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**TOPIC**

**A hybrid GRASP algorithm for the minimum weight  
vertex cover problem**

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## INTRODUCTION

The Minimum Weight Vertex Cover problem (MWVC) is a classical combinatorial optimization which can be stated as follows. Given a vertex-weighted undirected (with positive values) graph, the goal of the MWVC problem is to find a subset of the given vertices with minimum total weight with the property that at least one of the endpoints of each edge of the graph is contained in the subset. MWVC problem is a fundamental graph problem with many important real-life applications such as, for example, in wireless communication, circuit design and network flows [Bouamama 2012].

In theory of complexity, the Minimum Weight Vertex Cover Problem is an NP-complete problem [Garey 1979] where is unlikely to find a polynomial-time algorithm for solving it to optimality.

Therefore, most existing methods for dealing with this problem are based on heuristics for providing approximate solutions in a reasonable computation time.

In this work we firstly provide a comparison between different variations of a greedy heuristic for the MWVC problem. Besides, the greedy heuristics that have been used for the minimum weight dominating set problems [Potluri 2013][Bouamama 2016] have been also adopted for the MWVC problems. Secondly, the two best greedy heuristics obtained from the previous experiments have been investigated in the developments of a hybrid GRASP algorithm. The performance of our approach has been tested on well known dataset.

The thesis is organized as follows. In the first chapter we discuss graph theory basics, we give examples and some widely used algorithms. In the second chapter we talk about combinatorial optimization, its classes and we explain the Traveling Salesman Problem (TSP). The third chapter contains the interested problem (the minimum weight vertex cover or MWVC). The last two chapters discuss the proposed algorithms before and our algorithm, we compare between them in term of running-time and the minimum cost to see if our proposed algorithm is better than the previous.

### **The objective:**

We aim in our work to solve the MWVC problem with lower complexity (running-time) and lower cost than the previous proposed approaches.

## General conclusion

### GENERAL CONCLUSION:

The minimum weight vertex cover problem (MWVCP) is a well known optimization problem with a wide range of practical applications such as in wireless communication, circuit design, and network flows.

GRASP, which stands for greedy randomized adaptive search procedures is a multistart, or iterative, metaheuristic. GRASP was proposed before to tackle The capacitated clustering problem (CCP), and we modified it to tackle the minimum weight vertex cover problem (MWVCP).

The platform of our experiments was a personal computer with an Intel(R) Core(TM) i3-3110M CPU with 2.40GHz and 4 GB RAM. We used CodeBlocks as our IDE with g++ compiler and C++ as our programming language. The codes are available under CC license.

The obstacles that faced us was the difficulty of the good understanding of the problem at hand, how to select the proper language to code the algorithm, how to code it to obtain minimum running-time and results as possible as we could.

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## ملخص:

في مشروعنا هذا حاولنا برمجة برنامج يحل مشكل MWVC بأقل تكلفة ممكنة واستعملنا لهذا الغرض لغة البرمجة سي++ وبرنامج codeblocks كبيئة برمجية

## الكلمات المفتاحية:

codeblocks ، سي++ ، MWVCP

## Abstract :

In this project we tried to implement a program to solve MWVCP with minimum cost, to do that we used C++ language and codeblocks IDE.

## Key words:

MWVCP, C++, codeblocks

## Resume:

Dans ce projet, nous avons essayé de mettre en œuvre un programme pour résoudre MWVCP avec un coût minimum, pour le faire, nous avons utilisé le langage C ++ et le codebloccs IDE.

## Mots clés :

MWVCP , C++ , codeblocks