1}{r_2} = \frac{1}{r}\left(1 + \frac{2.a.cos\theta}{r}\right)^{-1/2}$

$$\Rightarrow \frac{1}{r_2} = \frac{1}{r}.\left(1 - \frac{a.cos\theta}{r}\right)$$

$\Rightarrow V_M = k.q\left(\frac{1}{r_1} - \frac{1}{r_2}\right) \quad \Rightarrow \quad V_M = k.q\frac{2.a.cos\theta}{r^2}$

III.1.2/ Calculating of Electric field $\vec{E}$ at point M

$\vec{E} = -\overrightarrow{grad}(V)$, so, in Cartesian coordinates, $\vec{E} = -\left(\frac{\partial V}{\partial x}\vec{i} + \frac{\partial V}{\partial y}\vec{j} + \frac{\partial V}{\partial z}\vec{k}\right)$, and in polar coordinates, $\vec{E} = -\frac{\partial V}{\partial r}\overrightarrow{U_r} - \frac{1}{r}.\frac{\partial V}{\partial \theta}\overrightarrow{U_\theta}$

we have, $\frac{\partial V}{\partial r} = -k.q\frac{4.a.\cos\theta}{r^3}$ and $\frac{\partial V}{\partial \theta} = -k.q\frac{2.a.\sin\theta}{r^2}$

$$\Rightarrow \vec{E} = k.q\frac{4.a.\cos\theta}{r^3}\overrightarrow{U_r} + k.q\frac{2.a.\sin\theta}{r^3}\overrightarrow{U_\theta}$$

$$\Rightarrow E = k.q\frac{2.a}{r^3}.(4.\cos\theta^2 + \sin\theta^2)^{1/2}$$

III.1.3 Electric Dipole Moment ($\vec{P}$)

Two equal and opposite charges separated by a small distance (2a) constitute a dipole. The product of the charge and distance between them is called the dipole moment.

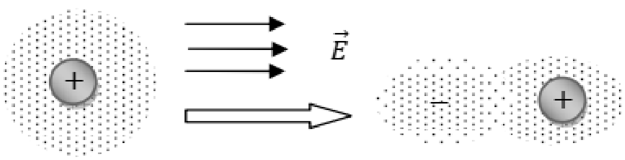
$\vec{p} = q.\vec{d} \Rightarrow p = 2.a.q$

Then, the expressions of potential V and electric field E at point P are as follow:

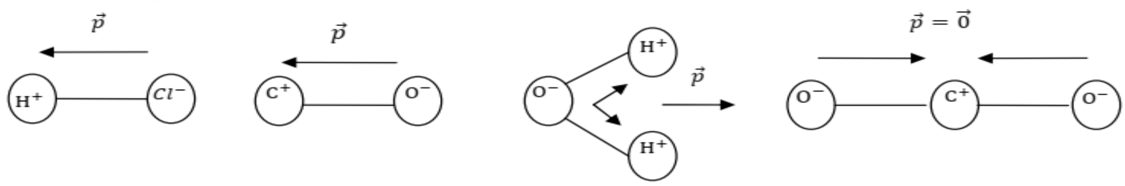
$$V_M = k.\frac{p.\cos\theta}{r^2}$$

$$E = k.\frac{p}{r^3}.(4.\cos\theta^2 + \sin\theta^2)^{1/2}$$

<u>**N.B:**</u> The study of dipoles is of great importance in the study of atoms or molecules placed in an external electric field, where the center of gravity of the atoms is displaced by a distance from the nucleus, so they become polarized and behave as dipoles.



Some molecules in nature, in the absence of an external electric field, appear as permanent poles called polar molecules, such as:



III.2/ Gauss's theorem

The electric field flux ϕ through a closed surface S is equal to the algebraic sum of the charges contained in this surface $\sum Q_{int}$ divided by the permittivity in vacuum ε_0.

$$\phi = \oint_S \vec{E}.\overrightarrow{dS} = \frac{\sum Q_{int}}{\varepsilon_0}$$

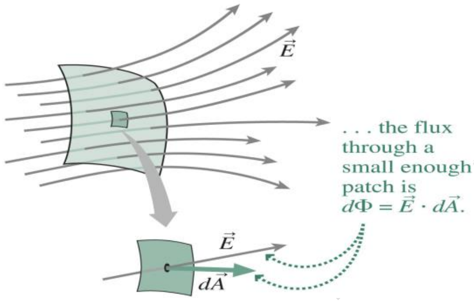
Electric flux ϕ is a measure of the field lines crossing a surface. Its IS unit is $N.m^2/C$.

The circle on the integral means that the surface is closed and qinside is the net charge inside this closed surface.

We now consider a point charge q located at a point O in space. The total flux of the electric field created by this charge across any oriented elementary surface dS is by definition:

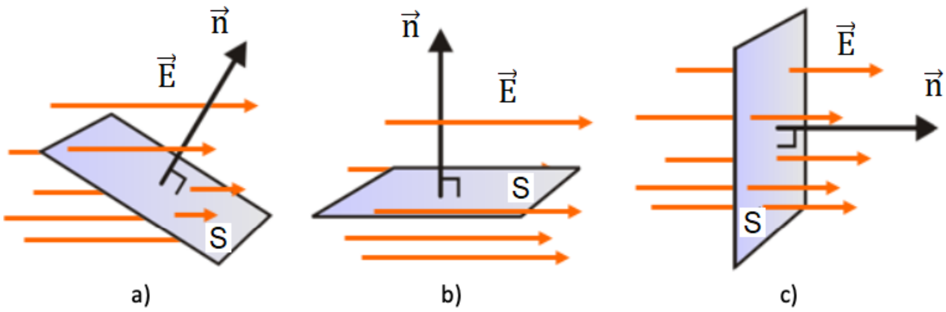
$$\phi = \oint d\phi = \oint_S \vec{E}.\overrightarrow{dS} = \oint_S \vec{E}.\vec{n}.dS = \oint_S E.dS.\cos\theta = \frac{\sum Q_{int}}{\varepsilon_0}$$

Where: $\vec{n}$ is the unit vector normal to the surface dS and θ is the angle between $\vec{n}$ (or $\overrightarrow{dS}$) and electric field line $\vec{E}$.



According to the figure below, the electric flux ϕ is dependent with angle between the directions of electric field $\vec{E}$ and normal to the surface S:

- $\phi = E.dS.\cos\theta$. (Fig. a)
- $\phi = E.dS.\cos 90° = 0$, electric flux is zero when the electric field lines are tangential to the surface S. (Fig. b)
- $\phi = E.dS.\cos 0° = E.dS$, electric flux is maximal when the electric field lines are perpendicular to the surface S. (Fig. c)



III.2.1/ Application of Gauss's law

The Gauss's theorem is used to calculate the electric field strength if the distribution of charges is sufficiently symmetrical (symmetric). A good choice of Gauss's surface ensures that the integration on this surface can be easily completed. This surface should satisfy the following conditions:

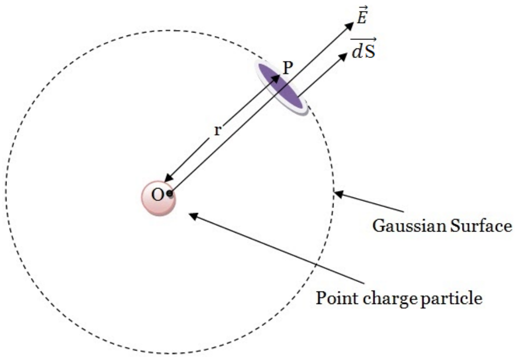
- A closed imaginary surface that includes the point at which the field is to be calculated.
- A surface that makes the scalar product $\vec{E}.\overrightarrow{dS}$ known at any point on it.
- A surface that makes the field intensity constant along its length.
- If there are no charges inside a Gauss's surface or the algebraic sum of the charges contained in this surface is equal to zero, then the electric field flux is zero.

Example 01 (Electric field resulting from a point charge q)

Find the electric field at a distance r from the charge (q), by using the Gauss's theorem.

Solution

Firstly, we select Gaussian surface, a sphere at distance r from the charge. At every point of this sphere the electric field has the same magnitude E and it is perpendicular to the surface itself. Hence, from the Gauss's law:



$$\oint_S \vec{E}.\overrightarrow{dS} = \frac{\sum Q_{int}}{\varepsilon_0}$$

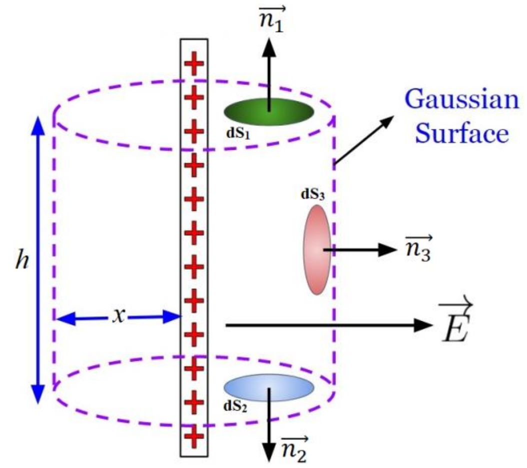
$$E.S = \frac{q}{\varepsilon_0}$$

$E.4\pi r^2 = \frac{q}{\varepsilon_0}$, so, $E = \frac{q}{4\pi r^2\,\varepsilon_0} = k.\frac{q}{r^2}$.

Example 02 (Electric field resulting from a spherical surface distribution of charges)

Calculate the electric field at distance x from a wire of length h and that having linear charge distribution (λ).

Solution



From the Gauss's law:

$$\oint_S \vec{E}.\overrightarrow{dS_1} + \oint_S \vec{E}.\overrightarrow{dS_2} + \oint_S \vec{E}.\overrightarrow{dS_3} = \frac{\sum Q_{int}}{\varepsilon_0}$$

$$\vec{E}.\overrightarrow{S_1} + \vec{E}.\overrightarrow{S_2} + \vec{E}.\overrightarrow{S_3} = \frac{\sum Q_{int}}{\varepsilon_0}$$

$$E.S_1.\cos 90 + E.S_2.\cos 90 + E.S_3.\cos 0 = \frac{\sum Q_{int}}{\varepsilon_0}$$

$$\Longrightarrow E.S_3 = \frac{\sum Q_{int}}{\varepsilon_0} \Longrightarrow E.2.\pi.x.h = \frac{\lambda.h}{\varepsilon_0}$$

We get, $E = \frac{\lambda}{2.\pi.x.\varepsilon_0}$.

III.3/ Exercises

Exercises with solution

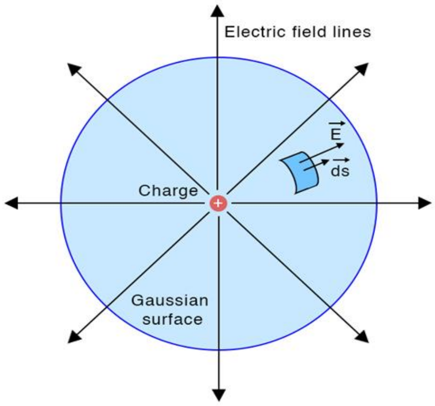
Exercise 01

1/ Using the Gauss's theorem, calculate the electric field created by a positive charge at distance r.

2/ Using the Gauss's theorem, calculate the electric field created by a negative charge at distance r.

Solution of Exercise 01

1/ For a positive point charge, the field lines are directed radially outward,

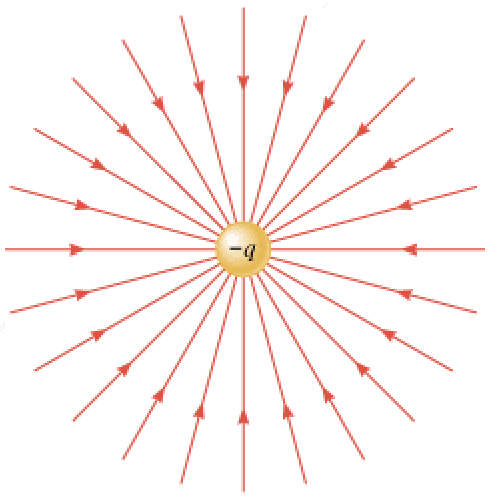


$$\oint_S \vec{E}.\vec{dS} = \frac{\sum Q_{int}}{\varepsilon_0}$$

$$E.S = \frac{q}{\varepsilon_0}$$

$E.4\pi r^2 = \frac{q}{\varepsilon_0}$, so, $E = \frac{q}{4\pi r^2\,\varepsilon_0} = k.\frac{q}{r^2}$.

2/ For a negative point charge, the field lines are directed radially inward,



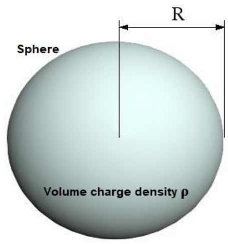
So, we get, $E = \frac{-q}{4\pi r^2\,\varepsilon_0} = k.\frac{-q}{r^2}$.

Exercise 02

1) Using the Gauss's theorem, calculate the electric field created by an uniformly charged sphere with radius R and having a volume charge density ρ at: * inside the sphere (with radius r, r<R), * and outside the sphere (r>R).

2) Deduce the potential at these positions.

3) Draw schematically E(r) and V(r).



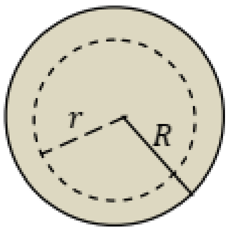
Solution of Exercise 02

1/ The electric field at every point in space is radial, depending only on r, which allows us to choose the surface of a sphere of radius r whose center coincides with the center of the charged sphere. The flux through the surface of the sphere is:

$$\phi = \oint_S \vec{E}.\overrightarrow{dS} = E.S = \frac{\sum Q_{int}}{\varepsilon_0}$$

$$\Rightarrow E.4\pi r^2 = \frac{\sum Q_{int}}{\varepsilon_0}$$

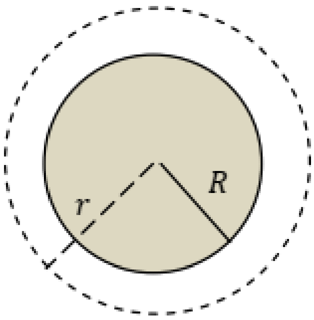
- r<R:



$E_1.4\pi r^2 = \frac{\rho.V_r}{\varepsilon_0} \Rightarrow E_1.4\pi r^2 = \frac{\rho.\frac{4}{3}\pi r^3}{\varepsilon_0}$

$\Rightarrow E_1 = \frac{\rho.r}{3\varepsilon_0}$

- r>R:



$$E_2.4\pi r^2 = \frac{\rho.V_R}{\varepsilon_0} \Rightarrow E_2.4\pi r^2 = \frac{\rho.\frac{4}{3}\pi R^3}{\varepsilon_0}$$

$$\Rightarrow E_2 = \frac{\rho.R^3}{3\varepsilon_0}.\frac{1}{r^2}$$

2/ Using the relation between $\vec{E}$ and V:

$$\vec{E} = -\overrightarrow{grad}\,V = -\frac{\partial V}{\partial r}\overrightarrow{U_r} - \frac{1}{r}.\frac{\partial V}{\partial \theta}\overrightarrow{U_\theta} - \frac{1}{r\sin\theta}.\frac{\partial V}{\partial \varphi}\overrightarrow{U_\varphi}$$

The field change only diagonally (with r), so: $\vec{E} = -\frac{\partial V}{\partial r}\overrightarrow{U_r} \Rightarrow E = -\frac{\partial V}{\partial r} \Rightarrow V = -\int E.dr$

- r<R:

$$V_1 = -\int E_1.dr = -\int \frac{\rho.r}{3\varepsilon_0}dr \Rightarrow V_1 = -\frac{\rho.r^2}{6\varepsilon_0} + C_1$$

- r>R:

$$V_2 = -\int E_2.dr = -\int \frac{\rho.R^3}{3\varepsilon_0}.\frac{1}{r^2}dr \Rightarrow V_2 = \frac{\rho.R^3}{3\varepsilon_0}.\frac{1}{r} + C_2$$

- Find the constants C_1 and C_2,

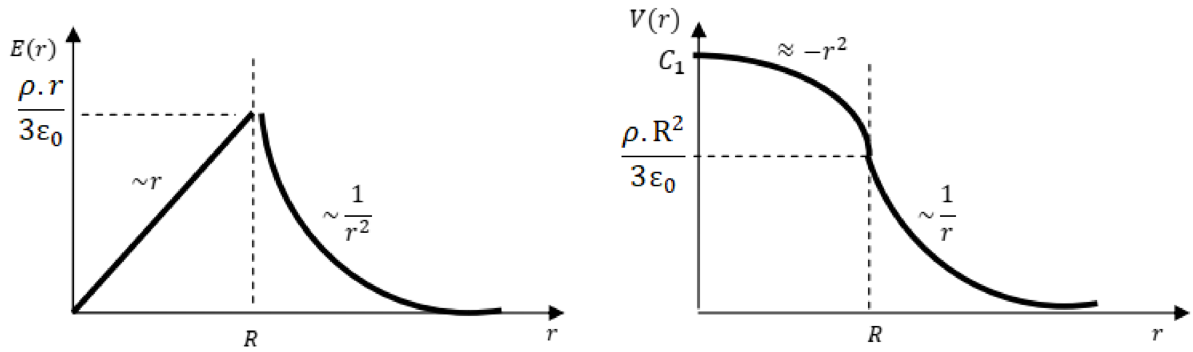
We use the limiting conditions: $V_2(\infty) = 0 \Rightarrow C_2 = 0 \Rightarrow V_2 = \frac{\rho.R^3}{3\varepsilon_0}.\frac{1}{r}$

According to the continuity of potential at r=R: $V_2(r = R) = V_1(r = R)$

$$\Rightarrow \frac{\rho.R^2}{3\varepsilon_0} = -\frac{\rho.R^2}{6\varepsilon_0} + C_1 \Rightarrow C_1 = \frac{\rho.R^2}{2\varepsilon_0}$$

So, $V_1 = -\frac{\rho.r^2}{6\varepsilon_0} + \frac{\rho.R^2}{2\varepsilon_0}$

3/

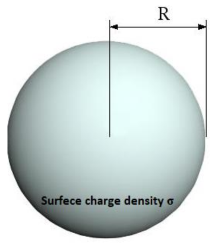


Exercise 03

1) Using the Gauss's theorem, calculate the electric field created by an uniformly charged sphere with radius R and having a surface charge density σ at: * inside the sphere (with radius r, r<R), * and outside the sphere (r>R).

2) Deduce the potential at these positions.

3) Draw schematically E(r) and V(r).



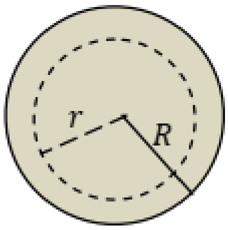
Solution of Exercise 03

1/ The electric field at every point in space is radial, depending only on r, which allows us to choose the surface of a sphere of radius r whose center coincides with the center of the charged sphere. The flux through the surface of the sphere is:

$$\phi = \oint_S \vec{E}.\overrightarrow{dS} = E.S = \frac{\sum Q_{int}}{\varepsilon_0}$$

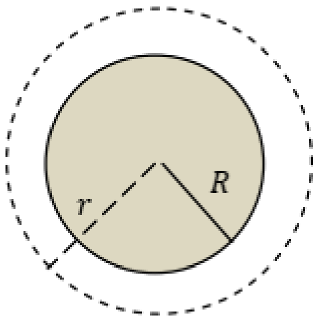
$$\Longrightarrow E.4\pi r^2 = \frac{\sum Q_{int}}{\varepsilon_0}$$

- r<R:



$E_1.4\pi r^2 = \frac{0}{\varepsilon_0} \Longrightarrow E_1 = 0$

- r>R:



$$E_2.4\pi r^2 = \frac{\rho.V_R}{\varepsilon_0} \Longrightarrow E_2.4\pi r^2 = \frac{\sigma.4\pi R^2}{\varepsilon_0}$$

$$\Longrightarrow E_2 = \frac{\sigma.R^2}{\varepsilon_0}.\frac{1}{r^2}$$

2/ Using the relation between $\vec{E}$ and V:

$\vec{E} = -\overrightarrow{grad}\,V = -\frac{\partial V}{\partial r}\overrightarrow{U_r} - \frac{1}{r}.\frac{\partial V}{\partial \theta}\overrightarrow{U_\theta} - \frac{1}{r\sin\theta}.\frac{\partial V}{\partial \varphi}\overrightarrow{U_\varphi}$

The field change only diagonally (with r), so: $\vec{E} = -\frac{\partial V}{\partial r}\overrightarrow{U_r} \Rightarrow E = -\frac{\partial V}{\partial r} \Rightarrow V = -\int E.dr$

- r<R:

$V_1 = -\int E_1.dr \Rightarrow V_1 = C_1$

- r>R:

$V_2 = -\int E_2.dr = -\int \frac{\sigma.R^2}{\varepsilon_0}.\frac{1}{r^2}dr \Rightarrow V_2 = \frac{\sigma.R^2}{\varepsilon_0}.\frac{1}{r} + C_2$

- Find the constants C_1 and C_2,

We use the limiting conditions: $V_2(\infty) = 0 \Rightarrow C_2 = 0$

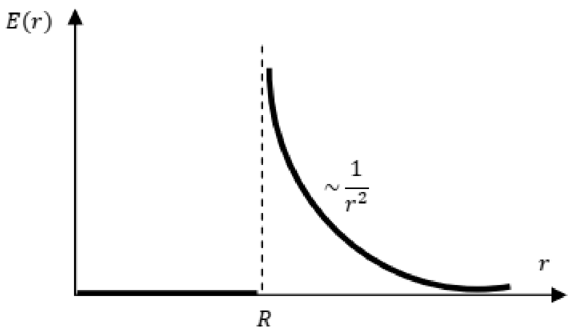
$\Rightarrow V_2 = \frac{\sigma.R^2}{\varepsilon_0}.\frac{1}{r}$

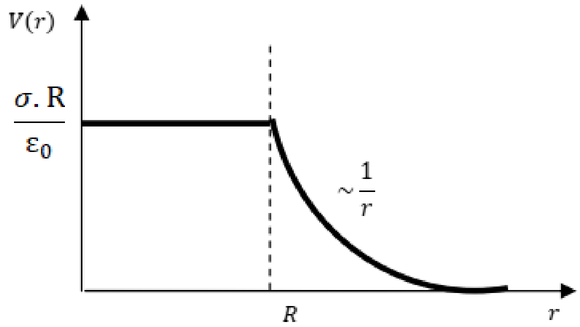
According to the continuity of potential at r=R: $V_2(r = R) = V_1(r = R)$

$\Rightarrow \frac{\sigma.R}{\varepsilon_0}. = C_1$

So, $V_1 = \frac{\sigma.R}{\varepsilon_0}$

3/





Exercise 04

1) Using the Gauss's theorem, calculate the electric field created by an uniformly charged infinite cylinder with radius R and having a volume charge density ρ at: * inside the sphere (with radius r, r<R), * and outside the sphere (r>R).

2) Deduce the potential at these positions.

Solution of Exercise 04

1/ The electric field is uniform at the Gauss' surface which takes a cylindrical form,

- Case I: r<R:

We have, $\oint_S \vec{E}.\overrightarrow{dS_1} + \oint_S \vec{E}.\overrightarrow{dS_2} + \oint_S \vec{E}.\overrightarrow{dS_3} = \frac{\sum Q_{int}}{\varepsilon_0}$

$$E.S_1.\cos 90 + E.S_2.\cos 90 + E.S_3.\cos 0 = \frac{\sum Q_{int}}{\varepsilon_0}$$

$$\oint_S \overrightarrow{E_1}.\overrightarrow{dS_3} = \frac{\sum Q_{int}}{\varepsilon_0} \Longrightarrow E_1.S_3.\cos 0 = \frac{\rho.V_r}{\varepsilon_0}$$

$$\Longrightarrow E_1.2\pi rh = \frac{\rho.\pi r^2 h}{\varepsilon_0} \Longrightarrow E_1 = \frac{\rho.r}{2\varepsilon_0}$$

- Case II: r=R:

$$\oint_S \overrightarrow{E_2}.\overrightarrow{dS_3} = \frac{\sum Q_{int}}{\varepsilon_0} \Longrightarrow E_2.S_3.\cos 0 = \frac{\rho.V_R}{\varepsilon_0}$$

$$\Longrightarrow E_2.2\pi Rh = \frac{\rho.\pi R^2 h}{\varepsilon_0} \Longrightarrow E_2 = \frac{\rho.R}{2\varepsilon_0}$$

- Case III: r>R:

$$\Longrightarrow \oint_S \overrightarrow{E_3}.\overrightarrow{dS_3} = \frac{\sum Q_{int}}{\varepsilon_0} \Longrightarrow E_3.S_3.\cos 0 = \frac{\rho.V_R}{\varepsilon_0}$$

$$\Longrightarrow E_3.2\pi rh = \frac{\rho.\pi R^2 h}{\varepsilon_0} \Longrightarrow E_3 = \frac{\rho.R^2}{2\varepsilon_0}.\frac{1}{r}$$

2/ The relation between $\vec{E}$ and V is: $\vec{E} = -\overrightarrow{grad}\,V \Longrightarrow E = -\frac{\partial V}{\partial r} \Longrightarrow V = -\int E.dr$

- Case I: r<R:

$$V_1 = -\int E_1.dr \Longrightarrow V_1 = -\int \frac{\rho.r}{2\varepsilon_0}.dr$$

$$\Longrightarrow V_1 = -\frac{\rho.r^2}{4\varepsilon_0} + C_1$$

- Case II: r=R:

$V_2 = -\int E_2.\mathrm{dr} \Rightarrow V_2 = -\int \frac{\rho.R}{2\varepsilon_0}.\mathrm{dr}$

$\Rightarrow V_2 = -\frac{\rho.R^2}{4\varepsilon_0} + C_2$

- Case III: r>R:

$V_3 = -\int E_3.\mathrm{dr} \Rightarrow V_3 = -\int \frac{\rho.R^2}{2\varepsilon_0}.\frac{1}{r}.\mathrm{dr}$

$\Rightarrow V_3 = -\frac{\rho.R^2}{2\varepsilon_0}.\ln(r) + C_3$

To find the constants C_1, C_2 and C_3, we consider that the electric potential is zero at the axis of the cylinder:

If $r = 0 \Rightarrow V_1 = 0 \Rightarrow C_1 = 0$

So, $\Rightarrow V_1 = -\frac{\rho.r^2}{4\varepsilon_0}$

According to the continuity of potential at r=R: $V_1(r = R) = V_2(r = R)$

$\Rightarrow -\frac{\rho.R^2}{4\varepsilon_0} = -\frac{\rho.R^2}{4\varepsilon_0} + C_2 \Rightarrow C_2 = 0$

$\Rightarrow V_2 = -\frac{\rho.R^2}{4\varepsilon_0}$

Also, according to the continuity of potential at r=R: $V_2(r = R) = V_3(r = R)$

$\Rightarrow -\frac{\rho.R^2}{4\varepsilon_0} = -\frac{\rho.R^2}{2\varepsilon_0}.\ln(R) + C_3 \Rightarrow C_3 = \frac{\rho.R^2}{2\varepsilon_0}.\ln(R) - \frac{\rho.R^2}{4\varepsilon_0}$

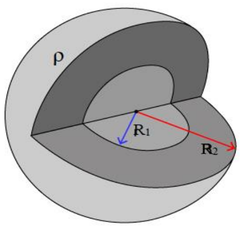
$\Rightarrow V_3 = -\frac{\rho.R^2}{2\varepsilon_0}.\ln(r) + \frac{\rho.R^2}{2\varepsilon_0}.\ln(R) - \frac{\rho.R^2}{4\varepsilon_0}$

$\Rightarrow V_3 = \frac{\rho.R^2}{2\varepsilon_0}.\ln(R/r) - \frac{\rho.R^2}{4\varepsilon_0}$

Exercise 05

1) Using the Gauss's theorem, calculate the electric field created by an uniformly charged hollow sphere with an inner radius R_1 and outer radius R_2 and having a volume charge density ρ.

2) Deduce the potential at these positions.



Solution of exercise 05

1/ The electric field at every point in space is radial, depending only on r, which allows us to choose the surface of a sphere of radius r whose center coincides with the center of the charged hollow sphere. The flux through the surface of the sphere is:

$$\phi = \oint_S \vec{E}.\overrightarrow{dS} = E.S = \frac{\sum Q_{int}}{\varepsilon_0}$$

- Case I: $r<R_1$:

$$\sum Q_{int} = 0 \Rightarrow E_1 = 0$$

- Case II: $R_1 < r < R_2$:

$$\oint_S \overrightarrow{E_2}.\overrightarrow{dS_2} = \frac{\sum Q_{int}}{\varepsilon_0} \Rightarrow E_2.S_2.cos0 = \frac{\rho.V}{\varepsilon_0} = \frac{\rho.(V_r - V_{R_1})}{\varepsilon_0}$$

$$\Rightarrow E_2.4\pi r^2 = \frac{\rho.\frac{4}{3}\pi(r^3 - R_1^3)}{\varepsilon_0} \Rightarrow E_2 = \frac{\rho.(r^3 - R_1^3)}{3\varepsilon_0}.\frac{1}{r^2}$$

$$\Rightarrow E_2 = \frac{\rho.(r^3 - R_1^3)}{3\varepsilon_0}.\frac{1}{r^2} = \frac{\rho}{3\varepsilon_0}.\left(r - \frac{R_1^3}{r^2}\right)$$

- Case III: $r>R_2$:

$$\oint_S \overrightarrow{E_3}.\overrightarrow{dS_3} = \frac{\sum Q_{int}}{\varepsilon_0} \Rightarrow E_3.S_3.cos0 = \frac{\rho.V}{\varepsilon_0} = \frac{\rho.(V_{R_2} - V_{R_1})}{\varepsilon_0}$$

$$\Rightarrow E_3.4\pi r^2 = \frac{\rho.\frac{4}{3}\pi(R_2^3 - R_1^3)}{\varepsilon_0} \Rightarrow E_3 = \frac{\rho.(R_2^3 - R_1^3)}{3\varepsilon_0}.\frac{1}{r^2}$$

2/ Deduce V:

The relation between $\vec{E}$ and V is: $\vec{E} = -\overrightarrow{grad}\,V \Rightarrow E = -\frac{\partial V}{\partial r} \Rightarrow V = -\int E.dr$

- Case I: $r<R_1$:

$$V_1 = -\int E_1.dr \Rightarrow V_1 = -\int 0.dr$$

$$\Rightarrow V_1 = C_1$$

- Case II: $R_1 < r < R_2$:

$$V_2 = -\int E_2.dr \Rightarrow V_2 = -\int \frac{\rho}{3\varepsilon_0}.\left(r - \frac{R_1^3}{r^2}\right).dr$$

$$\Rightarrow V_2 = -\frac{\rho}{3\varepsilon_0}.\left(\frac{r^2}{2} + \frac{R_1^3}{r}\right) + C_2$$

- Case III: r>R_2:

$$V_3 = -\int E_3.dr \Rightarrow V_3 = -\int \frac{\rho.(R_2^3 - R_1^3)}{3\varepsilon_0}.\frac{1}{r^2}.dr$$

$$\Rightarrow V_3 = \frac{\rho.(R_2^3 - R_1^3)}{3\varepsilon_0}.\frac{1}{r} + C_3$$

- Find the constants C_1, C_2 and C_3

We use the limiting conditions: $V_3(\infty) = 0 \Rightarrow C_3 = 0$

$$\Rightarrow V_3 = \frac{\rho.(R_2^3 - R_1^3)}{3\varepsilon_0}.\frac{1}{r}$$

From the continuity of potential at r=R_2: $V_3(R_2) = V_2(R_2)$

$$\Rightarrow \frac{\rho.(R_2^3 - R_1^3)}{3\varepsilon_0}.\frac{1}{R_2} = -\frac{\rho}{3\varepsilon_0}.\left(\frac{{R_2}^2}{2} + \frac{R_1^3}{R_2}\right) + C_2$$

$$\Rightarrow C_2 = \frac{\rho}{3\varepsilon_0}.\left(\frac{3R_2^2}{2}\right) = \frac{\rho}{2\varepsilon_0}.(R_2^2)$$

$$\Rightarrow V_2 = -\frac{\rho}{3\varepsilon_0}.\left(\frac{r^2}{2} + \frac{R_1^3}{r}\right) + \frac{\rho}{2\varepsilon_0}.(R_2^2)$$

From the continuity of potential at r=R_1: $V_2(R_1) = V_1(R_1)$

$$\Rightarrow -\frac{\rho}{3\varepsilon_0}.\left(\frac{{R_1}^2}{2} + \frac{R_1^3}{R_1}\right) + \frac{\rho}{2\varepsilon_0}.(R_2^2) = C_1$$

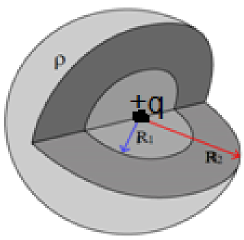
$$\Rightarrow C_1 = -\frac{\rho}{2\varepsilon_0}.(R_1^2) + \frac{\rho}{2\varepsilon_0}.(R_2^2) = V_1$$

Exercises without solution

Exercise 01

1) Using the Gauss theorem, calculate the electric field created by a positive charge located at the origin of a neutral hollow metallic sphere with an inner radius R_1 and outer radius R_2, in inside the inner sphere (with radius r, r<R_1), in the hollow sphere (R_1< r< R_2) and outside the outer sphere (r>R_2).

2) Deduce the potential at these positions.



Exercise 02

1) Using the Gauss's theorem, calculate the electric field created by an uniformly charged infinite cylinder with radius R and having a surface charge density σ at: * inside the sphere (with radius r, r<R), * and outside the sphere (r>R).

2) Deduce the potential at these positions.

IV.1/ Conductors

A conductor is a material in which charges move when an electrostatic force is applied to them.

IV.1.1/ Conductor in electrostatic equilibrium

A conductor is in electrostatic equilibrium if the mobile charges do not move, it follows that the charge distribution remains constant over time.

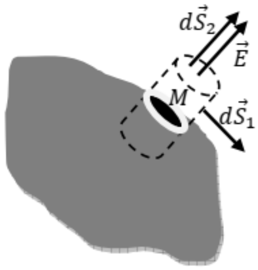
IV.1.2/ Properties of Conductor in electrostatic equilibrium

- The electrostatic field inside the conductor is zero $\vec{E} = \vec{0}$ because the charges are not moving and $\overrightarrow{F_{ext}} = \vec{0}$.
- A conductor constitutes an equipotential volume (V = Cte) because the electrostatic field inside the conductor is zero $\vec{E} = \vec{0}$.
- The surface of the conductor is an equipotential, because the electrostatic field lines are perpendicular to the equipotential surfaces.
- The total charge is zero in any region internal to the conductor.
- All excess charges are located on the surface of the conductor.
- The electrostatic field near the surface of a conductor is $E = \frac{\sigma}{\varepsilon_0}$.

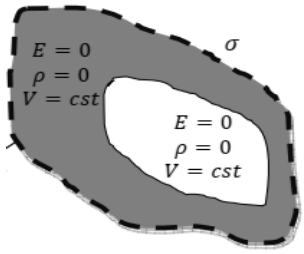
Demonstration: The surface charge density σ near point M that in the immediate vicinity of the conductor, the charge inside the Gauss's surface (the cylinder) is equal to: $dq = \sigma dS_2$, so the flux is,

$$d\phi = \vec{E}.\overrightarrow{dS_1} + \vec{E}.\overrightarrow{dS_2} = \frac{\sum Q_{int}}{\varepsilon_0}$$

Since, $\vec{E}.\overrightarrow{dS_1} = \vec{0}$, so, $E.dS_2 = \frac{\sigma dS_2}{\varepsilon_0} \Longrightarrow E = \frac{\sigma}{\varepsilon_0}$



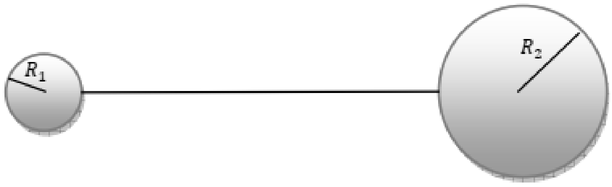
- The previous properties of the conductor remain true for a hollow conductor.



IV.1.3/ Power of sharp surfaces

Experimentally, it has been shown that the distribution of charges on the surface of the conductor does not correspond to a constant surface density. Rather, charges tend to accumulate in surface regions whose radius of curvature is small. This phenomenon is called the power of sharp surfaces. The surface density is large in sharp parts. The same applies to the electric field strength, which is large near the sharp head.

Demonstration: We have two spheres with radii of R_1 and R_2 ($R_2 > R_1$) connected by a wire and charged with q_1 and q_2 with densities of charge σ_1 and σ_2, respectively.



The spheres have the same potential:

$$V(R_1) = V(R_2)$$

$$\frac{kq_1}{R_1} = \frac{kq_2}{R_2} \Longrightarrow \frac{\sigma_1 4\pi \mathrm{R}_1^2}{R_1} = \frac{\sigma_2 4\pi \mathrm{R}_2^2}{R_2}$$

$$\sigma_1 . R_1 = \sigma_2 . R_2$$

The surface charge density is greater the smaller the radius of curvature of the surface.

IV.1.4/ Capacity of a conductor in electrical equilibrium

In the case of an isolated conductor of charge q, this results a potential V proportional to q:

$$\mathrm{C} = \frac{\mathrm{q}}{\mathrm{V}}$$

The constant C is the capacity of a conductor; its unit is Farad [F].

Example

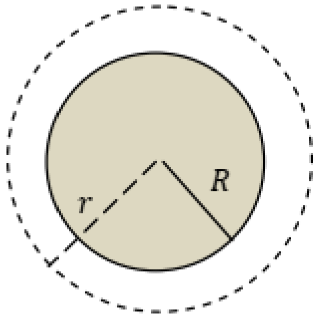
Calculate the capacity of a spherical conductor with radius R.

Solution

1/ Calculate E

Using the Gauss's theorem, $\phi = \vec{E}.\vec{S} = \frac{\sum Q_{int}}{\varepsilon_0} \Rightarrow E.4\pi.r^2 = \frac{q}{\varepsilon_0} \Rightarrow E = \frac{q}{4\pi\varepsilon_0 r^2}$

Dashed circle is the Gauss's surface.



2/ Calculate V

$$dV = -E.dr \Rightarrow V = \frac{q}{4\pi\varepsilon_0 r} + A$$

$V(\infty) = 0 \Rightarrow A = 0 \Rightarrow V = \frac{q}{4\pi\varepsilon_0 r}$

3/ Deducing C

$$C = \frac{q}{V} = 4\pi\varepsilon_0 r$$

At the surface, r = R, $\Rightarrow$ $C = 4\pi\varepsilon_0 R$

We notice from this example that the capacity of this conductor is related only to its radius.

IV.1.5/ Internal energy of Conductor (E_p)

Let's consider an isolated conductor. C is its capacitance, Q is its charge and V is its potential at equilibrium state. The internal energy of the conductor is defined as the work necessary to charge it by Q.

$$dE_p = qdV \Rightarrow E_p = \int_0^Q qdV = \frac{1}{C}\int_0^Q qdq \Rightarrow E_p = \frac{1}{2C}Q^2 = \frac{1}{2}CV^2 = \frac{1}{2}QV$$

IV.1.6/ Effect of an external electric field on an isolated neutral conductor

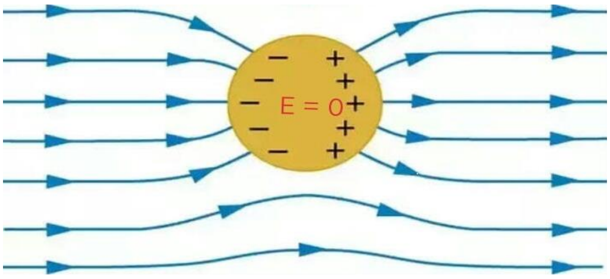
The figure below shows a spherical conductor in static equilibrium (i.e. $\overrightarrow{E_0} = \vec{0}$)aaffected by an originally uniform electric field $\overrightarrow{E_{ext}}$. Free charges move within the conductor, polarizing it, until the electric field lines are perpendicular to the surface. The field lines end on excess negative charge on one section of the surface and begin again on excess positive charge on the opposite side.

No electric field exists inside the conductor, since free charges in the conductor would continue moving in response to any field until it was neutralized.

The charge of the conductor did not change; all that happened was a redistribution of charges and a change in the potential.

The electric field at any point M inside the conductor is zero:

$$\overrightarrow{E_{int}}(M) = \overrightarrow{E_{ext}} + \overrightarrow{E_0} = \vec{0}$$

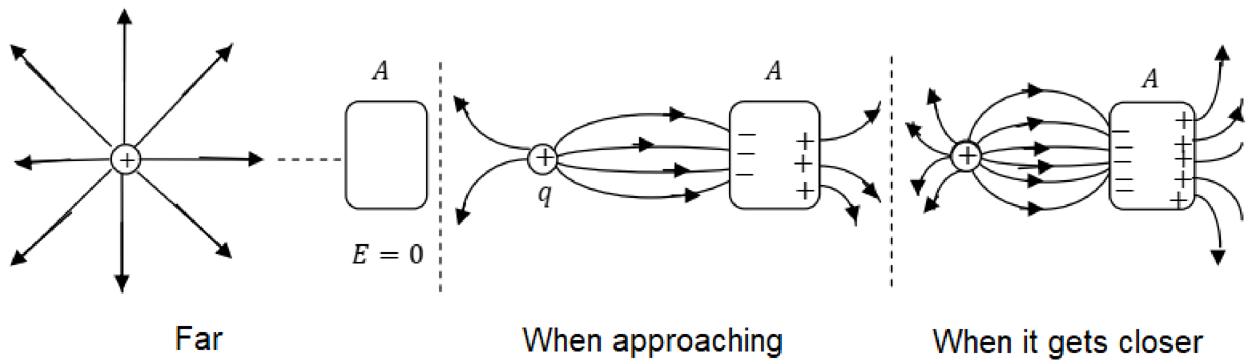


IV.1.7/ Influence phenomena between charged conductors

IV.1.7.1/ Partial influence of point charge on conductor

Let's see the effect of a positive charge on an isolated and neutral conductor.

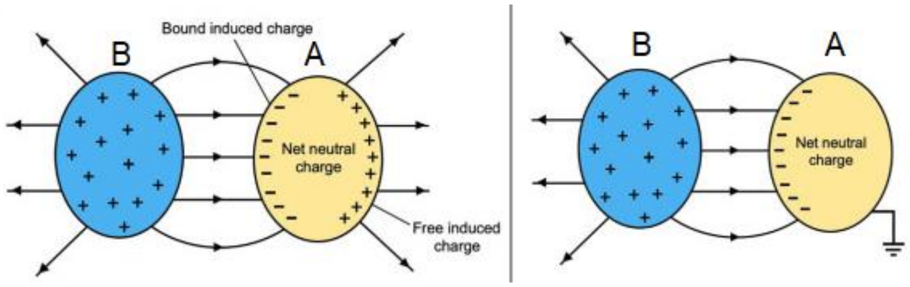
After approaching the charge q to the conductor, the free electrons of conductor A will migrate towards positive charge q under the action of the electrostatic field which results from electrification by influence, which contributes to the creation of an electric field inside the conductor. This phenomenon is called partial influence.



IV.1.7.2/ Mutual influence

Let's consider a neutral and isolated conductor A and another one charged positively. If we approach the two conductors, there will be a redistribution of charges inside them as follow. This phenomenon is called mutual influence.

If the conductor A is grounded (connected to the earth), they become as one body, the positive charges move into the earth, and the conductor potential remains zero and no line comes out of it.



IV.1.7.3/ Total influence

Two conductors are said to be in total influence if one (A) is inside the other (B). Any field line that goes out (A) can only go to (B). Let's consider:

Q_A: is the charge of conductor A.

Q_B: is the charge of conductor B.

Q_{Bi}: is the charge of internal surface of conductor B.

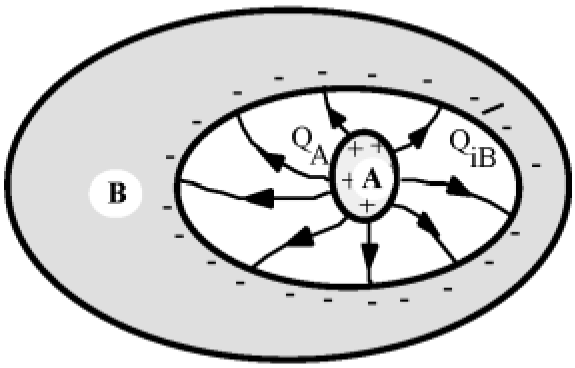
Q_{Be}: is the charge of external surface of conductor B.

- If (B) is isolated and neutral from the beginning, then according to the principle of charge conservation of the conductor B:

$$Q_{Bi} + Q_{Be} = 0 \Longrightarrow Q_{Be} = -Q_{Bi} = Q_A$$

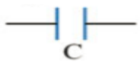
- If (B) is isolated and charged by Q_0 from the beginning, then according to the principle of charge conservation of the conductor B:

$$Q_0 = Q_{Bi} + Q_{Be} \Longrightarrow Q_{Be} = Q_0 + Q_A$$



IV.2 Capacitors

A capacitor is a device used to store electrical charge and electrical energy. Capacitors are generally with two electrical conductors separated by a distance.

The capacitor remains neutral overall, but with charges +Q and −Q residing on opposite conductors. It is schematized by: 

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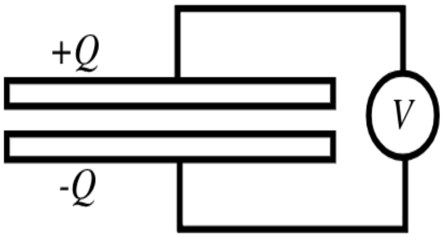


IV.2.1/ Capacitance (C)

It is a property of an electric conductor. The capacitance C is the ratio of the amount of charge q on either conductor to the potential difference V between the conductors:

$$C = Q/|V_A - V_B|$$

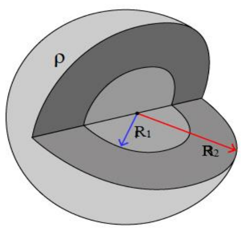
The unit of capacitance is the farad [F].

IV.2.2/ Calculating of Capacitance

- Determine the electric field $\vec{E}$ between the conductors.
- Find the potential difference between the conductors from, $V_B - V_A = -\int_A^B \vec{E}.\vec{dl}$.
- Obtained the capacitance from, $C = \frac{Q}{|V_A - V_B|}$.

Example

Calculate the capacitance C for a hollow spherical conductor with radii R_1 and R_2 ($R_1 < R_2$).



Solution

- Calculate E: Let's use the Gauss's theorem, $\oint_S \vec{E}.\vec{n}.dS = \frac{\sum Q_{int}}{\varepsilon_0}$

$$E.S = \frac{\sum Q_{int}}{\varepsilon_0} \Rightarrow E.4\pi.x^2 = \frac{Q}{\varepsilon_0} \Rightarrow E = \frac{Q}{4\pi.\varepsilon_0.x^2}$$

- Calculate $V_A - V_B$: We have, $\vec{E} = -\overrightarrow{\text{grad}}(V) \Rightarrow dV = -E.dx \Rightarrow V_B - V_A = -\int_{R_1}^{R_2} E.dx$

$\Rightarrow V_B - V_A = \frac{Q}{4\pi.\varepsilon_0}.\left[\frac{1}{x}\right]_{R_1}^{R_2} \Rightarrow V_B - V_A = \frac{Q}{4\pi.\varepsilon_0}\left[\frac{1}{R_2} - \frac{1}{R_1}\right]$

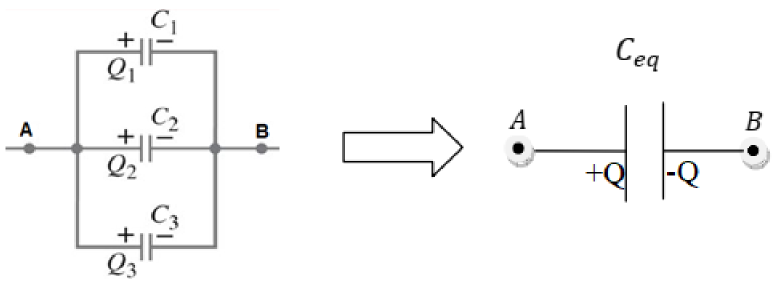
$$\Rightarrow V_A - V_B = \frac{Q}{4\pi.\varepsilon_0}\left[\frac{R_2 - R_1}{R_2 R_1}\right].$$

- Find the capacitance C: $C = \frac{Q}{|V_A - V_B|} \Rightarrow C = 4\pi.\varepsilon_0.\frac{R_2 R_1}{R_2 - R_1}$

IV.2.3/ Association of capacitors

IV.2.3.1/ Association of capacitors in parallel

Let's have three capacitors associated in parallel, as shown in below.



We have the differential potential for each capacitor is the same of $V_A - V_B$,

$V_1 = V_A - V_B = Q_1/C_1$

$V_2 = V_A - V_B = Q_2/C_2$

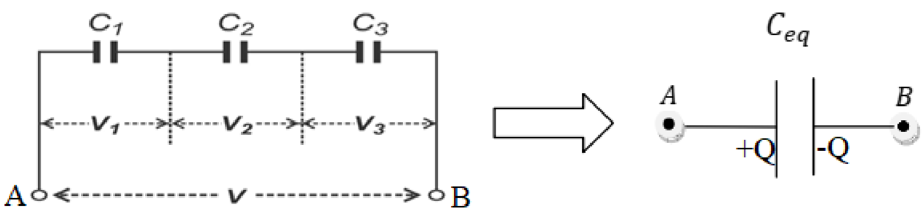
$V_3 = V_A - V_B = Q_3/C_3$

$Q_{eq} = Q_1 + Q_2 + Q_3 \Rightarrow C_{eq} = C_1 + C_2 + C_3$

So, for n capacitors associated in parallel, $C_{eq} = \sum_{i=1}^{n} C_i$

IV.2.3.2/ Association of capacitors in series

Let's have three capacitors associated in series, as shown in below.



$$V_{eq} = V_1 + V_2 + V_3 \Rightarrow \frac{Q_{eq}}{C_{eq}} = \frac{Q_1}{C_1} + \frac{Q_2}{C_2} + \frac{Q_3}{C_3}$$

We have $Q_{eq} = Q_1 = Q_2 = Q_3 \Rightarrow \frac{1}{C_{eq}} = \frac{1}{C_1} + \frac{1}{C_2} + \frac{1}{C_3}$

So, for n capacitors associated in series, $\frac{1}{C_{eq}} = \sum_{i=1}^{n} \frac{1}{C_i}$

IV.3 Exercises

Exercises with solution

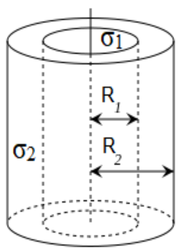
Exercise 01

Two infinite cylinders of negligible thicknesses with radius R_1 and R_2 ($R_1<R_2$) having surface charge densities σ_1 and σ_2, respectively. Using Gauss theorem:

1) Calculate the electric field in the positions: $r<R_1$, $R_1<r<R_2$ and $r>R_2$, where r is the radius of Gauss's sphere.

2) Deduce the potential at these positions.

3) Deduce the capacitance of the cylindrical capacitor between R_1 and R_2.



Solution of exercise 01

1/ Due to the symmetry, the electric field is radial and constant at every point from the surface of Gauss that takes the form of cylinder with radius r and height h,

So, from the Gauss's law:

$$\oint_S \vec{E}.\overrightarrow{dS_1} + \oint_S \vec{E}.\overrightarrow{dS_2} + \oint_S \vec{E}.\overrightarrow{dS_3} = \frac{\sum Q_{int}}{\varepsilon_0}$$

$$\vec{E}.\overrightarrow{S_1} + \vec{E}.\overrightarrow{S_2} + \vec{E}.\overrightarrow{S_3} = \frac{\sum Q_{int}}{\varepsilon_0}$$

$$E.S_1.\cos\left(\frac{\pi}{2}\right) + E.S_2.\cos\left(\frac{\pi}{2}\right) + E.S_3.\cos(0) = \frac{\sum Q_{int}}{\varepsilon_0}$$

$\Rightarrow E.S_3 = \frac{\sum Q_{int}}{\varepsilon_0} \Rightarrow E.2.\pi.r.h = \frac{\sum Q_{int}}{\varepsilon_0}$

- Case I: $r<R_1$:

Due to the surface distribution, we have, $\sum Q_{int} = 0 \Rightarrow E_1 = 0$

- Case II: $R_1 < r < R_2$:

$E_2.S_G = \frac{\sum Q_{int}}{\varepsilon_0} \Rightarrow E_2.2.\pi.r.h = \frac{\sigma_1.S_1}{\varepsilon_0} = \frac{\sigma_1.2.\pi.R_1.h}{\varepsilon_0}$

$$E_2 = \frac{\sigma_1 . R_1}{\varepsilon_0} . \frac{1}{r}$$

- Case III: $r > R_2$:

$$E_3 . S_G = \frac{\sum Q_{int}}{\varepsilon_0} \Longrightarrow E_3 . 2 . \pi . r . h = \frac{\sigma_1 . S_1 + \sigma_2 . S_2}{\varepsilon_0} = \frac{\sigma_1 . 2 . \pi . R_1 . h + \sigma_2 . 2 . \pi . R_2 . h}{\varepsilon_0}$$

$$\Longrightarrow E_3 = \left(\frac{\sigma_1 . R_1 + \sigma_2 . R_2}{\varepsilon_0} \right) . \frac{1}{r}$$

2/ Deduce V:

The relation between $\vec{E}$ and V is: $\vec{E} = -\overrightarrow{grad}\, V \Rightarrow E = -\frac{\partial V}{\partial r} \Rightarrow V = -\int E . dr$

- Case I: r<R_1:

$$E_1 = 0 \Rightarrow V_1 = C_1$$

- Case II: $R_1 < r < R_2$:

$$V_2 = -\int E_2 . dr = -\int \frac{\sigma_1 . R_1}{\varepsilon_0} . \frac{1}{r} . dr$$

$$\Rightarrow V_2 = -\frac{\sigma_1 . R_1}{\varepsilon_0} \ln(r) + C_2$$

- Case III: $r > R_2$:

$$V_3 = -\int E_3 . dr = -\int \left(\frac{\sigma_1 . R_1 + \sigma_2 . R_2}{\varepsilon_0} \right) . \frac{1}{r} . dr$$

$$\Rightarrow V_3 = -\left(\frac{\sigma_1 . R_1 + \sigma_2 . R_2}{\varepsilon_0} \right) \ln(r) + C_3$$

- Deduce the constants C_1, C_2 and C_3:

$$V_1(0) = 0 \Rightarrow C_1 = 0 = V_1$$

From the continuity of potential at r=R_1: $V_2(R_1) = V_1(R_1) = 0$

$$\Rightarrow -\frac{\sigma_1 . R_1}{\varepsilon_0} \ln(R_1) + C_2 = 0 \Rightarrow C_2 = \frac{\sigma_1 . R_1}{\varepsilon_0} \ln(R_1)$$

$$\Rightarrow V_2 = -\frac{\sigma_1 . R_1}{\varepsilon_0} \ln(r) + \frac{\sigma_1 . R_1}{\varepsilon_0} \ln(R_1) = \frac{\sigma_1 . R_1}{\varepsilon_0} \ln\left(\frac{R_1}{r} \right)$$

From the continuity of potential at r=R_2: $V_2(R_2) = V_3(R_2)$

$$\Rightarrow \frac{\sigma_1 . R_1}{\varepsilon_0} \ln\left(\frac{R_1}{R_2} \right) = -\left(\frac{\sigma_1 . R_1 + \sigma_2 . R_2}{\varepsilon_0} \right) \ln(R_2) + C_3$$

$$\Rightarrow C_3 = \frac{\sigma_1 . R_1}{\varepsilon_0} \ln(R_1) + \frac{\sigma_2 . R_2}{\varepsilon_0} \ln(R_2)$$

We get, $V_3 = -\left(\frac{\sigma_1 . R_1 + \sigma_2 . R_2}{\varepsilon_0} \right) \ln(r) + \frac{\sigma_1 . R_1}{\varepsilon_0} \ln(R_1) + \frac{\sigma_2 . R_2}{\varepsilon_0} \ln(R_2)$

3/ The relation of the capacitance is: $C = Q/|\Delta V|$

In this case, the charge Q consists of that contained in the interior cylinder, so, $Q = \sigma_1 . \mathrm{S}_1$.

The difference in potential $|\Delta V| = |V_2(\mathrm{R}_1 \rightarrow \mathrm{R}_2)| = \left|\frac{\sigma_1 . \mathrm{R}_1}{\varepsilon_0} \ln\left(\frac{R_1}{\mathrm{R}_2}\right)\right|$

So, $C = \frac{\sigma_1 . \mathrm{S}_1}{\left|\frac{\sigma_1 . \mathrm{R}_1}{\varepsilon_0} \ln\left(\frac{R_1}{\mathrm{R}_2}\right)\right|} = \frac{\sigma_1 . 2\pi . \mathrm{R}_1 . h}{\left|\frac{\sigma_1 . \mathrm{R}_1}{\varepsilon_0} \ln\left(\frac{R_1}{\mathrm{R}_2}\right)\right|} = 2\pi h \varepsilon_0 . \ln\left(\frac{R_2}{\mathrm{R}_1}\right)$

Exercise 02

A solid cylindrical conductor of radius a, length L and charge Q (with linear distribution λ) is coaxial with another cylindrical shell of negligible thickness with radius b (b>a) to form together a capacitor.

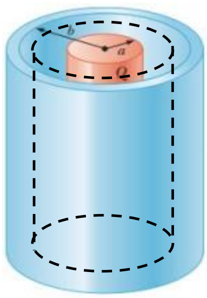
1) Find the capacitance.

Solution of exercise 02

1/ By influence the exterior cylinder is charged by (-Q), so we have a capacitor.

The relation of the capacitance is: $C = \frac{Q}{|\Delta V|}$

- In this case, the charge Q consists of that contained in the interior cylinder with radius a. Because the linear distribution in this cylinder, we get, $Q = \lambda . L$.
- ΔV is the difference of potential between the two cylinders and in order to get that we have to calculate the electric field E between the two cylinders by using the Gauss' theorem as follows,



The Gaussian surface is that represented by dashed line,

$\oiint E . S = \frac{\sum \mathrm{Q_{int}}}{\varepsilon_0} \Longrightarrow \mathrm{E}. 2\pi . r . L = \frac{\lambda . L}{\varepsilon_0}$

$$\Rightarrow \mathrm{E} = \frac{\lambda}{2\pi.\varepsilon_0.r}$$

Next, let us find the difference of potential between the two conductors, we have,

$$\vec{\mathrm{E}} = -\overrightarrow{\mathrm{grad}}\,\mathrm{V} \Rightarrow \mathrm{E} = -\frac{\partial \mathrm{V}}{\partial \mathrm{r}} \Rightarrow \Delta V = -\int_a^b \mathrm{E.dr}$$

$$\Rightarrow \Delta V = -\int_a^b \frac{\lambda}{2\pi\varepsilon_0 r}.\mathrm{dr} = -\frac{\lambda}{2\pi\varepsilon_0}\ln\left(\frac{b}{a}\right) = \frac{\lambda}{2\pi\varepsilon_0}\ln\left(\frac{a}{b}\right)$$

We get the capacitance, $C = \frac{Q}{|\Delta V|} = \frac{\lambda.L}{\left|\frac{\lambda}{2\pi\varepsilon_0}\ln\left(\frac{a}{b}\right)\right|} = 2\pi\varepsilon_0.L.\ln\left(\frac{b}{a}\right)$

Exercise 03

Let us consider a conducting sphere A of radius a carrying a positive charge $+2Q$ placed at the center of a conducting sphere B with inner radius b and outer radius c, carrying a charge $-Q$.

1) Calculate the charges Q_{Bint} and Q_{Bext} of the inner and outer surfaces of the spherical shell B at electrostatic equilibrium.
2) We connect the sphere B to the ground by a conductive wire. What are the new charges Q_{Bint} and Q_{Bext} after returning to electrostatic equilibrium?
3) Calculate the electric field at any point in space by using the theorem Gauss,
4) Deduce the electric potential at any point in space (knowing that it is zero at infinity).
5) Calculate the capacitance of the spherical capacitor consisting of the sphere of radius a and the interior face of the sphere B (radius b).

Solution of exercise 03

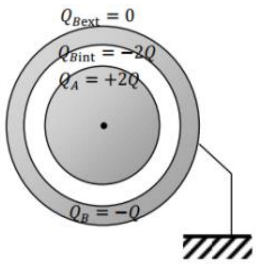
1/ At electrostatic equilibrium, and by influence, we have,

$Q_{Bint} = -Q_A = -2Q$

On the other hand, the principle of conservation of charges in conductor (B):

$Q_{Bint} + Q_{Bext} = -Q \Rightarrow Q_{Bext} = -Q - Q_{Bint} = -Q + 2Q$

$\Rightarrow Q_{Bext} = Q.$



2/ After we connect the conductor B to the ground (i.e. electrostatic equilibrium), there was a movement of charges of its outer surface into the ground, so, $Q_{Bext} = 0$ ($V_B = 0$, reference potential to the earth).

- On the inner surface of the conductor (B), the charge is maintained and remain connected by attraction to the those of surface A, so, $Q_{Bint} = -Q_A = -2Q$.

3/ $\oiint E.S = \frac{\sum Q_{int}}{\varepsilon_0}$, E is radial $\Rightarrow \mathrm{E.S} = \mathrm{E}.4\pi r^2 = \frac{\sum Q_{int}}{\varepsilon_0}$. We have 4 cases:

- $0 \leq r \leq a$: $\sum Q_{int} = 0 \Rightarrow E_1 = 0$
- $a \leq r \leq b$: $\sum Q_{int} = 2Q \Rightarrow E_2 = \frac{k2Q}{r^2}$
- $b \leq r \leq c$: $\sum Q_{int} = 2Q - 2Q \Rightarrow E_3 = 0$
- $c \leq r$: $\sum Q_{int} = 2Q - 2Q \Rightarrow E_4 = 0$

4/ The relation between $\vec{E}$ and V is: $\vec{E} = -\overrightarrow{grad}\, V \Rightarrow E = -\frac{\partial V}{\partial r} \Rightarrow V = -\int E.dr$

- $E_1 = 0 \Rightarrow V_1 = C_1$
- $E_2 = \frac{k2Q}{r^2} \Rightarrow V_2 = \frac{k2Q}{r} + C_2$
- $E_3 = 0 \Rightarrow V_3 = C_3$
- $E_4 = 0 \Rightarrow V_4 = C_4$

- Find the constants C_1, C_2, C_3 and C_4:

We use the limiting conditions: $V_4(\infty) = 0 \Rightarrow C_4 = 0 \Rightarrow V_4 = 0$

From the continuity of potential at r=c: $V_4(c) = V_3(c), \Rightarrow C_3 = 0 \Rightarrow V_3 = 0$

From the continuity of potential at r=b: $V_3(b) = V_2(b), \Rightarrow C_2 = -\frac{k2Q}{b} \Rightarrow V_2 = k2Q\left(\frac{1}{r} - \frac{1}{b}\right)$

From the continuity of potential at r=a: $V_2(a) = V_1(a), \Rightarrow C_1 = \frac{k2Q}{a} - \frac{k2Q}{b}$

$\Rightarrow V_1 = k2Q\left(\frac{1}{a} - \frac{1}{b}\right) = \frac{k2Q(b-a)}{ab}$.

5/ The capacitance of the spherical capacitor consisting of the sphere of radius a and the interior face of the sphere B (radius b) is:

$$C = \frac{q}{V} = \frac{2Q}{V_1 - V_3}$$

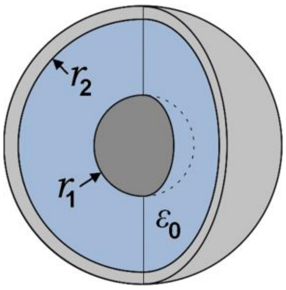
$\Rightarrow\; C = 4\pi.\varepsilon_0.\frac{a.b}{(b-a)}$.

Exercise 04

We consider a capacitor as sphere with inner radius r_1 and outer radius r_2. The inner sphere contains a charge Q, and by influence the outer one contains – Q.

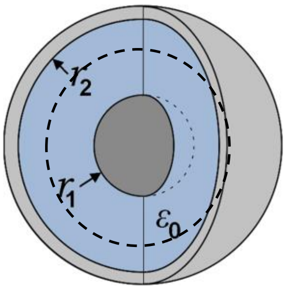
1) Calculate the electric field in the position between r_1 and r_2.

2) Deduce the potential and then the capacity C.



Solution of exercise 04

1/ Let us use the Gauss's theorem, $\oint_S \vec{E}.\overrightarrow{dS} = \frac{\sum Q_{int}}{\varepsilon_0}$



The surface of Gauss is that represented by dashed line,

The sphere with radius r_1 contains Q charge $\Longrightarrow E.S = \frac{\sum Q_{int}}{\varepsilon_0} \Longrightarrow E.4\pi.r^2 = \frac{Q}{\varepsilon_0}$

$$\Longrightarrow \quad E = \frac{Q}{4\pi.\varepsilon_0.r^2} = \frac{kQ}{r^2}$$

2/ We have, $\vec{E} = -\overrightarrow{\text{grad}}(V) \Longrightarrow dV = -E.dr \Longrightarrow V_{r_2} - V_{r_1} = -\int_{r_1}^{r_2} E.dr$

$\Longrightarrow V_{r_2} - V_{r_1} = \frac{Q}{4\pi.\varepsilon_0}.\left[\frac{1}{r}\right]_{r_1}^{r_2} \Longrightarrow V_{r_2} - V_{r_1} = \frac{Q}{4\pi.\varepsilon_0}\left[\frac{1}{r_2} - \frac{1}{r_1}\right]$

$$\Longrightarrow V_{r_1} - V_{r_2} = \frac{Q}{4\pi.\varepsilon_0}\left[\frac{r_2 - r_1}{r_2 r_1}\right].$$

- Find the capacitance C: $C = \frac{Q}{|V_{r_1} - V_{r_2}|} \Longrightarrow C = 4\pi.\varepsilon_0.\frac{r_2 r_1}{r_2 - r_1}$

We have, $r_1 \approx r_2 \Longrightarrow r_2 r_1 \approx r^2$ and $r_2 - r_1 = e$

So, $C = 4\pi.\varepsilon_0.\frac{r^2}{e} = .\frac{\varepsilon_0.S}{e}$

Where, S is the surface area of one of the corners and e is the distance between the two corners.

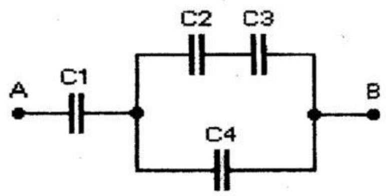
Exercise 05

Let consider the capacitors in figure as below:

Where: C_1= 6μF, C_2= 2μF, C_3= 4μF and C_4= 6μF.

1) Calculate the equivalent capacity C_{AB}.

2) If the potential V_{AB} = 2000V, calculate the charge and potential for each capacitor.



Solution of exercise 05

1/ We know that:

For n capacitors associated in parallel, $C_{eq} = \sum_{i=1}^{n} C_i$ and for n capacitors associated in series,

$\frac{1}{C_{eq}} = \sum_{i=1}^{n} \frac{1}{C_i}$,

$$\frac{1}{C'} = \frac{1}{C_2} + \frac{1}{C_3} \Longrightarrow C' = \frac{C_2 . C_3}{C_2 + C_3} = \frac{4}{3} \mu F$$

$$C'' = C_4 + C' = \frac{22}{3} \mu F$$

So, $\frac{1}{C_{AB}} = \frac{1}{C_1} + \frac{1}{C''} \Longrightarrow C_{AB} = \frac{C''.C_1}{C''+C_1} = 3.3 \mu F$.

2/ The charges that crossed C_1 are the same that crossed C_{AB}, so,

$\Longrightarrow Q_1 = Q_{AB} = C_{AB}.V_{AB} = 3.3 \times 10^{-3} \times 2000$

$\Longrightarrow Q_1 = 6.6\ mC$ and $V_1 = \frac{Q_1}{C_1} = 1100\ V$

We have, $V_4 = V_{AB} - V_1 = 900\ V$ and $Q_4 = C_4 . V_4 = 5.3\ mC$

The potential between C_2 and C_3 is V_4 and we have, $Q_2 = Q_3 = C' . V_4 = 1.2\ mC$

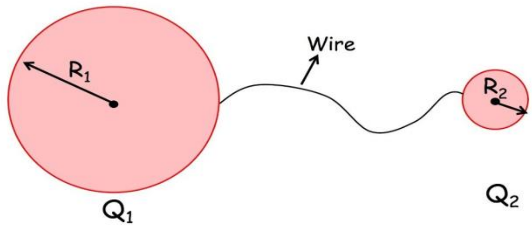
So, $V_2 = \frac{Q_2}{C_2} = 600\ V$ and $V_3 = \frac{Q_3}{C_3} = 300\ V$.

V.1/ Electrical current (I)

Consider two conductors A and B in electrostatic equilibrium and having V_A and V_B electrical potential, respectively. If we connect them by a wire, a temporary electric current appears. It ends once the conductors reach a state of electrostatic equilibrium.

This means that when the conductors come into contact, the charges move from one to the other.

$$Q_1 + Q_2 = Q_1' + Q_2' \quad (Conservation\ of\ charges)$$



Therefore, the electric current (I) can be defined as the amount of charges dQ passing through a section S during a time dt:

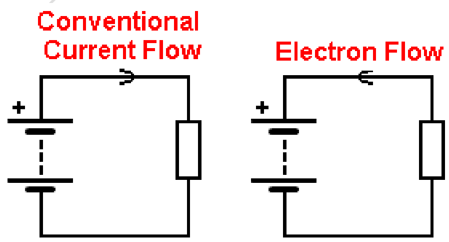
$$I = \frac{dQ}{dt}$$

The unit of electric current on international scale is the Ampere with symbol [A].

V.1.1/ Conventional sense of electric current

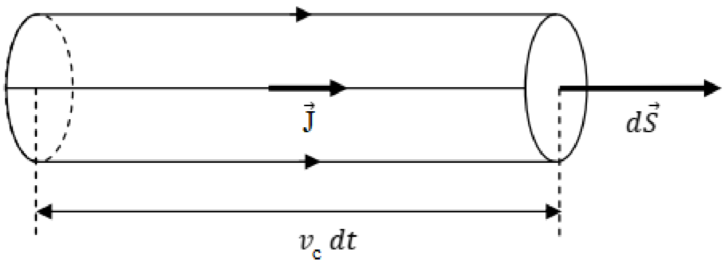
By convention, electric current always flows from the positive terminal + to the negative terminal − outside the generator (the opposite direction of electrons).

The direction of the current is represented by an arrow placed on a connection wire, and oriented according to the conventional direction of electric current.



V.1.2/ Vector of current density ($\vec{J}$)

Consider a conductor in the form of a cylinder, through which a linear current of intensity dI flows, and let's be $\vec{v_c}$ the average velocity of the free charges and ρ is its volume density. The volume of the cylinder is $dV = v_c.dt.dS$.



The amount of charge dq that passes section dS during a period of time dt is:

$$dq = \rho.dV = \rho.v_c.dt.dS \Rightarrow dI = \frac{dq}{dt} = \rho.v_c.dS \Rightarrow J = \frac{dI}{dS} = \rho.v_c$$

$$\Rightarrow \vec{J} = \rho.\vec{v_c} = n.q.\vec{v_c}$$

Where n is the number of free charges per unit of volume and the q is the algebraic value of each free charge.

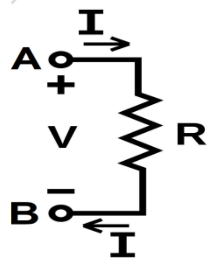
V.2/ Law of Ohm

The ratio of the potential difference V_{AB} between two points A and B of a homogeneous metallic conductor located at a constant temperature, to the electric current I is constant, and we call this constant the electrical resistance R (Its unit is Ohm, Ω).

So, we can write,

$$R = \frac{V}{I}\ or\ V = R.I$$

This expression is the macroscopic law of Ohm.



V.3/ Electric conductivity and resistivity

Consider a cylindrical conductor that has length ℓ and section S, exposed to electric field $\vec{E}$. The potential difference V between its two sides is, $V = \int_A^B \vec{E}.\vec{dl} = E.\ell$.

We have, $V = R.I = E.\ell \Rightarrow R.J.S = E.\ell$

We get a new expression of current density as,

$$J = \frac{\ell}{RS}.E = \sigma.E \quad \Rightarrow \quad \vec{J} = \sigma.\vec{E}$$

This expression is the microscopic Ohm's law.

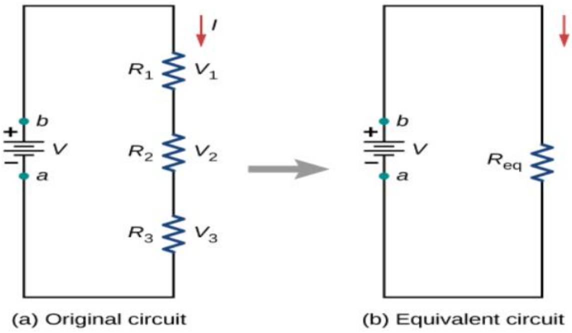
The constant $\sigma = \frac{\ell}{RS}$ is called the electric conductivity. Its unit is $\Omega^{-1}.\mathrm{m}^{-1}$.

The reciprocal of the conductivity is the resistivity ρ (its unit is Ω. m), where, $\rho = \frac{1}{\sigma} = \frac{RS}{\ell}$.

V.4/ Combination of resistances

V.4.1/ Series combination

Resistors are said to be in series when the current flows through the resistors is the same. Consider the following circuit:



$$V_a - V_b = V_1 + V_2 + V_3$$

$$\Longrightarrow R_{eq}I = R_1 I + R_1 I + R_1 I$$

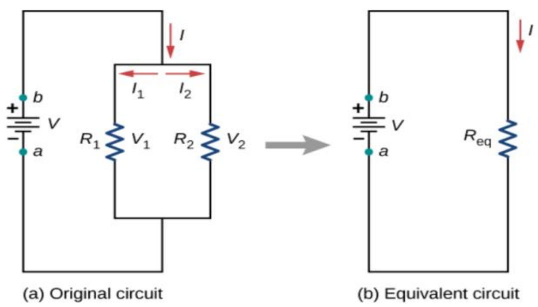
$$\Longrightarrow R_{eq} = R_1 + R_1 + R_1$$

The equivalent resistance of a set of resistors in a series connection is equal to the algebraic sum of the individual resistances. So for n resistors:

$$R_{eq} = \sum_{i=1}^{n} R_i$$

V.4.2/ Parallel combination

Resistors are in parallel when the potential drop across each resistor is the same. Consider the following circuit:



$$I_{eq} = I_1 + I_2$$

$$\Longrightarrow \frac{V_a - V_b}{R_{eq}} = \frac{V_1}{R_1} + \frac{V_2}{R_2}$$

$$\Longrightarrow \frac{1}{R_{eq}} = \frac{1}{R_1} + \frac{1}{R_2}$$

So for n resistors in a parallel connection:

$$\frac{1}{R_{eq}} = \sum_{i=1}^{n} 1/R_i$$

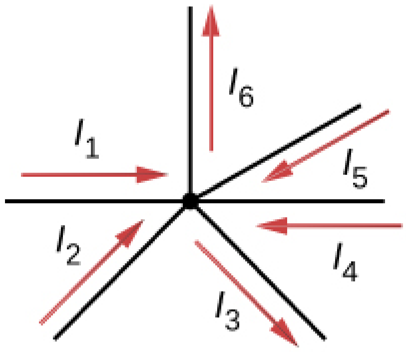
V.5/ Kirchhoff's Rules

Gustav Kirchhoff (1824-1887) had invented rules known by Kirchhoff's rules that can be used to determine the intensity of electric current in the different branches of complex circuit.

V.5.1/ Kirchhoff's junction Rule

The sum of all currents entering a junction must equal the sum of all currents leaving the junction:

$$\sum I_{int} = \sum I_{out}$$



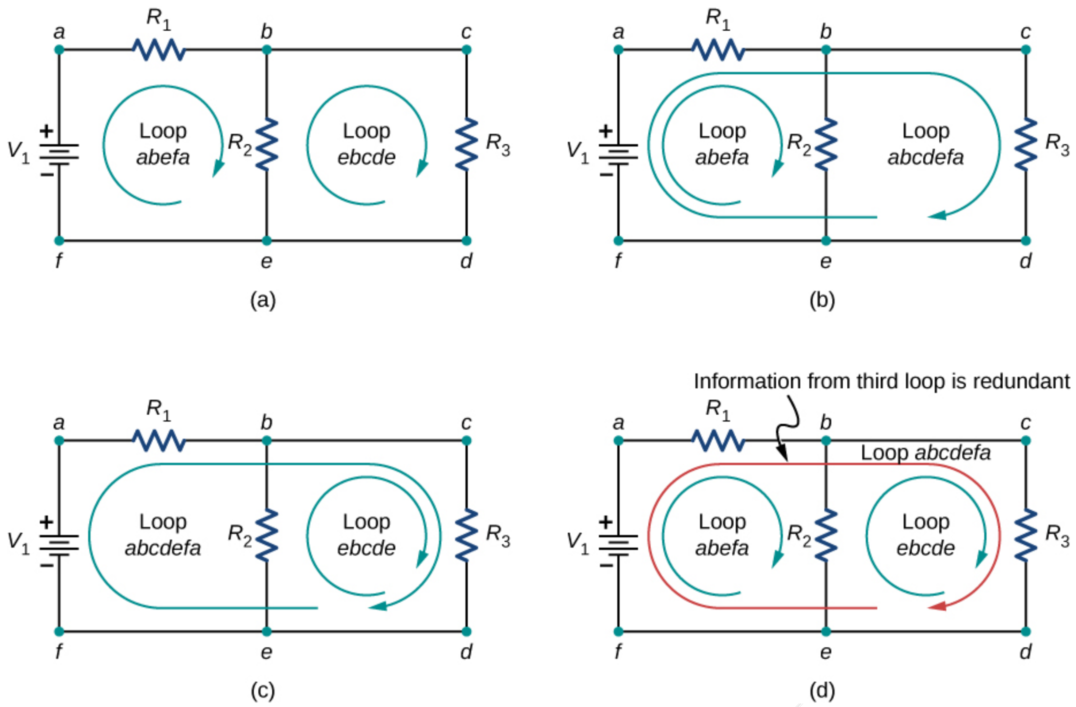
$$I_1 + I_2 + I_4 + I_5 = I_3 + I_6$$

V.5.2/ Kirchhoff's loop Rule

The algebraic sum of changes in potential around any closed circuit path (loop) must be zero:

$$\sum V = 0$$

In the figure below, panels (a)–(c) are sufficient to analyze of the circuit. The two loops shown in each case contain all the circuit elements necessary to solve the circuit completely.



V.6 Electric power

In an electric circuit, there is a loss in electric potential energy. The rate of energy loss is the power P delivered to the circuit elements. Its unit is Joule [J].

$$P = R.I^2 = I.V$$

V.7/ Exercises

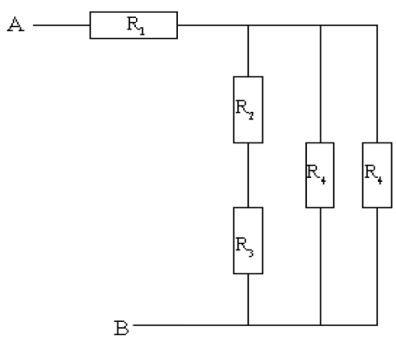
Exercises with solution

Exercise 01

1/ Calculate the equivalent resistance between points A and B for the combination in below.

2/ If the electric potential between A and B is V_{AB}= 12 V, calculate the electric current I_{AB}.

Where: R_1=R_3=100Ω, R_2=150Ω, R_4=500Ω.



Solution of exercise 01

1/ The two resistors R_2 and R_3 are connected in series, so, $R' = R_2 + R_3$,

The resistor R' and the other two resistors R_4 are connected in parallel, so, $\frac{1}{R''} = \frac{1}{R'} + \frac{1}{R_4} + \frac{1}{R_4}$

$$\Rightarrow R'' = \frac{R' R_4}{R_4 + 2R'}$$

Then, the resistors R_1 and R'' become connected in series, so, $R_{eq} = R_1 + R'' = 100.25\Omega$

2/ We have the Ohm's law, $V = R.I \Rightarrow V_{eq} = R_{eq}.I_{eq} \Rightarrow I_{eq} = V_{eq}/R_{eq}$

$$\Rightarrow I_{eq} = \frac{12}{100.25} = 0.120A = 120mA$$

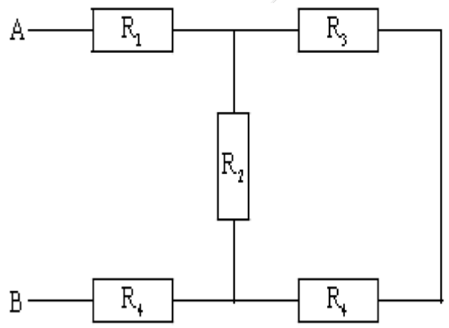
Exercise 02

Let us consider the following combination of resistors, as shown,

1/ Calculate the equivalent resistance between points A and B.

2/ Calculate the electric current I_{AB}, if, V_{AB}= 12 V.

Where: R_1=R_3=100Ω, R_2=150Ω, R_4=500Ω.



Solution of exercise 02

1/ The two resistors R_3 and R_4 are connected in series, so, $R' = R_3 + R_4$,

Now, the resistors R' and R_2 are connected in parallel, so, $\frac{1}{R''} = \frac{1}{R'} + \frac{1}{R_2} \Rightarrow R'' = \frac{R' R_2}{R_2 + R'}$

After that, the three resistors R_1, R'' and R_4 are connected in series, so, we get,

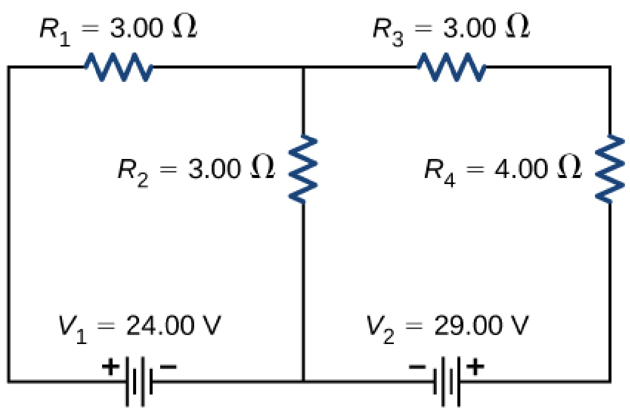
$$R_{eq} = R_1 + R'' + R_4 = 720\Omega$$

2/ We have the Ohm's law, $V = R.I \Rightarrow V_{eq} = R_{eq}.I_{eq} \Rightarrow I_{eq} = V_{eq}/R_{eq}$

$$\Rightarrow I_{eq} = \frac{12}{720} = 0.016A = 16mA$$

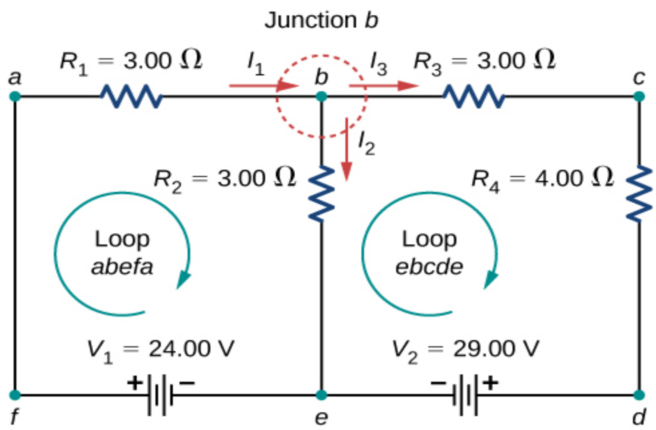
Exercise 03

Calculate the electric currents passing through all the branches in electrical circuit shown in the following figure,



Solution of exercise 03

- Firstly, we need to label the circuit from (a) to (f) to help us with the orientation.
- Then, we have to know the junctions; here, we have junction b and junction e.
- Finaly, we need to select/choose the loops; here, there are loop (abefa) and loop (ebcde).



- From Kirchhoff's junction rule, at junction (b),

$$I_1 = I_2 + I_3 \implies I_1 - I_2 - I_3 = 0$$

- From Kirchhoff's loops rule,

Loop abefa: $I_1R_1 + I_2R_2 = V_1$

Loop abcde: $I_2R_2 - I_3(R_3 + R_4) = V_2$

So, we get three equations with three unknown electric currents I_1, I_2 and I_3:

$I_1 - I_2 - I_3 = 0 \quad (1)$

$I_1R_1 + I_2R_2 = V_1 \quad (2)$

$I_2R_2 - I_3(R_3 + R_4) = V_2 \quad (3)$

Do the sum of eq.1 times R_2 to the eq.2, we get,

$(R_1 + R_2)I_1 - R_2I_3 = V_1 \implies 6I_1 - 3I_3 = 24 \quad (4)$

Then, substruct eq.3 from eq.2, we get,

$I_1R_1 + I_3(R_3 + R_4) = V_1 - V_2 \implies 3I_1 + 7I_3 = -5 \quad (5)$

Next, do the sum of three times of eq.5 with seven times of eq.4, we get,

$51I_1 = 153 \implies I_1 = 3\ A$

From eq.4, $I_3 = -2\ A$

Finaly, from eq.1, $I_2 = 5\ A$

We see that I_3 takes negative value, this is correct because its dirrection was taken in the opposite direction of conventional current flow.

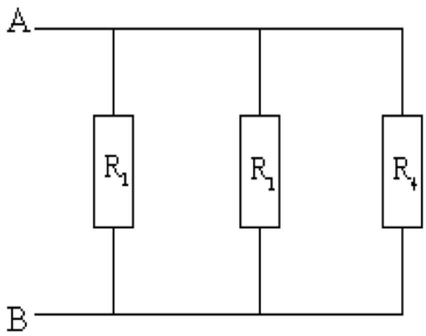
Exercises without solution

Exercise 01

1/ Calculate the equivalent resistance between points A and B for the combination in below.

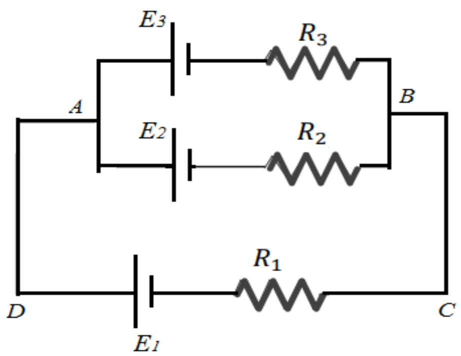
2/ If the electric potential between A and B is V_{AB}= 12 V, calculate the electric current I_{AB}.

Where: $R_1=R_3=100\Omega$, $R_2=150\Omega$, $R_4=500\Omega$.



Exercise 02

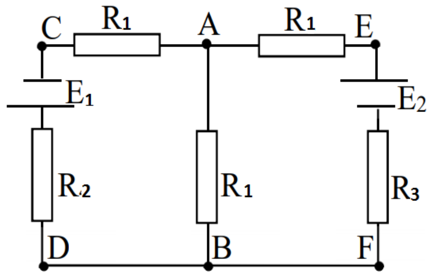
Calculate the electric currents passing through all the branches in electrical circuit shown in the following figure:



Where: E_1 =12V, E_2 =2V, E_3 =4V, $R_1=10R_2=2R_3=100\Omega$.

Exercise 03

Calculate the electric currents passing through all branches in electrical circuit shown in the following figure:



Where: $E_1=E_2=2V$, $R_1=5\Omega$, $R_2=4\Omega$, $R_3=6\Omega$.

References

1. Missoum Radjai ; Physics 2, Courses and exercises corrected, Faculty of Technology, University of Medea, Algeria, 2024/2025.
2. Edward, M, Percell and David, J, Morin ; Book : Electricity and Magnetism, Third edition, Cambridge University. UK, 2008.
3. Nouri Sabrina ; Physique 2, Electrostatique-électrocinétique, University of Science and technologies H. Bouemdiene, Algiers, 2016/2017, http://perso.usthb.dz/~snouri/.
4. Rabah Moussa ; Course of Physics II : Electrostatics, Departement of Physics, University of Djelfa, Algeria, June 2024.
5. شهرة ثورية، محاضرات في الفيزياء: الكهرباء والمغناطيس، جامعة قاصدي مرباح ورقلة، الجزائر.