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Par

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Sujet

Etude de certaines hyperstructures
(Study of certain hyperstructure)

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Contents

Introduction

Hyperstructures represent a natural extension of classical algebraic structures and they are introduced in 1934 by the French mathematician F. Marty. In a classical algebraic structure, the composition of two elements is an element, while in an algebraic hyperstructure, the result of this composition is a set.

The Stone's representation theorems proved that every Boolean algebra is isomorphic to a set of $\{I_a/a \in A\}$ (where I_a denotes the set of prime ideals of A not containing a). Since then, representation theorems for distributive lattices has known a vast development.

H. A. Priestley developed another kind of duality for bounded distributive lattices. Such representation theorems enable a deep and a concrete comprehension of the lattices as well as their structures. The duality is central in making the link between syntactical and semantic approaches to logic, also in theoretical computer science this link is central as the two sides correspond to specification languages and the space of computational states. This ability to translate faithfully between algebraic specification and spatial dynamics has often proved itself to be a powerful theoretical tool as well as a handle for making practical problems decidable. Topological duality for Boolean algebras and distributive lattices is a useful tool for studying relational semantics for propositional logics. Canonical extensions, provide a way of looking at these semantics algebraically. Priestley's duality for bounded distributive lattices has enjoyed growing attention and has been variously applied in the international literature since its inception in 1970.

In this work, we have tried to apply the notion of hyperlattice in order to obtain a representation theorem for bounded distributive hyperlattice.

The theory of hyperlattices was introduced by Konstantinidau and J. Mittas in 1977, and they have studied ideals and filter in these structures. The notion of hyperlattices is a generalization of the notion of lattices and there are some intimate connections between hyperlattices and lattices. In particular, Rasouli and Davvaz further studied the theory of hyperlattices and obtained some interesting results which enrich the theory of hyperlattices.

This memory is divided into three chapter:

In the first Chapter, we give some basic notions and definitions of ordered sets(Partially ordered set, lattices and distributive lattices), topology space(basis for topological spaces, compact spaces...), and we present the notion of hyperoperations.

The second Chapter, we recall some results concerning the Priestley duality for distributive lattices. Where we show their representation theorem and Priestley spaces isomorphisms.

In the third Chapter, we represent some notion of hyperlattices and their structures and hyperlattices isomorphisms. In addition to Priestley duality of bounded distributive hyperlattices.

Chapter 1

Preliminaries

In this chapter, we recall some useful notions on topological spaces, ordered sets, lattices, lattices isomorphisms and hyperoperation.

1.1 Ordered sets

Definition 1.1 (binary relation) *A binary relation on a set X is a part R of the set X^2 couples of X. The notation (x, y) $\in \Re$ (or $x \Re y$) means that the couple (x,y) belongs to the relation $\Re$. We write (x,y) $\in \Re$ (or $x \Re y$).*

Example 1.1 *if $E = \mathcal{P}(F)$, set of the parts of a set F, we can define the relation of inclusion between elements of E. If A and B are two parts of F, we say that "A is included in B " and write" $A \subset B$ " if the elements of A all belong to B.*

Example 1.2 *(1) The usual order relation on $\mathbb{R}$ (or on $\mathbb{Q}$).*
(2) The divisibility relation in $\mathbb{N}^*$ (or in $\mathbb{Z}^*$): $m \mid n$ if there exists $q \in \mathbb{N}^*$ (resp.$\mathbb{Z}^*$) such that $n = qm$.
(3) The inclusion relation between parts of a set E.
The last two examples are not total orders

1.1.1 Partial ordered sets

Definition 1.2 *Let P be a set. An order (or partial ordered) on a set P is a binary relation $\leq$ on P such that, for all $x,y,z \in P$*
(i) $x \leq x$,
(ii) $x \leq y$ and $y \leq x$ imply $x = y$,
(iii) $x \leq y$ and $y \leq z$ imply $x \leq z$.
These conditions are referred to, respectively, as reflexivity, antisymmetry, and transitivity. A set P equipped with an order relation $\leq$ is said to be an ordered set(or partially ordered set). We write $(P, \leq)$ is an ordered set.

Definition 1.3 *A poset $(P, \leq)$ is* ***totally ordered*** *if every x, $y \in P$ are comparable, that is,*

$$x \leq y \text{ or } y \leq x.$$

Example 1.3 *(1) Let $X=\{a,b,c,d,e\}$,$P=(X,\Re)$ be ordered set where $\Re$ is an order relation on X as in following table*

$\Re$	a	b	c	d	e
a	1	1	0	0	1
b	0	1	0	0	0
c	0	1	1	1	0
d	0	0	0	1	1
e	0	0	0	0	1

(2) For all set E, $(P(E),\subseteq)$ is a partially ordered sets, with $P(E)$ is a power sets of E.

Definition 1.4 *Let P be an ordered set and let $S \subseteq P$. An element $x \in P$ is an* ***upper bound*** *of S if $s \leq x$ for all $s \in S$.*
An element $x \in P$ is a ***lower bound*** *of S if $x \leq s$, for all $s \in S$.*
-The ***least upper bound*** *of S exists if and only if there exists $x \in P$ such that*

$$(\forall\ y \in P)[((\forall\ s \in S)\ s \leq y) \Longleftrightarrow x \leq y]$$

-The ***greatest lower bound*** *of S exists if and only if there exists $x \in P$ such that*

$$(\forall\ y \in P)[((\forall\ s \in S)\ y \leq s) \Longleftrightarrow y \leq x]$$

*Since least element and greatest element are unique, least upper bounds and greatest lower bounds are unique when they exist. The **least upper bound** of S is also called the **supremum** of S, and is denoted by sup S; the **greatest lower bound** of S is also called the **infimum** of S and is denoted by inf S.*

Lemma 1.1 (Zorn's lemma) *Let S be a partially ordered set. If every totally ordered subset of S has an upper bound, then S contains a maximal element*

1.1.2 Isomorphism and duality

In mathematics the notion of isomorphism between two structures is fundamental. It allows to show that two sets of objects of nature totally different can check the same properties. In the case of structures of order, the notion of anti-isomorphism (or duality) just as important.

Definition 1.5 *Let (X, $\leq$), (Y, $\leq$') to be tow ordered sets are so-called isomorphic (or of the same type) if there is a bijection f of X on Y checking:*

$$x \leq y \iff f(x) \leq' f(y)$$

The bijection f is called an isomorphism (order) between X and Y.

1.2 Lattices

We shall be particularly interested in ordered sets $(P,\leq)$ in which $x \vee y$ and $x \wedge y$ exist for all $x, y \in P$

Notation 1.1 *Looking ahead, we shall adopt the following neater notation: we write $x \vee y$(read as 'x **join** y') in place of sup{x,y} when it exists and $x \wedge y$(read as 'x **meet** y') in place of inf{x,y} when exists. Similarly, we write $\bigvee S$(the '**join** of S') and $\bigwedge S$(the '**meet** of S') instead of supS and infS when these exist.*

Definition 1.6 *A **meet semilattice** (**join semilattice**) is a poset (L,$\leq$) such that every two-element subset x,y has an infimum (a supremum); this element is usually denoted by $x\wedge y$ (by $x\vee y$) and called the **meet(join)** of x and y. A lattice is a poset which is both a **meet semilattice** and a **join semilattice**.*

Definition 1.7 (Order lattice) *Let P be a non-empty ordered set.*

(i) If $x \vee y$ and $x \wedge y$ exist for all x,$y \in P$, then P is called a ***lattice****.*

(ii) If $\bigvee S$ and $\bigwedge S$ exist for all $S \subseteq P$, then P is called a ***complete lattice****.*

Example 1.4 *-The ordered set $\mathcal{M}_n$ (for $n \geqslant 1$) (see Figure 1.1) is easily seen to be a lattice. Let x, $y \in \mathcal{M}_n$ with $x \parallel y$.*

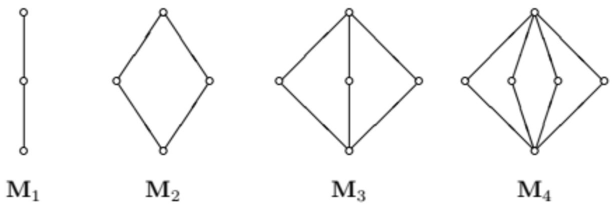


Figure 1.1

Definition 1.8 (Algebraic lattice) *Let T be a non-empty set and $\vee$, $\wedge$ be two binary operations on T. we call $(T,\vee,\wedge)$ a lattice if and only if $\vee$ and $\wedge$ satisfy, for all $a,b,c \in L$:*

1-$(a\vee b)\vee c=a\vee(b\vee c)$, $(a\wedge b)\wedge c=a\wedge(b\wedge c)$ (Associative)

2- $a\vee b=b\vee a$, $a\wedge b=b\wedge a$ (commutative)

3- $a\vee a=a$, $a\wedge a=a$ (idempotent)

4- $a\vee(a\wedge b)=a$, $a\wedge(a\vee b)=a$ (absorption)

Example 1.5 *The classic example of a lattice is $(P(E),\cap,\cup)$. The laws of union and classical intersection are well associative, commutative and idempotent and verify the law of absorption. Thus, $(P(E),\cap,\cup)$ is a lattice*

1.2.1 Ideals and filters

Ideals and filters are of fundamental importance in algebra. They have a variety of applications in logic and topology.

Definition 1.9 (Filter) *Let L be a lattice. A non-empty subset F of L is called a filter if*

(i) $a,b \in F$ implies $a \wedge b \in F$,

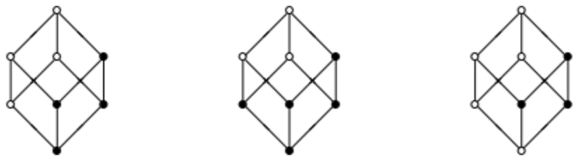
(ii) $a\in L$, $b \in F$ and $a \geq b$ imply $a \in F$.

Definition 1.10 (Ideal) *Let L be a lattice. A non-empty subset I of L is called an ideal if*

(i) $a,b \in I$ implies $a \vee b \in I$,

(ii) $a \in L$, $b \in I$ and $a \leq b$ imply $a \in I$.

Example 1.6 *In this example we give some cases of ideals. Let*



shaded elements an ideal   shaded elements not an ideal   shaded elements not an ideal

Figure 1.2

Definition 1.11 (Prime ideal) *A proper ideal I of L is said to be prime if: $a \wedge b \in I$ implies $a \in I$ or $b \in I$ for any $a,b \in L$.*

Definition 1.12 (Prime filter) *An filter F of L is a prime filter if: $a \vee b \in F$ implies $a \in F$ or $b \in F$ for any $a,b \in L$.*

Example 1.7 *(1) Let H =\{0, a, b, 1\} be a lattice, I=\{0\} is a ideal and P = \{0, a\} is a prime ideal, while I is not prime.*

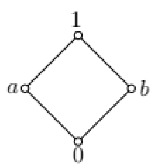


Figure 1.3

Lemma 1.2 *([11]) Let F be a filter and I an ideal of a bounded distributive lattice L. If $F \cap I = \phi$, then there is a prime filter P of L such that $F \subseteq P$ and $P \cap I = \phi$.*

Proof 1.1 *Let $\mathcal{F}$ be the set of all filters of L containing F and disjoint from I. Then $\mathcal{F}$ is nonempty since it contains F. Therefore, by Zorn's lemma, $\mathcal{F}$ contains a maximal element P. We prove that P is prime. Suppose that $a \vee b \in P$. Consider the filters F_1 and F_2 generated by $P \cup \{a\}$ and $P \cup \{b\}$, respectively. Suppose that a, $b \notin P$. Then P is properly contained in both F_1 and F_2, therefore, F_1 and F_2 are not elements of F. Thus, $F_i \cap I$ is nonempty for each i, let $x_i \in F_i \cap I$. By construction of F_i , there is $p_i \in P$ with $p_1 \wedge a \leq x_1$ and $p_2 \wedge b \leq x_2$. Therefore,*

$$x_1 \vee x_2 \geq (p_1 \wedge a) \vee (p_2 \wedge b) = (p_1 \vee p_2) \wedge (p_1 \vee a) \wedge (p_2 \vee b) \wedge (a \vee b)$$

All four terms are in P, so their meet is in P. This implies that $x_1 \vee x_2 \in P \cap I$, a contradiction. Thus, either $a \in P$ or $b \in P$, so P is a prime filter.

1.2.2 Distributive Lattices

Lemma 1.3 *([8]) Let L be a lattice. Then the following are equivalent:*
(D1) $\forall$ a, b, c $\in$L: a $\vee$ (b $\wedge$ c) = (a $\vee$ b) $\wedge$ (a $\vee$ c).
(D2) $\forall$ a, b, c $\in$L: a $\wedge$ (b $\vee$ c) = (a $\wedge$ b) $\vee$ (a $\wedge$ c).

Example 1.8 *Every toset (totaly ordered sets) is a distributive lattice. Here x$\wedge$y = min(x, y) and x$\vee$y =max{x,y}, i.e, the least and greatest element of the set {x,y}, respectively.*
However, non-distributive lattices do exist, e.g:
1) The pentagon, i.e, the five-element lattice {0, a, b, c, l} where a $\parallel$ b and b $\parallel$ c while a $<$ c;

1.2.3 Lattices isomorphisms

Definition 1.13 *([8]) Let L and K be lattices. A map f: L $\longrightarrow$ K is said to be a **homomorphism** (or, for emphasis, **lattice homomorphism**) if f is **join-preserving** and **meet-preserving**, that is, for all a,b $\in$ L,*

$$f(a \vee b) = f(a) \vee f(b) \text{ and } f(a \wedge b) = f(a) \wedge f(b)$$

*A bijective homomorphism is a (**lattice**) **isomorphism**. If f: L $\longrightarrow$ K is a one-to-one homomorphism and onto.*

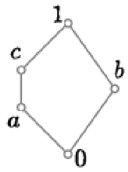


Figure 1.4

2) The diamond, i.e. the five-element lattice having three pairwise incomparable elements.

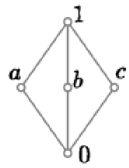


Figure 1.5

Definition 1.14 *If L and L' are bounded lattices, a lattice homomorphism* $f:$ *L* $\longrightarrow$ *L' is said to be a bounded-lattice homomorphism if it preserves 0 and 1, i.e. f(0) = 0 and f(1) = 1*

Remark 1.1 *For bounded lattices L and K it is often appropriate to consider homomorphisms* $f:$ *L* $\longrightarrow$ *K such that f(0) = 0 and f(1) = 1. Such maps are called {0,1}-homomorphisms.*

1.3 Topological spaces

Definition 1.15 *Let X be a nonempty set. A topology on X is a collection* τ *of subsets of X satisfies the following conditions:*

• τ *contains* ϕ *and X, i.e,* $\phi \in \tau$ *and* $X \in \tau$

• τ *is closed under arbitrary unions, i.e. If* $(U_i)_{i\in I} \subset \tau$*, then* $\cup_{i\in I} U_i \in \tau$*.,*

• τ *is closed under finite intersections, i.e. If* $U_1 \in \tau$ *and* $U_2 \in \tau$*, then* $U_1 \cap U_2 \in \tau$*.*

Let $\mathcal{C}$ be the family of closed sets of the topological space (X,τ). Then $\mathcal{C}$ satisfies the following conditions:

(C1) $\phi \in \mathcal{C}$ and X $\in \mathcal{C}$.

(C2) If $F_1 \in \mathcal{C}$ and $F_2 \in \mathcal{C}$, then $F_1 \cup F_2 \in \mathcal{C}$.

(C3) If $(F_i)_{i\in I} \subset \mathcal{C}$, then $\cap_{i\in I} F_i \in \mathcal{C}$.

Then τ is called a topology on X and the pair (X, τ) is called a topological space. Every element of (X, τ) is called a point. Every element of τ is called an open set of X. If {x} $\in \tau$, then the point x is called an isolated point of X. The complement of an open set is called a closed set of X or closed in X.

If a set is open and closed in a topological space, then it is called open-and-closed or closed-and-open (or clopen for short).

Example 1.9 *(1) Let X be a non empty set. The collection* $\{\phi, X\}$ *is a topology on X called the trivial topology or indiscrete topology.*
(2) Let E be a non-empty set, then $\mathcal{O} = \mathcal{P}(E)$ *define a topology, called discrete topology. Note that the topology is discrete if and only if all the singletons of E (i.e. the parts of E consist of a single element) are an opens.*
(3) For the usual topology of $\mathcal{R}$*, [a,b] is closed and the intervals [a,b[,]a,b] and]a,b[are not closed.*

Example 1.10 *Let a set E =*$\{1, 2, 3\}$ *It may be provided with 29 different topologies given by the following list.*

$$\mathcal{O}_1 = \{\phi, E\},\ \mathcal{O}_2 = \{\phi,\{1\}, E\},\ \mathcal{O}_3 = \{\phi,\{1\},\{1, 2\}, E\},\ \mathcal{O}_4 =\{\phi,\{1\},\{1, 3\}, E\},\ \mathcal{O}_5 =\{\phi,\{1\},\{2, 3\}, E\},\ \mathcal{O}_6 =\{\phi,\{1\},\{1, 3\}, \{1, 2\}, E\},\ \mathcal{O}_7 =\{\phi,\{2\}, E\},\ \mathcal{O}_8 =\{\phi,\{2\},\{1, 2\}, E\},\ \mathcal{O}_9 =\{\phi,\{2\}, \{1, 3\}, E\},\ \mathcal{O}_{10} =\{\phi,\{2\}, \{2, 3\}, E\},\ \mathcal{O}_{11} =\{\phi,\{2\}, \{2, 3\}, \{1, 2\}, E\},\ \mathcal{O}_{12} =\{\phi,\{3\}, E\},\ \mathcal{O}_{13} =\{\phi,\{3\},\{1, 2\}, E\},\ \mathcal{O}_{14} =\{\phi,\{3\}, \{1, 3\}, E\},\ \mathcal{O}_{15} =\{\phi,\{3\}, \{2, 3\}, E\},\ \mathcal{O}_{16} =\{\phi,\{3\}, \{1, 3\}, \{3, 2\}, E\},\ \mathcal{O}_{17} =\{\phi,\{1\}, \{2\}, \{1, 2\}, E\},\ \mathcal{O}_{18} =\{\phi,\{1\}, \{2\}, \{1, 2\}, \{1, 3\}, E\},\ \mathcal{O}_{19} =\{\phi,\{1\}, \{2\}, \{1, 2\}, \{2, 3\}, E\},\ \mathcal{O}_{20} =\{\phi,\{1\}, \{3\}, \{1, 3\}, E\},\ \mathcal{O}_{21} =\{\phi,\{1\}, \{3\}, \{1, 3\}, \{1, 2\}, E\},\ \mathcal{O}_{22} =\{\phi,\{1\}, \{3\}, \{1, 3\}, \{2, 3\}, E\},\ \mathcal{O}_{23} =\{\phi,\{2\}, \{3\}, \{2, 3\}, E\},\ \mathcal{O}_{24} =\{\phi,\{2\}, \{3\}, \{2, 3\}, \{1, 2\}, E\},\ \mathcal{O}_{25} =\{\phi,\{2\}, \{3\}, \{2, 3\}, \{1, 3\}, E\},\ \mathcal{O}_{26} =\{\phi,\{1\}, \{2\}, \{3\}, \{1, 2\}, \{1, 3\}, \{2, 3\}, E\},\ \mathcal{O}_{27} =\{\phi, \{1, 2\}, E\},\ \mathcal{O}_{28} =\{\phi, \{1, 3\}, E\},\ \mathcal{O}_{29} =\{\phi, \{2, 3\}, E\}.$$

Definition 1.16 *Let A be a family of parts of E. The intersection of all the topologies of E containing A is a topology on E containing A. It is called the topology generated by the family A, and we the note* O_A.

Definition 1.17 *A mapping* $f : X \longrightarrow Y$ *between two topological spaces is called continuous if for every* $U \subseteq Y$ *open in Y the inverse image* $f^{-1}(U)$ *is open in X. We also say that f is a map.*

Proposition 1.1 *The identity mapping is continuous. A composition of two continuous maps is continuous.*

Definition 1.18 *A homeomorphism $f : X \longrightarrow Y$ is a continuous bijection with the reciprocal image is continuous.*

1.3.1 Basis for a topological spaces

Definition 1.19 (Basis) *If X is a set and $\mathcal{B}$ be a collection of open set. A **basis** for a topology on X is a collection $\mathcal{B}$ of subsets of X (called basis element) such that:*
1. For each $x \in X$, there is at least one basis element B containing x.
2. If x belongs the intersection of two basis elements B_1 and B_2, then there is a basis element B_3 containing x such that $B_3 \subset B_1 \cap B_2$.

If $\mathcal{B}$ satisfies these two conditions, then we define the topology τ generated by $\mathcal{B}$ as follows: A subset U of X is said to be open in X (that is, to be an element of τ) if for each element x $\in$ U, there is a basis element B $\in \mathcal{B}$ such that x $\in$ B and B $\subset$ U. Note that each basis element is itself an element of τ.

Definition 1.20 (sub-basis) *A subset B of τ is called a **sub-basis** of the topology if the set $\mathcal{B}$ obtained by taking all the finite element intersections of B is a base of the topology.*

Example 1.11 *If X is topology space with the discrete topology, then the collection of open sets*

$$\mathcal{B} = \{ \ \{x\} \ x \in X \ \}$$

is a basis for the discrete topology.

1.3.2 Induced topology

Proposition 1.2 *$(A \cap O)_{O \in \mathcal{O}}$ defines a topology on A called topology induced on A by the topology of E.*

A $=$ A $\cap$ E and $\phi = \phi \cap$ A, so $(A \cap O)_{O \in \mathcal{O}}$ contains A and ϕ. If $(A \cap O_i)_{i \in I}$ is a family of $(A \cap O)_{O \in \mathcal{O}}$, then $\cup_{i \in I} (A \cap O_i) =$ A $\cap$ $(\cup_{i \in I} \cap O_i)$ therefore $(A \cap O)_{O \in \mathcal{O}}$ is stable by any meeting. Similarly, $(A \cap O)_{O \in \mathcal{O}}$ is stable by finite intersection.

1.3.3 Topological subspaces

Definition 1.21 *Let X be a topological space and $A \subseteq X$ its subset. The subspace topology on A is given by the collection*

$$\{A \cap U \ / \ U \text{ open in } X\}$$

Thus a subset $V \subseteq A$ is open in this topology if and only if there exists an open subset $U \subseteq X$ such that $V = A \cap U$. We also call A endowed with the subspace topology a subspace of X.

Definition 1.22 *Let X be a topological space with topology τ. If Y is a subset of X, the collection*

$$\tau_Y = \{Y \cap U \ / \ U \in \tau\}$$

is a topology on Y. It is called the induced topology, the subspace topology, or the relative topology. With this topology, Y is called a subspace of X; its open sets consist of all intersections of open sets of X with Y.

Lemma 1.4 *If $\mathcal{B}$ is a basis for the topology X, then the collection:*

$$\mathcal{B}_Y = \{B \cap Y \ / B \in \mathcal{B}\}$$

is a basis for the subspace topology on X.

Example 1.12 *Let (X, τ) be a topological space and Y a subset of. Then*

$$S = \{H \subset Y \ / \ H = G \cup Y \text{ for some } G \in \tau \}$$

is a topology on X, the open sets in Y are the intersections of open sets in X with Y. This topology is called induced or relative topology of Y on X, and (Y, S) is called a topological of a subspace of (X, τ).

1.3.4 Finite products of topological spaces

Let X and Y be two topological spaces, and consider the cartesien product X $\times$ Y.

Definition 1.23 *Let X and Y be topological spaces. The product topology on $X \times Y$ is given by*

$$\{W \subseteq X \times Y / (\exists (x, y) \in W)\ (\exists\ U \subseteq X,\ V \subseteq Y \text{ open})\ U \times V \subseteq W\}$$

Definition 1.24 *The product topology on $X \times Y$ is the topology having as basis the collection $\mathcal{B}$ of all sets of the form $U \times V$, where U is an open set of X and V is an open set of Y.*

Theorem 1.1 *If $\mathcal{B}$ is a basis for the topology X and $\mathcal{C}$ is a basis for the topology Y, then the collection*

$$\mathcal{D} = \{B \times C \ /\ B \in \mathcal{B} \text{ and } C \in \mathcal{C}\}$$

is a basis for the topology of $X \times Y$.

1.3.5 Separate topology

Definition 1.25 *We say that the topology is separated having given $x \neq y$ in X, there exist open disjoint U_1 and U_2 such that $x \in U_1$ and $y \in U_2$.*

Example 1.13 *If E is provided with discrete topology then $\{x\}$ and $\{y\}$ are disjoint open if x and y are distinct, so E is separated.*

1.3.6 Compact spaces

Definition 1.26 *A collection U of open subsets of a topological space X is called an (open) cover if its union is the whole of X, i.e. $\bigcup \mathcal{U} = \bigcup U_{U\mathcal{U}} = X$. A subcollection $\mathcal{U}' \subseteq U$ is called a sub-cover if it is itself a cover.*

Definition 1.27 *Let A be a subset of the topological space X. An open cover of A is a collection $\mathcal{O}$ of open sets whose union contains A. A subcover derived from the open cover $\mathcal{O}$ is a subcollection $\mathcal{O}'$ of $\mathcal{O}$ whose union contains A.*

Definition 1.28 *Let E be a topological space and $X \subset E$. We say that X is a compact part of E if and only if X with the induced topology (by that of E) is compact.*

Theorem 1.2 *Let E be a separate topological space and $X \subset E$.*

1) If X is compact then X is a closed of E.

2) If X is a closed E and E is compact then X is compact.

In particular, if E is compact then X ?? E is compact if and only if X is closed in E.

Definition 1.29 *A topological space X is compact provided that every open cover of X has a finite subcover.*

This says that however we write X as a union of open sets, there is always a finite subcollection $\{\mathcal{O}_i\}_{i=1}^{i=n}$ of these sets whose union is X. A subspace A of X is compact if A is a compact space in its subspace topology. Since relatively open sets in the subspace topology are the intersections of open sets in X with the subspace A, the definition of compactness for subspaces can be restated as follows.

Definition 1.30 *A subspace A of X is compact if and only if every open cover of A by open sets in X has a finite subcover*

Example 1.14 *(1) $\mathcal{R}$ is not compact because $F_n = [n, + \propto [$ is a decreasing sequence of closed non-empty $\mathcal{R}$ whose intersection is empty.*
(2) X =]0, 1] is not compact because $O_n =]\frac{1}{n}n, 1]$ is a series of open X covering X and from which one can not extract any finite overlay.

1.4 Hyperoperations

Algebraic hyperstructures are a suitable generalization of classical algebraic structures. In a classical algebraic structure, the composition of two elements is an element, while in an algebraic hyperstructure, the composition of two elements is a set. More exactly, if H is a nonempty set and P*(H) is the set of all nonempty subsets of H.

Definition 1.31 *([15]) Let H be a non-empty set and a binary operation $\cdot$ where $\cdot : H \times H \longrightarrow P^*(H)$ is said to be a hyperoperation. where $P^*(H)$ is denoted the set of all non-empty subsets of H. The image of the pair (x, y) is denoted by $x \cdot y$(or xy).*

Definition 1.32 *([15]) Let H be a set and $\cdot : H{\times}H \longrightarrow P^*(H)$ be a hyperoperation.*
*The $(\cdot)$ in H is called **associative** if*

$$(x \cdot y) \cdot z = x \cdot (y \cdot z) \text{ for all } x,y,z \text{ in } H$$

Definition 1.33 *([16]) Let H be a set and $\cdot : H{\times}H \longrightarrow P^*(H)$ be a hyperoperation.*
*The $(\cdot)$ is called **commutative** if*

$$x \cdot y = y \cdot x \text{ for all x,y in } H$$

Definition 1.34 *([7])Let H be a set and $\cdot : H \times H \longrightarrow P^*(H)$ be a hyperoperation. An element $e \in H$ is called an **identity** (or unit) in (H, $\cdot$) if*

$$x = e \cdot x = x \cdot e \text{ for all } x \in H$$

Definition 1.35 *([7]) Let H be a set and $\cdot : H \times H \longrightarrow P^*(H)$ be a hyperoperation. An element $x^{'} \in H$ is called an **inverse** of $x \in H$ if there is an identity $e \in H$, such that*

$$x \cdot x' = x' \cdot x = e \text{ for all } x \in H$$

Example 1.15 *(1) Suppose that a hyperoperation $\circ$ on H={0,a,b,1} is defined by the following tables*

$\circ$	0	a	b	1
0	{0}	{a}	{b}	{1}
a	{a}	{0, a}	{1}	{b, 1}
b	{b}	{1}	{0, b}	{a, 1}
1	{1}	{b, 1}	{a, 1}	H

(2) Consider L = {a, b, c} and define hyperoperation $*$ on L by the following table:

$*$	a	b	c
a	{a}	{c, a}	{b, a}
b	{a, c}	{b, a}	{a}
c	{b, a}	{a}	{c, a}

(3) Let L = {a, b, c, d} be a set. Define the hyperoperations '$\otimes$ ' and '$\oplus$' on L with the following Cayley table :

$\otimes$	a	b	c	d
a	a	a	a	a
b	a	b	a	{a,b}
c	a	a	c	c
d	a	{a,b}	c	d

$\oplus$	a	b	c	d
a	a	b	$\{c,d\}$	d
b	b	b	d	d
c	$\{c,d\}$	d	$\{c,d\}$	d
d	d	d	d	d

Chapter 2

Priestly duality for bounded distributive lattices

A Priestley space is a partially ordered Stone space (X, $\leq$) in which, whenever x $\neq$ y, there is a clopen up-set U containing x and missing y. The well-known Priestley duality establishes that the category of bounded distributive lattices and bounded lattice homomorphisms is dually equivalent to the category of Priestley spaces and continuous order-preserving maps.

Definition 2.1 *([12]) Let (X, $\leq$) be a partially ordered set. A subset A of X is said to be increasing (decreasing) if $x \in A$, $y \in X$ and $x \leq y$ ($y \leq x$) imply $y \in A$.*

Definition 2.2 *([12]) An ordered topological space is a triple (X, τ, $\leq$) such that (X, τ) is a topological space and (X, $\leq$) is a poset. A τ-clopen set in a topological space is a set which is both open and closed. The ordered topological space is said to be totally disconnected for the order ($\leq$) if for every x,y $\in$ X such that $x \not\leq y$ there exists an increasing τ-clopen U and a decreasing τ-clopen v such that $U \cap V = \phi$ with $x \in U$ and $y \in V$.*

Lemma 2.1 *([4]) Let X be an ordered compact space. And let F_0, F_1 be two closed sets in X, such that $F_0 \cap F_1 = \phi$, F_0 is increasing and F_1 is decreasing. Then there exist two open sets A_0, $A_1 \subseteq X$ such that $A_0 \cap A_1 = \phi$, $F_0 \subseteq A_0$, $F_1 \subseteq A_1$, A_0 is increasing and A_1 is decreasing.*

Definition 2.3 *A topological structure X = (X, $\leq$, τ) is a Priestley space if*

i) (X, $\leq$) is an ordered set,

ii) τ is a compact topology on X, *iii) for all x, $y \in X$ with $x \neq y$, there is a clopen down-set A of X such that $x \notin A$ and $y \in A$. The category of Priestley spaces.*

2.1 The representation theorem

We shall consider only distributive lattices which have zero and identity elements. Lattice homomorphisms are consequently defined to be maps preserving these elements in addition to preserving joins and meets. To obtain the representation of a distributive lattice A we shall use homomorphisms from A into a two element distributive lattice (with elements denoted by 0, 1) in preference to Stone's construction using prime ideals. The two approaches are equivalent, but the topology we require is more easily defined in the context of homomorphisms. Hence let X be the set of all lattice homomorphisms f : A $\longrightarrow$ {0,1}; X is closed in the topology τ induced by the product topology of $\{0,1\}^A$ and hence compact. We order X by defining, for f, $g \in$ X, $f \leq g$ if and only if f(a) $\leq g$(a) for all a $\in$ A.

Definition 2.4 *([4]) A topological space endowed with a partially ordered $\leq$ is called a* ***totally disconnected ordered topological space*** *provided $x \nleq y$ imply there exist an increasing clopen set A such that $x \in A$ and $y \notin A$. If, moreover, the space is compact, then it is said to be a Priestley space.*

Definition 2.5 *([12]) A Priestley space is a compact totally disconnected ordered topological space.*

If A is a distributive lattice then its dual space is defined to be T(A) = (X, τ, $\leq$) , where X is the set of homomorphisms from A onto {0, 1} preserving 0 and 1, τ be the topology induced by the product topology of $\{0,1\}^A$, and $\leq$ is the partial order: $f \leq g$ in X if and only if
f(a) $\leq$ g(a) for all a $\in$ A.
T (A) = (X, τ, $\leq$) is compact totally order disconnected, i.e., a Priestley space.

Lemma 2.2 *([13]) (X, τ, $\leq$) is compact totally order disconnected.*

Proof 2.1 *Let f,g $\in$ X, $f \ngeq g$. Then there exists a $\in$ A such that g(a) = 1, f(a) = 0. The set U={h $\in$ X/ h(a)= 1} is τ-clopen, increasing and contains g,* *X part of a compact space.*

1) $(h_n)_n \in X$, $\lim\limits_{n\to+\infty} h_n(a) = 1 = h(a)$, *then* $h \in U$.

2) 0-1 homomorphism

(a) $h_n(a \wedge b) = h_n(a) \wedge h_n(b)$

$\lim\limits_{n\to+\infty} h_n(a \wedge b) = h(a \wedge b)$

$$\begin{aligned}\lim_{n\to+\infty} (h_n(a) \wedge h_n(b)) &= \lim_{n\to+\infty} h_n(a) \wedge \lim_{n\to+\infty} h_n(b)\\ &= h(a) \wedge h(b).\end{aligned}$$

Then $h(a \wedge b) = h(a) \wedge h(b)$.

(b) $h_n(a \vee b) = h_n(a) \vee h_n(b)$

$\lim\limits_{n\to+\infty} h_n(a \vee b) = h(a \wedge b)$

$$\begin{aligned}\lim_{n\to+\infty} (h_n(a) \vee h_n(b)) &= \lim_{n\to+\infty} h_n(a) \vee \lim_{n\to+\infty} h_n(b)\\ &= h(a) \vee h(b).\end{aligned}$$

Then $h(a \vee b) = h(a) \vee h(b)$.

(c) $h_n(0) = 0 \Rightarrow \lim\limits_{n\to+\infty} h_n(0) = 0$

$\Rightarrow h(0) = 0$.

(d) $h_n(1) = 1 \Rightarrow \lim\limits_{n\to+\infty} h_n(1) = 1$

$\Rightarrow h(1) = 1$.

and $X—U = \{k \in X/\ k(a) = 0\}$ *is* τ*-clopen, decreasing and contains f.*

1) $(k_n)_n \in X$, $\lim\limits_{n\to+\infty} k_n(a) = 0 = k(a)$, *then* $k \in X\text{-}U$.

2) 0-1 homomorphism

(a) $k_n(a \wedge b) = k_n(a) \wedge k_n(b)$

$\lim\limits_{n\to+\infty} k_n(a \wedge b) = k(a \wedge b)$

$$\begin{aligned}\lim_{n\to+\infty} (k_n(a) \wedge k_n(b)) &= \lim_{n\to+\infty} k_n(a) \wedge \lim_{n\to+\infty} k_n(b)\\ &= k(a) \wedge k(b).\end{aligned}$$

Then $k(a \wedge b) = k(a) \wedge k(b)$.

(b) $k_n(a \vee b) = k_n(a) \vee k_n(b)$

$\lim\limits_{n\to+\infty} k_n(a \vee b) = k(a \wedge b)$

$$\begin{aligned}\lim_{n\to+\infty} (k_n(a) \vee k_n(b)) &= \lim_{n\to+\infty} k_n(a) \vee \lim_{n\to+\infty} k_n(b)\\ &= k(a) \vee k(b).\end{aligned}$$

Then $k(a \vee b) = k(a) \vee k(b)$.

(c) $k_n(0) = 0 \Rightarrow \lim\limits_{n\to+\infty} k_n(0) = 0$

$\Rightarrow k(0) = 0.$

(d) $k_n(1) = 1 \Rightarrow \lim_{n\to+\infty} k_n(1) = 1$

$\Rightarrow k(1) = 1.$

Lemma 2.3 *Let A be a bounded distributive lattice. The map $F_A : A \mapsto L(T(A))$ defined by*

$$F_A(a) = \{f \in X/\ f(a) = 1\}$$

is a lattice isomorphism.

Theorem 2.1 *([13]) Let A be a distributive lattice with zero and identity and let $(X, \tau, \leq)$ be its dual space. Then A is isomorphic to the dual lattice of $(X, \tau, \leq)$.*

Proof 2.2 *Let $L(T(A))$ be the dual lattice of $(X, \tau, \leq)$ and define F_A by*

$$F_A : A \longrightarrow L(T(A))$$
$$a \longmapsto F_A(a) = \{f \in X/f(a) = 1\}$$

For each $a \in A$, $F_A(a)$ is a clopen increasing subset of X, and a standard computation shows that $F_A : A \longrightarrow L(T(A))$ is a homomorphism.

1- $F_A(a \vee b) = F_A(a) \cup F_A(b)$;

$F_A(a \vee b) = \{f \in X/\ f(a \vee b) = 1\}$

$= \{f \in X/\ f(a) \vee f(b)) = 1\}$

$= \{f \in X/\ f(a)= 1$ *ou* $f(b)= 1\}$

$= \{f \in X/\ f(a)= 1\}$ *ou* $\{f \in X/\ f(b)= 1\}$

$= F_A(a) \cup F_A(b)$

2- $F_A(a \wedge b) = F_A(a) \cap F_A(b)$;

$F_A(a \wedge b) = \{f \in X/\ f(a \wedge b) = 1\}$

$= \{f \in X/\ f(a) \vee f(b)) = 1\}$

$= \{f \in X/\ f(a)= 1$ *et* $f(b)= 1\}$

$= \{f \in X/\ f(a)= 1\}$ *et* $\{f \in X/\ f(b)= 1\}$

$= F_A(a) \cap F_A(b)$

3- $F_A(a) = \phi$;

$F_A(0) = \{f \in X/\ f(0) = 1\} = \phi$

4- $F_A(a) = X$.

$F_A(1) = \{f \in X/ f(1) = 1\} = X$

It remains to prove that F is onto A'. We define

$$\mathcal{R} = \{F(a) \ / \ a \in A\}, \ \mathcal{S} = \{X\text{-}F(a) \ / \ a \in A\},$$

and note that these families are closed under finite unions and intersections. Let U be clopen increasing in X; we require to show that $U \in \mathcal{R}$. *If* $f \in U$, $g \in X\text{-}U$, *then* $g \not\geq f$. *It follows that there exist disjoint sets* $R(f,g) \in \mathcal{R}$, $S(f,g) \in \mathcal{S}$ *with* $f \in R(f,g)$, $g \in S(f,g)$. *The compactness of X— U enables this set to be expressed as a finite union of sets* $S(f,g_i)$, $g_i \in X$ *-* U, i=l,...,m. *Letting*

$$R(f) = \bigcup_{1\leqslant n\leqslant m} R(f,g_i)$$

we have

$$\begin{aligned} U &= \bigcup_{f\in U} R(f) \\ &= \bigcup_{i=1}^{n} R(f_i) \\ &= \bigcup_{i=1}^{n} F(a_i) \\ &= F(\bigvee_{i=1}^{n} a_i) \\ &= F(a). \end{aligned}$$

Compactness of U allows this to be reduced to a finite union of elements of $\mathcal{R}$, *that is an element of* $\mathcal{R}$.

2.2 Priestley spaces isomorphisms

Definition 2.6 *([5]) Let* $(X,\tau,\leq)$, *and* $(X_0, \tau_0,\preceq)$ *be two Priestley spaces.*

1. A function $f : X \to X_0$ *is called increasing if for all* $x, y \in X$, $x \leq y$) $f(x) \preceq f(y)$.

2. Let $f : X \to X_0$ *be a function between Priestley spaces. Then f is called a Priestley spaces homomorphism if is increasing and continuous.*

If f is a bijection, then f is said to be Priestley spaces isomorphism.

Lemma 2.4 *If* $\delta = (X,\tau,\leq)$ *is a Priestley space. Then* $(L(\delta),\cap,\cup,\phi,X)$ *is a bounded distributive lattice. Where* $L(\delta) = \{Y \subseteq X/X$ *is increasing and* τ*-clopeng*$\}$

Lemma 2.5 *([12]) If* $\delta = (X,\tau,r)$ *is a Priestley space, then the map* $G_\delta : \delta \mapsto T(L(\delta))$ *defined by*

$$G_\delta(x)(Y) = \begin{cases} 1 \; if \; x \in Y, \\ 0 \; if \; x \notin Y. \end{cases}$$

for all $Y \in L(\delta)$ *is an isomorphism of Priestley space, i.e., a bijection, continuous and increasing map.*

Proof 2.3 *Let* $G_\delta : \delta \mapsto T(L(\delta))$ *defined by*

$$G_\delta(x)(Y) = \begin{cases} 1 \; if \; x \in Y, \\ 0 \; if \; x \notin Y. \end{cases}$$

To prove the surjection, take $f \in T(L(\delta))$ *and setting* $U = \{Y \in L(\delta) : f(Y) = 1\}$, $V = \{Z \in L(\delta) : f(Z) = 0\}$, $A = \cap_{Y \in U\}} Y$ *and* $B = \cup_{Z \in V} Z$. *To show that* $A \;\; B \neq \phi$, *suppose that* $A \;\; B = \phi$, *it follows that* $(\cap_{Y \in U\}} Y) \cap (\cup_{Z \in V} Z)^C = \phi$, *then* $(\cap_{Y \in U\}} Y) \;\; (\cup_{Z \in V} Z^C) = \phi$, *since X is compact we have* $(\cap_{i=1}^n Y_i) \cap (\cap_{j=1}^m Z_j^C) = \phi$, *it follows that* $\cap_{i=1}^n Y_i \subseteq \cup_{j=1}^m Z_j$, *then* $f(\cup_{j=1}^m Z_j) = 1$,

contradiction since $f(\cup_{j=1}^m Z_j) = \vee_{j=1}^m f(Z_j) = 0$, *hence* $A\text{-}B \neq \phi$. *Then, there exists* $x \in A\text{-}B$ *such that* $G_(\delta)(x) = f$, *therefore* $G_(\delta)(x)(Y) = 1 \Leftrightarrow x \in Y \Leftrightarrow Y \in U \Leftrightarrow f(Y) = 1$; *hence* $G_(\delta)(x)$ *is surjective. Let* $x_1, x_2 \in \delta, x_1 \neq x_2 \Rightarrow x_1 \nleq x_2 or x_2 \nleq x_1$. *If* $x_2 \nleq x_1$, *then there exists* $Y_0 \in L(\delta)$ *such that* $x_1 \in Y_0$ *and* $x_2 \notin Y_0$, *hence* $G_\delta(x_1)(Y_0) \neq G_\delta(x_2)(Y_0)$. *If* $x_2 \nleq x_1$, *then there exists* $Y_1 \in L(\delta)$ *such that* $x_2 \in Y_1$ *and* $x_1 \notin Y_1$, *hence* $G_\delta(x_2)(Y_1) \neq G_\delta(x_1)(Y_1)$. *It follows that* $x_1 \neq x_2 \Rightarrow G_\delta(x_1)(Y) \neq G_\delta(x_2)(Y)$, *hence* G_δ *is injective. To prove that* G_δ *is continuous, let Z be a* τ*-clopen of* $T(L(\delta))$. *Then, there exists* $y \in L(\delta)$ *such that* $Y = F_{L(\delta)}(y)$.

$$\begin{aligned} G_\delta^{-1}(Y) &= G_\delta^{-1}(F_{L(\delta)}(y)) \\ &= \{x \in X / G_\delta(x) \in F_{L(\delta)}(y)\} \\ &= \{x \in X / G(x)(y) = 1\} \\ &= \{x \in X / x \in y\} \\ &= X \cap y \\ &= y(\tau - clopen). \end{aligned}$$

Hence, G_δ *is continuous. Note further that, since* $Y \in L(\delta)$ *are increasing,* $x \leq y$ *implies* $G_\delta(x)(Y) \leq G_\delta(y)(Y)$.

Lemma 2.6 *If* $h : \delta_1 \mapsto \delta_2$ *is an homomorphism of Priestley space, then the map* $L(h)$*:* $L(\delta_2) \mapsto L(\delta_1)$ *defined by* $L(h)(y) = h^{-1}(y)$ *for every* $y \in L(\delta_2)$ *is a lattices homomorphism.*

Proof 2.4 *For all* $y \in L(\delta_2)$ *we have* $L(h)(y) \in L(\delta_1)$*.*
For all $y,\ z \in L(\delta_2)$ *since* h^{-1} *commutes with set-theoretical operations we have,*

$$\begin{aligned} L(h)(y \cup z) &= h^{-1}(y \cup z) \\ &= h^{-1}(y) \cup h^{-1}(z) \\ &= L(h)(y) \cup L(h)(z). \end{aligned}$$

And

$$\begin{aligned} L(h)(y \cap z) &= h^{-1}(y \wedge z) \\ &= h^{-1}(y) \cap h^{-1}(z) \\ &= L(h)(y) \cap L(h)(z). \end{aligned}$$

And for all $y,\ z \in L(\delta_2)$

$$\begin{aligned} y \subseteq z &\Rightarrow h^{-1}(y) \subseteq h^{-1}(z) \\ &\Rightarrow L(h)(y) \subseteq L(h)(z). \end{aligned}$$

Hence, $L(h)$ *is lattices homomorphism.*

Theorem 2.2 *([12]) If* $f : A_1 \longrightarrow A_2$ *is a lattice homomorphism, then*

$$L(T(f)) \circ F_{A_1} = F_{A_2} \circ f :$$

$$\begin{tikzcd} A_1 \arrow[r, dashed, "f"] \arrow[d, dashed, "F(A_1)"'] & A_2 \arrow[d, dashed, "F(A_2)"] \\ L(T(A_1)) \arrow[r, dashed, "L(T(f))"'] & L(T(A_2)) \end{tikzcd}$$

Proof 2.5 *For all* $a \in A_1$*,*

$$\begin{aligned} (L(T(f)) \circ F_{A_1})(a) &= L(T(f))(F_{A_1}(a)) \\ &= T^{-1}(f)(F_{A_1}(a)) \\ &= \{g \in T(A_1) : T(f)(g) \in F_{A_1}(a)\} \\ &= \{g \in T(A_2)/g \circ f \in F_{A_1}(a)\} \\ &= \{g \in T(A_2)/g(f(a)) = 1\} \end{aligned}$$

$$
\begin{aligned}
&= F_{A_2}(f(a)) \\
&= F_{A_2} \circ f(a).
\end{aligned}
$$

Theorem 2.3 *If $h : \delta_1 \longrightarrow \delta_2$ is an homomorphism of Priestley space, then*

$$T(L(h)) \circ G\delta_1 = G\delta_2 \circ h:$$

$$
\begin{tikzcd}
\delta_1 \arrow[r, dashed, "h"] \arrow[d, dashed, "G\delta_1"'] & \delta_2 \arrow[d, dashed, "G\delta_2"] \\
L(T(\delta_1)) \arrow[r, dashed, "L(T(h))"'] & L(T(\delta_2))
\end{tikzcd}
$$

Proof 2.6 *For all $f \in \delta_1$,*

$$
\begin{aligned}
(T(L(h)) \circ G_{\delta_1})(f) &= T(L(h))(G_{\delta_1}(f)) \\
&= G_{\delta_1}(f) \circ L(h)(since T(f)(g) = g \circ f)
\end{aligned}
$$

hence for all $y \in L(\delta_2)$,

$$
\begin{aligned}
(T(L(h)) \circ G_{\delta_1})(f)(y) &= (G_{\delta_1}(f) \circ L(h))(y) \\
&= G_{\delta_1}(f)()h^1(y)) \\
&= \begin{cases} 1 \ if \ f \in h^{-1}(y), \\ 0 \ if \ f \notin h^{-1}(y). \end{cases} \\
&= \begin{cases} 1 \ if \ h(f) \in y, \\ 0 \ if \ h(f) \notin y. \end{cases} \\
&= G_{\delta_2}(h(f))(y) \\
&= (G_{\delta_2} \circ h)(f)(y).
\end{aligned}
$$

Example 2.1 *(1) Let A={0, a, b, 1} be a distributive lattices.*

x	f_1	f_2
0	0	0
a	0	1
b	1	0
1	1	1

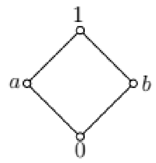


Figure 2.1

T(A) = The set of 0-1 homomorphisms from A onto {0,1} $= \{f_1,\ f_2\}$.

And its bidual is: L(T(A)) $= \{\phi,\ \{f_1\},\{f_2\},\ \{f_1,\ f_2\}\}$.

(2) Let H={0, a, b, c, d, e, f, 1} be a distributive lattices.

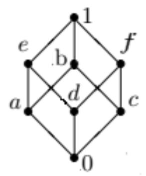


Figure 2.2

x	f_1	f_2	f_3
0	0	0	0
a	0	0	0
b	0	0	1
d	0	1	1
c	0	0	1
e	1	0	1
f	0	1	1
1	1	1	1

T(H) = The set of 0-1 homomorphisms from A onto {0,1} $= \{f_1, f_2, f_3\}$.

And its bidual is: L(T(H)) $= \{\phi,\ \{f_1\},\{f_2\},\ \{f_3\},\ \{f_1,\ f_2\},\ \{f_2,\ f_3\},\ \{f_1,\ f_3\},\ X\ \}$.

(3) Let K={0, a, b, c, d, e, f, 1} be a distributive lattices.

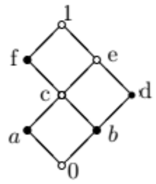


Figure 2.3

x	f_1	f_2	f_3	f_4
0	*0*	*0*	*0*	*0*
a	*0*	*0*	*0*	*1*
b	*0*	*0*	*1*	*0*
d	*0*	*1*	*1*	*0*
c	*0*	*0*	*1*	*1*
e	*1*	*0*	*1*	*1*
f	*0*	*1*	*1*	*1*
1	*1*	*1*	*1*	*1*

T(K) = The set of 0-1 homomorphisms from A onto {0,1} $= \{f_1, f_2, f_3, f_4\}$.

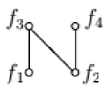


Figure 2.4

And its bidual is: L(T(K)) $= \{\phi, \{f_3\}, \{f_4\}, \{f_3, f_4\}, \{f_1, f_2, f_3\}, \{f_2, f_3, f_4\}, \{f_1, f_3, f_4\}, X\}$.

Chapter 3

Hyperlattices

Hyperlattices generalize both Boolean hyperalgebras by removing the complements requirement, and lattices by replacing the meet $\vee$ *with a hyperoperation* $\sqcup$.

Let X *be a nonempty set and* $P^*(X)$ *denotes the set of all nonempty subsets of* X. *Maps* $f\colon X \times X \longrightarrow P^*(X)$, *are called hyperoperations.*

Definition 3.1 *([3]) Let* L *be a nonempty set,* $\wedge$ *be a binary operation and* $\sqcup$ *be a hyperoperation on* L. L *is called a hyperlattice if for all* a, b, $c \in L$ *the following conditions hold:*

(i) $a \in a \sqcup a$, *and* $a \wedge a = a$;

(ii) $a \sqcup b = b \sqcup a$, *and* $a \wedge b = b \wedge a$;

(iii) $a \in [a \wedge (a \sqcup b)] \cap [a \sqcup (a \wedge b)]$;

(iv) $a \sqcup (b \sqcup c) = (a \sqcup b) \sqcup c$, *and* $a \wedge (b \wedge c) = (a \wedge b) \wedge c$;

(v) $a \in a \sqcup b \Rightarrow a \wedge b = b$.

Where for all nonempty subsets A *and* B *of* L,

$A \sqcup B = \bigcup\{ a \sqcup b / a \in A,\ b \in B\}$;

$A \wedge B = \vee\{a \wedge b / a \in A,\ b \in B\}$.

The converse of condition (v) in Definition 1 is true. Indeed using (iii) in Definition 1, we obtain $a \in a \sqcup b$ *by taking* $b = a \wedge b$. *Hence, we can define an order on* L *by:*

$$a \leq b \Leftrightarrow b \in a \sqcup b \Leftrightarrow a \wedge b = a:$$

The binary relation $\leq$ *is reflexive, antisymmetric and transitive. Thus* $(L, \leq)$ *is a poset.*

Definition 3.2 *([2]) A hyperlattice L is called bounded if there exist 0,1 $\in$ L such that for all a $\in$ L, 0 $\leq$ a $\leq$ 1, and we say that 0 is the least element and 1 is the greatest element of L.*

Definition 3.3 *([1]) Let L be a hyperlattice, L is distributive if for all a,b,c $\in$ L,*

$$a \wedge (b \sqcup c) = (a \wedge b) \sqcup (a \wedge c)$$

Definition 3.4 *([1]) Let L be a hyperlattice, L is dual distributive if for all a,b,c $\in$ L,*

$$a \sqcup (b \wedge c) = (a \sqcup b) \wedge (a \sqcup c)$$

Example 3.1 *(1) Let L = {0, a, 1}. Consider the following Cayley tables*

$\wedge$	0	a	1
0	0	0	0
a	0	a	a
1	0	a	1

$\sqcup$	0	a	1
0	{0,a,1}	{a,1}	{1}
a	{a,1}	{a}	{1}
1	{1}	{1}	{1}

Then (L, $\sqcup$, $\wedge$, 0, 1) is a distributive hyperlattice.
(2) Let L = {a, b}. Consider the following Cayley tables

$\wedge$	a	b
a	a	b
b	b	b

$\sqcup$	a	b
a	{a, b}	{a}
b	{a}	{b}

Then (L, $\sqcup$, $\wedge$) is a distributive hyperlattice, but since b $\sqcup$ (a $\wedge$ a) = {a}, and (b $\sqcup$ a) $\wedge$ (b $\sqcup$ a) = {a, b}, L is not dual distributive.

Consider a lattice (L, $\wedge$, $\sqcup$). We define the Nakano hyperoperation $\sqcup$ on L by x $\sqcup$ y = {z $\in$ L/z $\vee$ x = z $\vee$ y = x $\vee$ y}, for all x, y $\in$ L.

Lemma 3.1 *([3]) If (L, $\wedge$, $\vee$)is a distributive lattice, then (L, $\wedge$, $\sqcup$) is a distributive hyperlattice where a $\sqcup$ b = {x $\in$ L/ a $\vee$ b = a $\vee$ x = b $\vee$ x}, for all a,b $\in$ L.*

Example 3.2 *Let L = {0, a, b, 1} be a distributive lattice, where* $\wedge$ *and* $\vee$ *is defined as in the following table*

$\wedge$	0	a	b	1
0	0	0	0	0
a	0	a	0	a
b	0	0	b	b
1	0	a	b	1

$\vee$	0	a	b	1
0	0	a	b	1
a	a	a	1	1
b	b	1	b	1
1	1	1	1	1

We use the Nakano hyperoperation, and we obtain the following table

$\sqcup$	0	a	b	1
0	{0}	{a}	{b}	{1}
a	{a}	{0,a}	{1}	{b,1}
b	{b}	{1}	{0,b}	{a,1}
1	{1}	{b,1}	{a,1}	L

Then (L, $\wedge$*,* $\sqcup$*) is a hyperlattice.*

Lemma 3.2 *([3]) Let L = {0,1}. Then ({0,1},* $\wedge$*,* $\sqcup$*) is a bounded distributive hyperlattice, where*

$\wedge$	0	1
0	0	0
1	0	1

and

$\sqcup$	0	1
0	{0}	{1}
1	{1}	{0,1}

Proof 3.1 *By definition 1*

i) $0 \in 0 \sqcup 0$, *because* $0 \sqcup 0 = \{0\}$, *and* $0 \wedge 0 = 0$;

ii) $0 \sqcup 1 = 1 \sqcup 0 = \{1\}$, *and* $0 \wedge 1 = 1 \wedge 0 = 0$;

iii) $0 \in [0 \wedge (0 \sqcup 1)] \bigcap [0 \sqcup (0 \wedge 1)] \Leftrightarrow 0 \in [(0 \wedge 0) \sqcup (0 \wedge 1)] \bigcap [0 \sqcup (0 \wedge 1)]$

$\Leftrightarrow 0 \in [0 \sqcup 0] \bigcap [0 \sqcup 0]$

$\Leftrightarrow 0 \in \{0\} \bigcap \{0\}$

$\Leftrightarrow 0 \in \{0\}$

$1 \in [1 \wedge (1 \sqcup 0)] \bigcap [1 \sqcup (1 \wedge 0)] \Leftrightarrow 1 \in [(1 \wedge 1) \sqcup (1 \wedge 0)] \bigcap [1 \sqcup (1 \wedge 0)]$

$\Leftrightarrow 1 \in [1 \sqcup 0] \bigcap [1 \sqcup 0]$

$\Leftrightarrow 1 \in \{1\} \bigcap \{1\}$

$$\Leftrightarrow 1 \in \{1\}$$

iv)

a	b	c	$a \sqcup b$	$b \sqcup c$	$a \sqcup (b \sqcup c)$	$(a \sqcup b) \sqcup c$
0	0	0	{0}	{0}	{0}	{0}
0	0	1	{0}	{1}	{1}	{1}
0	1	0	{1}	{1}	{1}	{1}
0	1	1	{1}	{0,1}	{0,1}	{0,1}
1	0	0	{1}	{0}	{1}	{1}
1	0	1	{1}	{1}	{0,1}	{0,1}
1	1	0	{0,1}	{1}	{0,1}	{0,1}
1	1	1	{0,1}	{0,1}	{1,{0,1}}	{1,{0,1}}

We then have $a \sqcup (b \sqcup c) = (a \sqcup b) \sqcup c$.

v)

a	b	$a \sqcup b$
0	0	{0}
0	1	{1}
1	0	{1}
1	1	{0,1}

3.1 Ideals and filters in hyperlattices

Definition 3.5 *([10]) Let L be a hyperlattice and I be a nonempty subset of L. I is called an ideal if the following conditions hold:*

(i) If $a,b \in I$, *then* $a \sqcup b \subseteq I$;

(ii) If $a \in I$, $b \leqslant a$, *and* $b \in L$, *then* $b \in I$.

Definition 3.6 *([1]) Let L be a hyperlattice. A proper Ideal I is called a prime ideal if* $a \wedge b \in I$; *then* $a \in I$ *or* $b \in I$ *for all* $a,b \in L$

Definition 3.7 *([10]) Let L be a hyperlattice and F be a non-empty subset of L. F is called a filter if the following conditions hold:*

(i) If $a,b \in F$, *then* $a \wedge b \in F$;

(ii) If $a \in F$, $a \leqslant b$, *and* $b \in L$, *then* $b \in F$.

Definition 3.8 *([1]) Let L be a hyperlattice. A proper filter F is called prime filter if it satisfies* $\forall$ *a,b* $\in$ *L; (a* $\sqcup$ *b)* $\cap$ *F* $\neq \phi \Rightarrow$ *a* $\in$ *F or b* $\in$ *F.*

Theorem 3.1 *([1]) Let L be a hyperlattice. If P is a prime ideal of L, then L/P is a prime filter of L.*

Proof 3.2 *Let a, b* $\in$ *L/P. We show that a* $\wedge$ *b* $\in$ *L* $\setminus$ *P. It is clear that a, b* $\notin$ *P.*

If a $\wedge$ *b* $\in$*P, then a* $\in$ *P or b* $\in$ *P because P is prime filter, which is a contradiction. So a* $\wedge$ *b* $\in$ *L* $\setminus$ *P. Assume x* $\in$ *L, a* $\in$ *L* $\setminus$ *P such that a* $\leq$ *x, we show that x* $\in$ *L* $\setminus$ *P. It is clear that a* $\notin$ *P. Suppose x* $\notin$ *L* $\setminus$ *P, so x* $\in$ *P. We have a* $\leq$ *x and P is an ideal, therefore a* $\in$ *P that is a contradiction. So, L* $\setminus$ *P is a filter. It is enough to show that L* $\setminus$ *P is a prime filter. Suppose a, b* $\in$ *L and (a* $\sqcup$ *b)* $\cap$ *(L* $\setminus$ *P)* $\neq \phi$*. So there exists x* $\in$ *L such that x* $\in$ *a* $\sqcup$ *b and x* $\in$ *L* $\setminus$ *P. If a, b* $\notin$ *L* $\setminus$ *P, then a, b* $\in$ *P, therefore a* $\sqcup$ *b subseteq P. Hence (a* $\sqcup$ *b)* $\cap$ *(L* $\setminus$ *P) =* ϕ*, which is a contradiction.*

Theorem 3.2 *([1]) Let L be a hyperlattice. If F be a prime filter of L, then L* $\setminus$ *F is a prime ideal of L.*

Proof 3.3 *Let a, b* $\in$ *L* $\setminus$ *F. If a* $\sqcup$ *b* $\nsubseteq$ *L* $\setminus$ *F, then there exists x* $\in$ *a* $\sqcup$ *b such that x* $\notin$ *L* $\setminus$ *F. So x* $\in$ *a* $\sqcup$ *b and x* $\in$ *F. Hence x* $\in$ *(a* $\sqcup$ *b)* $\cap$ *F and it implies that (a* $\sqcup$ *b)* $\cap$ *F* $\neq \phi$*. Thus a* $\in$ *F or b* $\in$ *F, which is a contradiction. Let a* $\in$ *L* $\setminus$ *F, x* $\leq$ *a. Assume x* $\notin$ *L* $\setminus$ *F. Thus we have: x* $\in$ *F and x* $\leq$ *a. Since F is a filter, a* $\in$ *F, which is a contradiction. Therefore, L* $\setminus$ *F is an ideal. We must show that L* $\setminus$ *F is prime. Assume a, b* $\in$ *L such that a* $\wedge$ *b* $\in$ *L*$\setminus$*F. If a* $\notin$ *L* $\setminus$ *F and b /*$\notin$ *L* $\setminus$ *F, then a* $\in$ *F and b* $\in$ *F. Since F is a filter, a* $\wedge$ *b* $\in$ *F. So a* $\wedge$ *b* $\notin$ *L* $\setminus$ *F, which is a contradiction. Thus L* $\setminus$ *F is a prime ideal.*

Example 3.3 *Let L = {0, a, 1} and* $\wedge$ *and* $\sqcup$ *are given with Table below. Consider F = {1}, F is a filter of L, but it is not prime. We have L* $\setminus$ *F = {0, a}, 0* $\sqcup$ *0 = L, and L* $\nsubseteq$ *L* $\setminus$ *F, so L* $\setminus$ *F is not a ideal of L.*

$\wedge$	0	a	1
0	0	0	0
a	0	a	a
1	0	a	1

$\sqcup$	0	a	1
0	{0, a, 1}	{a, 1}	{1}
a	{a, 1}	{a}	{1}
1	{1}	{1}	{1}

For every nonempty subset δ of X, the smallest filter of X (with respect to the inclusion) which contains δ is said to be the filter generated by δ and will be denoted by $\langle\delta\rangle$.

Proposition 3.1 *([3]) If δ is a non-empty subset of a hyperlattice L, then*

$$\langle\delta\rangle = \{x \in L/a_1 \wedge ... \wedge a_n \leq x; \text{ for some } a_1...a_n \in \delta\}.$$

Proof 3.4 *Let $\langle\delta\rangle = \{x/ \wedge_{i=1}^{n} a_i \leq x, a_1, a_2, ..., a_n \in \delta\}$.*
First, we prove that $\langle\delta\rangle$ is non-empty. Let $a \in \delta$, since $a \leq a$, then $a \in \langle\delta\rangle$, hence $\langle\delta\rangle \neq \phi$. To proof that $\langle\delta\rangle$ is a filter. Let $x \in \langle\delta\rangle$, $y \in X$ such that $x \leq y$, there exist $a_1, a_2, ..., a_n$ such that $\wedge_{i=1}^{n} a_i \leq x$: Then, $\wedge_{i=1}^{n} a_i \leq y$, then $y \in F$.
On the other hand, let x, $y \in \langle\delta\rangle$, there exist $a_1, a_2, ..., a_n, b_1, b_2, ..., b_n$ such that $\wedge_{i=1}^{n} a_i \leq x$ and $\wedge_{j=1}^{m} b_j \leq y$. Then, $(\wedge_{i=1}^{n} a_i) \wedge (\wedge_{j=1}^{m} b_j) \leq x \wedge y$. Therefore, $x \wedge y \in \langle\delta\rangle$.
Next, let $a \in \delta$, since $a \leq a$, we have $a \in \langle\delta\rangle$. Then $\delta \subseteq \langle\delta\rangle$.
Finally, suppose that F is a filter with $\delta \subseteq F$. Then for any $x \in \langle\delta\rangle$, then there exists $a_1, a_2, ..., a_n$ such that:
$\wedge_{i=1}^{n} a_i \leq x$, then $x \in F$. Therefore $\langle\delta\rangle \subseteq F$. If $\delta = a$, we write $\langle\delta\rangle = \uparrow a = \{x \in L/a \leq x\}$, we shall call $\uparrow a$ a principal filter generated by a.

Proposition 3.2 *([3]) Let (L, $\sqcup$, $\wedge$) be a distributive hyperlattice. If $a \in L$ then $I = \downarrow a = \{x \in L/ x \leq a\}$ is an ideal.*

Proof 3.5 *Let x, $y \in L$, such that $x \leq y$ and $y \in I = \downarrow a$. Since $x \leq y$ and $y \leq a$, we have $x \in I$. furthermore, if p, $q \in I$, then $p \leq a$ and $q \leq a$. By the distributivity of L, we have $a \wedge(p \sqcup q)=(a\ ) \sqcup (a \wedge q)$. Thus for every $x \in p \sqcup q$, there exists $y \in p \sqcup q$ such that $x = a \wedge y \leq a$, proving $p \sqcup q \subseteq I$*

Theorem 3.3 *([3]) Let X be a distributive hyperlattice, F be a filter and I an ideal of X. If $F \cap I = \phi$, then there is a prime filter P such that $F \subseteq P$ and $P \cap I = \phi$.*

Proof 3.6 *Let G be the family of those filters F' which satisfy $F \subset$ F' and F' $\cap\, I = \phi$.*
It follows from the Zorn's lemma that G has a maximal element P. Since $P \in G$ it remains to prove that the filter P is prime. Since $P \cap I = \phi$, P is proper. Suppose P is not prime. Then there exist a, $b \in X$ such that $(a \sqcup b) \cap P \neq \phi$, and $a \notin P$ and $b \notin P$. Let $a_0 \in (a \sqcup b) \cap P$ and let $\delta = P \cup \{a\}$. Then $\langle\delta\rangle \cap I \neq \phi$, otherwise

$P \subseteq \langle\delta\rangle \in G$ *contradicting the maximality of* P. *Take* $x \in \langle\delta\rangle \cap I$. *This implies easily there exists* $p \in P$ *that* $p \wedge a \leq x$, *it follows that* $a_0 \wedge p \wedge a \leq x$ *and since* $x \in I$, *it follows that* $a_0 \wedge p \wedge a \in I$. *Similarly* $a_0 \wedge q \wedge b \in I$. *Then,* $a_0 \wedge m \wedge a \in I$ *and* $a_0 \wedge m \wedge b \in I$ *such that* $m = p \wedge q$, *it follows that* $(a_0 \wedge m \wedge a) \sqcup (a_0 \wedge m \wedge b) \subseteq I$, *which implies* $a_0 \wedge m \in (a_0 \wedge m) \wedge (a \sqcup b) \subseteq I$, *and* $a_0 \wedge m \in P$, *therefore* $I \cap P \neq \phi$, *which is a contradiction.*

Corollary 3.1 *Let L be a distributive hyperlattice. If I is an ideal and* $a \in L$ *- I, then there exists a prime filter P such that* $a \in P$ *and* $P \cap I = \phi$.

Proof 3.7 *Let I be an ideal,* $a \in L$ *- I and take* $F = \langle a\rangle$, *it fommows* $F \cap I = \phi$. *By Theorem 3.3, there is a prime filter P such that* $F \subseteq P$ *and* $P \cap I = \phi$.

3.2 Hyperlattices isomorphisms

Definition 3.9 *([3]) Let L and L' be two hyperlattices and* $f: L \longrightarrow L'$ *be a mapping.*
1. f is said to be a hyperlattices homomorphism if $f(x \wedge y) = f(x) \wedge f(y)$ *and* $f(x \sqcup y) \subseteq f(x) \sqcup f(y)$, *for all* $x, y \in L$.
2. f is said to be a strong homomorphism of a hyperlattices, if $f(x \wedge y) = f(x) \wedge f(y)$ *and* $f(x \sqcup y) = f(x) \sqcup f(y)$, *for all* $x, y \in L$. *If f is a bijection, then f is said to be a hyperlattices isomorphism (strong isomorphism).*

Example 3.4 *Let L be the hyperlattice in the Example 3.2 and f:* $L \to L$ *is defined by:* $f(0) = 0$, $f(a) = a$, *and* $f(1) = a$. *Then f is a homomorphism. Since* $f(0 \sqcup a) = \{a\}$, *and* $f(0) \sqcup f(a) = \{a, 1\}$, *so* $f(0 \sqcup a) \subset f(0) \sqcup f(a)$.

Proposition 3.3 *Let* $(L, \wedge, \sqcup)$ *be a hyperlattice,* $(\{0,1\}, \wedge, \sqcup)$ *be the hyperlattice in Lemma 3.2 and F be a subset of L. If F is a prime filter, then there is a surjective hyperlattices homomorphism* $f: L \longrightarrow \{0,1\}$, *such that* $F = f^{-1}(\{1\})$.

Proof 3.8 *Let*

$$f(x) = \begin{cases} 1 \; if \;\; X \subseteq F, \\ 0 \; if \;\; X \subseteq L/F. \end{cases}$$

The surjection of f follows from $\phi \neq F \neq L$. And since $(x \sqcup y) \cap F \neq \phi$, $(x \in F$ or $y \in F)$, then

$f(x \sqcup y) = 1 \Rightarrow x \sqcup y \subseteq F$

$\Rightarrow (x \sqcup y) \cap F \neq \phi$

$\Rightarrow x \in F$ or $y \in F$

$\Rightarrow f(x) = 1$ or $f(y) = 1$

Hence, $f(x \sqcup y) \subseteq f(x) \sqcup f(y)$.

If $f(x \sqcup y) = 0$, we have $x \sqcup y \nsubseteq F$, it follows that $(x \sqcup y) \cap F \neq \phi$, which implies $x \notin F$ and $y \notin F$, it follows that $f(x) = 0$ and $f(y) = 0$, which implies $f(x) \sqcup f(y) = 0$.

Therefore, $f(x \sqcup y) \subseteq f(x) \sqcup f(y)$.

For the second homomorphism axiom, we have

$f\ (x \wedge y) = 0 \Leftrightarrow x \wedge y \subseteq L/F$

$\Leftrightarrow x \in L/F$ or $y \in L/F$ (L/F is prime ideal)

$\Leftrightarrow f\ (x) = 0$ or $f\ (y) = 0$

$\Leftrightarrow f\ (x) \wedge f\ (y) = 0$

Hence, $f\ (x \wedge y) = f\ (x) \wedge f\ (y)$.

Corollary 3.2 *Let L be a distributive hyperlattice. If $a, b \in X$ are such that $a \nleq b$ there is a prime filter F such that $a \in F$ and $b \notin F$.*

Proof 3.9 *Take $I = \downarrow b$ in Corollary 3.1*

Example 3.5 *Let L be the hyperlattice, i.e. Let $L = \{0, a, b, 1\}$ and $\wedge$ and $\sqcup$ are given with Table bellow:*

$\wedge$	0	a	b	1
0	0	0	0	0
a	0	a	0	a
b	0	0	b	b
1	0	a	b	1

$\sqcup$	0	a	b	1
0	{0}	{a}	{b}	{1}
a	{a}	{0, a}	{1}	{b,1}
b	{b}	{1}	{0,b}	{a,1}
1	{1}	{b,1}	{a,1}	1

Then $(L, \wedge, \sqcup, 0, 1)$ is a distributive hyperlattice. Let moreover $f : L \to L$ be defined by: $f(a) = b$, $f(b) = a$, $f(0) = 0$, $f(1) = 1$, then f is an isomorphism and $\mathcal{K}er(f) = \{1\}$.

3.3 Priestley duality for bounded distributive hyperlattices

Definition 3.10 *A Priestley space is a compact totally order-disconnected topological space.*

If A is a bounded distributive hyperlattice, then its dual space is defined to be $T(A) = (X, \tau, \nleq)$, where X is the set of homomorphisms from A onto $(\{0,1\}, \wedge, \sqcup)$, preserving 0 and 1, τ is the product topology induced from $\{0,1\}^A$, and $\nleq$ is the partial order defined by $f \nleq g$ in X if and only if $f(a) \nleq g(a)$ for all $a \in A$.

$T(A) = (X, \tau, \leq)$ is compact, and it is also totally order disconnected, i.e., a Priestley space.

3.3.1 Priestley spaces isomorphisms

Lemma 3.3 *([3]) If $\delta = (X, \tau, \leq)$ is a Priestley space, then there exists a hyperoperation $\sqcup$ such that $(L(\delta), \cap, \sqcup, \phi, X)$ is a bounded distributive hyperlattice. Where $L(\delta) = \{Y \subseteq X / Y$ is increasing and τclopeng $\}$ and $\sqcup$ is defined by*

$$A \sqcup B = \{X \in L(\delta) / A \cup B = A \cup X = B \cup X\}$$
$$\text{for all } A, B \in L(\delta).$$

Example 3.6 *Let $E = \{a, b, c\}$, and $\mathcal{P}(E) = \{\phi, \{a\}, \{b\}, \{c\}, \{a,b\}, \{a,c\}, \{b,c\}, E\}$ be a family sets of E. Let $A = \{a,b\}$, and $B = \{a,c\}$ an subsets of $\mathcal{P}(E)$.*

$A \sqcup B = \{X \in \mathcal{P}(E) / \{a,b\} \cup X = \{a,c\} \cup X = \{a, b, c\}\}$

$\quad = \{\{b,c\}, E\}$.

Proof 3.10 *Since (L(δ), $\cap$, $\cup$, ϕ, X) is a bounded distributive lattice,*

then (L(δ),$\cap$, $\sqcup$, ϕ, X) is a bounded distributive hyperlattice by Lemma 3.1.

Lemma 3.4 *([3]) Let A be a bounded distributive hyperlattice. The map $F_A : A \mapsto L(T(A))$ defined by $F_A(a)$ = {f $\in$ X/f(a) = 1}is a hyperlattice isomorphism.*

Proof 3.11 *For all a,b $\in$ A we have*

$$
\begin{aligned}
F_A(a \wedge b) &= \{f \in X/f(a \wedge b) = 1\} \\
&= \{f \in X/f(a) = 1\} \cap \{f \in X/f(b) = 1\} \\
&= F_A(a) \sqcap F_A(b) \ ;
\end{aligned}
$$

and

$$
\begin{aligned}
F_A(a \sqcup b) &= \{F_A(t)/t \in a \sqcup b\} \\
&= \{f \in X/f(a \sqcup b) = 1\} \\
&\subseteq \{\{f \in X/f(a) = 1\} \cup \{f \in X/f(b) = 1\}\} \\
&= \{F_A(a) \cup F_A(b)\} \\
&\subseteq F_A(a) \sqcup F_A(b)
\end{aligned}
$$

Suppose that $a \neq b$, it follows $a \nleq b$ or $b \nleq a$.

If $a \nleq b$ then, there exist a prime filter F such that $a \in F$ and $b \notin F$ (By Corollary 3.2), by Proposition 3.3, there is a surjective hyperlattices homomorphism $f : A \to \{0,1\}$ such that $a \in f^{-1}(\{1\})$ and $b \notin f^{-1}(\{1\})$, hence f(a) = 1 and f(b) = 0, i.e., $F_A(a) \nleq F_A(b)$.

Similarly if $b \nleq a$ we have $F_A(b) \nleq F_A(a)$. Hence, $a \neq b$ imply $F_A(b) \neq F_A(a)$ i.e., F_A is injective.

To prove that F_A is surjective let U $\in$ L(T(A)), for all $f \in U$ and g $\in$ L(T(A)) U, since U is increasing it follows that $g < f$, it follows that $a_{fg} \in A$ such that $f(a_{fg})$ = 1 and $g(a_{fg})$ = 0. Then, $f \in F_A(a_{fg})$ and g $\in$ L(T(A)) $F_A(a_{fg})$. For a fixed $f \in U$ we have g $\in$ L (T(A))U $\subseteq$ $\bigcup_{i=1}^n (L(T(A))F_A(a_{fgi})) = L(T(A))F_A(\wedge_{i=1}^n a_{fgi})(L(T(A))U$ is compact). Setting $\wedge_{i=1}^n a_{fgi} = a_f$, it follows $F_A(a_f) = F_A(\wedge_{i=1}^n a_{fgi}) \subset U$, on the other hand $f(a_f) = 1$, then $f \in F_A(a_f)$. Therefore, U = $\cup_{f \in U} f \in F_A(a_f)$, we find again a finite covering U = $F_A(a_{fj})$, it follows that {U} $\supseteq \sqcup_{i=1}^n F_A(a_{fj}) \supseteq F_A(\sqcup_{i=1}^n a_{fj})$, (since B $\subset$ B' $\Rightarrow$ $F_A^1(B) \subset F_A^1(B')$ and F_A injective) $\Rightarrow$ $F_A^1(F^A(\sqcup_{i=1}^n a_{fj}))$ = $\sqcup_{i=1}^n a_{fj}$

We have $\sqcup_{i=1}^n a_{fj} \subseteq F_A^1(U)$ (since $\sqcup_{i=1}^n a_{fj} \in \mathcal{P}*(A)$, i.e., $\phi \neq \sqcup_{i=1}^n a_{fj} \subseteq A$, i.e., F_A is surjective, since F_A is injective, there exists $a \in A$ such that U = $F_A(a)$. Therefore, F_A is a *hyperlattice isomorphism.*

Lemma 3.5 *If $f : A_1 \mapsto A_2$ is a hyperlattice homomorphism, then the map $T(f) : T(A_1) \mapsto T(A_1)$ defined by $T(f)(g) = g \circ f$ is a homomorphism of Priestley space, i.e., a continuous and increasing map .*

Proof 3.12 *For all $g_1, g_2 \in T(A_1)$, $g_1 \leq g_1 \Rightarrow g_1 \circ f \leq g_2 \circ f$, hence $T(f)$ is increasing.*

The continuity of $T(f)$ follows from the fact that for every $a \in A_1$;

$$\begin{aligned} T^{-1}(f)(F_{A_1}(a)) &= \{g \in T(A_2)\ /T(f)(g) \in F_{A_1}(a)\} \\ &= \{g \in T(A_2)\ /g \circ f(a) = 1\} \\ &= \{g \in T(A_2)\ /g(f(a)) = 1\} \\ &= F_{A_2}(f(a)). \end{aligned}$$

Hence, $T\ (f)$ is continuous.

Lemma 3.6 *If $h : \delta_1 \mapsto \delta_2$ is a homomorphism of Priestley space, then the map $L(h) : L(\delta_1) \mapsto L(\delta_2)$ defined by $L(h)(y) = h^{-1}(y)$ for every $y \in L(\delta_2)$ is a hyperlattices homomorphism.*

Proof 3.13 *For all $y \in L(\delta_2)$ we have $L(h)(y) \in L(\delta_1)$. For all $y, z \in L(\delta_2)$ since h^{-1} commutes with set-theoretical operations we have,*

$$\begin{aligned} L(h)(y \sqcup z) &\subseteq \{h^{-1}(x)/h^{-1}(y \sqcup z) = h^{-1}(y \sqcup x) = h^{-1}(z \sqcup x)\} \\ &= \{h^{-1}(x) = h^{-1}(y) \cup h^{-1}(z) = h^{-1}(y) \cup h^{-1}(x) = h^{-1}(z) \cup h^{-1}(x)\} \\ &\subseteq \{L(h)(y) \sqcup L(h)(z)\}. \end{aligned}$$

and

$$\begin{aligned} L(h)(y \cap z) &= h^{-1}(y \cap z) \\ &= h^{-1}(y) \cap h^{-1}(z) \\ &= L(h)(y) \cap L(h)(z). \end{aligned}$$

Hence, $L(h)$ is hyperlattices homomorphism.

Theorem 3.4 *([3]) If $f : A_1 \mapsto A_2$ is a hyperlattice homomorphism, then*

$$L(T(f)) \circ F_{A_1} = F_{A_2} \circ f.$$

$$
\begin{tikzcd}
A_1 \arrow[r, dashed, "f"] \arrow[d, dashed, "F(A_1)"'] & A_2 \arrow[d, dashed, "F(A_2)"] \\
L(T(A_1)) \arrow[r, dashed, "L(T(f))"'] & L(T(A_2))
\end{tikzcd}
$$

Proof 3.14 *For all* $a \in A_1$,

$$
\begin{aligned}
(L(T(f)) \circ F_{A_1})(a) &= L(T(f))(F_{A_1}(a)) \\
&= T^{-1}(f)(F_{A_1}(a)) \\
&= \{g \in T(A_2)/T(f)(g) \in F_{A_1}(a)\} \\
&= \{g \in T(A_2)/g \circ f \in F_{A_1}(a)\} \\
&= \{g \in T(A_2)/g(f(a)) = 1\} \\
&= F_{A_2}(f(a)) \\
&= F_{A_2} \circ f(a).
\end{aligned}
$$

Example 3.7 *Let L = {0, a, b, 1}. Consider the following Cayley tables*

$\wedge$	0	a	b	1
0	0	0	0	0
a	0	a	0	a
b	0	0	b	b
1	0	a	b	1

$\sqcup$	0	a	b	1
0	{0}	{a}	{b}	{1}
a	{a}	{0,a}	{1}	{b,1}
b	{b}	{1}	{0,b}	{a,1}
1	{1}	{b,1}	{a,1}	L

Then (L, $\wedge, \sqcup$*, 0, 1) is a bounded distributive hyperlattice.*

T(A) = The set of 0 - 1 homomorphisms from A onto {0,1}=$\{f_1, f_2\}$

A	f_1	f_2
0	0	0
a	1	0
b	0	1
1	1	1

and its bidual is: L(T(A)) = \{ ϕ, $\{f_1\}$, $\{f_2\}$, $\{f_1, f_2\}$ \}, where $\cap$ and $\sqcup$ are given by

$\cap$	ϕ	$\{f_1\}$	$\{f_2\}$	$\{f_1, f_2\}$
ϕ	ϕ	ϕ	ϕ	ϕ
$\{f_1\}$	ϕ	$\{f_1\}$	ϕ	$\{f_1\}$
$\{f_2\}$	ϕ	ϕ	$\{f_2\}$	$\{f_2\}$
$\{f_1, f_2\}$	ϕ	$\{f_1\}$	$\{f_2\}$	$\{f_1, f_2\}$

$\sqcup$	ϕ	$\{f_1\}$	$\{f_2\}$	$\{f_1, f_2\}$
ϕ	$\{\phi\}$	$\{\{f_1\}\}$	$\{\{f_2\}\}$	$\{\{f_1, f_2\}\}$
$\{f_1\}$	$\{\{f_1\}\}$	$\{\phi, \{f_1\}\}$	$\{\{f_1, f_2\}\}$	$\{\{f_2\}, \{f_1, f_2\}\}$
$\{f_2\}$	$\{\{f_2\}\}$	$\{\{f_1, f_2\}\}$	$\{\phi, \{f_2\}\}$	$\{\{f_1\}, \{f_1, f_2\}\}$
$\{f_1, f_2\}$	$\{\{f_1, f_2\}\}$	$\{\{f_2\}, \{f_1, f_2\}\}$	$\{\{f_1\}, \{f_1, f_2\}\}$	$\{\phi, \{f_2\}, \{f_1, f_2\} \}$

Then (L(T(A))), $\cap$, $\sqcup$, ϕ, X) is a bounded distributive hyperlattice where $X = \{f_1, f_2\}$.

Finally, F_A: A $\mapsto$ L(T(A)) is given by

A	$F_A(a_1)/\ i = 1$ to 4
0	ϕ
a	$\{f_1\}$
b	$\{f_2\}$
1	$\{f_1, f_2\}$

Example 3.8 *Let D(30) = \{1, 2, 3, 5, 6, 10, 15, 30\} be the set of positive divisors of 30 and (D(30), $\wedge$; $\vee$) the lattice where, for all a, b $\in$ D(30), $a \wedge b$ and $a \vee b$ are respectively the greatest common divisor of a and b and the least common multiplier of a and b. Define on D (30) the hyperoperation by:*

$a \sqcup b = \{x \in D(30)/a \vee b = a \vee x = b \vee x\}$, for all a, b $\in$ L. We have the following Cayley tables:

$\sqcup$	1	2	3	5	6	10	15	30
1	{1}	{2}	{3}	{5}	{6}	{10}	{15}	{30}
2	{2}	D(2)	{6}	{10}	{3,6}	{5,10}	{30}	{15,30}
3	{3}	{6}	D(3)	{15}	{2,6}	{30}	{5,15}	{10,30}
5	{5}	{10}	{15}	D(5)	{30}	{2,10}	{3,15}	{6,30}
6	{6}	{3,6}	{2,6}	{30}	D(6)	{15,30}	{10,15}	{5,10,15,30}
10	{10}	{5,10}	{30}	{2,10}	{15,30}	D(10)	{6,30}	{3,6,15,30}
15	{15}	{30}	{5,15}	{3,15}	{10,30}	{6,30}	D(15)	{2,6,10,30}
30	{30}	{15,30}	{10,30}	{6,30}	{5,10,15,30}	{3,6,15,30}	{2,6,10,30}	D(30)

(D(30), $\wedge$, $\sqcup$, 1, 30) is a bounded distributive hyperlattice.

T(D(30)) = The set of 0-1 homomorphisms from A onto {0,1} = $\{f_1, f_2, f_3\}$

D(30)	f_1	f_2	f_3
1	0	0	0
2	1	0	0
3	0	1	0
5	0	0	1
6	1	1	0
10	1	0	1
15	0	1	1
30	1	1	1

and its bidual is: $L(T(A)) = \{\phi,\{f_1\}, \{f_2\},\{f_3\},\{f_1, f_2\}, \{f_1, f_3\}, \{f_2, f_3\}, X\}$,
where $X = \{f_1, f_2, f_3\}$.

$\cap$	ϕ	$\{f_1\}$	$\{f_2\}$	$\{f_3\}$	$\{f_1, f_2\}$	$\{f_1, f_3\}$	$\{f_2, f_3\}$	X
ϕ	ϕ	ϕ	ϕ	ϕ	ϕ	ϕ	ϕ	ϕ
$\{f_1\}$	ϕ	$\{f_1\}$	ϕ	ϕ	$\{f_1\}$	$\{f_1\}$	ϕ	$\{f_1\}$
$\{f_2\}$	ϕ	ϕ	$\{f_2\}$	ϕ	$\{f_2\}$	ϕ	$\{f_2\}$	$\{f_2\}$
$\{f_3\}$	ϕ	ϕ	ϕ	$\{f_3\}$	ϕ	$\{f_3\}$	$\{f_3\}$	$\{f_3\}$
$\{f_1, f_2\}$	ϕ	$\{f_1\}$	$\{f_2\}$	ϕ	$\{f_1, f_2\}$	$\{f_1\}$	$\{f_2\}$	$\{f_1, f_2\}$
$\{f_1, f_3\}$	ϕ	$\{f_1\}$	ϕ	$\{f_3\}$	$\{f_1\}$	$\{f_1, f_3\}$	$\{f_3\}$	$\{f_1, f_3\}$
$\{f_2, f_3\}$	ϕ	ϕ	$\{f_2\}$	$\{f_3\}$	$\{f_2\}$	$\{f_3\}$	$\{f_2, f_3\}$	$\{f_2, f_3\}$
X	ϕ	$\{f_1\}$	$\{f_2\}$	$\{f_3\}$	$\{f_1, f_2\}$	$\{f_1, f_3\}$	$\{f_2, f_3\}$	X

$\sqcup$	ϕ	$\{f_1\}$	$\{f_2\}$	$\{f_3\}$
ϕ	$\{\phi\}$	$\{\{f_1\}\ \}$	$\{\{f_2\}\}$	$\{\{f_3\}\}$
$\{f_1\}$	$\{\{f_1\}\}$	$\{\{\phi,\ f_1\}\}$	$\{\{f_1, f_2\}\}$	$\{\{f_1, f_3\}\}$
$\{f_2\}$	$\{\{f_2\}\}$	$\{\{f_1, f_2\}\}$	$\{\{\phi,\ f_2\}\}$	$\{\{f_2, f_3\}\}$
$\{f_3\}$	$\{\{f_3\}\}$	$\{\{f_1, f_3\}\}$	$\{\{f_2, f_3\}\}$	$\{\{\phi,\ f_3\}\}$
$\{f_1, f_2\}$	$\{\{f_1, f_2\}\}$	$\{\{f_2\},\{f_1, f_2\}\}$	$\{\{f_1\},\{f_1, f_2\}\}$	$\{X\}$
$\{f_1, f_3\}$	$\{\{f_1, f_3\}\}$	$\{\{f_3\},\{f_1, f_3\}\}$	$\{X\}$	$\{\{f_3\},\{f_1, f_3\}\}$
$\{f_2, f_3\}$	$\{\{f_2, f_3\}\}$	$\{X\}$	$\{\{f_3\},\{f_2, f_3\}\}$	$\{\{f_2\},\{f_1, f_3\}\}$
X	$\{X\}$	$\{\{f_2, f_3\},\ X\}$	$\{\{f_1, f_3\},\ X\}$	$\{\{f_1, f_2\},\ X\}$

$\sqcup$	$\{f_1, f_2\}$	$\{f_1, f_3\}$
ϕ	$\{\{f_1, f_2\}\}$	$\{\{f_1, f_3\}\}$
$\{f_1\}$	$\{\{f_2\},\{f_1, f_2\}\}$	$\{\{f_3\},\{f_1, f_3\}\}$
$\{f_2\}$	$\{\{f_1\},\{f_1, f_2\}\}$	$\{X\}$
$\{f_3\}$	$\{X\}$	$\{\{f_1\},\{f_1, f_3\}\}$
$\{f_1, f_2\}$	$\{\phi,\ \{f_1\},\ \{f_2\},\ \{f_1, f_2\}\}$	$\{\{f_2, f_3\},\ X\}$
$\{f_1, f_3\}$	$\{\{f_2, f_3\},\ X\}$	$\{\phi,\ \{f_1\},\ \{f_3\},\ \{f_1, f_3\}\}$
$\{f_2, f_3\}$	$\{\{f_1, f_3\},\ X\}$	$\{\{\{f_1, f_2\},\ X\}$
X	$\{\{f_3\},\ \{f_1, f_3\},\ \{f_2, f_3\},\ X\}$	$\{\{f_2\},\ \{f_1, f_2\},\ \{f_2, f_3\},\ X\}$

$\sqcup$	$\{f_2, f_3\}$	X
ϕ	$\{\{f_2, f_3\}\}$	$\{X\}$
$\{f_1\}$	$\{X\}$	$\{\{f_2, f_3\}, X\}$
$\{f_2\}$	$\{\{f_3\},\{f_2, f_3\}\}$	$\{\{f_1, f_3\}, X\}$
$\{f_3\}$	$\{\{f_2\},\{f_2, f_3\}\}$	$\{\{f_1, f_2\}, X\}$
$\{f_1, f_2\}$	$\{\{f_1, f_3\}, X\}$	$\{\{f_3\}, \{f_1, f_3\}, \{f_2, f_3\}, X\}$
$\{f_1, f_3\}$	$\{\{f_1, f_2\}, X\}$	$\{\{f_2\}, \{f_1, f_2\}, \{f_2, f_3\}, X\}$
$\{f_2, f_3\}$	$\{\phi, \{f_2\}, \{f_3\}, \{f_2, f_3\}\}$	$\{\{f_1\}, \{f_1, f_2\}, \{f_1, f_3\}, X\}$
X	$\{\{f_1\}, \{f_1, f_2\}, \{f_1, f_3\}, X\}$	$L(T(A))$

Then $(L(T(A)), \cap, \sqcup, \phi, X)$is a distributive hyperlattice.

Finally, F_A: $A \mapsto L(T(A))$ is given by

$D(30)$	$F_A(a_1)/\ i = 1$ to 8
1	$\{\phi\}$
2	$\{f_1\}$
3	$\{f_2\}$
5	$\{f_3\}$
6	$\{f_1, f_2\}$
10	$\{f_1, f_3\}$
15	$\{f_2, f_3\}$
30	X

Example 3.9 *Let $(X, \tau, \leq)$ be a Priestly space, where $X = \{a, b, c\}$ and $\leq$ is given by*

$\leq$	a	b	c
a	1	0	1
b	0	1	1
c	0	0	1

and $L(X) = \{\phi, \{c\}, \{a, c\}, \{b, c\}, X\}$, where

$\cap$	ϕ	$\{c\}$	$\{a,\ c\}$	$\{b,\ c\}$	X
ϕ	ϕ	ϕ	ϕ	ϕ	ϕ
$\{c\}$	ϕ	$\{c\}$	$\{c\}$	$\{c\}$	$\{c\}$
$\{a,\ c\}$	ϕ	$\{c\}$	$\{a,\ c\}$	$\{c\}$	$\{a,\ c\}$
$\{b,\ c\}$	ϕ	$\{c\}$	$\{c\}$	$\{b,\ c\}$	$\{b,\ c\}$
X	ϕ	$\{c\}$	$\{a,\ c\}$	$\{b,\ c\}$	X

and

$\sqcup$	ϕ	$\{c\}$	$\{a,\ c\}$	$\{b,\ c\}$	X
ϕ	$\{\phi\}$	$\{\{c\}\}$	$\{\{a,c\}\}$	$\{\{b,c\}\}$	$\{X\}$
$\{c\}$	$\{\{c\}\}$	$\{\phi,\ \{c\}\ \}$	$\{\{a,c\}\}$	$\{\{b,c\}\}$	$\{X\}$
$\{a,\ c\}$	$\{\{a,\ c\}\}$	$\{\{a,\ c\}\}$	$L(X)/\{\{b,\ c\},\ X\}$	$\{X\}$	$\{\{b,\ c\},\ X\}$
$\{b,\ c\}$	$\{\{b,\ c\}\}$	$\{\{b,\ c\}\}$	$\{X\}$	$L(X)/\{\{a,\ c\},\ X\}$	$\{\{a,\ c\},\ X\}$
X	$\{X\}$	$\{X\}$	$\{\{b,\ c\},\ X\}$	$\{\{a,\ c\},\ X\}$	$L(X)$

$\leqslant$	ϕ	$\{c\}$	$\{a,\ c\}$	$\{b,\ c\}$	X
ϕ	1	1	1	1	1
$\{c\}$	0	1	1	1	1
$\{a,\ c\}$	0	0	1	0	1
$\{b,\ c\}$	0	0	0	1	1
X	0	0	0	0	1

and $T(L(A)) = \{f_1, f_2, f_3\}$ *such that*

$L(X)$	f_1	f_2	f_3
ϕ	0	0	0
$\{c\}$	0	0	1
$\{a,\ c\}$	1	0	1
$\{b,\ c\}$	0	1	1
X	1	1	1

The isomorphism $G_X : X \to T(L(A))$ *is defined as follows*

X	$G_X(X_i)/X_i \in X$
a	f_1
b	f_2
c	f_3

Conclusion

In this work, we recall a way to represente distributive hyperlattices. It is shown that the dual of the category of Priestley spaces is equivalent to the category of bounded distributive hyperlattices.

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الملخص

ثنوية بريستلي تنشئ تكافئ طبيعي بين الشبكات التوزيعية المغلقة و ثنوي فضاءات بريستلي.
في هذا العمل، حاولنا تطبيق تقنيات بريستلي من أجل الحصول على نظرية متشابهة تخص فوق الشبكات التوزيعية (Hyperlattices).

Abstract

Priestley's duality establishes a natural equivalent between the bounded distributive lattices and the Priestley spaces.
In this work, we have traied to apply Priestley technis in order to obtain a similar representation theorem for bounded distributive hyperlattice.

Résumé

La dualité de Priestley établit une équivalence naturelle entre les treillis distributifs fermés et le dual des espaces de Priestley.
Dans ce travail, nous avons essayé d'appliquer les technique de Priestley afin d'obtenir un théorème de représentation similair pour un hyper-treillis distributifs fermés.