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Fuzzy topology spaces

Presented by :

Ramzi Zourig

Defended on: ....

Before the jury composed of:

Chair :	*Ferahtia* Nassim	M.C.B, Med Boudiaf University-M'Sila
Supervisor :	*Kheir* Saadaoui	M.C.A, Med Boudiaf University-M'Sila
Examiner :	*Abdelaziz* Amroune	Prof, Med Boudiaf University-M'Sila

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Gratitude

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General Introduction

The mathematical study of sets and spaces is extended to accommodate the notion of partial membership by Fuzzy topology, which is a generalization of classical topology. Fuzzy topology, first presented in 1965 by Lotfi Zadeh as part of the development of Fuzzy set theory, and later defined by C.L. Chang in 1968, offers a potent framework for the modeling and analysis of events involving imprecision, ambiguity, and vagueness

When scholars started examining how to expand the ideas of conventional topological spaces to incorporate derivations of open and closed set theory in spaces with varying degrees of Fuzziness, Fuzzy topological spaces historically first appeared in the 1970s. The development of these spaces was aided by the idea of Fuzzy membership, which reflects degrees of group membership rather than binary values (yes or no). Nowadays, Fuzzy topological spaces are a vibrant area of study in mathematics and applied sciences, with applications ranging from control theory to artificial intelligence to image processing.

Fuzzy topological spaces: An important consideration[15] There are numerous applications where Fuzzy topological spaces are crucial, but the following are the most well-known: Applications in engineering and physics: inaccurate data encountered by engineering and physics procedures is represented by Fuzzy topological spaces. They enable the explanation of things and occurrences that don't easily fit into conventional mathematical frameworks. Information theory and artificial intelligence: Machine learning models and Fuzzy logic systems are developed in part by the use of Fuzzy topological spaces in artificial intelligence to reflect imprecise knowledge and unusual decisions.

Fuzzy logic and Fuzzy control: Fuzzy topological space is used in Fuzzy control theory to characterize non-stationary constraints and imprecise variables in dynamic systems, which aids in the creation of control techniques that are adaptive and resistant to distortion [16]. Applications in biology and medicine: Fuzzy topological spaces can be used to describe erroneous medical information in the fields of biology and medicine, such as when interpreting medical imaging and diagnosing illnesses. Fuzzy topological spaces have practical applications in the industrial and engineering domains. They are employed in the design and analysis of engineering systems and structures in the domains of mechanical, electrical, and civil engineering, where they can enhance efficiency and cut expenses.

Generally speaking, [3] using Fuzzy topological spaces is a useful method of handling imprecision and uncertainty in data, mathematical models, and different computational and engineering applications.

A total of two chapters comprise the memoir: Fuzzy subset theory's foundational ideas, its relationship to classical set theory, and the fundamentals of Fuzziness will all be covered in Chapter 1. Algebraic subgroups and regulations Reciprocal image of a Fuzzy subset, computations, and direct form. ideas on composition, some classes of Fuzzy relations, ambiguous relations, their fundamental characteristics, and various algebraic operations. Moreover, we will examine Fuzzy topology as a concept and some of its fundamental characteristics in the second chapter. Since an ambiguous relationship produces ambiguous topology, which is our concern in this case, we will look at several forms of this topology.

Chapter 1

Generalizations on Fuzzy Relations and Sets

Fuzzy sets were introduced by Zadeh as a generalization of crisp sets. this chapter reviews the concepts and notations of sets, and then introduces the concepts of Fuzzy sets. the concept of Fuzzy sets is a generalization of the crisp sets.

1.1. Generalities on Fuzzy Sets and Relations

Before starting the definition of Fuzzy subset, we first take car of the classical set and its properties.for more informations see [1, 2, 15, 16, 18]

1.1.1. Crisp Sets

Crisp set is a group of objects that define the precious and definite feature which employs bi-valued (yes/no) logic. That is whether each particular element can either within or not belong to a set S, $S \subseteq X$ It is mainly a classical set which is label by a special type of fuzzy sets. Crisp set can be denoted by
$\mathcal{X}_s : X \to \{0, 1\}$[[4]]

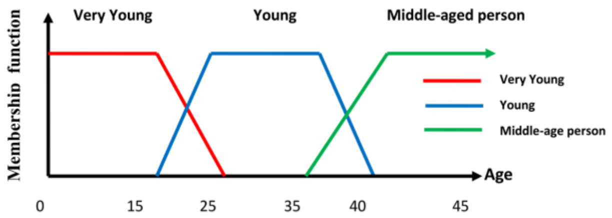

Figure 1.1: fuzzy sets

1.1.2. The Classic Set

A collection of individual object identified as the member or elements of the set that can be distinguished from one another and which follows some basic property is known as classical set. It is defined in such a way that, each element of the set is spitted either member or non-member groups. i.e. for a set A either $a \in A$ or $a \neq A$. There is no chance of existence of partial member ship.[4]

Example 1.1. *For instance,* $2 \in \{1, 2, 3, 4, 5, 7, 9\}$ *, and* $5 \neq \{1, 2, 3\}$.

1.1.2.1. Characteristic Function of Crisp Sets

Suppose the set of universal set is denoted by X and A is the subset of X, i.e. $\emptyset$.[4]
Then for each $x \in X$ its characteristic function is denoted by $x_A(x)$ or 1_s is defined as,

$$\chi_A(x) = \begin{cases} 0; & \text{if} \quad x \notin A \\ 1; & \text{if} \quad x \in A \end{cases}$$

i,e when $\chi_A(x) = 1 \Rightarrow x \in A$, and $\chi_A(x) = 0 \Rightarrow x \notin A$
Figure 1.2 show the graphical representation of the above example.

1.1.2.2. Characteristic Function of The Complement

For a set A the complement of the characteristic function is denoted by $\chi_{\overline{A}}(x)$ and for each $x \in X$ defined as[4]

$$\chi_{\overline{A}}(x) = \begin{cases} 0; & if \quad x \in S \\ 1; & if \quad x \notin S \end{cases}$$

Example 1.2. *let* $\chi_{\overline{A}}(x) \begin{cases} 1, & if \quad 3 \leqslant x \leqslant 10 \\ 0, & otherwise \end{cases}$

Therefore,

$$\chi_{\overline{A}}(x) = \begin{cases} 0, & if \quad 3 \leqslant x \leqslant 10 \\ 1, & 0 \leqslant x < 3 \quad or \quad 10 < x < \infty \end{cases}$$

1.3 express the graphical representation of the above example

1.1.2.3. Characteristic Functions (Union and Intersection)

[4] Let P and Q be two sets, the characteristic functions of the union and intersection are denoted by $\chi_{U\cup Q}$ and $\chi_{P\cap Q}$ can also be obtain by pertaining the formulas:

$$\chi_{U\cup Q}(x) = max\left[\chi_P(x), \chi_Q(x)\right] \quad \text{and} \quad \chi_{P\cap Q}(x) = min\left[[\chi_P(x), \chi_Q(x)\right]$$

espectively.

Example 1.3. *Let,* $P = \{x : 5 \leq x \leq 15\}$,$Q = \{x : 10 \leq x \leq 20\}$

$$\chi_P(x) = \begin{cases} 1, & if \quad 5 \leq x \leq 15 \\ 0, & otherwise \end{cases} \qquad and \qquad \chi_Q(x) = \begin{cases} 1, & if \quad 10 \leq x \leq 20 \\ 0, & otherwise \end{cases}$$

So

$$\chi_{U\cup Q}(x) = \begin{cases} 1, & if \quad 5 \leq x \leq 10 \\ 0, & otherwise \end{cases} \qquad and \qquad \chi_{P\cap Q}(x) = \begin{cases} 1, & when \quad 10 \leq x \leq 15 \\ 0, & otherwise \end{cases}$$

1.4 displays the graphical representation of the above example.

$$\chi_A(x) \begin{cases} 0; & if \quad x \notin S \\ 1; & if \quad x \in S \end{cases}$$

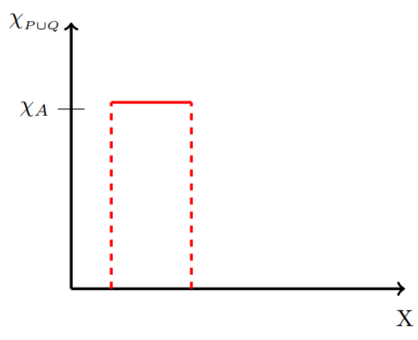

Figure 1.2: Where "0" and "1" denote excluded and included values respectively.

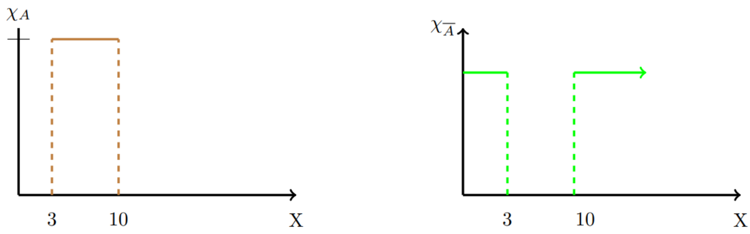

Figure 1.3: Characteristic function of the complement.

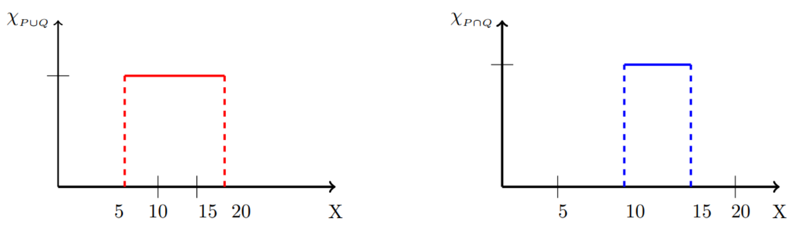

Figure 1.4: Characteristic function of $1 - A \cap B$ and $1 - A \cup B$ respectively.

1.1.2.4. Operation on Crisp Sets

i) **Union:** the union of two sets A and B is given as :

$$A \cup B = \{x/x \in A \; or \; x \in B\}$$

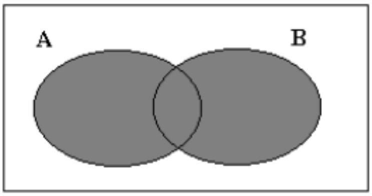

Figure 1.5: Union Operation on crisp sets

ii) **Intersection**: the intersection of two sets A and B is given as:

$$A \cap B = \{x/x \in A \; and \; x \in B\}$$

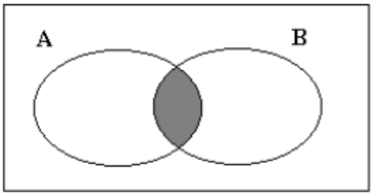

Figure 1.6: Intersection Operation on crisp sets

iii) **Complement**: it is denoted by A and is defined as

$$\overline{A}\{x/x \; does \; not \; belongs \; A \; and \; x \in \mathbf{X}\}$$

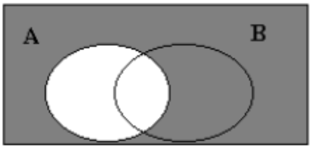

Figure 1.7: Complement Operation on crisp sets

iv) **Difference:**

$$A - B = A \setminus B = \{x/x \in A \; and \; x \notin B\} = A - (A \cap B)$$

.

$$B - A = B \setminus A = \{x/x \notin A \text{ and } n \in B\}.$$

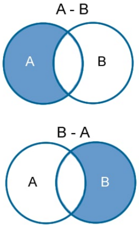

Figure 1.8: Difference Operation on crisp sets

1.1.3. Fuzzy Sets

fuzzy set is the more universal concept of classical set which is an impending tool for handling indistinctness and uncertainties. It is typically characterized in the form of membership function, whereas a member ship function is characterized by the universal set Ũ to the set ranging between 0 and 1. [4]
More exclusively, let $I = [0, 1]$ be the unit interval and X be a null set, where x be any particular element in X. Subsequently, a function $\mu : X \to I$, where $x \to \mu(x)$ is described as fuzzy set in X. Where,$\mu(x)$ is defined as the "Grade of membership of $x \in X$ in μ " ,for more information see [1, 2, 15, 16, 18]

Example 1.4. *In the example three consequences Fuzzy sets are explain that stand for the perception of very young, young and middle-age person in a country which also mentioned graphically in 1.1. The membership functions* $\alpha, \beta,$ *and* $\mathbf{x}$ *are stand for these concepts defined on the interval* $[0, 50]$ *as follows:*

$$\alpha(x) = \begin{cases} 1, & if \quad x \leqslant 15 \\ \frac{(25-x)}{10}, & if \quad 15 < x < 25 \\ 0, & otherwise \end{cases}$$

$$\beta(x) = \begin{cases} 0, & if \quad x \leq 15 \quad or \quad x > 40 \\ \frac{(25-x)}{10}, & if \quad 15 < x < 25 \\ 1, & if \quad 25 \leq x \leq 40 \end{cases}$$

$$\mathbf{x}(x) = \begin{cases} 0, & if \quad x \leqslant 35 \\ \frac{(25-x)}{10}, & if \quad 35 < x < 45 \\ 0, & otherwise \end{cases}$$

1.1.4. Operation on Fuzzy Sets

Let $F(x)$ denote the collection of all Fuzzy sets on a given universe of discourse X.[8]

The basic connectives in Fuzzy set theory are inclusion, union, intersection, and com-plementation. When Zadeh introduced these operations, he based union and intersection connectives on the max and min operations .

- **Inclusion** : Let $A, B \in F(X)$, We say that the set A is included in B if

$$\mu_A(x) \leq \mu_B(x), \quad \forall x \in X$$

The empty (Fuzzy) set $\emptyset$ is defined as $\emptyset(x) = 0$, $\forall x \in X$, and the total set x

is $X(x) = 1, \forall x \in$.

- **Intersection**: Let $A, B \in F(X)$. The intersection of A and B is the Fuzzy set C with

$$C(x) = \min\{A(x), B(x)\} = mu_A(x) \wedge \mu_B(x), \forall x \in X$$

We denote $C = A \wedge B$.

- **Union** : Let $A, B \in F(X)$. The union of A and B is the Fuzzy set D with

$$D(x) = \max\{A(x), B(x)\} = mu_A(x) \vee \mu_B(x), \forall x \in X$$

We denote $D = A \vee B$.

- **Complementation** : Let $A, B \in F(X)$ be a Fuzzy set. The complementation of A is the Fuzzy set B given by

$$\mu_B(x) = 1 - \mu_A(x), \quad \forall x \in X$$

We denote $B = \overline{A}$.

Example 1.5. *Let A and Btwo Fuzzy sets in the universe of discourse X and $(x_1, x_2, x_3, x_4, x_5, x_6) \in X)$ defined as followers:*

$$A = \{\frac{0}{x_1} + \frac{1}{x_2} + \frac{0.7}{x_3} + \frac{0.4}{x_4} + \frac{0.2}{x_5} + \frac{0}{x_6}\}$$
$$B = \{\frac{0}{x_1} + \frac{0.4}{x_2} + \frac{0.7}{x_3} + \frac{0.8}{x_4} + \frac{1}{x_5} + \frac{0}{x_6}\}$$

The union of Fuzzy sets A and B using the max membership fonction is:

$$C_{max} = A \cup B = \{\frac{0}{x_1} + \frac{0.4}{x_2} + \frac{0.7}{x_3} + \frac{0.8}{x_4} + \frac{1}{x_5} + \frac{0}{x_6}\}$$

.

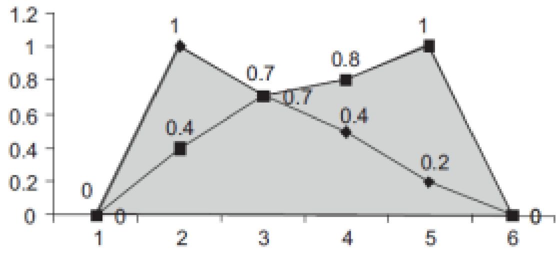

Figure 1.9: Union Operation

the intersection of Fuzzy sets A and B using the min membership fonction is:

$$C_{min} = A \cap B = \{\frac{0}{x_1} + \frac{1}{x_2} + \frac{0.7}{x_3} + \frac{0.4}{x_4} + \frac{0.2}{x_5} + \frac{0}{x_6}\}$$

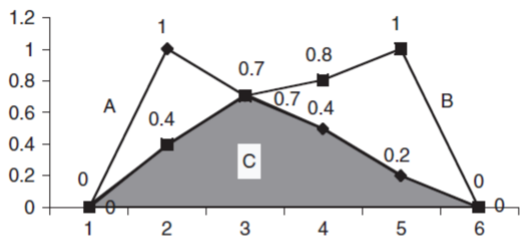

Figure 1.10: Intersection Operation

Let C be a Fuzzy set in the universe of discoverse and $(x_1, x_2, x_3, x_4, x_5, x_6, x_7, x_8) \in X)$ defined as followers:

$$C = \{\frac{1}{x_1} + \frac{1}{x_2} + \frac{0.9}{x_3} + \frac{0.8}{x_4} + \frac{0.7}{x_5} + \frac{0.3}{x_6} + \frac{0.1}{x_7} + \frac{0}{x_8}\}$$

the complement of Fuzzy set is C:

$$\widehat{C} = \{\frac{0}{x_1} + \frac{0}{x_2} + \frac{0.1}{x_3} + \frac{0.2}{x_4} + \frac{0.3}{x_5} + \frac{0.7}{x_6} + \frac{0.9}{x_7} + \frac{1}{x_8}\}$$

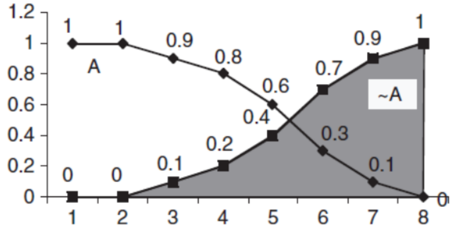

Figure 1.11: Complement Operation

Proposition 1.1 (**Fundamental properties of set operations**). *Let* $A, B,$ *and* $C \in \mathcal{F}(X)$*, we have the following properties:*

Involution	$\overline{\overline{A}} = A$
Commutativity	$A \cup B = B \cup A, \quad A \cap B = B \cap A$
Associativity	$(A \cup B) \cup C = A \cup (B \cup C)$ and $(A \cap B) \cap C = A \cap (B \cap C)$
Distributivity	$\begin{cases} A \cap (B \cup C) = (A \cap B) \cup (A \cap C) \\ A \cup (B \cap C) = (A \cup B) \cap (A \cup C) \end{cases}$
Absorption	$A \cup (A \cap B) = A,\ A \cap (A \cup B) = A$
Idempotence	$A \cup A = A, A \cap A = A$
Absorption by X and $\emptyset$	$A \times X = X.A \cap \emptyset = \emptyset$
Identity	$A \cup \emptyset = A$
Law of contradiction	$A \cap \overline{A} \neq \emptyset$
Law of excluded middle	$A \cup \overline{A} \neq X$
De Morgan's laws	$\overline{A \cap B} = \overline{A} \cup \overline{B}$ and $\overline{A \cup B} = \overline{A} \cap \overline{B}$

1.1.5. A Fuzzy sets characteristics.

In this section, we will give definitions for characteristics of Fuzzy sets support, kernel, height and cardinality of a Fuzzy subset, and we will give an example and proposition.[3]

Definition 1.1 (**Support of Fuzzy subset**). *Let* A *be a Fuzzy set on a set* X*. The support of* A *is the crisp subset on* X *given by*

$$Supp(A) = \{x \in X \mid \mu_A(x) > 0\}$$

Definition 1.2 (Kernel of a Fuzzy subset)**.** *Let A be a Fuzzy set on a set X. The kernel of A is the crisp subset on X given by*

$$ker(A) = \{x \in X \mid \mu_A(x) = 1\}$$

Definition 1.3 (Height of Fuzzy subset)**.** *Let A be a Fuzzy set on a set X. The height of A is the highest value taken by its membership function given by*

$$H(A) = \sup\{\mu_A(x) \mid x \in X\}$$

Definition 1.4. *A Fuzzy subset A is said to be normal whenever $Ht(A) = 1$.*

Definition 1.5 (Cardinality of a Fuzzy subset)**.** *The cardinality of a finite Fuzzy subset A denoted $|A|$ is defined by*

$$|A| = \sum_{x \in X} (\mu_A(x))$$

Example 1.6. *Let $X = [a, b]$ with $\alpha, \beta \in \mathbb{R}$ and let $a, b \in \mathbb{R}$. We define the Fuzzy set A on X by*

$$\mu_A(x) = \begin{cases} 0 & si \quad x < a - \alpha \quad or \quad b + \beta < x; \\ 1 & if \quad a < x < b; \\ 1 + \left(\frac{x-a}{\alpha}\right) & if \quad a - \alpha < x < a; \\ 1 - \left(\frac{b-x}{\beta}\right) & if \quad b < x < b + \beta. \end{cases}$$

Then $ker(A) = [a, b]$, $Supp(A) = [a - \alpha, b + \beta]$ and $H(A) = 1$.

Example 1.7. *Let $X = \{x, \mu_A(x)\}\{1, 2, \ldots, 6\}$, and A be a Fuzzy set of X given by:*

$$A = \{\langle 1, 0.2\rangle; \langle 2, 0.0\rangle; \langle 3, 0.8\rangle; \langle 4, 1.0\rangle; \langle 5, 0.5\rangle; \langle 6, 1.0\rangle\}$$

Then $Supp(A) = \{1, 3, 4, 5, 6\}$, $ker(A) = \{4, 6\}$, $H(A) = \{1\}$, et $\mid A \mid= 3.5$.

Proposition 1.2. *Let A a Fuzzy subset of X. The kernel and support of a Fuzzy subset verify the following properties:*

$$Supp(A^c) = (ker(A))^c$$

$$ker(A^c) = (Supp(A))^c$$

1.1.6. Cartesian product and projection of Fuzzy subsets.

Definition 1.6 (**Cartesian product**). *in[2] the use of The Cartesian product in the following is a definition of n Fuzzy sets: suppose that* $\mu_{A_1}(x), \mu_{A_2}(x), \mu_{A_3}(x), \ldots, \mu_{A_n(x)}$ *is the membership function of* $A_1, A_2, A_3, \ldots, A_n$. *Then, the membership function of* $(A_1, A_2, A_3, \cdots, A_n)$ *the membershipe degree of* $(x_1, x_2, ..., x_n) \in X_1 \times \cdots \times X_n$ *on the fuzzy sets* $A_1 \times \cdots \times A_n$ *therfor:*

$$\mu_{A_1 \times A_2 \times \cdots \times A_n}(x) = \min\left[\mu_{A_1}(x_1), \mu_{A_2}(x_2), ..., \mu_{A_n}(x_n)\right]$$

Example 1.8. *Lets* $X = \{x_1, x_2, x_3\}, Y = \{y_1, y_2\}$ *and lets* A_1, A_2 *are two Fuzzy subsets respectively defined on X and Y given by* $A_1 = \{\langle x_1, 0.1\rangle, \langle x_2, 0.4\rangle, \langle x_3, 0.75\rangle\}$ *, and* $A_1 = \{\langle y_1, 0.2\rangle, \langle y_2, 0.6\rangle\}$. *So, we find:*

$$\mu_{A_1 \times A_2} = \{\langle (x_1, y_1), 0.1\rangle, \langle (x_1, y_2), 0.1\rangle, \langle (x_2, y_1), 0.2\rangle, \langle (x_2, y_2), 0.4\rangle,$$
$$\langle (x_3, y_1), 0.2\rangle, \langle (x_3, y_2), 0.6\rangle\}$$

Definition 1.7 (**Fuzzy subsets Projection**). *the Fuzzy set* $Proj_{X_1}(A)$ *of* X_1 *is the projection on* X_1 *of the Fuzzy set A of* $X_1 \times X_2$, *whose the membership. the definition of the function is given by:*

$$\forall x_1 \in X_1, Proj_{X_1}(A) = \sup_{x_2 \in X_2} \mu_A(x_1, x_2)$$

1.1.7. Triangular Standards and Conformances

The intuitionistic operations of intersection, union and complementation of subsets usually employed can be replaced by other operations constructed using operators introduced in the domain of random metric spaces and we call on them when the usual operators do not prove satisfactory.[2]

Definition 1.8 (**Triangular standard**). *A triangular norm (t-norm) is a function* $T : [0,1] \times [0,1] \longrightarrow [0,1]$ *checking:*

(i) ***Montony:*** $\forall x, y, z \in [0,1], (x \leq y) \Rightarrow T(x,z) \leq T(y,z)$;

(ii) ***Commutativity*** : $\forall x, y \in [0,1], T(x,y) = T(y,x)$;

(iii) ***Associativity*** : $\forall x, y, z \in [0,1], T(x, T(y,z)) = T(T(x,y), z)$;

(iv) ***Neutral element 1*** : $\forall x \in [0,1], T(x,1) = x$.

Definition 1.9 (Triangular conformal). *triangular conorm (t-conorm) is a functionnot* $S : [0,1] \times [0,1] \longrightarrow [0,1]$ *checking:*

(i) ***Neutral element 1*** : $\forall x \in [0,1], S(x,0) = x$

(ii) ***Montony:*** $\forall x, y, z \in [0,1], (x \leq y) \Rightarrow S(x,z) \leq S(y,z)$

(iii) ***Commutativity*** *:* $\forall x, y \in [0,1], S(x,y) = S(y,x)$

(iv) ***Associativity*** *:* $\forall x, y, z \in [0,1], S(x, S(y,z)) = S(S(x,y), z)$

Example 1.9 (**Different standards and triangular conrms).**

t-norm	*t-conform*	*name*
$\min(x,y)$	$\max(x,y)$	*Zadeh*
$\max(x+y-1,0)$	$\min(x+y,1)$	*Lukasiewicz*
$\dfrac{xy}{\gamma+(1-\gamma)(x+y-xy)}$	$\dfrac{x+y-xy-(1-\gamma)xy}{1-(1-\gamma)xy}$	*Hamacher* $(\gamma>0)$
xy	$x+y-xy$	*Probabilistic*
$\max(1-((1-x)^p+(1-y)^p)^{\frac{1}{p}},0)$	$\min((x^p+y^p)^{\frac{1}{p}},1)$	*Yager* $(p>0)$
$\max((x+y-1+\lambda xy)/(1+\lambda),0)$	$\min(x+y+\lambda xy,1)$	*Weber* $(\lambda>-1)$
$\begin{cases} x & if \quad y=1; \\ y & if \quad x=1; \\ 0 & if \quad non. \end{cases}$	$\begin{cases} x & if \quad y=0; \\ y & if \quad x=0; \\ 1 & if \quad no. \end{cases}$	*drastic*

Remark 1.1. *Either T triangular standard, the application S defined as*

$$S : [0,1] \times [0,1] \longrightarrow [0,1]$$

$$S(x,y) = 1 - T(1-x, 1-y)$$

Is the dual t-complies with T.

Remark 1.2. *Every t-norm is an intersection operator, that is to say we can define $A \cap_T B$ by its membership function in the following way:*

$$\forall x \in X,\ \mu_{A\cap_T B}(x) = T\left(\mu_A(x), \mu_B(x)\right)$$

Any t-conformity is a union operator, that is to say we can define $A \cup_T B$ by its membership function as follows:

$$\forall x \in X,\ \mu_{A\cup_T B}(x) = S\left(\mu_A(x), \mu_B(x)\right)$$

Example 1.10. *We define the intersection and the union respectively by:*

$$\forall x \in X,\ \mu_{A\cap_T B}(x) = \max\left(\mu_A(x), \mu_B(x) - 1, 0\right)$$

$$\forall x \in X,\ \mu_{A \cup_T B}(x) = \min\left(\mu_A(x), \mu_B(x), 1\right)$$

1.2. Fuzzy Relations

First we will talk about the basics of a classic relationship with some examples. for more information [1, 2, 10, 12]

Definition 1.10 ((Classical relationship))**.** *Let X and Y be two non-empty sets, a binary relationship R between two sets X and Y is a part of XxY. For $(x, y) \in \mathbb{R} \subseteq X_x Y$, note xRy.*

Definition 1.11 ((Properties of a classical relation))**.** *Let R be a binary relation on a non-empty set X (R a part of XxX)*

(i) $\mathcal{R}$ is reflexive $\Leftrightarrow \forall x \in X,\ x\mathcal{R}x$;

(ii) $\mathcal{R}$ is symmetrical $\Leftrightarrow \forall x, y \in X,\ x\mathcal{R}y \Rightarrow y\mathcal{R}x$;

(iii) $\mathcal{R}$ is antisymmetric $\Leftrightarrow \forall x, y \in X,\ (x\mathcal{R}y \text{ et } y\mathcal{R}x) \Rightarrow x = y$;

(iv) $\mathcal{R}$is transitive $\Leftrightarrow \forall x, y, z \in X,\ (x\mathcal{R}y \text{ et } y\mathcal{R}z) \Rightarrow x\mathcal{R}z$;

If R is reflexive, antisymmetric and transitive, $\mathcal{R}$ is an order relation on X. This order relation is said to be partial and is noted $\preceq$, if there are at least two $x, y \in X$ elements such as $x \npreceq y$ and $y \npreceq x$, and the pair $(X, \preceq)$ is called partially ordered. If for all $x, y \in X$, $x \preceq y$ or $y \preceq x$ the relation of order is said to be total or linear, and the couple $(X, \preceq)$ is said to be totally ordered together.

If R is reflexive, symmetric and transitive, R is an equivalence relation on X

Example 1.11. *1. The divisibility relationship ($x\mathcal{R}y \Leftrightarrow x \mid y$) is a partial order on N^*;*

2. The usual order $<$ is a total order in the integers $N, \mathbb{Z}, \mathbb{Q}, \mathbb{R}$;

3. For any set the equality relation ($xRy \Leftrightarrow x = y$) is reflexive, symmetric, antisymmetric and transitive.

1.2.1. Basic Definitions of Fuzzy Relation

In what follows we will discuss the definition of a fuzzy relationship and its inverse with some examples[2]

Definition 1.12 (fuzzy relations)**.** *A fuzzy relation R between two sets of references X and Y is a fuzzy subset of Cartesian product $X \times Y$, belonging function pn: $\mu_{\mathcal{R}} : X \times Y \longrightarrow [0, 1]$, or simply $\mathcal{R}(x, y)$ which is called the degree of relation between x and y.*

$$\mathcal{R} = \{\langle (x,y), \mathcal{R}(x,y)\rangle \mid (x,y) \in X \times Y\}.$$

Special case:

1. If $X = y$, a Fuzzy relation $\mathcal{R}$ définit on the two universes X and Y est a Fuzzy binary relation defines on X;
2. If X et Y sont finishes, a Fuzzy relation Rdéfinit on the two X et Y universes can be described by the matrix of values of its membership function, the coefficients of $\mathcal{M}_{\mathcal{R}}$.indicated on the line x and the y ayant column for value $\mathcal{R}(x,y)$, for all x de X et x de Y.

Example 1.12 ((Finite case))**.** *Let* $X = \{1,2,3\}$*, the relationship* $\mathcal{R}$ *"Approximately equal to" can be defined by:*

$$\mathcal{R} : \{1,2,3\} \times \{1,2,3\} \longrightarrow [0,1]$$

$$(x,y) \longmapsto \mathcal{R}(x,y) = \begin{cases} 1, & si\ x = y\ ; \\ 0.8, & if \mid x-y \mid = 1\ ; \\ 0.3, & if \mid x-y \mid = 2\ ; \end{cases}$$

It can be represented by the following table

$\mathcal{R}$	*1*	*2*	*3*
1	*1*	0.8	0.3
2	0.8	*1*	0.8
3	0.3	0.8	*1*

Example 1.13 ((Infinite case))**.** *Fuzzy relation R"x approximately equal to* 3" *can be set to RxR by the parent function:*

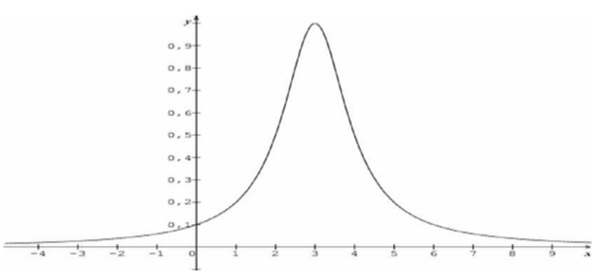

Figure 1.12: " xapproximatively equal to 3"

Definition 1.13 ((The inverse of a Fuzzy relation))**.** *Let* $\mathcal{R}$ *be a Fuzzy relation* *between X and Y, we define $\mathcal{R}^t$ between Y and X by*

$$\mathcal{R}^t(y,x) = \mathcal{R}(x,y).$$

$\mathcal{R}^t$ the inverse relationship of $\mathcal{R}$.

Special case:

If X et Y sont finish, the matrix MRIassociée in the opposite of the fuzzy relation Rest the transposed matrix $M - n-$

Example 1.14. *The inverse relationship R^t the Fuzzy binary relationship $\mathcal{R}$ which is defined in Example 1.12 on $X = \{1,2,3\}$ we will define this matrix form as below*

$\mathcal{R}^t$	*1*	*2*	*3*
1	*1*	0.8	0.3
2	0.8	*1*	0.8
3	0.3	0.8	*1*

Remark 1.3. *Since Fuzzy relationships are special cases of Fuzzy sets, all properties and definitions that relate to Fuzzy sets are applicable.*

1.2.2. Operations on Fuzzy Relation

Given two Fuzzy relations $\mathcal{R}$ and $\mathcal{S}$ of $X \times Y$.[1]

Definition 1.14. *Let $\mathcal{R}$ and $\mathcal{S}$ be two Fuzzy relations, for all x, y in $X \times Y$ we can define*

1. $\mathcal{R} = \mathcal{S} \Leftrightarrow \mathcal{R}(x,y) = \mathcal{S}(x,y)$;
2. $\mathcal{R} \subseteq \mathcal{S} \Leftrightarrow \mathcal{R}(x,y) \leq \mathcal{S}(x,y)$;
3. $\mathcal{R} \cup \mathcal{S} = \{\langle (x,y), \max(\mathcal{R}(x,y), \mathcal{S}(x,y))\rangle\} = \{\langle (x,y), \mathcal{R}(x,y) \vee \mathcal{S}(x,y)\rangle\}$;
4. $\mathcal{R} \cap \mathcal{S} = \{\langle (x,y), \min(\mathcal{R}(x,y), \mathcal{S}(x,y))\rangle\} = \{\langle (x,y), \mathcal{R}(x,y) \wedge \mathcal{S}(x,y)\rangle\}$;
5. $\mathcal{R}^c = \{\langle (x,y), 1 - \mathcal{R}(x,y)\rangle\}$.

Example 1.15. *Let $\mathcal{R}$ and $\mathcal{S}$ be two Fuzzy relations on $X \times X$ such as $X = \{x,y,z\}$, represented by its $\mathcal{M}_\mathcal{R}$ and $\mathcal{M}_\mathcal{S}$ matrices witch are defined by the following tables*

$\mathcal{R}$	x	y	z
x	*1*	*0.6*	*0.3*
y	*0.4*	*1*	*0.2*
z	*0*	*0.5*	*1*

$\mathcal{S}$	x	y	z
x	*0.2*	*0.4*	*0*
y	*0.8*	*0.1*	*1*
z	*0.4*	*0*	*1*

The matrices $\mathcal{M}_{\mathcal{R}\cup\mathcal{S}}$ and $\mathcal{M}_{\mathcal{R}\cap\mathcal{S}}$ correspond to the Fuzzy relations $R \cup S$ and $R \cap S$ are

$R \cup S$	x	y	z
x	1	0.6	0.3
y	0.8	1	1
z	0.4	0.5	1

$R \cap S$	x	y	z
x	0.2	0.4	0
y	0.4	0.1	0.2
z	0	0	1

The complementary relationships is given by the following table

$\mathcal{R}^{\mathcal{C}}$	x	y	z
x	0	0.4	0.7
y	0.6	0	0.8
z	1	0.5	0

Proposition 1.3. *Let $\mathcal{R}, \mathcal{S}, \mathcal{Q}$ be three Fuzzy relations of $X \times Y$ then*

1. $\mathcal{R} \subseteq \mathcal{S} \Rightarrow \mathcal{R}^t \subseteq \mathcal{S}^t$;
2. $(\mathcal{R} \cup \mathcal{S})^t = \mathcal{R}^t \cup \mathcal{S}^t$;
3. $(\mathcal{R} \cap \mathcal{S})^t = \mathcal{R}^t \cap \mathcal{S}^t$;
4. $(\mathcal{R}^t)^t = \mathcal{R}$;
5. $\mathcal{R} \cap (\mathcal{S} \cup \mathcal{Q}) = (\mathcal{R} \cap \mathcal{S}) \cup (\mathcal{R} \cap \mathcal{Q})$ *et* $\mathcal{R} \cup (\mathcal{S} \cap \mathcal{Q}) = (\mathcal{R} \cup \mathcal{S}) \cap (\mathcal{R} \cup \mathcal{Q})$;
6. $\mathcal{R} \subseteq \mathcal{R} \cup \mathcal{S}$, $\mathcal{S} \subseteq \mathcal{R} \cup \mathcal{S}$, $\mathcal{R} \cap \mathcal{S} \subseteq \mathcal{R}$, $\mathcal{R} \cap \mathcal{S} \subseteq \mathcal{S}$;
7. *if* $\mathcal{S} \subseteq \mathcal{R}$ *and* $\mathcal{Q} \subseteq \mathcal{R}$ *then* $\mathcal{S} \cup \mathcal{Q} \subseteq \mathcal{R}$,

   if $\mathcal{R} \subseteq \mathcal{S}$ *and* $\mathcal{R} \subseteq \mathcal{Q}$ *then* $\mathcal{R} \subseteq \mathcal{S} \cap \mathcal{Q}$.

Proof. 1. If $\mathcal{R} \subseteq \mathcal{S}$, then $\mathcal{R}(x,y) \leq \mathcal{S}(x,y)$, and we have $\mathcal{R}^t(y,x) = \mathcal{R}(x,y) < \mathcal{S}(x,y) = \mathcal{S}^t(y,x)$ for all x, y of $X \times Y$. So, $\mathcal{R}^t \subseteq \mathcal{S}^t$.

2. We have

$$\begin{aligned} (\mathcal{R} \cup \mathcal{S})^t(y,x) &= (\mathcal{R} \cup \mathcal{S})(x,y) \\ &= \mathcal{R}(x,y) \vee \mathcal{S}(x,y) \\ &= \mathcal{R}^t(y,x) \vee \mathcal{S}^t(y,x) \\ &= (\mathcal{R}^t \cup \mathcal{S}^t)(y,x). \end{aligned}$$

Similarly, we demonstrate $(\mathcal{R} \cap \mathcal{S})^t = \mathcal{R}^t \cap \mathcal{S}^t$ and $(\mathcal{R}^t)^t = \mathcal{R}$.

3. We use that operations $\vee$, $\wedge$ satisfied the distribution properties, we get

$$\begin{aligned}
\mathcal{R} \cap (\mathcal{S} \cup \mathcal{Q}) &= \mathcal{R}(x,y) \wedge (\mathcal{S} \cup \mathcal{Q}(x,y) \\
&= \mathcal{R}(x,y) \wedge (\mathcal{S}(x,y) \vee \mathcal{Q}(x,y)) \\
&= (\mathcal{R}(x,y) \wedge \mathcal{S}(x,y)) \vee (\mathcal{R}(x,y) \wedge \mathcal{Q}(x,y)) \\
&= (\mathcal{R} \cap \mathcal{S})(x,y) \cup (\mathcal{R} \cap \mathcal{Q})(x,y) \\
&= ((\mathcal{R} \cap \mathcal{S}) \cup (\mathcal{R} \cap \mathcal{Q}))(x,y).
\end{aligned}$$

In the same way we demonstrate $\mathcal{R} \cup S \supseteq \mathcal{R}$, $\mathcal{R} \cup S \supseteq S$

$\mathcal{R} \cap S \supseteq \mathcal{R}$, $\mathcal{R} \cap S \supseteq S$ and also

If $\mathcal{R} \supseteq S$ and $\mathcal{R} \supseteq \mathcal{Q}$ then $\mathcal{R} \supseteq S \cup \mathcal{Q}$,

If $\mathcal{R} \subseteq S$ and $\mathcal{R} \subseteq \mathcal{Q}$ then $\mathcal{R} \subseteq S \cap \mathcal{Q}$.

$\square$

Chapter 2

Fuzzy Topological Spaces

Compared to conventional set theory, which generalizes different topological ideas, Fuzzy set theory is perceived as offering us a wider framework. The topological structure and ordered structure are combined by Fuzzy topology. This field of mathematics was initially introduced from a pure mathematical perspective by the renowned mathematician Ehrenman, who included the two most prominent aspects of topology on lattice, that influence one another. Chang was the one who initially created fuzzy topology notation. Many researchers carried out additional research in this field in the future. For further details, see [11, 13], where we noted that fuzzy topology is thought of as a specific example of generic point set topology, where memberships functions are currently by characteristic functions.

2.1. Basic Definitions.

Foundational Terminologies Chang started the first major effort to construct the Fuzzy counterpart of basic topology back in 1968. From Chang's perspective, on a set X

A family $F = \{\mu : \mu :$ is Fuzzy set in $X\}$ of Fuzzy subsets (i.e. $F \in I^x$) that complies with the subsequent three axioms is said to have a Fuzzy topology: [4]

(i) $0, 1 \in F$;

(ii) $\mu_1, \mu_2 \in F$, then $\mu_1 \vee \mu_2 \in F$;

(ii) If$\{\mu_i : i \in j\} \subset F$, where j denotes an index set, then $\vee_J \mu_i \in f$

F is described as a Fuzzy topology for$X$ and the pair (X, F) is named as a Fuzzy topological space or in short f.t.s. The members of F are defined as F-open Fuzzy set. If the complement of ρ , denoted by ρ^c , is F-open then an element $\rho \in [0, 1]^x$ is said to be a closed Fuzzy set.

Indiscrete Fuzzy topology
ust like in general Fuzzy topology, only Fuzzy sets 0 and 1 are present in indiscrete Fuzzy topology.

Example 2.1. *Indiscrete Fuzzy topological* (X, F) $.F = \{\rho \in I^x : \text{ a constant Fuzzy set}\}$
i.e. for all, $\rho \in F, if\, \rho = constant$
thn $\rho(x) = 0, \forall x.$
$\rho(x) = 1, \forall x.$
Wherever,I^x *is the Fuzzy set on* X

Discrete Fuzzy topology
The set that contained every Fuzzy set is known as discrete Fuzzy topology.

Example 2.2. *The variables* $x, y, z, \alpha, \beta,$ *and* δ *are defined as follows:* $\alpha = (0.8, 0.9, 0.7), \delta = (0.3, 0.3, 0.2), \beta = (0.6, 0.5, 0.4)$ *and* $F = \{0, \alpha, \beta, \delta, 1\}$ *and. Right now*

$$\alpha \wedge \beta = (0.8, 0.9, 0.7) \wedge (0.6, 0.5, 0.4) = (0.6, 0.5, 0.4) = \beta \in F$$

$$\alpha \wedge \delta = (0.8, 0.9, 0.7) \wedge (0.3, 0.3, 0.2) = (0.3, 0.3, 0.2) = \delta \in F$$

$$\beta \wedge \delta = (0.6, 0.5, 0.4) \wedge (0.3, 0.3, 0.2) = (0.3, 0.3, 0.2) = \delta \in F$$

and

$$\alpha \wedge \beta = (0.8, 0.9, 0.7) \wedge (0.6, 0.5, 0.4) = (0.6, 0.5, 0.4) = \beta \in F$$

$$\alpha \wedge \delta = (0.8, 0.9, 0.7) \wedge (0.3, 0.3, 0.2) = (0.3, 0.3, 0.2) = \delta \in F$$

$$\beta \wedge \delta = (0.6, 0.5, 0.4) \wedge (0.3, 0.3, 0.2) = (0.3, 0.3, 0.2) = \delta \in F$$

$$\begin{aligned} \alpha \vee \beta \vee \delta &= (0.8, 0.9, 0.7) \vee (0.6, 0.5, 0.4) \vee (0.3, 0.3, 0.2) \\ &= (0.8, 0.9, 0.7) \\ &= \alpha \in F \end{aligned}$$

Clearly, it is $0, 1 \in F$ *.*
As a result, a Fuzzy topological space is (X, F)*.*

Example 2.3. *Let* X *be an abstract set that has the Fuzzy set family* F_1 *attached to it.* $F_1 = \left\{\nu : \nu(x) \geq \frac{1}{2}, \forall x \in X\right\} \cup \left\{\alpha : \alpha \text{ is a constant} < \frac{1}{2}\right\}$ *Next, based on the qualities of the Fuzzy topological space, we may deduce that:*

1. $0, 1 \in F_1$
2. *If* $\rho_1, \rho_2 \in F_1$*; then* $\rho_1(x) \wedge \rho_2(x) = (\rho_1 \wedge \rho_2)(x) \geq \frac{1}{2} \Rightarrow (\rho_1 \wedge \rho_2) \in F_1$*; that is, the finite intersection belongs to* F_1*.*
3. *If* $\rho_i \in F_1$ *for each* $i \in J$*, where* J *is an index set, then* $\bigvee_{i \in J} \rho_i \geq \frac{1}{2} \Rightarrow \bigvee_{i \in J} \rho_i \in F_1$*.*

Thus, we conclude that the pair (X, F_1) is a topological space.

2.2. Neighborhood of Fuzzy Topological Space

Given a Fuzzy topological space (X, F), let us assume that $\mu, \lambda \in [0,1]^X$ and $\mu \geq \lambda$. If μ is real, then μ is said to be in a Fuzzy neighborhood (or, to put it another way, n.b.d.) of λ if $\gamma \in F$ exists and $\mu \geqslant \gamma \geq \lambda$. [4]

Example 2.4. *Given a Fuzzy topological space (X, F) and $\alpha, \beta \in F$, we can conclude that β is the fuzzy n.b.d. of α if and only if $\alpha \sim \beta$.*

Let's now consider:

$$X = \{u, v, w\}, F = \{0, 1, \alpha, \beta, \gamma\}$$

$$\alpha = (0.5, 0.1, 1.0), \beta = (0.7, 0.1, 0), \gamma = (0.5, 0.1, 0).$$

$$F_{1\alpha} \equiv (0.5, 0.1, 0.1) \leq \alpha$$

$$F_{2\alpha} \equiv (0.0, 0.3, 0.0) \leq \alpha$$

$$F_{3\alpha} \equiv (0.0, 0.0, 0.2) \leq \alpha$$

where $F_{1\alpha}, F_{2\alpha}$ and $F_{3\alpha}$ stand for Fuzzy points.

In this case, $F_{1\alpha} \vee F_{2\alpha} \vee F_{3\alpha} = \alpha$, but $F_{1\alpha} \vee F_{2\alpha} \vee F_{3\alpha} \neq \beta$.

Furthermore, since

$$F_{1\alpha} \leq \beta$$

$$F_{2\alpha} \leq \beta$$

$$F_{3\alpha} \leq \beta$$

Thus, we deduce that $\alpha \leqslant \beta$.

Therefore, the Fuzzy n.b.d. of α is β.

2.3. Closure of Fuzzy Topological Space .

The smallest closed Fuzzy set containing μ is known as the closure of the Fuzzy topological space, and it is represented by the symbol $\overline{\mu}$ for each μ. To define equivalent $\overline{\mu}$, use the formula below:

$$\overline{\mu} = \{\alpha : \alpha \, isF - closed \; and\alpha \geqslant \mu$$

Thus, it follows that $\overline{\mu}$) is always F-close.

We have now covered and demonstrated a few of the Fuzzy closure operator's features.[4]

2.3.1. Properties of Closure Operator .

If a map $\mu \rightarrow \overline{\mu}$ from $[0,1]^x$ into $[0,1]^x$ meets the following four criteria for all $\mu, \lambda \in [0,1]^X$, it is considered a closure operation:

- $\mu \leqslant \overline{\mu}$;;
- $\overline{\mu \vee \lambda} = \overline{\mu} \vee \overline{\lambda}$;
- $\overline{0} = 0$;
- $\overline{\overline{\mu}} \leqslant \overline{\mu}$; (i.e., the closure operation is idempotent);

Proof. We can infer that

$$\overline{\mu} = \wedge\{v : v \text{ closed } \& \ v \geqslant \mu\}.$$

from the definition of the closure operator, which means that$\mu \leqslant \overline{\mu}$. □

Proof. Given that $\overline{\overline{\mu}}$ is the smallest closed set that contains $\overline{\mu}$ and that $\overline{\mu}$ is closed, it follows that $\overline{\overline{\mu}} = \overline{\mu}$. □

Proof. Clearly $\overline{\alpha} \vee \overline{\beta}$ is closed.
Again,

$$\overline{\alpha} \vee \overline{\beta} \geqslant \alpha \vee \beta \tag{2.1}$$

$$\Rightarrow \overline{\overline{\alpha} \vee \overline{\beta}} \geqslant \overline{\alpha \vee \beta}$$

$$\Rightarrow \overline{\alpha} \vee \overline{\beta} \geqslant \overline{\alpha} \vee \overline{\beta} \tag{2.2}$$

[Sine $\overline{\alpha} \vee \overline{\beta}$ is closed ,So $\overline{\overline{\alpha} \vee \overline{\beta}}$]
Again,

$$\alpha \vee \beta \geqslant \alpha \cdots\cdots (a)$$

$$\Rightarrow \overline{\alpha \vee \beta} \geqslant \overline{\alpha}$$

$$\text{Similarly } \overline{\alpha \vee \beta} \geqslant \overline{\beta} \cdots\cdots (b)$$

From 2.3.1 and 2.3.1 we have,

$$\overline{\alpha \vee \beta} \geqslant \overline{\alpha} \vee \overline{\beta} \tag{2.3}$$

So from (2.2) and (2.3), we have,

$$\overline{\alpha} \vee \overline{\beta} = \overline{\alpha \vee \beta}$$

□

Proof. Since the whole space1 $\in F$ is open, then its complement i.e. $1^c = 0$, is closed
Also, $\overline{0}$ is closed.
So, we can write $\overline{0} = 0$ □

Definition 2.1. *Fuzzy topology generated by closure operator is denoted by F_x and is define by :*

$$F_x = [\mu \in [0,1]^x : \overline{1-\mu} = 1 - \mu$$

Then (XF_X) is called closure of Fuzzy topological space (f.t.s.) generated by closure operator.

Theorem 2.1. *Suppose$\alpha\beta$ are Fuzzy sets in X and (X, F) is a Fuzzy topological space. Then prove that $\overline{\alpha} \wedge \overline{\beta} = \overline{\alpha \wedge \beta}$*

Proof. Clearly , $\overline{\alpha} \wedge \overline{\beta}$ is open.Again

$$\begin{aligned} \overline{\alpha} \wedge \overline{\beta} &\leqslant \alpha \wedge \beta \\ \Rightarrow \overline{\overline{\alpha} \wedge \overline{\beta}} &\leqslant \overline{\alpha \wedge \beta} \\ \Rightarrow \overline{\alpha} \wedge \overline{\beta} &\leqslant \overline{\alpha} \wedge \overline{\beta} \end{aligned} \tag{2.4}$$

Again

$$\alpha \wedge \beta \leqslant \alpha \tag{2.5}$$

$$\Rightarrow \overline{\alpha \wedge \beta} \leqslant \overline{\alpha} \tag{2.6}$$

Similarly,

$$\overline{\alpha \wedge \beta} \leqslant \overline{\beta} \tag{2.7}$$

From 2.6 and 2.7 we have,

$$\overline{\alpha \wedge \beta} \leqslant \overline{\alpha} \wedge \overline{\beta} \tag{2.8}$$

From 2.5 and 2.8, we have,

$$\overline{\alpha} \wedge \overline{\beta} = \overline{\alpha \wedge \beta}$$

□

2.4. Interior of a Fuzzy Topologically Space.

The smallest superior bound of all interior Fuzzy sets of μ is called the interior of μ , and is denoted by μ° .evidently, $\mu^\circ \in F^X$. So, μ° is F-open.
Suppose (X, F) be a Fuzzy topological space and $I = [0,1]$, and $\mu, \lambda \in [0,1]^x$ where $\mu \geqslant \lambda$. Then λ is defined as an interior Fuzzy set of μ iff for $\rho \in F_X$ such that $\mu \geqslant \rho \geqslant \lambda$.[4]

Theorem 2.2. *Assume (X, F) be a Fuzzy topological space. Then*

i) $0^\circ = 0, 1^\circ = 1$

ii) $\mu^\circ \leqslant \mu$

iii) $\mu^\circ = \mu^\circ$

iv) $(\mu \wedge \upsilon)^\circ = \mu^\circ \wedge \upsilon^\circ$

Proof. Since, the interior of any set joining of all open subset contained in this set. Now, the empty set 0 and the whole space 1 of an f.t.s. is open. Thus $0^\circ = 0, 1^\circ = 1$.

□

Proof. From the definition of interior of a set μ , the combination of all open subsets included μ , denoted μ, **i.e**.

$$\mu^\circ = \vee\{\upsilon \text{ open } \& \ \upsilon \leqslant \mu$$

Hence we conclude $\mu^\circ \leqslant \mu$. □

Proof. Since $(\mu \wedge \lambda)^\circ \leqslant \lambda^\circ$
So

$$(\mu \wedge \lambda)^\circ \leqslant \mu^\circ \wedge \lambda^\circ$$

On the other hand,
$\mu^\circ \wedge \lambda^\circ \leqslant \mu \wedge \lambda$, of which the open set $\mu^\circ \wedge \lambda^\circ$ hold in $\mu \wedge \lambda$;
Hence, $\mu^\circ \wedge \lambda^\circ$ must be contained in the largest open set $(\mu \wedge \lambda)^\circ$

$$\textbf{i,e. } \mu^\circ \wedge \lambda^\circ \leqslant (\mu \wedge \lambda)^\circ$$

From (7) and (8) we conclude that

$$(\mu \wedge \lambda)^\circ = \mu^\circ \wedge \lambda^\circ$$

□

Theorem 2.3. *Assume α and β be two fuzzy sets in an f.t.s.. Then $\alpha^\circ \vee \beta^\circ = (\alpha \vee \beta)^\circ$.*

Proof. α° is open and β° is open .
SO $\alpha^\circ = \alpha$, and $\beta^\circ = \beta$
therfore $\alpha^\circ \vee \beta^\circ = (\alpha \vee \beta)$
then $\alpha^\circ \vee \beta^\circ = (\alpha \vee \beta)^\circ$
Again $\alpha^\circ \vee \beta^\circ$ is open ,so $(\alpha^\circ \vee \beta^\circ)^\circ = \alpha^\circ \vee \beta^\circ$
therefore, $\alpha^\circ \vee \beta^\circ = (\alpha \vee \beta)^\circ$
Hence, $\alpha^\circ \vee \beta^\circ = (\alpha \vee \beta)^\circ$ (Proved). □

2.5. Boundary of Fuzzy Topological Space .

The fuzzy boundary for $\mu \in [0,1]^X$ is represented by μ^b), which is defined as the minimum of all F-closed sets ρ with the attribute $\rho(x) \geqslant \overline{\mu}(x)$,$\forall x \in X$, for which we have $[\overline{\mu} \wedge (1-\mu)](x) > 0$.
μ^b is obviously F-closed, and $\mu^b \leqslant \overline{\mu}$
.

Fuzzy point
The Fuzzy point for any fuzzy set, $\mu_\beta \in [0,1]^X$, can be expressed as follows:

$$\mu_\beta(x) = \begin{cases} \beta \; if \; x = x_\circ \\ 0 \; otherwise \end{cases}$$

where $x_\circ$ is the Fuzzy point's support.

2.6. Topology Generated by Fuzzy Relation

Knoblauch introduced the concept of a topology formed by binary relations. We now elaborate on this concept within the framework of Fuzzy topology.[5]

Definition 2.2. *Let $\mathbf{R}$ be an ambiguous relation on X. The Fuzzy sets L_x and R_x, which are defined as $L_x(y) = \mathbf{R}(y,x)$ for any $y \in X$, are then for .*

$$R_x(y) = \mathbf{R}(x,y), \; for \; all \; y \in X$$

The elements $x \in X$ have lower and upper contours, respectively, denoted by $R_x(y) = R(x,y)$ for every $y \in X$.

τ_1 and τ_2, respectively, represent the Fuzzy topology produced by the collection S_1 of all lower contours (i.e., $S_1 = L_x : x \in X$) and the collection S_2 of all upper contours (i.e.,($S_2 = R_x : x \in X$).

Definition 2.3. *Indicated by $\tau_{\mathbf{R}}$., is the Fuzzy topology produced by the subbase* $S = \{L_x\}_{x\in X} \cup \{R_x\}_{x\in X}$.

Example 2.5. *Let* $\mathbf{R}$ *be a Fuzzy relation on* $X = \{x, y\}$, *given by*

R	x	y
x	*0.5*	*0.7*
y	*0.3*	*0.4*

Then, L_x, L_y, R_x, R_y *are the Fuzzy sets in* X *given by*
$L_x = \frac{0.5}{x} + \frac{0.3}{y}$,$L_y = \frac{0.7}{x} + \frac{0.4}{y}$,$R_x = \frac{0.5}{x} + \frac{0.7}{y}$,$R_y = \frac{0.3}{x} + \frac{0.4}{y}$.
therfore

$$\tau_1 = \{0_X, 1_X, L_x, L_y\},$$
$$\tau_2 = \{0_X, 1_X, R_x, R_y\},$$
$$\tau_3 = \{0_X, 1_X, L_x, L_y, R_x, R_y, \frac{0.5}{x} + \frac{0.4}{y}, \frac{0.3}{x} + \frac{0.3}{y}, \frac{0.7}{x} + \frac{0.7}{y}\}.$$

Example 2.6. *Let* $\mathbf{R}$ *be a Fuzzy relation on* $X = \{x, y\}$, *given as follows:*

R	x	y
x	*0.7*	*0.8*
y	*0.7*	*0.5*

Then the Fuzzy topology τ_R *is generated by the following subbase* $\mathbf{S}$*:*

$$\mathbf{S} = \{L_x, L_y, R_x, R_y\},$$

where L_x, L_y, R_x, R_y *are given by:*
$L_x = \frac{0.7}{x} + \frac{0.7}{y}$,$L_y = \frac{0.8}{x} + \frac{0.5}{y}$,$R_x = \frac{0.7}{x} + \frac{0.8}{y}$,$R_y = \frac{0.7}{x} + \frac{0.5}{y}$.
therefore, $\tau_{\mathbf{R}} = \{0_X, 1_X, L_x, L_y, R_x, R_y, \frac{0.8}{x} + \frac{0.7}{y}, \frac{0.8}{x} + \frac{0.8}{y}\}$. *and since for* $x, y \in X$ *such that* $x \neq y$, *there exists* $L_y \in \tau_{\mathbf{R}}$ *such that* $L_y(x) \neq L_y(y)$, *so* $(X, \tau_{\mathbf{R}})$ *is fuzzy* T_0.

Example 2.7. *Let* $\mathbf{R}$ *be a Fuzzy relation on* $X = \{x, y\}$, *given as follows:*

R	x	y	z
x	*1*	*0.5*	*0*
y	*0*	*1*	*0.8*
z	*0.7*	*0*	*1*

Then the Fuzzy topology τ_R *is generated by the following subbase* $\mathbf{S}$*:*

$$\mathbf{S} = \{L_x, L_y, L_z, R_x, R_y, R_z\},$$

where $L_x, L_y, L_z, R_x, R_y, R_z$ *are given by:*

$$L_x = \frac{1}{x} + \frac{0}{y} + \frac{0.7}{z}, L_y = \frac{0.5}{x} + \frac{1}{y} + \frac{0}{z}, L_z = \frac{0}{x} + \frac{0.8}{y} + \frac{1}{z},$$

$$R_x = \frac{1}{x} + \frac{0.5}{y} + \frac{0}{z}, R_y = \frac{0}{x} + \frac{1}{y} + \frac{0.8}{z}, R_z = \frac{0.7}{x} + \frac{0}{y} + \frac{1}{z},$$

Note that (X, τ_R) *is Fuzzy* T_1*, since for the Fuzzy points* x_r*,* y_s *in* X*, there exist Fuzzy open sets* $U = L_x \cup R_z$ *and* $V = L_z \cup R_y$ *such that* $x_r \in U$*,* $x_r \notin V$*,* $y_s \notin U$*,* $y_s \in V$*, for the Fuzzy points* y_r*,* z_s *in* X*, there exist Fuzzy open sets* $U = R_x \cup L_y$ *and* $V = L_x \cup R_z$ *such that* $y_r \in U$*,* $y_r \notin V$*,* $z_s \notin U$*,* $z_s \in V$ *and for Fuzzy points* x_r*,* z_s *in* X*, there exist Fuzzy open sets* $U = R_x \cup L_y$ *and* $V = L_z \cup R_y$ *such that* $x_r \in U$*,* $x_r \notin V$*,* $z_s \notin U$*,* $z_s \in V$

Example 2.8. *Let* R *be a Fuzzy relation on* $X = \{x, y, z\}$*, which is given as follows:*

R	x	y	z
x	1	0.3	0.5
y	0	1	0
z	0	0.9	1

Then the Fuzzy topology τ_R *is generated by the following subbase* S*:*

$$\mathbf{S} = \{L_x, L_y, L_z, R_x, R_y, R_z\}$$

where $L_x, L_y, L_z, R_x, R_y, R_z$ *are given by:*

$$L_x = \frac{1}{x} + \frac{0}{y} + \frac{0}{z}, \quad L_y = \frac{0.3}{x} + \frac{1}{y}, \frac{0.9}{z} \quad L_z = \frac{0.5}{x} + \frac{0}{y} + \frac{1}{z}$$

$$R_x = \frac{1}{x} + \frac{0.3}{y} + \frac{0.5}{z},, \quad R_y = \frac{0}{x} + \frac{1}{y} + \frac{0}{z}, \quad R_z = \frac{0}{x} + \frac{0.9}{y} + \frac{1}{z}.$$

Note that (X, τ_R) *is Fuzzy* T_2*, since for the Fuzzy points* x_r*,* y_s *in* X*, there exist Fuzzy open sets* $U = L_x$ *and* $V = R_y$ *such that* $x_r \in U$*,* $y_s \in V$*,* $U \cap V = 0_X$*. For the Fuzzy points* y_r*,* z_s *in* X*, there exist Fuzzy open sets* $U = R_y$ *and* $V = L_z \cap R_z$ *such that* $y_r \in U$*,* $z_s \in V$*,* $U \cap V = 0_X$*. And for the Fuzzy points* x_r*,* z_s *in* X*, there exist Fuzzy open sets* $U = L_x$ *and* $V = L_z \cap R_z$ *such that* $x_r \in U$*,* $z_s \in V$*,* $U \cap V = 0_X$*.*

Definition 2.4. ***Definitions of Fuzzy Relations***

On a set X*, a Fuzzy relation* R *is known as:*

1. ***Reflexive*** *for each* $x \in X$ *if* $R(x, x) = 1$*;*

2. ***Irreflexive*** *if* $R(x,x) \neq 1$ *for some* $x \in X$*;*

3. ***Antireflexive*** *if* $R(x,x) = 0$ *for each* $x \in X$*;*

4. ***Symmetric*** *if, for each* $(x,y) \in X \times X$, $R(x,y) = R(y,x)$*;*

5. ***Transitive*** *if, for each* $x,y,z \in X$, $R(x,z) \geq \min\{R(x,y), R(y,z)\}$*;*

6. ***Asymmetric*** *if, for each* $(x,y) \in X \times X$, $\min\{R(x,y), R(y,x)\} = 0$*;*

7. ***Antisymmetric*** *if, for each* $(x,y) \in X \times X$ *such that* $x \neq y$, $\min\{R(x,y), R(y,x)\} = 0$*;*

8. ***Negatively transitive*** *if, for each* $x,y,z \in X$*;,* $\max\{R(x,y), R(y,z)\} \geq R(x,z)$*;*

9. ***Total*** *if, for each* $x,y \in X$, $\max\{R(x,y), R(y,x)\} = 1$*;*

10. ***Connecting*** *if, for each* $(x,y) \in X \times X$ *such that* $x \neq y$, $\max\{R(x,y), R(y,x)\} = 1$.

We note here that definitions 9 and 10 have been referred to as "complete" and "strongly complete," respectively, while definition 3 has been referred to as "irreflexive."

Definition 2.5. *A Fuzzy relation is called a Fuzzy preorder relation if it is both reflexive and transitive.*

Definition 2.6. *Fuzzy partial order relation is the name given to a fuzzy connection that is reflexive, transitive, and antisymmetric.*

Definition 2.7. *A Fuzzy connection is called a similarity relation if it is reflexive, symmetric, and transitive.*

We now show:

Proposition 2.1. $\tau_1 = \tau_2$ *if* R *is a symmetric Fuzzy relation.*

Proof. Since R is a symmetric Fuzzy relation, so $R(x,y) = R(y,x)$, for each $x,y \in X$. This implies that $R_x(y) = L_x(y)$, for each $x,y \in X$ and hence $R_x = L_x$, for each $x \in X$. Thus the topologies τ_1 and τ_2, which are generated by $\{L_x : x \in X\}$ and $\{R_x : x \in X\}$ respectively, are same. □

Proposition 2.2. *If* R *is a Fuzzy preorder relation, then*

1. *If* $A \in \tau_1$, *then* $A \supseteq \bigcup_{x:A(x)=1} R_x$.

2. *If* $A \in \tau_2$, *then* $A \supseteq \bigcup_{x:A(x)=1} L_x$.

Proof. 1. To show that $A \supseteq \bigcup_{x:A(x)=1} L_x$, let $y_r \in \bigcup_{x:A(x)=1} L_x$. This implies that there exists some x such that $A(x) = 1$ and $y_r \in L_x$. So $r < R(y, x)$. Now since A is open and $A(x) = 1$, so $x_r \in A$ and there exists a basic fuzzy open set $\bigcap_{i=1}^{n} L_{x_i}$ such that

$$x_r \in \bigcap_{i=1}^{n} L_{x_i} \subseteq A$$

$$\Rightarrow r < R(x, x_i), \text{ for each } i = 1, 2, \ldots, n$$

$$\Rightarrow r < \min\{R(y, x), R(x, x_i)\} \leq R(y, x_i), \text{ for each } i = 1, 2, \ldots, n$$

$$\Rightarrow y_r \in L_{x_i}, \text{ for each } i = 1, 2, \ldots, n$$

$$\Rightarrow y_r \in \bigcap_{i=1}^{n} L_{x_i} \subseteq A$$

$$\Rightarrow y_r \in A$$

$$\Rightarrow A \supseteq \bigcup_{x:A(x)=1} L_x$$

2. The proof is similar to that of part 1.

□

Theorem 2.4. *Let (X, τ) be a Fuzzy topological space. Then, a Fuzzy relation R has a subbase $\{U_x, V_x : x \in X\}$ such that $U_y(x) = V_x(y)$ for each $x, y \in X$, if and only if the Fuzzy topology τ is formed by that relation.*

Proof. First assume that τ is generated by some Fuzzy relation R, then obviously it has a subbase $\{U_x, V_x : x \in X\}$, where $U_x = L_x$ and $V_x = R_x$, for each $x \in X$ such that $U_y(x) = V_x(y)$, for each $x, y \in X$.
Conversely, assume that τ has a subbase $\{U_x, V_x : x \in X\}$ such that $U_y(x) = V_x(y)$, for each $x, y \in X$. Now to show that τ is generated by some Fuzzy relation R, define a Fuzzy relation $R : X \times X \to I$ by $R(x, y) = U_y(x) = V_x(y)$, for each $(x, y) \in X \times X$. Then for $x \in X$, $L_x(y) = R(y, x) = U_x(y)$ and $R_x(y) = R(x, y) = V_x(y)$, for each $y \in X$ which implies that $L_x = U_x$ and $R_x = V_x$, for each $x \in X$. So from the hypothesis of the theorem, we have that the family $\{L_x, R_x : x \in X\}$ is a subbase for τ. Hence τ is generated by the Fuzzy relation R. □

Theorem 2.5. *Let (X, τ) be a topological space that is Fuzzy. Assume that for any $x, y \in X$, τ has a subbase $\{U_x, V_x : x \in X\}$ such that $U_y(x) = V_x(y)$. For every $O \in \tau$ and $r \in (0, 1)$, let $a, b \in X$ be such that $a_r \in O \Rightarrow b_r \in O$,. Next, $U_a \subseteq U_b$ and $V_a \subseteq V_b$.*

Proof. Let $z_r \in U_a$, for some $r \in (0,1)$.

$$\Rightarrow r < U_a(z) = V_z(a)$$

$$\Rightarrow a_r \in V_z$$

$$\Rightarrow b_r \in V_z \quad (\text{Since } V_z \in \tau)$$

$$\Rightarrow r < V_z(b) = U_b(z)$$

$$\Rightarrow z_r \in U_b$$

$$\Rightarrow U_a \subseteq U_b$$

Thus far, our findings indicate that τ is produced by a Fuzzy relation R if and only if τ displays a subbase $\{U_x, V_x : x \in X\}$ such that, for any$x, y \in X$, $U_y(x) = V_x(y)$.

Based on this outcome, given the Fuzzy topology τ and the aforementioned subbase,

One can immediately acquire the Fuzzy relation R that producesτ ,(i.e., $\tau = \tau_R$) by declaring $R : X \times X \to I$. In this situation, the lower contour L_x and the upper contour R_x of the element $x \in X$ are the same as U_x and V_x, respectively. That is, $R(x,y) = U_y(x) = V_x(y)$ for each $(x,y) \in X \times X$.

The Fuzzy relation R that produces τ will satisfy certain extra features if we enforce the following theorem a few requirements for the subbase elements V_x and U_x .

□

Let (X,τ) be a Fuzzy topological space with a subbase $\{U_x, V_x : x \in X\}$ such that, for every $x, y \in X$, $U_y(x) = V_x(y)$. Take into account the Fuzzy relation $R : X \times X \to I$. For each$(x,y) \in X \times X$ is defined as $R(x,y) = U_y(x) = V_x(y)$. Next, the following properties hold:

1. For every $x \in X$, R is **reflexive** if and only if $U_x(x) = 1$.
2. For some $x \in X$, R is **irreflexive** if and only if $U_x(x) \neq 1$.
3. For any $x \in X$, R is **antireflexive** if and only if $U_x(x) = 0$.
4. R can be considered **symmetric** if and only if $U_x = V_x$ for every $x \in X$.
5. For any $x \in X$, R is **asymmetric** if and only if $U_x \cap V_x = 0_X$.
6. R **is antisymmetric** if and only if, for each $x, y \in X$ such that $x \neq y$.,$(U_x \cap V_x)(y) = 0$
7. For every$x, y, z \in X$., R is **transitive** if and only if $U_z(x) \geq (V_x \cap U_z)(y)$

8. For each $x, y, z \in X$, R is **negatively transitive** if and only if $(V_x \cup U_z)(y) \geq U_z(x)$
9. R is **total** if and only if $U_x \cup V_x = 1_X$, for each $x \in X$.
10. R is **connected** if and only if, for each $x, y \in X$ such that $x \neq y$., $(U_x \cup V_x)(y) = 1$

The reasoning behind these properties is straightforward.

Definition 2.8. *Assume that R is a Fuzzy partial order relation or a Fuzzy preorder relation. Next, for any $(x, y) \in X \times X$, the corresponding asymmetric Fuzzy relation R_1 is defined as follows:*

$$R_1(x, y) = \max\{R(x, y) - R(y, x), 0\}.$$

The asymmetric portion of the Fuzzy preorder or Fuzzy partial order R is referred to as R_1.

*Similarly to [**?**], we now define a preorderable Fuzzy topology.*

Definition 2.9. *Let (X, τ) be a Fuzzy topological space. A complete fuzzy partial order relation is said to be preorderable (or **orderable**) on X if its asymmetric half produces the Fuzzy topology τ .*

Example 2.9. *Let R be a Fuzzy relation on $X = \{x, y\}$, given by*

R	x	y
x	1	0.7
y	0.6	1

It is easy to verify that R is a Fuzzy preorder relation and its associated asymmetric part R_1 is the Fuzzy relation on $X = \{x, y\}$, given by

R_1	x	y
x	0	0.1
y	0	0

Now, $L_x^{R_1}$, $L_y^{R_1}$, $R_x^{R_1}$, $R_y^{R_1}$ are the Fuzzy sets in X, given by

$$L_x^{R_1} = \frac{0}{x} + \frac{0}{y}, \quad L_y^{R_1} = \frac{0.1}{x} + \frac{0}{y}, \quad R_x^{R_1} = \frac{0}{x} + \frac{0.1}{y}, \quad R_y^{R_1} = \frac{0}{x} + \frac{0}{y}.$$

Therefore, the preorderable Fuzzy topology τ_{R_1} on X is given by

$$\tau_{R_1} = \{0_X, 1_X, L_y^{R_1}, R_x^{R_1}, \frac{0.1}{x} + \frac{0.1}{y}\}.$$

Example 2.10. *Let R be a Fuzzy relation on $X = \{x, y\}$, given by*

R	x	y
x	*1*	*0.3*
y	*0*	*1*

It is easy to verify that R is a Fuzzy partial order relation and its associated asymmetric part R_1 is the fuzzy relation on $X = \{x, y\}$, given by

R_1	x	y
x	*0*	*0.3*
y	*0*	*0*

Now, $L_x^{R_1}$, $L_y^{R_1}$, $R_x^{R_1}$, $R_y^{R_1}$ are the Fuzzy sets in X, given by

$$L_x^{R_1} = \frac{0}{x} + \frac{0}{y}, \quad L_y^{R_1} = \frac{0.3}{x} + \frac{0}{y}, \quad R_x^{R_1} = \frac{0}{x} + \frac{0.3}{y}, \quad R_y^{R_1} = \frac{0}{x} + \frac{0}{y}.$$

Therefore, the orderable Fuzzy topology τ_{R_1} on X is given by

$$\tau_{R_1} = \{0_X, 1_X, L_y^{R_1}, R_x^{R_1}, \frac{0.3}{x} + \frac{0.3}{y}\}.$$

Generally Speaking

This memoir examines the concept of Fuzzy topology that he, Professor Zhang, and Lowen introduced. We see several definitions and instances of this theory, which is a generalization of the idea of classical topology. While we looked at A particular kind of hazy Fuzzy topology, which arises from the relation, is topology.

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ملخص

في هذه المذكرة، استكشفنا الطوبولوجيا الغامضة كتعميم للطوبولوجيا الكلاسيكية. علاوة على ذلك، فقد بحثنا في نوع مميز من الطوبولوجيا المبهمة التي ترتكز على مفهوم الطوبولوجيا القائمة على العلاقات.

الكلمات المفتاحية : الطوبولوجيا المبنية على العلاقات، المجموعات الغامضة، الطوبولوجيا.

Abstract

In this memoir, we explored fuzzy topology as a generalization of classical topology. Furthermore, we delved into a distinctive type of fuzzy topology grounded in the concept of relation-based topology.

Keywords: relation-based topology, fuzzy sets, topology.

Résumée

Dans ce mémoire, nous avons exploré la topologie floue comme une généralisation de la topologie classique. De plus, nous avons exploré un type distinctif de topologie floue fondée sur le concept de topologie basée sur les relations.

Mots clés : topologie basée sur les relations, ensembles flous, topologie.