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**Optimal sliding mode control: application to
a Bioreactor**

Presented in front of the jury composed by:

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Dedication

الحمد لله وكفى و الصلاة والسلام على الحبيب المصطفى أما بعد

الحمد لله الذي وفقنا لإنجاز هذا العمل الذي عاد علينا بالكثير من المعرفة

I would like to dedicate my work to my family. who made me want to learn and make sacrifices to have access to quality education and reach this level. Also, dedication to my close friends who supported me throughout my student years.

Acknowledgments

This work would not have been possible without his support, so I would like to express my sincere gratitude to my supervisor Mr OUBABAS Hocine. Above all, He was too patient and helpful in guiding me in the realization of this Master thesis. I would also like to thank the members of the jury who will do me the honor of evaluating this modest work.

ملخص:

الهدف من عملنا هو تصميم طريقة تحكم مثالية و قوية ، في هذه الحالة (Optimal Sliding Mode Control OSMC) للقيام بذلك ، يتم تقديم تذكير بـ Sliding Mode Control و Linear Quadratic Control في الفصل الأول والفصل الثاني من هذه المخطوطة. تم وصف Optimal Sliding Mode Control في الفصل الثالث. يتم تنفيذ مثال توضيحي لعناصر التحكم الثلاثة (OSMC, SMC, QLC) على نظام حقيقي ، في هذه الحالة المفاعل الحيوي. تظهر نتائج المحاكاة متانة الطريقة المقترحة فيما يتعلق بتغير المعاملات ورفض الاضطرابات.

Summary:

The objective of our work is to design a robust optimal control method, in this case the optimal Sliding Mode Control (OSMC). To do this, a reminder of the Sliding Mode Control (SMC) and the linear Quadratic Control (LQC) is presented in chapter 1 and chapter 2 of this manuscript. A synthesis of the Optimal Sliding Mode Control is described in the third chapter. an example illustration of the three controls (LQ, SMC and OSMC) is carried out on a real system, in this case the bioreactor. The simulation results show the robustness of the proposed method with respect to parameter variation and disturbance rejection.

Résumé :

L'objectif de notre travail consiste à concevoir une méthode de command optimale robuste, en l'occurrence la Commande à Mode Glissant Optimale (OSMC). Pour ce faire un rappel sur la Commande a Mode Glissant (SMC) et la Commande Linéaire Quadratique (LQC) est présentée dans le chapitre 1 et le chapitre 2 de ce manuscrit. Une synthèse de La Commande Optimale a Mode Glissant est décrite dans le troisième chapitre. Un exemple d'illustration des trois commande (LQ, SMC et OSMC) est effectué sur un système réel, en l'occurrence le bioréacteur. Les résultats de simulation montrent la robustesse de la méthode proposée vis-à-vis de la variation des paramètres et du rejet de perturbations.

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List of Abbreviations and Symbols

Abbreviations

PID: Proportional -Integral–Derivative (controller)

LQR: Linear Quadratic Regulator

LQC: Linear Quadratic Control

LQ: Linear Quadratic

LTl: Linear Time Invariant

SMC: Sliding Mode Control

OSMC: Optimal Sliding Mode Control

CSTR: Continuous Stirred Tank Reactor

VSC: Variable Structure Control

Symbols

C_a, C_b, C_c : Concentration states (mol/m^3)

Q : Feed rate (m^3/s)

V : Volume (m^3)

$C_{A,in}$: Input Concentration (mol/m^3)

K_1, K_2 : Kinetic Constants (s^{-1})

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GENERAL INRODUCTION

A control system plays an important role in the people's daily life. We can find that many applications in our daily life are inseparable from control systems, such as the industrial development, the regulation of temperature for air conditioning, the adjustment of the elevator lifting speed, and automated aircraft landings.

On the other hand, control systems are not only limited to people's daily lives, but are also widely used in scientific research and industry. Some examples include the balanced control of a rocket while landing and robotic automation in manufacturing cells. In 2017, R. Lungu and M. Lungu performed research that used different control methods to achieve safe landing of an aircraft. In the research, they mentioned several control methods, which include Proportional-Integral-Derivative control (PID), Fuzzy control and Linear Quadratic Optimal Control. Conventional control laws such as PID are highly effective in linear systems with constant parameters. For non-linear systems or systems having non-constant parameters, these control laws may be insufficient because they are not robust, especially when the requirements on the accuracy and other dynamic characteristics of the system are strict. We need to use control laws that are not sensitive to parameter variations, disturbances, or non-linearities. The so-called laws regulating varying structures are a solution to these problems.[1].

Optimal control is an important area in modern control theory. Optimal control methodology provides a systematic approach to design a controller which satisfies predefined performance criteria like time minimization, cost minimization and error minimization. For designing an optimal controller for linear systems, linear quadratic regulator (LQR) technique is widely used because of its simplicity and systematic structure of design.[2]

Linear Quadratic Control, known as LQ control, is a method for calculating the optimal gain matrix of a control by state feedback. Linear Quadratic Control consists in minimizing a performance criterion whose objectives are tracking and energy. The system is linear[3].

Linear Quadratic Control (LQC), also known as Linear Quadratic Regulator (LQR), is a widely used control technique for linear time-invariant (LTI) systems. LQC aims to design optimal control strategies that minimize a quadratic cost function while considering the system dynamics and constraints.

In reality, almost all physical systems are affected by uncertainty caused by modelling error, parameter variation and external disturbances. Designing an optimal controller for uncertain systems is a highly challenging task. Continuing research in that area has led to design of robust optimal controllers.

The SMC is an ideal controller to ensure robustness. Though Optimal Control and SMC are two different types of control strategy, it is possible to combine them. Combination of optimal control with SMC has given rise to optimal sliding mode control (OSMC). Active research is continuing to design optimal sliding mode control as evident in literature.

Optimal Sliding Control offers several advantages, including robustness to uncertainties and disturbances, insensitivity to parameter variations, and the ability to handle nonlinear and time-varying systems. It has been successfully applied in various domains.

Optimal Sliding Control offers several advantages, including robustness to uncertainties and disturbances, insensitivity to parameter variations, and the ability to handle nonlinear and time-varying systems. It has been successfully applied in various domains.

The key idea behind optimal sliding control is to drive the system state into a predefined sliding surface, where the dynamics are simplified and more controllable. Once on the sliding surface, the control law is designed to keep the system state on this surface, minimizing the effects of disturbances and uncertainties.

This work presents the design of SMC and LQR for a Bioreactor. The performance of the designed SMC is compared with the LQR. The SMC can give a better performance if the switching function selection and tuning of the controller parameters are done precisely. The main contribution of this work is the development of an optimized SMC employing the LQR approach, which can asymptotically stabilize all states of the system with better performance even in the presence of external disturbances. In this work, the LQR/SMC controller is proposed to stabilize the Bioreactor system and achieve a desirable tracking[4].

Today, chemical reactors are the most influential and important units that chemists could find to carry out successive processes. Every type is used for specific tasks. One of its reactors is the perfectly stirred reactor, Continuous Stirred Tank Reactor (CSTR), this reactor is used in the first degree for the biological treatment of wastewater, it designates an axis active research for

control engineers and chemists, because of the need to fight against pollution and find other sources of water[5].

The memory is organized as follows.

Chapter 1: This chapter presents a first order sliding mode control method for linear systems affected by matched and mismatched uncertainties. It also describes the theory principle of the variable structure control

Chapter 2: Describes the Linear Quadratic Regulator based state feedback control law design. It describes the logic used in weight selection of the weighted matrices key to the LQR design.

Chapter 3: In this chapter an optimal sliding mode controller (OSMC) is proposed for linear uncertain systems. The optimal controller is designed for the nominal linear system using LQR technique. A sliding variable is designed to integrate the optimal controller with SMC to tackle the matched uncertainty affecting the linear system. The chapter then goes on to illustrate the performances of the proposed OSMC. Simulation results confirm effectiveness of the proposed controller. We end this Work by conclusion and aptly suggests the scope of future work.

CHAPTER

I

Variable Structure Control

I.1 Introduction

Variable Structure Control (VSC) was first introduced by Russian mathematician, Vladimir Utkin in the late 1970s. Utkin's work was motivated by the need for robust control methods for systems with uncertain or time-varying parameters. He proposed a control approach based on the concept of "sliding mode control," which he believed could provide a simple and effective solution to this problem [6].

The main concept behind sliding mode control is to create a sliding surface within the system's state space. The sliding surface is a hyperplane or manifold that defines a desired dynamic behavior or performance criterion for the system. The goal of SMC is to drive the system's state trajectory onto this sliding surface and then keep it on the surface during operation[7].

The key idea behind sliding mode control is the use of a discontinuous control signal. The control law consists of two main components: the "reaching law" and the "control discontinuity".

The reaching law determines the direction and magnitude of the control action to drive the system towards the sliding surface. The control discontinuity ensures that the system remains on the sliding surface by applying a switching control signal that forces the system to move along the surface[8].

One of the main advantages of sliding mode control is its robustness to uncertainties and disturbances.

In this chapter, we are going to show the principle of sliding mode control, the deferent definition relies to the structure variable theory and we present some other deferent technique and structures of sliding mode control (SMC).

I.2 Sliding mode control theory

The theory of systems with variable structure and the associated sliding modes is a nonlinear control technique, it is characterized by the discontinuity of the control at passages through a surface of switching called sliding surface. The technique of sliding modes consists in bringing the state trajectory of a system towards the sliding surface and to make it commute using an appropriate commutation around it to the point of equilibrium, hence the slippage phenomenon. In short, a sliding regime control is divided into two parts shown on (Figure I.1)[9] :

- ❖ Determining a state space region such that once the system is in that region, it has the desired behavior.
- ❖ Definition of a control law which leads the system to this region of the state space.

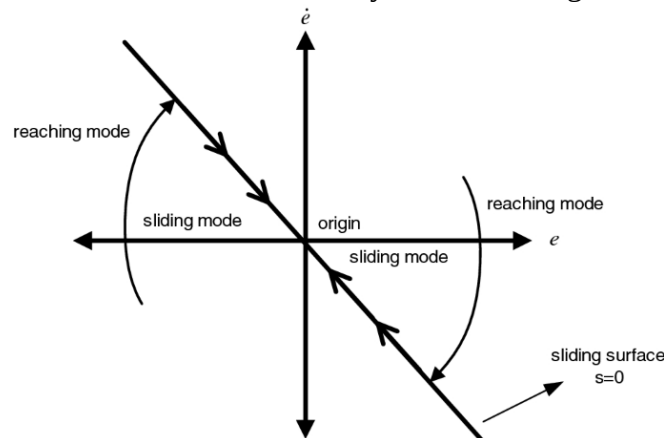


Figure I-1. Convergence of the sliding system

- **Reaching mode (RM):** This is the mode in which the variable to be tuned moves from any initial point in the phase plane, and tends towards the switching surface $S(x)=0$. This mode is characterized by the control law and the convergence criterion,
- **Sliding mode (SM):** It is the mode during which the state variable has reached the sliding surface and tends towards the origin of the phase plane. The dynamics of this mode is characterized by the choice of the sliding surface $S(x)=0$,
- **Steady state mode (SSM):** This mode is added for the study of the response of the system around its point of equilibrium (origin of the phase plane), it is characterized by the quality and the performance of the command.

Before starting the methods used in the synthesis of control system by sliding mode, we will explain some basic notions of the sliding regime[10].

I.2.1 Principle of control by sliding modes

The principle of sliding mode control is to constrain the system to reach a given surface called sliding surface according to the objectives of control, fixing the dynamics in closed loop: it is the mode of convergence, then by the synthesis of a discontinuous control which allows the trajectories of the system to reach and, then, to stay on this surface: this is the sliding mode[11].

I.2.2 Sliding mode existence condition

The second important aspect, after the design of the switching surface, is to ensure the existence of the sliding mode, hence the condition for obtaining the sliding regime:

$$\dot{S}(x)S(x) < 0 \quad (\text{I.1})$$

The existence of a sliding mode requires the stability of the state trajectory at the surface of slip $S(x,t)=0$.

The movement along the line, which no longer represents the trajectory of both structures, screams slip mode.

The switching phenomenon along this slip line can be demonstrated using of the following (Figure I.2)[12]:

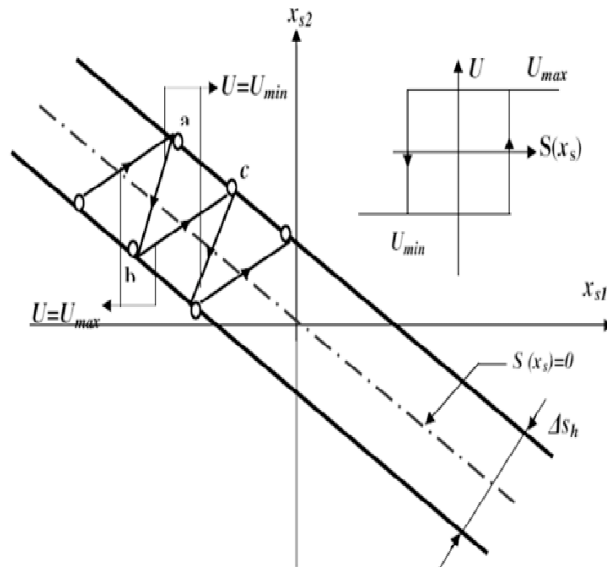


Figure I-2. Slide mode proof.

I.3 Sliding Mode Control Design

The design of regulators by sliding modes takes care of the problems of stability and desired performance in a systematic way. The implementation of this ordering method mainly requires three steps[13]:

1. The choice of surface.
2. The establishment of the conditions for the existence of convergence.
3. The determination of the control law.

I.3.1 Choice of sliding surfaces

Consider the system described by the following differential equation (2):

$$\dot{x} = f(x, t) + g(x, t)u \quad (I.2)$$

Where f and g are nonlinear functions, g is assumed to be invertible.

$u(t)$ The system entrance.

$x(t)$ System status.

Let x_d be the desired setpoint and e the following error defined by:

$$e = x - x_d \quad (I.3)$$

The general formula of the sliding surface is defined according to the order of the system as following:

$$S(x) = \left(\frac{\partial}{\partial t} + \lambda \right)^{r-1} e(x) \quad (I.4)$$

Where r : the relative degree of the system with respect to the output $y(t)$.

It represents the minimum number many times it is necessary to differentiate the output $y(t)$ with respect to time, to see the input appear there[14].

I.3.2 Convergence conditions

Convergence conditions allow the dynamics of the system to converge towards sliding surfaces. We retain from the literature two conditions, these correspond to the mode of convergence of the state of the system[13].

I.3.2.1 The direct switching conditions

This is the first convergence condition it is in the form:

$$\dot{S}(x)S(x) < 0 \quad (I.5)$$

I.3.2.2 Lyapunov's function

It is a question of formulating a positive scalar function $V(x) > 0$ for the state variables of the system and to choose a control law which will cause this function to decrease $\dot{V}(x) < 0$.

By defining the Lyapunov function:

$$V(x) = \frac{1}{2} S^2(x) \quad (I.6)$$

Its derivative will be:

$$\dot{V}(x) = S(x)\dot{S}(x) \quad (I.7)$$

For the Lyapunov function to decrease, it suffices to ensure that:

$$\dot{V}(x) = S(x)\dot{S}(x) < 0 \quad (I.8)$$

It is used to estimate the performance of the control, the study of robustness and guarantees the stability of the nonlinear system[15].

I.4 Definition of sliding regime

Sliding mode technology consists of bringing the state trajectory of a system towards the sliding surface and switching it with the help of an appropriate switching logic around it until the point of equilibrium, hence the sliding phenomenon.

I.4.1 Definition of the ideal sliding regime

The ideal sliding regime corresponds to an oscillation of infinite frequency (∞) and zero amplitude (0)[1].

I.4.2 Definition of the real sliding regime

In practice, the switching device is made from relays which presents imperfections such as the switching delay, so the slippery regime trajectory cannot ideally oscillate at an infinite frequency (Figure I.3). Actual slip finite switching frequency, oscillatory amplitude (Figure I.4)[1].

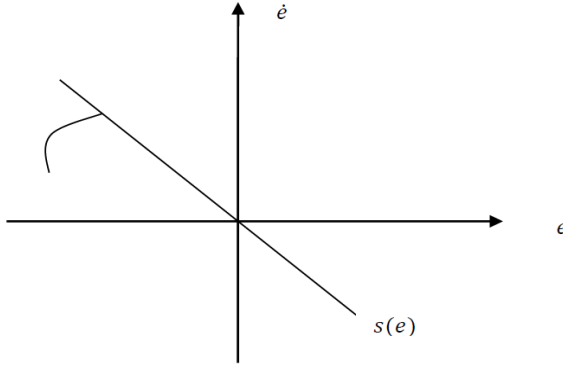


Figure I-3. Ideal sliding regime

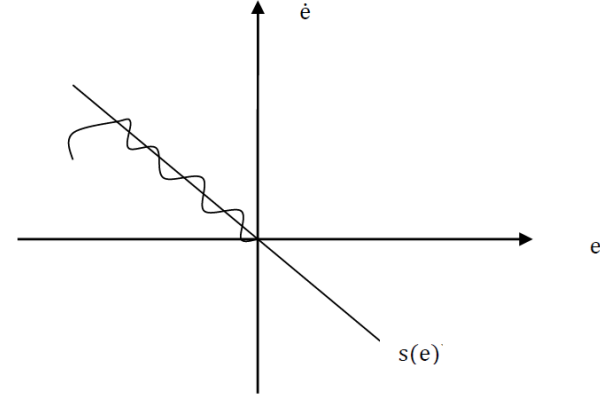


Figure I-4. Real slippery regime

I.5 law of command

Once the sliding surface is chosen, as well as the convergence criterion, it remains to determine the command needed to pull the variable to be tuned to the surface, then to its point equilibrium, while maintaining the existence condition of the sliding mode. When there is a sliding regime, the dynamics of the system is independent of the control law. Therefore, one can introduce a continuous part to reduce the amplitude of the discontinuity, we will therefore have:

$$u(t) = u_{eq}(t) + u_{dis}(t) \quad (I.9)$$

u_{eq} corresponds to the equivalent command proposed by Filipov and Utkin. It can be considered as the continuous average value that the control takes during a rapid switch between two values u^+ and u^- (Figure I.5). u_{eq} corresponds to the command that guarantees the attractiveness of the variable to be controlled towards the surface and satisfy the condition:

$$S(x)\dot{S}(x) < 0 \quad (I.10)$$

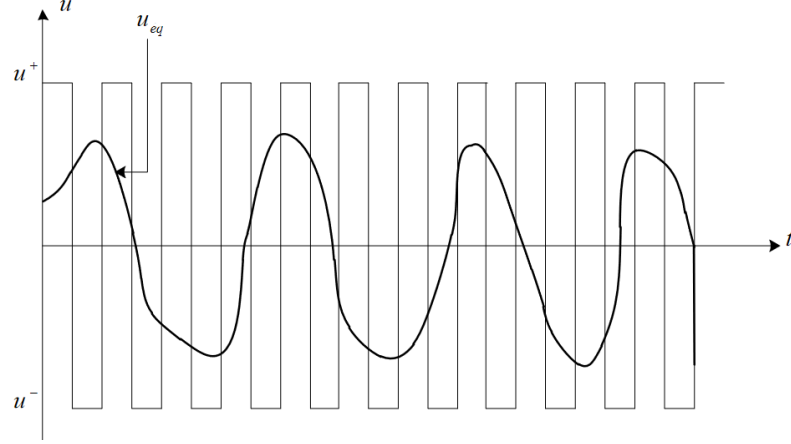


Figure I-5. Continuous value u taken by the control when switching between u^+ and u^-

To highlight the previous development, we consider a system defined in the state space by equation (I.2). It is a question of finding the analogical expression of the command u .

$$\dot{S}(t) = \frac{\partial S}{\partial t} = \frac{\partial S}{\partial x} \frac{\partial x}{\partial t} \quad (\text{I.11})$$

By replacing (I.2) and (I.9) in (I.10), we find:

$$\dot{S}(t) = \frac{\partial S}{\partial x} \left(f(x, t) + g(x, t)u_{eq}(t) \right) + \frac{\partial S}{\partial x} g(x, t)u_{dis}(t) \quad (\text{I.12})$$

I.5.1 Equivalent command

During the slip mode and the steady state, the surface is zero, and therefore its derivative and the discontinuous part are also zero. From which we deduce the expression of the command equivalent:

$$u_{eq}(t) = - \left(\frac{\partial S}{\partial x} g(x, t) \right)^{-1} \frac{\partial S}{\partial x} f(x, t) \quad (\text{I.13})$$

For the equivalent command to take a finite value, it is necessary that:

$$\frac{\partial S}{\partial x} g(x, t) \neq 0 \quad (\text{I.14})$$

I.5.1.1 Linear case

A system whose dynamics is linear is given by the following model:

$$\dot{x} = Ax(t) + Bu(t) \quad (\text{I.15})$$

The sliding surface is:

$$S(x) = \lambda x \quad (\text{I.16})$$

By differentiating with respect to time, we get:

$$\dot{S}(x) = \lambda Ax + \lambda Bu_{eq}(t) = 0 \quad (\text{I.17})$$

If the matrices C and B are invertible, we obtain the following equivalent command[16]:

$$u_{eq} = -(\lambda B)^{-1}CAx \quad (\text{I.18})$$

I.5.2 Discontinuous command

During convergence mode, and replacing the equivalent command by its expression in (I.11), we find the new expression for the derivative of the surface:

$$\dot{S}(t) = \frac{\partial S}{\partial x} g(x, t) u_{dis}(t) \quad (\text{I.19})$$

And the attractiveness condition expressed by (I.6) becomes:

$$S(x, t) \frac{\partial S}{\partial x} g(x, t) u_{dis}(t) < 0 \quad (\text{I.20})$$

In order to satisfy this condition, the sign of u_{dis} must be opposite to that of $S(x, t) \frac{\partial S}{\partial x} g(x, t)$. The simplest function that can be used to accomplish this condition is defined by:

$$u_{dis}(t) = k_s \text{sign}(s(x)) \quad (\text{I.21})$$

Where:

$$\text{sign}(s(x)) = \begin{cases} 1, & \text{if } s(x) > 0 \\ 0, & \text{if } s(x) = 0 \\ -1, & \text{if } s(x) < 0 \end{cases} \quad (\text{I.22})$$

The sign of k_s must be different from that of $\frac{\partial S}{\partial x} g(x, t)$.

However, the sign function generates on the sliding surface, a phenomenon called chattering which is generally undesirable because it adds to the spectrum of the component control high frequencies. (Figure I-6) represent sign function[17].

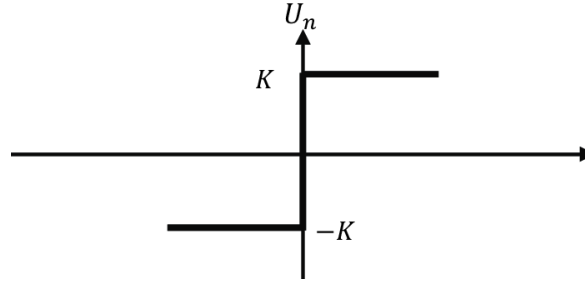


Figure I-6. Sign function

I.6 Phenomenon of reluctance (Chattering):

One of the problems related to the implementation of sliding mode control is the reluctance phenomenon. This phenomenon results in high-frequency oscillations around the sliding surface. This is due to modeling errors, but also to the fact that control by sliding mode requires switching of the control member at a theoretical infinite frequency, which in practice is not feasible (Figure I.8) represents the chattering phenomenon [18].

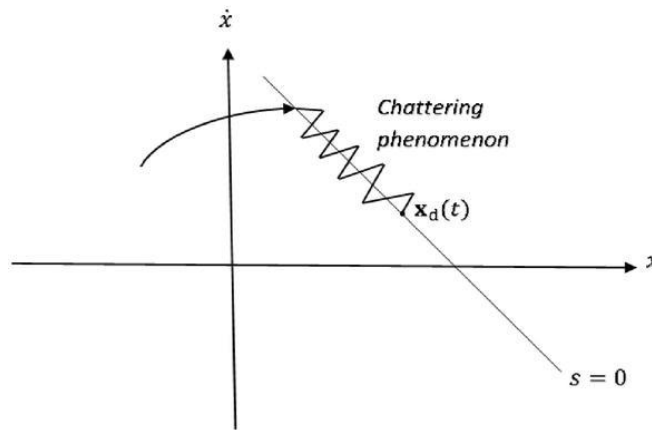


Figure I-7. Phenomenon of reluctance (Chattering)

I.7 Elimination of the chattering phenomenon (Chattering)

To achieve a reduction in this problem, several solutions have been proposed. The most popular methods are to make the smoother switching function around the surface or the increase in the order of the sliding mode. For a sliding mode of order r , the control algorithm must make not only the sliding surface S tend towards 0 but also the derivatives sequences of this surface up to the order [18].

In order to reduce these oscillations, several solutions have been provided, for example, the replacement of the "sign" function by a so-called Smooth function, (Figure I.8).

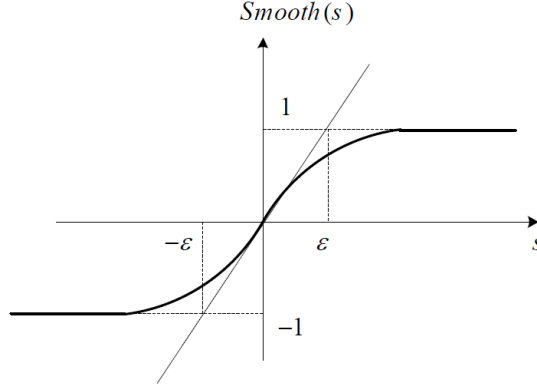


Figure I-8 Smooth function.

The mathematical expression of the Smooth function is given by[17]:

$$smooth(s) = \tanh(s) = \frac{e^x - e^{-x}}{e^x + e^{-x}} = \frac{s(x)}{|s(x)| + \varepsilon} \quad (I.23)$$

I.8 Different Structures of Sliding Mode Control:

In the literature there are three basic configurations for the synthesis of the different commands. The first corresponds to the simplest structure where switching is at the level of the control device itself. It will be called structure by commutation at the level of the control device. The second structure involves switching at the level of a state feedback, the last is a regulation structure with the addition of the equivalent command[19].

I.8.1 Structure by switching at the level of the control unit

The diagram of a switching structure at the level of the control unit is given in (Figure I.9), This command structure is the most classic and the most used and by the following switching law.

$$U_{cm}(t) = \begin{cases} U_{max} , & \text{to } S(X_s) > 0 \\ U_{min} , & \text{to } S(X_s) < 0 \end{cases} \quad (I.19)$$

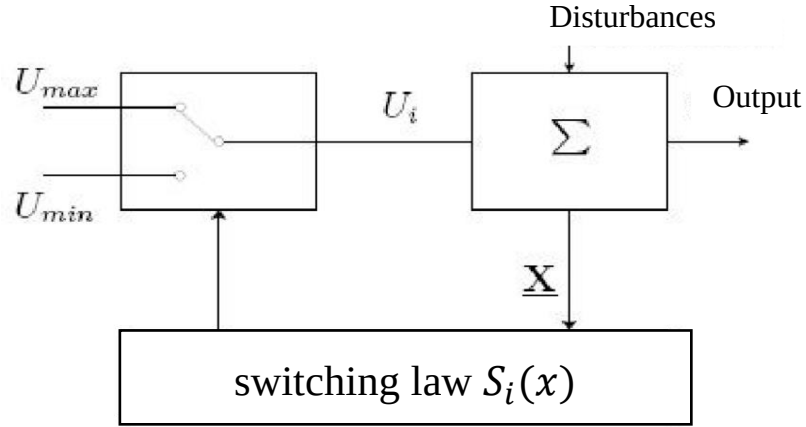


Figure I-9 Control structure by switching at the level of the control unit.

This structure corresponds to the on/off operation of the power switches associated in the vast majority of applications with variable speed drives. It was used for controlling the reactors[20].

I.8.2 Structure by commutation at the level of a state feedback

This structure is based on the choice of the status return by " $-K_1^T$ " or " $-K_2^T$ " which is done using the following condition[1]:

$$U_{cm}(t) = \begin{cases} U_{cm1}(t) = -K_1^T X_s, & \text{to } S(X_s) > 0 \\ U_{cm2}(t) = -K_2^T X_s, & \text{to } S(X_s) < 0 \end{cases} \quad (\text{I.20})$$

The diagram of this configuration is represented by (Figure I.11).

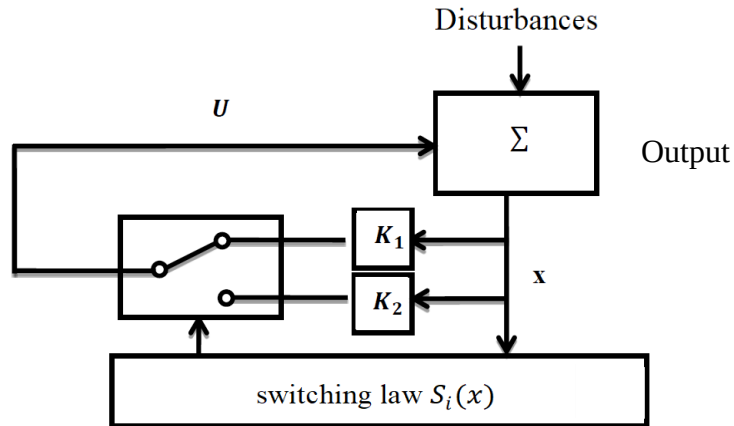


Figure I-10. VSC with change of structure by switching a variable state feedback.

I.8.3 Regulation structure with addition of the equivalent command

Such a structure, the principle of which is shown in (Figure I.12), has a real advantage. It makes it possible to pre-condition the future state of the system by the equivalent command which is nothing other than the will of the system to be in steady state.

The control device is much less stressed but we are more dependent on the parametric variations because of the expression of this equivalent control[19].

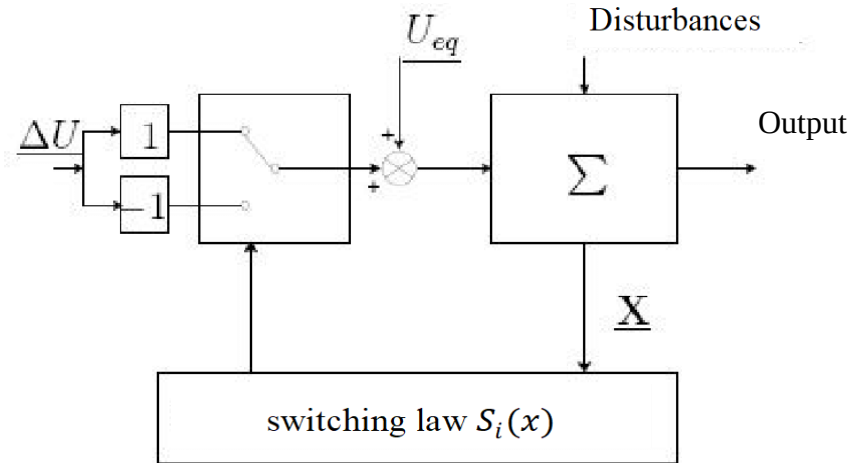


Figure I-11. Regulation structure by adding the equivalent command.

I.9 Control by sliding modes of order 1

Sliding mode control of order 1 (SMC) is a variable structure control that can change structure and switch between two values according to a very specific switching logic $S(x)$.

The principle of sliding mode control is to force the system to reach a given surface called the sliding surface and to remain there until equilibrium.

I.10 Control by higher-order sliding modes

Higher-order SMs were introduced to overcome the chattering problem while keeping the finite-time convergence and robustness properties of classical sliding mode controls. In this approach, the discontinuous term no longer appears directly in the expression of the synthesized command but in one of its higher derivatives, which has the merit of reducing chattering[21].

I.10.1 Control by sliding modes of order 2

The main goal of this technique is to generate a sliding regime of order two on a surface defines S and the convergence of $\dot{S} = \frac{dS}{dt} = 0$ in a finite time.

In the literature, several sliding mode algorithms of order 2 have been introduced.

Among these algorithms are[21]:

- ✓ twisting algorithm;
- ✓ Super twisting algorithm;
- ✓ prescribed convergence law algorithm.

I.11 Conclusion

As a summary, sliding mode control is subdivided into two parts, the equivalent command which is the connected part it is used to determine the future state of the system and the discontinuous command is responsible for keeping the system dynamics confined to the sliding surface,

The sliding mode control approach used in VSC has some advantages over other control methods. For example, it is robust to parameter variations and disturbances, and it can be used to design controllers that are insensitive to measurement noise. Additionally, the control law can be designed to achieve a fast response and good tracking performance.

CHAPTER

II

Quadratic linear optimal control

II.1 Introduction

Quadratic Linear Control is a type of control system design that involves optimizing a quadratic cost function subject to linear constraints. The goal of Quadratic Linear Control is to design a control law that minimizes a quadratic cost function while satisfying linear constraints on the system inputs and states.

In the 1950s and 1960s, researchers began to develop techniques for solving optimal control problems for linear systems with quadratic cost functions. These techniques, which involved solving a set of linear differential equations known as the Riccati equations, became known as linear quadratic optimal control[22].

Today, Quadratic Linear Control techniques are widely used in a wide range of applications, including aerospace, automotive, and process control. The ability of these techniques to handle nonlinear dynamics and uncertain system parameters has made them a popular choice for control system designers in many different fields.

The minimum of Pontryagin is a concept in optimal control theory that provides a necessary condition for finding the optimal control law for a dynamical system subject to certain constraints. It is named after the Russian mathematician Lev Pontryagin, who made significant contributions to the development of optimal control theory in the 1950s and 1960s[23].

In this chapter we will present to you principle of the minimum of Pontryagin and illustrated by an application in the case of a finite and infinite horizon.

II.2 Principle of minimum Pontryagin

The Minimum Principle is a generalization of the calculus of variations. This principle gives us a necessary condition of the first order, of local optimality, which must verify any potential solution of an optimal control problem[24].

Consider a system:

$$\dot{x} = f(x(t), u(t), t) \quad (\text{II.1})$$

The principle of the minimum consists in determining, if it exists, a command u^* which minimizes, on a set u of admissible commands, a criterion J of the form:

$$J = h(x(t_f), t_f) + \int_{t_0}^{t_f} L(x(t), u(t), t) dt \quad (\text{II.2})$$

The Hamiltonian is a mathematical function that depends on the state of the system, the control inputs, and the cost function[25]:

$$h(x(t_f), t_f) = \int_{t_0}^{t_f} \frac{dh(x(t), t)}{dt} dt + h(x(t_0), t_0) \quad (\text{II.3})$$

By replacing equation (II.3) in equation (II.2) we obtain:

$$J = h(x(t_0), t_0) + \int_{t_0}^{t_f} \left(L(x(t), u(t), t) + \frac{dh(x(t), t)}{dt} \right) dt \quad (\text{II.4})$$

Using partial derivatives, we get:

$$J = h(x(t_0), t_0) + \int_{t_0}^{t_f} \left(L(x(t), u(t), t) + \frac{\partial h(x(t), t)}{\partial x} \dot{x} + \frac{\partial h(x(t), t)}{\partial t} \right) dt \quad (\text{II.5})$$

We assume that t_0 and x_0 are constant, then $h(x(t_0), t_0)$ is constant.

The optimal solution $u^*(t)$ which minimizes the criterion (II.1), also minimizes the following criterion:

$$J_1 = \int_{t_0}^{t_f} \left(L(x(t), u(t), t) + \frac{\partial h(x(t), t)}{\partial x} \dot{x} + \frac{\partial h(x(t), t)}{\partial t} \right) dt \quad (\text{II.6})$$

the Lagrange multiplier method is used. This adjointed vector is used to form a new criterion J_2 which takes into account the criterion to be optimized J_1 and the constraints of the equality type which take the form:

$$f(x(t), u(t), t) - \dot{x} = 0 \quad (\text{II.7})$$

The new J_2 criterion is written as:

$$J_2 = h(x(t_f), t_f) + \int_{t_0}^{t_f} (L(x(t), u(t), t) + \lambda^T (f(x(t), u(t), t) - \dot{x}(t))) dt \quad (\text{II.8})$$

We define the Hamiltonian function (Hamiltonian) as follows:

$$H(x, u, \lambda, t) = L(x(t), u(t), t) + \lambda^T(t) f(x(t), u(t), t) \quad (\text{II.9})$$

Then the equation (II.8) is rewritten in the form:

$$J_2 = h(x(t_f), t_f) + \int_{t_0}^{t_f} (H(x, u, \lambda, t) - \lambda^T(t) \dot{x}(t)) dt \quad (\text{II.10})$$

Pontriagin's principle states that if $u^* \in U$ is an optimal order, then:

$$H(x^*, u^*, \lambda^*, t) \leq H(x^*, u, \lambda^*, t) \quad \forall u^* \in U, \forall t \in [t_0, t_f] \quad (\text{II.11})$$

where $x^*(t)$ and $\lambda^*(t)$ are respectively the optimal state and conjoint state.

Using Leibnitz's rule, the variation of J_2 as a function of the variation of x, u, λ and t , gives:

$$dJ_2 = \left(\frac{\partial h}{\partial x} \right)^T \partial x_f + \frac{\partial h}{\partial x} \partial t_f + (H - \lambda^T \dot{x}) \partial t_f + \int_{t_0}^{t_f} \left[\left(\frac{\partial H}{\partial x} \right)^T \partial x + \left(\frac{\partial H}{\partial u} \right)^T \partial u - \lambda^T \partial \dot{x} + \left(\frac{\partial H}{\partial \lambda} - \dot{x} \right)^T \partial \lambda \right] dt \quad (\text{II.12})$$

In eliminates the variation in $\dot{x}$, by integrating by parts:

$$\int_{t_0}^{t_f} [-\lambda^T \partial \dot{x}] dt = -\lambda^T \partial x_f + \int_{t_0}^{t_f} \lambda^T \partial x dt \quad (\text{II.13})$$

Knowing that: $dx_f = \partial x_f + \dot{x}_f dt_f$

$$dJ_2 = \left(\frac{\partial h}{\partial x} - \lambda \right)^T \partial x_f + \left(\frac{\partial h}{\partial x} + H \right) \partial t_f + \int_{t_0}^{t_f} \left[\left(\frac{\partial H}{\partial x} + \dot{\lambda} \right)^T \partial x + \left(\frac{\partial H}{\partial u} \right)^T \partial u + \left(\frac{\partial H}{\partial \lambda} - \dot{x} \right)^T \partial \lambda \right] dt \quad (\text{II.14})$$

The equation $dJ_2 = 0$ gives the following optimality conditions:

$$1. \quad \dot{x} = \frac{\partial H}{\partial \lambda} \quad \text{The state equation.} \quad (\text{II.15})$$

$$2. \quad \dot{\lambda} = -\frac{\partial H}{\partial x} \quad \text{The adjoint state equation.} \quad (\text{II.16})$$

$$3. \quad \frac{\partial H}{\partial u} = 0 \quad \text{stationarity condition.} \quad (\text{II.17})$$

$$4. \quad \left(\frac{\partial h}{\partial x} - \lambda \right)^T \partial x_f + \left(\frac{\partial h}{\partial x} + H \right) \partial t_f \quad \text{Transversality conditions.} \quad (\text{II.18})$$

According to the definition of the values of the final conditions, we have different problems of optimal control[26].

- ❖ If $x(t_f)$ is fixed and t_f is free, then $\partial x_f = 0$, $\partial t_f \neq 0$, $\frac{\partial h}{\partial x} + H = 0$
- ❖ If $x(t_f)$ is free and t_f is fixed, then $\partial x_f \neq 0$, $\partial t_f = 0$, $\frac{\partial h}{\partial x} - \lambda = 0$
- ❖ If $x(t_f)$ and t_f both fixed, then $\partial x_f = 0$, $\partial t_f = 0$, then both terms of (II.18) are zero. There are no final conditions to be determined by (II.18).
- ❖ If $x(t_f)$ and t_f both free, then $\partial x_f \neq 0$, $\partial t_f \neq 0$, $\frac{\partial h}{\partial x} + H = 0$, $\frac{\partial h}{\partial x} - \lambda = 0$.

II.3 Notions of controllability and observability

The concepts of controllability and observability play a very important role in the study of the problems of control and filtering. The concept of controllability studies the possibility of adjusting the behavior of the considered system in order to force the state to take the desired values for a finite time. On the other hand, the observability of a system can be defined as being the possibility of predicting the state of the system at any time included in the interval of the operating time[27].

Controllability and observability are important because they determine the limitations and possibilities of designing effective control systems. A system that is both controllable and observable can be fully controlled and monitored, allowing for precise regulation and optimization[28].

II.3.1 Controllability

We study the controllability of the linear system of dimension n defined in the state space by the system of equations of the following form:

$$\begin{cases} \dot{x}(t) = Ax(t) + Bu(t) \\ y(t) = Cx(t) + Du(t) \end{cases} \quad (\text{II.19})$$

- $x \in \mathbb{R}^{n \times 1}$ is the state vector.

- $A \in \mathbb{R}^{n \times n}$ is the state matrix.

- $B \in \mathbb{R}^{n \times m}$ is the matrix of control.

- $u \in \mathbb{R}^{m \times 1}$ is the control vector.

- $y \in \mathbb{R}^{p \times 1}$ is the output vector.
- $C \in \mathbb{R}^{p \times n}$ is the output matrix.
- $D \in \mathbb{R}^{p \times m}$ is the matrix of direct drive.

With p represents the output number.

Matrices A and B are time-independent matrices. In this case, the dynamics of the system is said to be invariant in time. The time interval going from present time ($t = 0$) and tending towards infinity is then the interval of interest on which the controllability of the system is studied.

the controllability matrix given by:

$$M_{con} = [B \ AB \ A^2B \ \dots \ A^{n-1}B] \quad (\text{II.20})$$

there are several equivalent conditions which guarantee the controllability of the system (I.19), we used the following condition:

$$\text{if } \text{rank}(M_{con}) = n$$

Then the system (I.19) is controllable[29].

II.3.2 Observability

The observability matrix given by:

$$M_{ob} = \begin{bmatrix} C \\ CA \\ CA^2 \\ \vdots \\ CA^{n-1} \end{bmatrix} \quad (\text{II.21})$$

like the controllability concept, observability condition is given by:

$$\text{if } \text{rank}(M_{ob}) = n$$

Then the system (I.19) is observable[28].

II.4 Quadratic Linear Control Law

The quadratic linear control, known as LQ control, is a method which makes it possible to calculate the matrix of gains of a control by state feedback. The system is linear and the control quadratic. The optimal command is a state feedback[30].

The diagram of this configuration is represented by (Figure III.1).

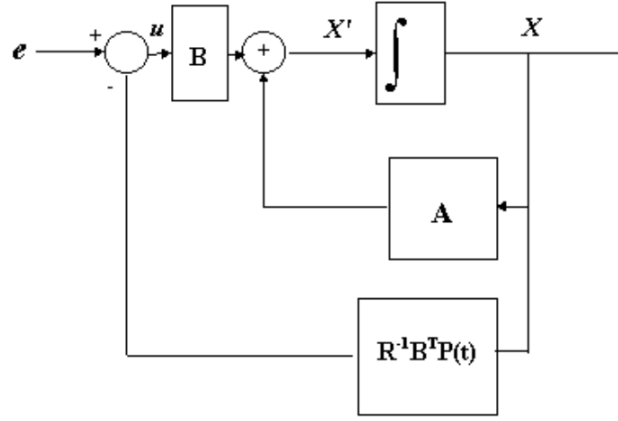


Figure II-1. diagram of an LQ control.

II.4.1 Principle of finite horizon LQ control

Consider the linear system [23, 31]:

$$\begin{cases} \dot{x}(t) = Ax(t) + Bu(t) \\ y(t) = Cx(t) + Du(t) \end{cases} \quad (\text{II.22})$$

The control by state feedback which stabilizes the system and minimizes the LQ criterion based on the minimization of the following criterion:

$$J(x, t, u) = \frac{1}{2} x_f^T S x_f + \int_{t_0}^{t_f} \frac{1}{2} (x^T Q(t) x + u^T R(t) u) dt \quad (\text{II.23})$$

With:

$$Q \geq 0, R \geq 0, \text{ And } S > 0$$

The Hamiltonian is written:

$$H(x, p, t, u) = p^T A(t) x + p^T B(t) u + \frac{1}{2} (x^T Q(t) x + u^T R(t) u) \quad (\text{II.24})$$

The Hamiltonian, verifies the following conditions:

- Adjunct state equation:

$$\dot{\lambda} = -\frac{\partial H}{\partial x} = -A^T(t)\lambda - Q(t)x \quad (\text{II.25})$$

- Transversality condition:

$$\lambda(t_f) = Sx_f \quad (\text{II.26})$$

- No constraint on the control:

$$\frac{\partial H}{\partial u} = \lambda B^T(t) + R(t)u = 0 \quad (\text{II.27})$$

From equation (II.24) we get the control input u is given by:

$$u = -R^{-1}(t)B^T(t)\lambda \quad (\text{II.28})$$

Therefore, the dynamic equation of the system can be written as follows:

$$\dot{x} = A(t)x - B(t)R^{-1}(t)B^T(t)\lambda \quad (\text{II.29})$$

Let's assume that: $\lambda = Px$.

And when we derive: $\dot{\lambda} = \dot{P}x + P\dot{x}$.

Then we will replace the equation (II.27) on it:

$$\dot{\lambda} = \dot{P}x + PA(t)x + PB(t)R^{-1}(t)B^T(t)Px \quad (\text{II.30})$$

From the equation (II.23) we find:

$$\dot{\lambda} = -A^T(t)Px - Q(t)x \quad (\text{II.31})$$

From equation (II.28) and equation (II.29) find that:

$$\left(\dot{P} + PA(t) - PB(t)R^{-1}(t)B^T(t)P + A^T(t)P + Q(t) \right) x = 0 \quad (\text{II.32})$$

The solution is obtained by solving Riccati's differential equation given as:

$$\dot{P} + PA(t) - PB(t)R^{-1}(t)B^T(t)P + A^T(t)P + Q(t) = 0 \quad (\text{II.33})$$

With:

P : the solution to Riccati's equation.

λ : assistant state.

Thereafter, the cost criterion is written as follows:

$$J(x_0, t_0, u) = \frac{1}{2}(x_f^T S x_f - \int_{t_0}^{t_f} \frac{d}{dx}(x^T P x) dt) \quad (\text{II.34})$$

By applying the condition of transversality $\lambda(t_f) = S$ on the criterion, the minimum of the criterion is therefore:

$$J_0(x_0, t_0, u) = \frac{1}{2}x_0^T P(t_0)x_0 \quad (\text{II.35})$$

$$\text{And:} \quad u = -Kx \quad (\text{II.36})$$

$$\text{Or:} \quad K = -R^{-1}B^T \lambda \quad (\text{II.37})$$

the matrix $P(t)$ remains dependent on time in the case of a finite time criterion, in the present case, K varies as a function of time, even in the case of a system and a criterion with invariant time (that is to say if the matrices A , B , Q and R do not depend on time) [23, 31].

II.4.2 Principle of infinite horizon LQ control

At the infinite horizon, the criterion can be written as follows[32]:

$$J(x, t, u) = \int_0^\infty \frac{1}{2}(x^T Q(t)x + u^T R(t)u) dt \quad (\text{II.38})$$

In steady state:

$$\lim_{t \rightarrow \infty} P(t) = P \xrightarrow{\frac{d}{dt}} \dot{P} = 0$$

The Riccati equation in steady state is written in the form:

$$PA(t) - PB(t)R^{-1}(t)B^T(t)P + A^T(t)P + Q(t) = 0 \quad (\text{II.39})$$

II.4.3 Choice of weighting matrices

The weighting matrices are generally chosen diagonal where:

$$Q = \text{diag}(q_1, q_2, q_3 \dots q_q) = \begin{bmatrix} q_1 & 0 & \cdot & 0 \\ 0 & q_2 & \cdot & \cdot \\ \cdot & \cdot & \cdot & \cdot \\ 0 & \cdot & 0 & q_q \end{bmatrix} \quad (\text{II.40})$$

$$R = \text{diag}(r_1, r_2, r_3 \dots r_r) = \begin{bmatrix} r_1 & 0 & \cdot & 0 \\ 0 & r_2 & \cdot & \cdot \\ \cdot & \cdot & \cdot & \cdot \\ 0 & \cdot & 0 & r_r \end{bmatrix} \quad (\text{II.41})$$

To make an initial evaluation of Q and R, we will rely on the physical calibration of the corresponding inputs-outputs, taking:

$$q_i = \left[\frac{1}{\sup(x_i)} \right]^2, i = 1, 2, 3 \dots q \quad (\text{II.42})$$

$$r_i = \left[\frac{1}{\sup(u_i)} \right]^2, i = 1, 2, 3 \dots r \quad (\text{II.43})$$

This is Bryson's rule.

We can then refine the choice of weightings by trial and error in simulation.

If we increase Q with respect to R, we will globally have fast responses at the cost of a more energetic control[33, 34].

II.5 Application

II.5.1 Example

Consider the system:

$$\begin{aligned} \dot{x}(t) &= \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix} x(t) + \begin{bmatrix} 0 \\ 1 \end{bmatrix} u(t) \\ y(t) &= [1 \quad 0] x(t) \end{aligned}$$

Calculate the optimal command that stabilizes the system knowing that: $Q = \begin{bmatrix} \alpha^2 & 0 \\ 0 & 0 \end{bmatrix}$, $R=1$ and $S=0$.

✓ Checking the controllability:

$$M_{con} = [B \quad AB] = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}$$

$$\det(M_{con}) = -1 \neq 0 \text{ So } \text{rank}(M_c) = 2$$

our system is controllable.

✓ Checking the observability:

$$M_{ob} = \begin{bmatrix} C \\ CA \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

$$\det(M_{ob}) = 2 \neq 0 \text{ So } \text{rank}(M_{ob}) = 2$$

our system is observable.

P: solution Riccati's equation.

$$PA(t) - PB(t)R^{-1}(t)B^T(t)P + A^T(t)P + Q(t) = 0$$

$$\begin{bmatrix} P_1 & P_2 \\ P_2 & P_3 \end{bmatrix} \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix} + \begin{bmatrix} 0 & 0 \\ 1 & 0 \end{bmatrix} \begin{bmatrix} P_1 & P_2 \\ P_2 & P_3 \end{bmatrix} - \begin{bmatrix} P_1 & P_2 \\ P_2 & P_3 \end{bmatrix} \begin{bmatrix} 0 \\ 1 \end{bmatrix} \begin{bmatrix} 0 & 1 \end{bmatrix} \begin{bmatrix} P_1 & P_2 \\ P_2 & P_3 \end{bmatrix} + \begin{bmatrix} \alpha^2 & 0 \\ 0 & 0 \end{bmatrix} = 0$$

$$\begin{bmatrix} 0 & P_1 \\ 0 & P_2 \end{bmatrix} + \begin{bmatrix} 0 & 0 \\ P_1 & P_2 \end{bmatrix} - \begin{bmatrix} P_2 \\ P_3 \end{bmatrix} [P_2 \quad P_3] + \begin{bmatrix} \alpha^2 & 0 \\ 0 & 0 \end{bmatrix} = 0$$

$$\begin{bmatrix} P_2^2 + \alpha^2 & P_1 - P_2 P_3 \\ P_1 - P_2 P_3 & 2P_2 - P_3^2 \end{bmatrix} = 0$$

Then we find that:

$$\begin{cases} P_2^2 + \alpha^2 = 0 \\ P_1 - P_2 P_3 = 0 \\ 2P_2 - P_3^2 = 0 \end{cases} \rightarrow \begin{cases} P_2^2 = -\alpha^2 \rightarrow P_2 = \pm \alpha \\ P_1 = P_2 P_3 \rightarrow P_1 = \pm \alpha \sqrt{2} \alpha \\ 2P_2 = P_3^2 \rightarrow P_3 = \pm \sqrt{2} \alpha \end{cases}$$

first case: $P_2 = +\alpha$, $P_3 = +\sqrt{2} \alpha$

$$P = \begin{bmatrix} \alpha & \sqrt{2} \alpha & \alpha \\ \alpha & \sqrt{2} \alpha & \alpha \end{bmatrix}.$$

By definition: $u = -R^{-1}B^T P x$

By replacing R, B and P we find: $u = -[0 \quad 1] \begin{bmatrix} \alpha \sqrt{2} \alpha & \alpha \\ \alpha & \sqrt{2} \alpha \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix}$

We find that optimal control: $u^* = -\alpha x_1 - x_2 \sqrt{2} \alpha$

II.6 Conclusion

In this chapter, the optimal control theory based on Pontryagin's minimum principle is presented. Linear quadratic control is a method used in control theory to design a control system capable of regulating a dynamic system. The basic idea of linear quadratic control is to minimize a cost function that penalizes deviations from the desired setpoint, while accounting for the control effort needed to achieve the desired output. This approach is illustrated with an example.

In order to make the optimal controller robust against uncertainties affecting the system, a sliding mode control (SMC) method is combined with the optimal controller using a sliding surface. This will be the subject of the next chapter.

CHAPTER

III

Optimal sliding mode control

III.1 Introduction

Optimal Sliding Mode Control (OSMC) is a control strategy that combines the concepts of sliding mode control and Linear Quadratic Control (LQR) to achieve robust and optimal control of a system.

Optimal Sliding Mode Control using Linear Quadratic Control offers robustness against uncertainties and disturbances while achieving optimal control performance.

The convergence of OSMC and LQR to form OSMC-LQR took place in the late 1980s and early 1990s. Researchers recognized the complementary nature of sliding mode control and LQR and sought to combine their benefits[35].

In this chapter we will present general information about bioreactor CSTR and applicate them for deferent types of controls, the interest is to benefit from the performance of each approach. A comparative study between the three approaches will be illustrated on a real system, in this our case the bioreactor.

III.2 Optimal sliding mode control design

The control input $u(t)$ is divided into two parts. At first, the optimal controller $u_{eq}(t)$ is designed for the nominal part of the system using LQR technique. In the second part, $u_{disc}(t)$ is the sliding mode control to tackle uncertainties.[4].

$$u(t) = u_{eq}(t) + u_{disc}(t) \quad (III.1)$$

III.2.1 Optimal control design

Consider the following system:

$$\dot{x}(t) = Ax(t) + Bu(t) \quad (III.2)$$

The performance index to be minimized for the optimal control is defined as:

$$J(x, t, u) = \int_0^\infty \frac{1}{2} (x^T(t) Q x(t) + u_{eq}^T(t) R u_{eq}(t)) dt \quad (\text{III.3})$$

Where $Q \in \mathbb{R}^{n \times n}$ and $R \in \mathbb{R}^{n \times n}$ are positive definite weighing matrices.

The optimal control law $u_{eq}(t)$ is constructed as:

$$u_{eq}(t) = -R^{-1} B^T P x(t) = -K x(t) \quad (\text{III.4})$$

Where $K = R^{-1} B^T P$ and P is the solution of the algebraic Riccati equation [4].

$$P A(t) - P B(t) R^{-1}(t) B^T(t) P + A^T(t) P + Q(t) = 0 \quad (\text{III.5})$$

III.2.2 Sliding mode control design

The discontinuous part of sliding mode control arises from the use of a signum function or other non-smooth function to generate the control signal. This non-smooth function produces a control signal that switches abruptly between two values as the system state crosses the sliding surface.

$$u_{disc}(t) = -K \text{sign}(s) \quad (\text{III.6})$$

where k is a constant gain and $\text{sign}(s)$ is the sign of the sliding variable.

The discontinuous part of the control law causes the control signal to switch abruptly between two values as the system state crosses the sliding surface. This provides robustness to uncertainties and disturbances in the system, as the control signal can rapidly adapt to changes in the system state[36].

III.3 About Continuous Stirred Tank Reactor

A CSTR (Continuous Stirred Tank Reactor) bioreactor is a type of bioreactor commonly used for industrial applications in the production of biomolecules, such as enzymes, proteins, and vaccines. In a CSTR bioreactor, a liquid culture of microorganisms is continuously stirred in a tank, and fresh nutrient medium is continuously added to the reactor, while spent medium and biomass are continuously removed[37].

Control of the bioreactor is critical for ensuring optimal growth and productivity. Common control variables include temperature, pH, dissolved oxygen, agitation rate, and nutrient feed rate.

Control systems for CSTR bioreactors typically involve feedback control loops that adjust the inputs to the reactor to maintain the desired setpoints for the control variables.

Overall, CSTR bioreactors are a versatile and widely used tool for the production of biomolecules in industrial biotechnology. Their ability to maintain a uniform culture and optimal growth conditions through continuous stirring and nutrient feed make them an attractive option for high-level production of bioproducts[25].

III.3.1 How Continuous Stirred Tank Reactor works

A Continuous Stirred Tank Reactor (CSTR) is a type of chemical reactor used in industrial processes for continuous mixing and reaction of liquids, gases, or slurries. It consists of a well-mixed tank or vessel equipped with an impeller or agitator to ensure thorough mixing of reactants and a system for maintaining a constant volume[38].

Here's a general overview of how a CSTR reactor works:

1. **Reactant Feed:** The reactants are continuously fed into the reactor through inlet pipes. The flow rates of the reactants are controlled to maintain a desired concentration and reaction rate.
2. **Mixing:** The reactor contains an impeller or agitator, which is usually driven by a motor. The impeller continuously stirs the contents of the reactor to ensure uniform mixing of the reactants. Proper mixing is crucial to achieve an efficient reaction and maintain a homogeneous reaction mixture.
3. **Reaction:** Once the reactants are mixed, the desired chemical reaction takes place. The reaction can be exothermic (releases heat) or endothermic (absorbs heat). The heat generated or required during the reaction affects the reactor's design and operation.
4. **Heat Transfer:** In exothermic reactions, the heat generated must be controlled to prevent temperature buildup, which could negatively impact the reaction or safety. Heat transfer mechanisms like cooling jackets or coils are employed to remove excess heat from the reactor. Conversely, in endothermic reactions, external heating may be necessary to maintain the desired reaction temperature.

5. **Residence Time:** The residence time refers to the average time a reactant spends inside the reactor. It is calculated by dividing the reactor volume by the volumetric flow rate of the reactants. The residence time determines the extent of reaction and is an essential parameter for reactor design and optimization.
6. **Product Withdrawal:** The reaction mixture, now containing the desired products, exits the reactor through an outlet pipe. Depending on the process requirements, the products may undergo further treatment or separation steps.
7. **Control and Monitoring:** Various process parameters such as temperature, pressure, and reactant concentrations are continuously monitored and controlled to ensure optimal reactor performance and product quality. Automated control systems adjust the reactant flow rates, agitation speed, and heat transfer to maintain the desired conditions.

CSTRs are widely used in the chemical, pharmaceutical, and biochemical industries due to their simplicity, versatility, and the ability to achieve continuous operation. They are suitable for reactions that require good mixing and a long residence time. However, CSTRs may not be suitable for reactions with complex kinetics or reactions that involve solids or heterogeneous catalysts, which may require different types of reactors, (Figure III.1) represent main idea of CSTR principle [39, 40].

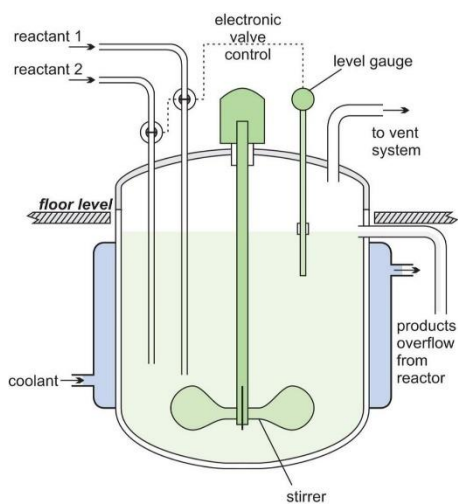


Figure III-1. Overview about Continuous Stirred Tank Reactor (CSTR)

Inputs:

1. **Reactants:** The reactants, which can be liquids, gases, or slurries, are continuously fed into the reactor. The flow rates and concentrations of the reactants are controlled to achieve the desired reaction conditions.
2. **Catalysts or additives:** In some cases, catalysts or additives may be added to the reactor to enhance the reaction rate or selectivity. These substances facilitate the desired chemical transformation.
3. **Heat transfer fluids:** If the reaction is exothermic (generates heat) or endothermic (requires heat), heat transfer fluids may be introduced into the reactor to control the temperature and maintain optimal reaction conditions.
4. **Utilities:** Various utilities such as cooling water, heating systems, or inert gases may be supplied to the reactor to assist in maintaining the desired temperature, pressure[41].

Outputs:

1. **Reaction products:** The reaction products are the desired output of the reactor. These products can be different chemical compounds resulting from the desired chemical reaction. The composition and quantity of the products depend on the reaction conditions and the specific reaction being carried out.
2. **By-products or waste:** In addition to the desired products, there may be some by-products or waste generated in the reactor. These can be undesired chemical compounds or materials that are formed as side reactions or impurities.
3. **Effluent:** The effluent is the mixture of reaction products, by-products, and unreacted reactants that exit the reactor. It is typically collected and processed further to separate and purify the desired products from the undesired components.

It's important to note that the specific inputs and outputs of a CSTR reactor will vary depending on the process and reaction being conducted. Different reactions may require specific input conditions or yield different outputs. Therefore, it's necessary to consider the specific reaction and process requirements when discussing the inputs and outputs of a CSTR reactor[42].

III.4 Application

Consider the dynamic model of the bioprocess given by the following system of equations[43]:

$$\begin{cases} \dot{C}_A = -\frac{Q}{V}(C_{A.in} - C_A) - K_1 C_A \\ \dot{C}_B = -\frac{Q}{V}C_B + K_1 C_A - K_2 C_B \\ \dot{C}_C = -\frac{Q}{V}C_C + K_2 C_B \end{cases} \quad (III.7)$$

where C_A , C_B , C_C are the concentrations in (mol/m^3), Q denotes the feed rate in (m^3/s), V the volume of the bioprocess in (m^3), K_1 and K_2 , the kinetic constants in (s^{-1}) and $C_{A.in}$ input concentration in (mol/m^3).

Then System can be expressed by:

$$\begin{cases} \dot{x}(t) = Ax(t) + BU(t) + Dv(t) \\ y(t) = Cx(t) \end{cases} \quad (III.8)$$

With:

$$A = \begin{bmatrix} -\frac{Q}{V} - K_1 & 0 & 0 \\ K_1 & -\frac{Q}{V} - K_2 & 0 \\ 0 & K_2 & -\frac{Q}{V} \end{bmatrix}, B = \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}, C = [1 \quad 0 \quad 0], D = \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}$$

Where $x(t) = [C_A \quad C_B \quad C_C]^T, y = C_C$.

Simulations are performed for the numerical values given in Table below.

Parameter	Value	Variable	Initial condition
$\frac{Q}{V}(\text{s}^{-1})$	2	$x(0)(\text{mol}/\text{l})$	(0.83;1.2;3)
$K_1(\text{s}^{-1})$	10		
$K_2(\text{s}^{-1})$	5		
$C_{A.in}(\text{mol}/\text{l})$	5		

Table 1. Numerical values used in the simulation in Example [43]

III.4.1 Open loop study

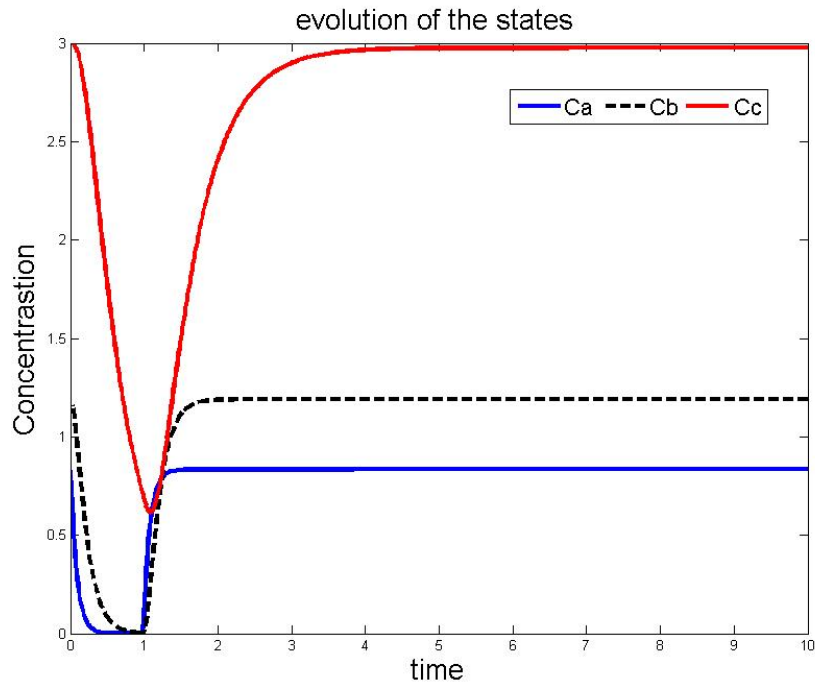


Figure III-2. Evolution of the states C_A , C_B and C_C

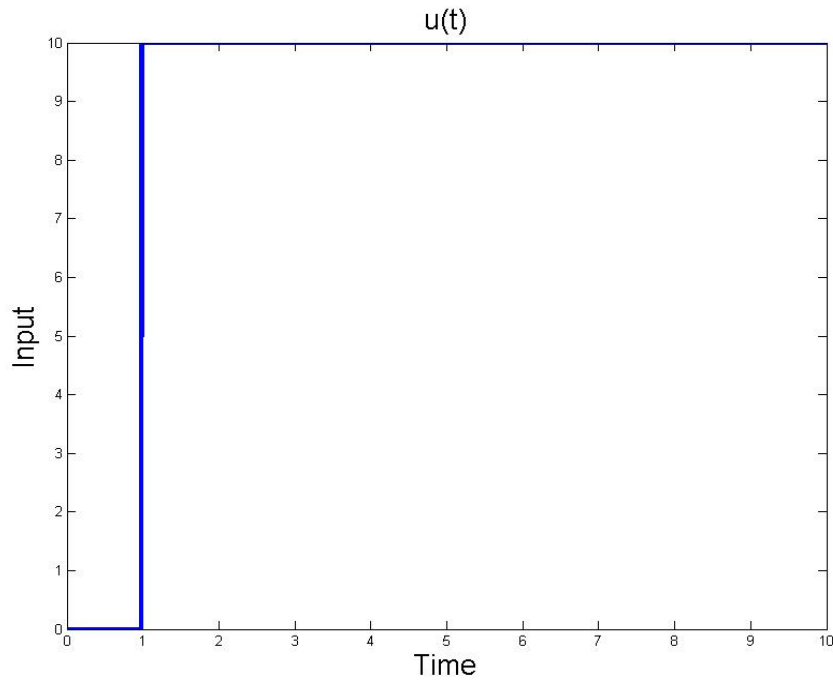


Figure III-3. Evolution of the control signal U .

The figures III-2 and Figure III-3 describes the evolution of states and control signal of system on open loop.

III.4.2 Applying SMC, OSMC, QLC without Disturbances

III.4.2.1 Sliding mode control design

Consider the following system:

$$\dot{x}(t) = Ax(t) + Bu(t) \quad (\text{III.9})$$

We choose the following sliding surface:

$$S = \lambda x \quad (\text{III.10})$$

Define control law:

$$u(t) = u_{eq}(t) + u_{dis}(t) \quad (\text{III.11})$$

Starting with u_{eq} by definition:

$$\dot{S} = 0 \quad (\text{III.12})$$

From equation (III.10) we find:

$$\dot{S} = \lambda \dot{x} = 0 \quad (\text{III.13})$$

Replacing equation (III.9) on equation (III.13):

$$u_{eq}(t) = -(\lambda B)^{-1} \lambda A x(t) \quad (\text{III.14})$$

Discontinues part given by:

$$u_{dis}(t) = K \text{sing}(S) \quad (\text{III.15})$$

Then we got the control law:

$$u(t) = -(\lambda B)^{-1} \lambda A x(t) - K \text{sing}(S) \quad (\text{III.16})$$

λ by definition is given by:

$$\lambda = B_{right}^{-1} \quad (\text{III.17})$$

We find that:

$$\lambda = [1 \quad 0 \quad 0] \quad (\text{III.18})$$

Convergence condition (Lyapunov's function)

The stability and the convergence in finite time is guaranteed by the attractivity condition

$$\dot{V} < 0.$$

$$V = \frac{1}{2}S^2 \quad (III.19)$$

By derivation we find:

$$\dot{V} = S^T \dot{S} < 0 \quad (III.20)$$

Substitute Equation (III.13) into Equation (III.18):

$$\dot{V} = S^T(\lambda \dot{x}) < 0 \quad (III.21)$$

Substitute Equation (III.9) into Equation (III.19):

$$\dot{V} = S^T[\lambda(Ax + Bu)] < 0 \quad (III.22)$$

Control equalization compensation:

$$\dot{V} = S^T[\lambda Ax - \lambda B(\lambda B)^{-1}\lambda Ax - K \text{sing}(S)] < 0 \quad (III.23)$$

The convergence condition stands for:

$$\|K\| > 0 \quad (III.24)$$

Choosing that $K=15$.

III.4.2.2 Linear quadratic control design

Choice of weighting matrices

By definition (II.40):

$$Q = \text{diag}(q_1, q_2, q_3 \dots q_q) = \begin{bmatrix} q_1 & 0 & . & 0 \\ 0 & q_2 & . & . \\ . & . & . & 0 \\ 0 & . & 0 & q_q \end{bmatrix}$$

And (II.42):

$$q_i = \left[\frac{1}{\sup(x_i)} \right]^2, i = 1, 2, 3 \dots q$$

Knowing that:

$\sup(x_1) = 0.833$, $\sup(x_2) = 1.2$ and $\sup(x_3) = 0.333$

Obtained:

$$Q = \text{diag}(q_1, q_2, q_3) = \begin{bmatrix} 1.44 & 0 & 0 \\ 0 & 0.694 & 0 \\ 0 & 0 & 0.111 \end{bmatrix} \times 10^3 \quad (\text{III.25})$$

We consider that $R = 1$.

Notice: Increasing of R cause the elimination of chattering phenomenon.

Simulation:

Knowing that $x_d = 0$ we obtain the following results:

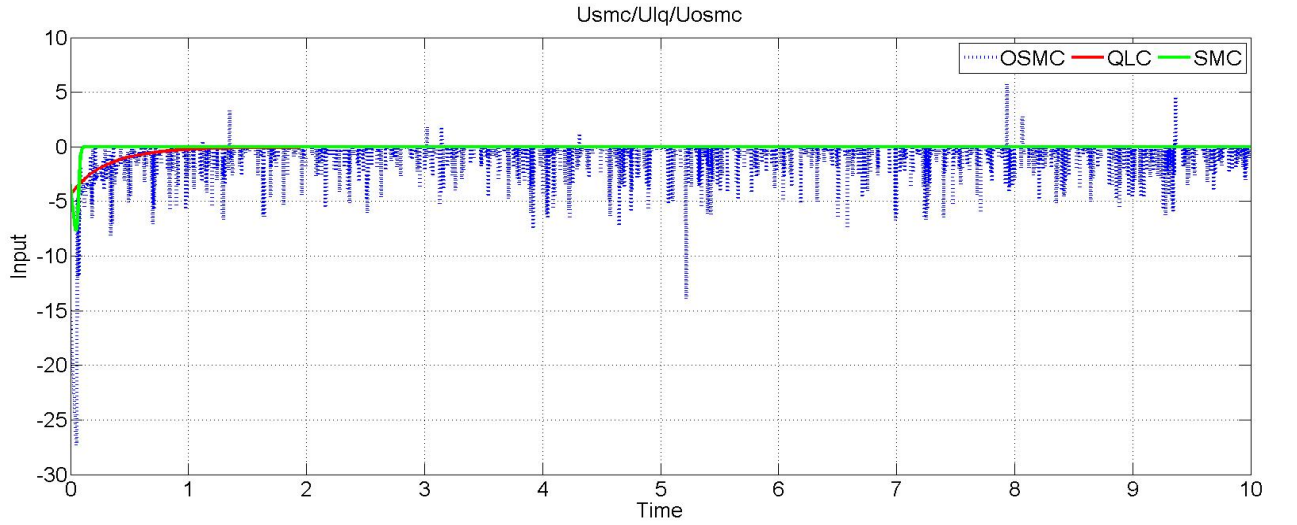


Figure III-4. Evolution of the control U_{osmc} , U_{smc} and U_{qlc}

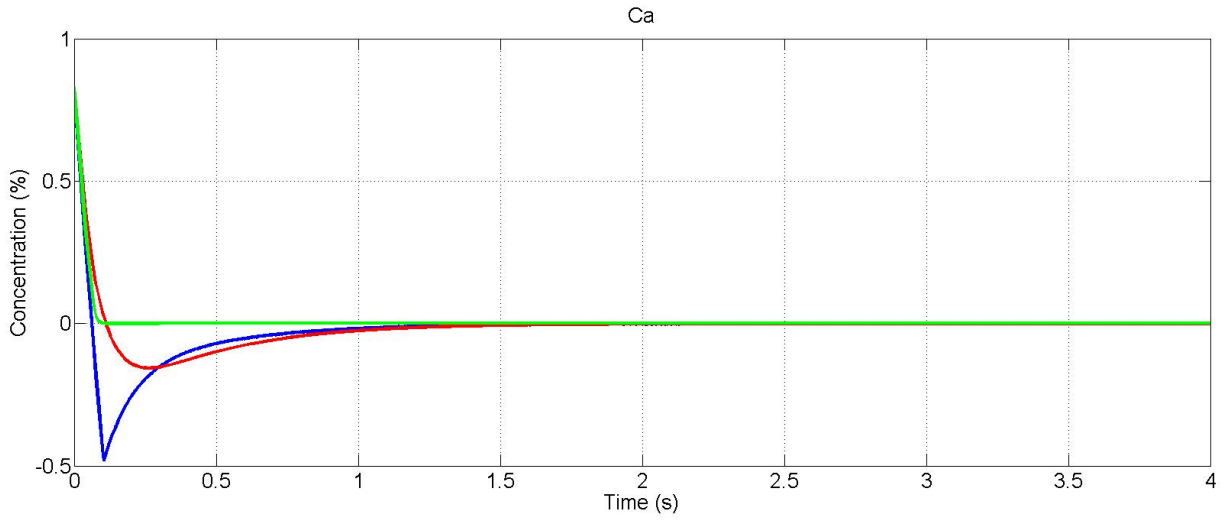


Figure III-5. Evolution of concentration C_a

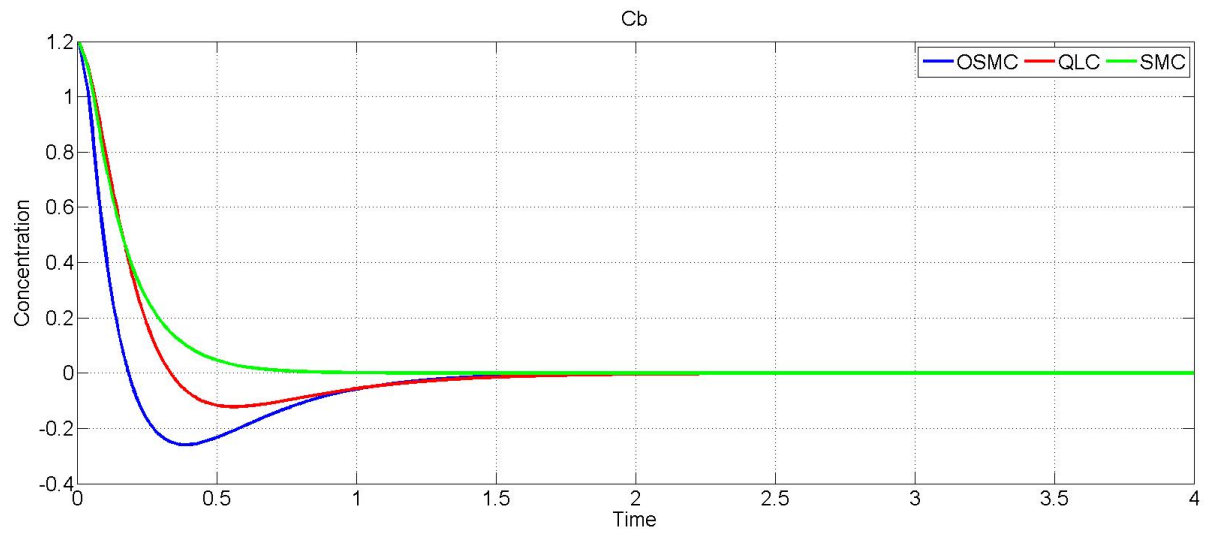
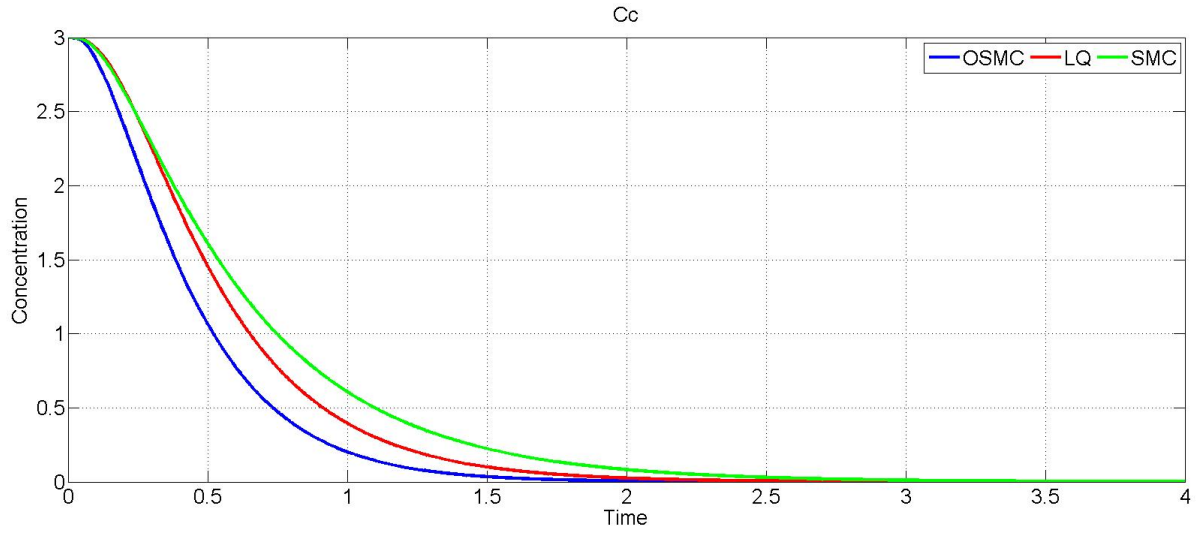


Figure III-6. Evolution of concentration C_b

Figure III-7. Evolution of concentration C_c

Results and discussion

The state trajectories are converged towards the desired values $x_d = 0$.

Figure III-4 describes the evolution of the various command signals in this case OSMC, LQR and SMC.

Figure III-5, Figure III-6 and Figure III-7 represent the evolution of the system states. For the three cases of controls the convergence towards the desired state is guaranteed, we can see that the OSMC control is faster than SMC and LQC.

III.4.3 Applying SMC, OSMC, QLC with variation of parameters

Simulation:

Knowing that: $\frac{Q}{V}(s^{-1}) = 6$, $K_1 = 15$ and $K_2 = 30$.

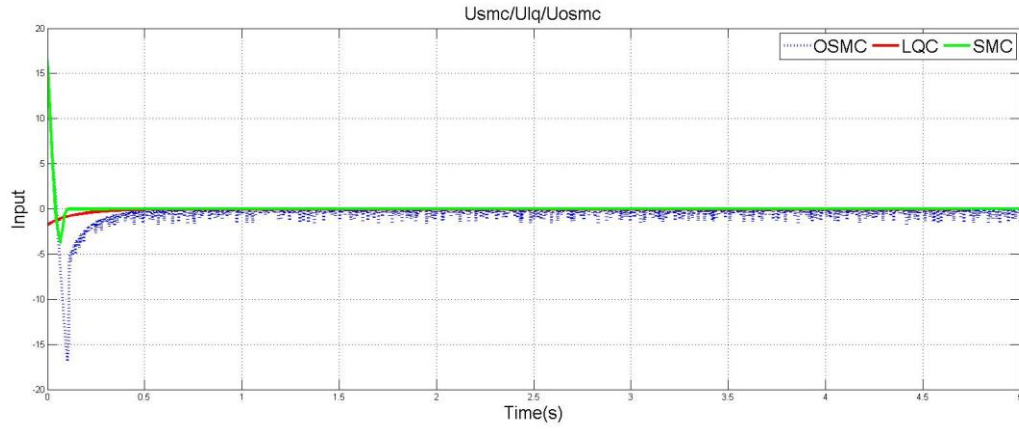


Figure III-8. Evolution of the three signals control U_{osmc} , U_{smc} and U_{qlc}

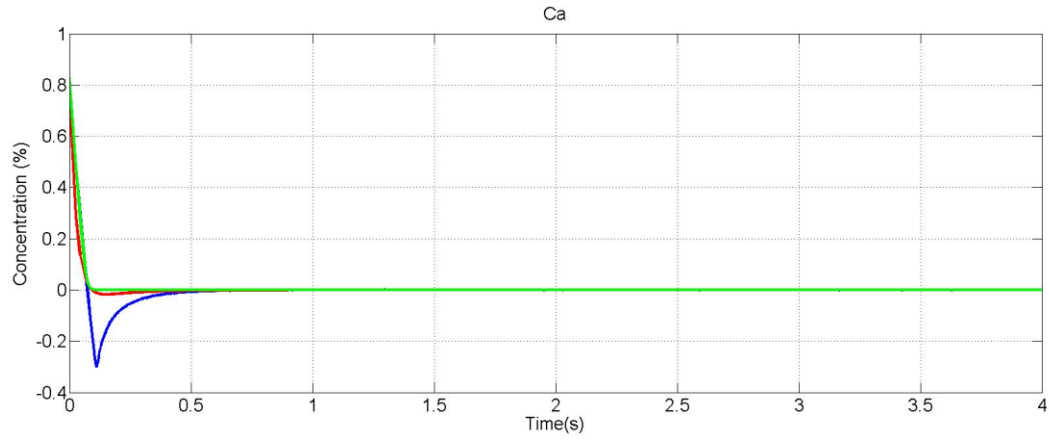


Figure III-9. Evolution of concentration Ca

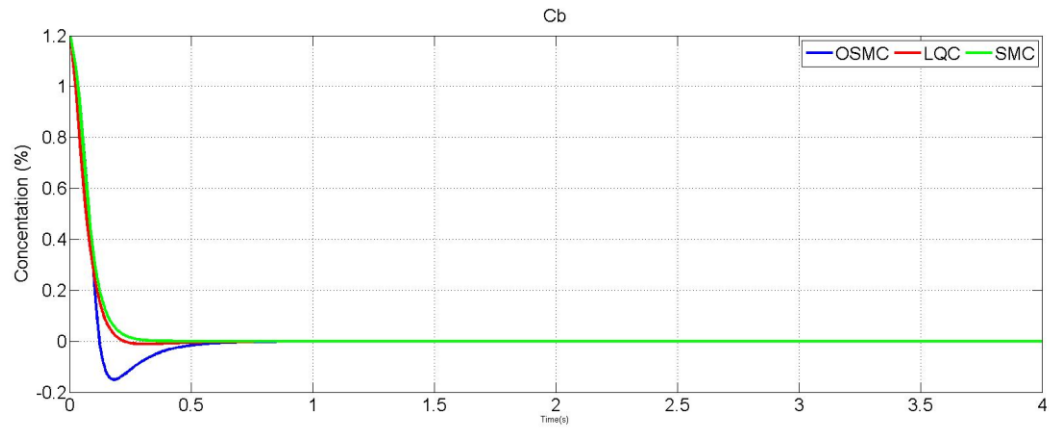
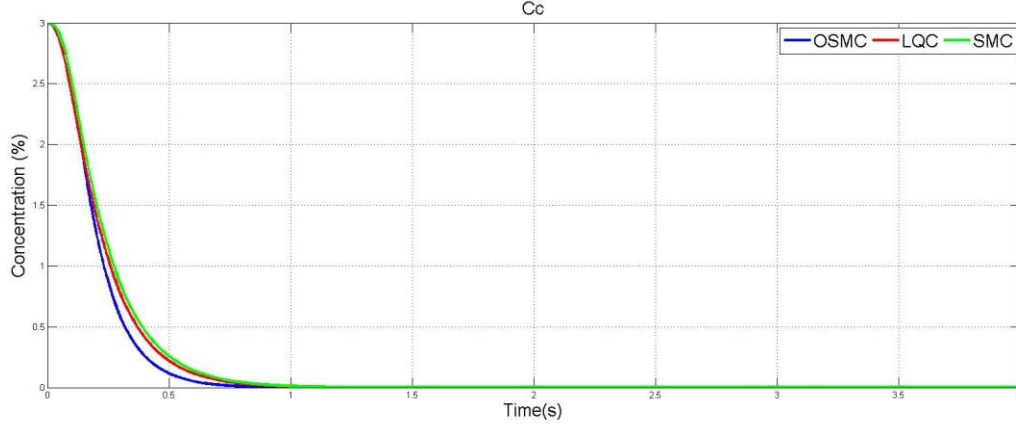


Figure III-10. Evolution of concentration Cb


 Figure III-11. Evolution of concentration C_c

Results and discussion

Figure III-8 describes the evolution of the various command signals in this case OSMC, LQR and SMC.

Figure III-9, Figure III-10 and Figure III-11 represents the evolution of the system states. For the three cases of controls the convergence towards the desired state is always guaranteed, these results confirm the robustness of the three control methods to the variation of parameters.

III.4.4 Applying SMC, OSMC, QLC with Disturbances

A disturbance has been added on the following interval time unities: [4 6].

Consider the following system:

$$\dot{x}(t) = Ax(t) + Bu(t) + Dv(t) \quad (\text{III.26})$$

We choose the following sliding surface:

$$S = \lambda x \quad (\text{III.27})$$

The control law given by:

$$u = -(\lambda B)^{-1} \lambda A x - Dv - K \text{sing}(S) \quad (\text{III.28})$$

Convergence condition (Lyapunov's function).

The stability and the convergence in finite time is guaranteed by the attractivity condition

$$\dot{V} < 0.$$

$$\dot{V} = S^T \dot{S} < 0 \quad (\text{III.29})$$

Knowing that:

$$\dot{S} = \lambda(Ax + Bu + Dv) \quad (\text{III.30})$$

We find:

$$\dot{V} = S^T [\lambda(Ax + Bu + Dv)] < 0 \quad (\text{III.31})$$

Control equalization compensation:

$$\dot{V} = S^T [\lambda(Ax + B(-(\lambda B)^{-1} \lambda Ax - Dv - K \text{sing}(S)) + Dv)] < 0 \quad (\text{III.32})$$

We find:

$$\dot{V} = S^T [(-\lambda B + \lambda) Dv - \lambda B K \text{sing}(S)] < 0 \quad (\text{III.33})$$

We find that:

$$\|K\| > \frac{\|\lambda B\| - \|\lambda\|}{\|\lambda B\|} \|Dv\| \quad (\text{III.34})$$

Simulation:

Choosing that $K=25$.

Knowing that $x_d = 0$, $v(t) = 3\text{squar}(t)$ we obtain the following results:

Remark: A square wave is a type of periodic waveform that alternates between two distinct levels, typically high and low, with a constant period. It is called a "square wave" because its shape resembles a series of square pulses as shown (Figure III-12).

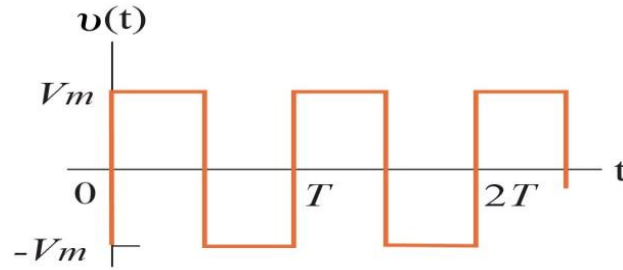


Figure III-12. square function

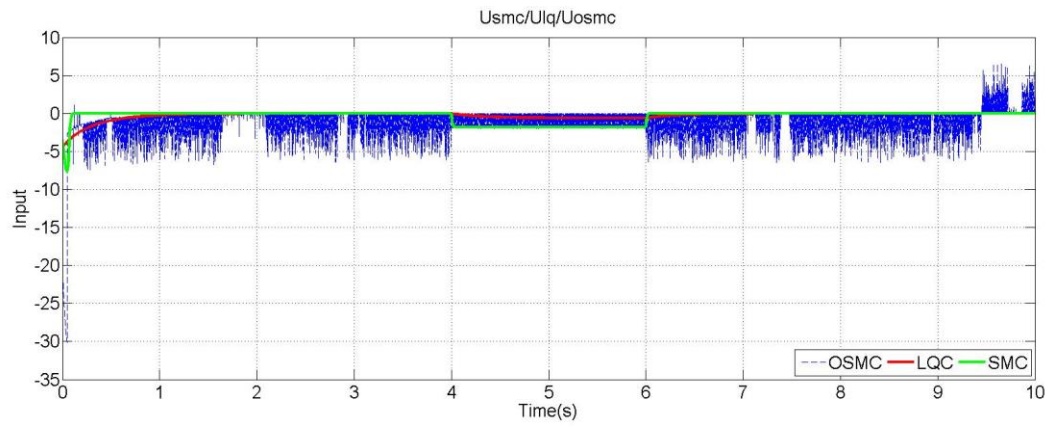


Figure III-13. Evolution of the control U_{osmc} , U_{smc} and U_{qlc}

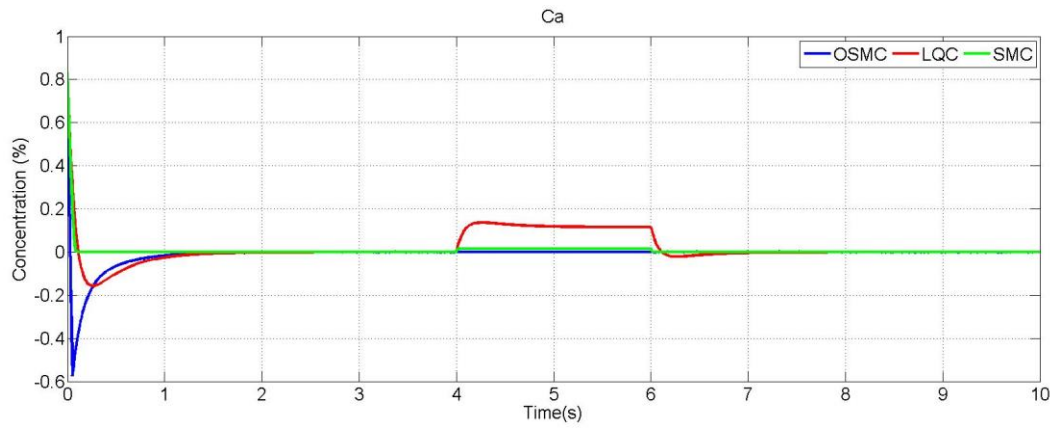


Figure III-14. Evolution of concentration Ca

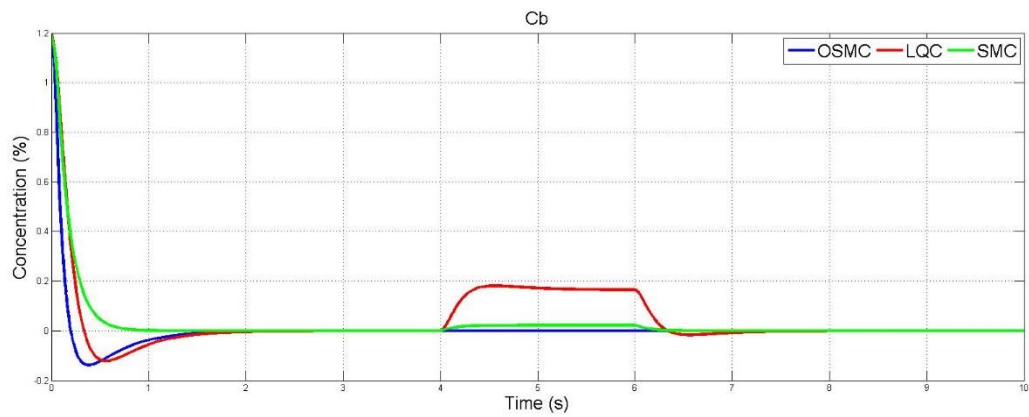
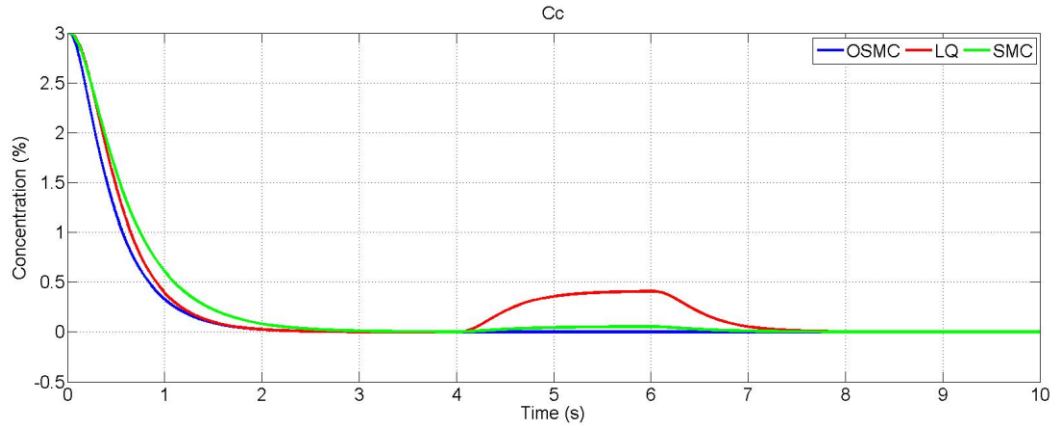


Figure III-15. Evolution of concentration Cb

Figure III-16. Evolution of concentration C_c

Results and discussion

Figure III-13 describes the evolution of the various command signals in this case OSMC, LQR and SMC.

Figure III-14, Figure III-15 and Figure III-16 represents the evolution of the system states.

We can see that OSMC and SMC give better performance in the presence of disturbance we also see that OSMC is better than SMC, as a summary, these results show the robustness of the OSMC and SMC controls with respect to the rejection of disturbance and the non-robustness of the LQC.

III.5 Conclusion

In this chapter, we have given a detailed study on the design of the optimal sliding mode control, resulting from the combination of the sliding mode control and the quadratic linear control, the study including the two cases with presence and without presence. of disturbances.

First of all, we presented the mathematical model of the bioreactor in this case the continuous stirred tank reactor (CSTR) and its operation and the main information on this subject.

In addition, we introduced the design of sliding mode control, including the choice of sliding surface and the choice of gain matrix by moving to the convergence condition and stability analysis (Lyapunov function).

In the third section, we introduced the linear quadratic control design including the choice of weighting matrices.

In the first case (without disturbances) the simulation results show that the three controls SMC, OSMC and LQC, give good results.

However, in the other case (with perturbations) the simulation results show that the optimal sliding mode control gives us better performance compared to the other controls. we also notice that the quadratic linear control is not robust to disturbances.

GENERAL CONCLUSION

The interest of the study presented in this end-of-study thesis consists in approaching the techniques of control by optimal sliding modes which constitute an alternative to the traditional controls, the control by sliding modes offers the robustness with respect to disturbances and parameter variation, insensitivity to model accuracy, and ease of implementation, making it a valuable control technique in various industrial applications. Linear quadratic control is an optimal control, with stabilization capabilities, performance tradeoffs, ... These advantages make LQC a popular and effective control technique in various applications, ranging from robotics and aerospace to control of industrial processes. Optimal sliding mode control (OSMC) combines the advantages of sliding mode control and optimal control techniques.

Sliding mode control is known to exhibit chattering, which refers to high frequency oscillations around the sliding surface. OSMC incorporates optimization techniques and control laws to reduce or eliminate chatter. By damping oscillations, OSMC can provide a smoother control action, which reduces actuator wear and improves system performance.

It can handle strong disturbances using the sliding mode principle and optimization techniques. OSMC is able to provide stable and optimal control for a wide range of systems.

Overall, optimal sliding mode control offers advantages of robustness, optimal performance, fast response, ... These advantages make OSMC a valuable control technique in various applications where robustness and optimality are desired.

The design of the LQR/SMC hybrid controller has been detailed in the third chapter of this manuscript. This controller has combined the properties of both LQR and SMC commands to get a new controller. To verify the performances of the proposed LQR/SMC controller, the Bioreactor model, which is an uncertain system, was chosen as the application. The efficiency of the proposed LQR/SMC controller was examined by performing a variation of the system parameters and in the presence of external disturbances. The results showed the superiority of the LQR/SMC controller compared to the SMC controller and the LQR controller.

As perspectives, the work presented in this thesis can be extended to the design of controllers for nonlinear systems and in the discrete domain.

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