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Thème

Sur les applications multilinéaires continues entre espaces asymétriques normés

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إهداء

يسرني تقديم هذا الشكر لأبي وأمي اللذان سهرا على تربيتي

وتعليمي منذ أن بدأت حياتي وأشكر

كل من درسني وساهم في تدريسي من دكاترة جامعة مسيلة كما

يسرني أن أوجه شكري لكل من

نصحني وأرشدني وساهم في إعداد هذا البحث على وجه الخصوص

أستاذي لطرش فايز وعبد الكبير سعد.

Contents

0.1 Introduction

The main topic treated in this memory is the study of the continuity of multilinear mappings between asymmetric normed spaces, following the classical scheme of linear operators. It is difficult to localize the first moment when asymmetric norms were used (see, [4],[5] and [7]).

Our main motivation is to show that the results that worked for the case of linear operators between asymmetric asymmetric normed spaces could be extended to the multilinear operators ,so the main goal of this memory is to introduce and study the continuity of multilinear mappings on asymmetric normed spaces . As far as we know that is a first attempt in this regard. Let $X_1,\ldots,X_m$ and Y be vector spaces and T a mapping form $X_1\times\ldots\times X_m$ into Y.We may fix m–1 coordinates and so obtain a mapping form an X_i into Y.If such a mapping is linear for each X_i ,than T is said to be m-linear (multilinear).We give some characterizations, for the class of continuous multilinear operators, by asymmetric norm inequalities similar to linear case and using the notion of N-asymmetric norm .Also we prove some fundamental theorems concerning this mappings in the framwork of asymmetric normed spaces (see, [6],[9] and [11]).

The memory consists of two chapters. In chapter 1 we establish the notation of the memory. We introduce some important results concerning asymmetric norm and continuous linear mappings between asymmetric norms spaces and we recall the main definitions and properties of the continuous multilinear mappings on normed spaces.

In chapter 2 of this memory we give a result that gives the characterization of the continuous multilinear mappings between asymmetric norms spaces. We study the completeness properties of the asymmetric normed semi-vector space of these mappings.

Chapter1

Asymmetric normed spaces

1 .1 Quasi-metric spaces

In the following,let X be a non-empty set **.**

Definition 1.1.1

A quasi-metric onX is a function $d: X \times \mathrm{X} \rightarrow \mathbb{R}^{+}$with thefollowing properties

1 .for all $x, y \in X$we have $\mathrm{d}(\mathrm{x}, \mathrm{y}) = \mathrm{d}(\mathrm{y}, \mathrm{x}) = 0$ if and only if$\mathrm{x} = \mathrm{y}$.

2.for all $\mathrm{x}, \mathrm{y}, \mathrm{z} \in X$ we have $d(x, y) \leq d(x, y) + d(x, y)$(triangle inquality) .

The pair $(\mathrm{X}, \mathrm{d})$is called quasi-metric space **.**

If $\mathrm{d}(\mathrm{x}, \mathrm{y}) = \mathrm{d}(\mathrm{y}, \mathrm{x}) = 0$ does not imply $\mathrm{x} = \mathrm{y}$ for some$\mathrm{x}, \mathrm{y} \in X$,the function d is called a quasi-semimetric, and the pair(X, d) is called a quasi-semi metric space .

Remark1.1.2

If d is a quasi-metric on X,then the function $\bar{d}$ definedon $X \times X$ by

$$\bar{\mathrm{d}}(\mathrm{x}, \mathrm{y}) = \mathrm{d}(\mathrm{x}, \mathrm{y}), \qquad \mathrm{x}, \mathrm{y} \in \mathrm{X}$$

Is a quasi-metric on X called the conjugate of d .

And the function d^{s} defined on $X \times X$ by

$$\mathrm{d}^{\mathrm{s}}(\mathrm{x}, \mathrm{y}) = \max\{\mathrm{d}(\mathrm{x}, \mathrm{y}), \bar{\mathrm{d}}(\mathrm{x}, \mathrm{y})\}, \mathrm{x}, \mathrm{y} \in \mathrm{X}$$

Is a metric onX .

Example 1.1.3 [see 12]

the $d: \mathbb{R} \times \mathbb{R} \longrightarrow \mathbb{R}^{+}$application defined by

$$\begin{cases} d(x, y) = y - x & if\ x \leq y \\ d(x, y) = 1 & if\ x > y \end{cases}$$

Is a quasi-metric on$\mathbb{R}$,indeed for all x,y and $z \in \mathbb{R}$ we have

1. $d(x, y) = d(y, x) = 0$ If and only if $y - x = 0$ if and only if $x = y$
2. We have the following cases

- If $x \leq y\ and\ x \leq z\ and\ z \leq y$ so we have

  $d(x, y) = y - x \leq z - x = d(x, z) \leq d(x, z) + d(z, y)$

- If $x \leq y\ and\ x \leq z\ and\ z < y$, so

  $d(x, y) = y - x = z - x + y - z = d(x, z) + d(z, y)$

- If $x \leq y\ and\ z < x$,so

  $d(x, y) = y - x \leq y - z = d(z, y) \leq d(x, z) + d(z, y)$

- If $y < x \text{ and } x \leq z, so$

  $d(x,y) = 1 \leq d(x,z) + d(z,y) = d(x,z) + 1$
- If $y < x \text{ and } y \leq z \text{ and } z < x$,so we have

$$d(x,y) = 1 \leq d(x,z) + d(z,y) = 1 + d(z,y)$$

- If $y < x \text{ and } z < y, so$

  $d(x,y) = 1 \leq d(x,z) + d(z,y) = 1 + d(z,y)$

In the following we give a condition so that $d^s$is a metric .

Definition 1.1.4

(Balls) let (X , d) be a quasi-metricspace ,$x_0 \in X \text{ and }$ r>0 .the open ball,of radius r centered at x_0,is the set defined by

$$B_d(x_0,r) = \{x \in X: d(x_0,x) < r\}$$

The closed ball of radius r centered at x_0 is defined by

$$B_d[x_0,r] = \{x \in X: d(x_0,x) \leq r\}$$

Let x$\in X$, we say that a set v$\subset X$is a neighborhood of the point x if and only if

$$\exists\, r > 0: B_d(x,r) \subset v$$

The collection neighborhoods of the point x is denoted by $v_d(x)$

The topologyτ_d of a quasi-metric space (X ,d) can be defined starting from the family $v_d(x)$.

Definition1.1.5

The convergence of a sequence$(x_n)_n$ to x with respect to τ_d,called d-convergence, in symbols $x_n \overset{d}{\rightarrow} x$,can be characterized in the following way :

$$x_n \overset{d}{\rightarrow} x \Leftrightarrow d(x,x_n) \rightarrow 0$$

Proposition 1.1.6[see 1]

Let $(x_n)_n$ be a sequence in a quasi-metric space (X ,d)

1.If$(x_n)_n$ is d-convergent to x and $\bar{d}$-convergent to y, then $d(x,y) = 0$.

2.If $(x_n)_n$ is d-convergent to x and $d(y,x) = 0$, then $(x_n)_n$ is also d-convergent to y .

Proof .[see 12]

1. By inequality

$d(x,y) \leq d(x,x_n) + d(x_n,y), for\ all\ n \in \mathbb{N}$

And like

$\lim_{n\to+\infty} d(x,x_n) = 0\ and\ \lim_{n\to+\infty} d(x_n,y) = \lim_{n\to+\infty} \bar{d}(y,x_n) = 0$

We obtain $d(x,y) = 0$

2. By inequality

$d(y,x_n) \leq d(y,x) + d(x,x_n), for\ all\ n \in \mathbb{N}$

We obtain

$d(y,x_n) \leq d(x,x_n), for\ all\ n \in \mathbb{N}$

And this implies that

$\lim_{n\to+\infty} d(y,x_n) \leq \lim_{n\to+\infty} d(x,x_n) = 0$

So $\lim_{n\to+\infty} d(y,x_n) = 0$,There fore$(x_n)_n is\ d$-convergent to y.

1.2 Asymmetric normed spaces

For the general theory of asymmetric normed spaces we refer the reader to the monograph [1]. In the following, let X be a real vector space .

Definition1.2.1

A function $p: X \to \mathbb{R}^+$ is an asymmetric norm on the real vecter space X if for every x,y$\in X$ and $\alpha \in \mathbb{R}^+$ we have

1.$p(x) = p(-x) = 0$ if and only ifx=0

2.$p(\alpha x) = \alpha p(x)$

3.$p(x+y) \leq p(x) + p(y)$

We say that the pair (X ,p) is an asymmetric normed space .

If $p(x) = p(-x) = 0$ does not imply x=0 for some $x \in X$ (i.e satisfies only the condition (2) and (3) ,the function p is called a asymmetric semi norm, and the pair (X,p)is called an asymmetric semi normed vector space .

Asymmetric norm conjugate and symmetrization

The asymmetric norm conjugate to p is the function $\bar{P}: X \to \mathbb{R}^+$ defined by $\bar{p}(x) = p(-x)$. As a consequence, the asymmetric norm p induces a norm p^sdefined on X by the formula $p^s(x) = max\{p(x), p(-x)\}$, this norm is referred to as the asymmetrization of the asymmetric norm p .

Topology of asymmetric norm

Evrey asymmetric norm p,on a vector spaces X, induces a quasi-metric d_p on $X \times X$defined by

$d_p(x, y) = p(y - x), \qquad x, y \in X$

If p is an asymmetric norm on X,then the topology τ_{dp}will be simply denoted by τ_p and we will say that τ_p is the topology induced by p.

τ_p is a T_0 topology on X,that is for any pair x,y of distinct points in X,at least one of them has a neighborhood not containing the other .The lopolgyτ_p is generated by the asymmetric open balls

$B_p(x, \varepsilon) = \{y \in X: p(y - x) < \varepsilon\}$,where $\varepsilon > 0$.

Moreover the collection $\{B_p(x, \varepsilon): \varepsilon > 0\}$forms a fundamental system of neighborhoods for the topology τ_p .However, in general this topology is not Hausdorff (see[2]).

Proposition 1.2.2 (see[1])

If (X, p) is an asymletric space, then any ball $B_p(x_0, r)$is open in the topology τ_p and any ball$B_p[x_0, r]$is closed in the topology $\tau_{\bar{p}}$.

Also, the following inclusions hold :

$B_{p^s}(x_0,r) C B_p(x_0,r)\, and\, B_{p^s}(x_0,r) C B_{\bar{p}}(x_0,r),$

With similar inclusions for the closed balls.

Convergence in asymmetric space

A sequence$(x_n)_n$ in an asymmetric normed space (X, p) is convergent to $x \epsilon X$ with respect to $\tau_p$if and only if $\lim_{n \to +\infty} p(x_n - x) = 0$.Form this we obtain the following result (see [3],remark1.1).

Proposition 1.2.3

let **Z** be a linear subspace of X .Then **Z** is closed in (X,p) if and only if it is closed in $(X, \bar{p})$.

Completeness in asymmetric normed spaces

There are several notions of Cauchy sequence and more notions of completeness in asymmetric normed spaces(see[1]and[4]) .We present only the following notions :

- A sequence $(x_n)_n$ of elements of X is said to be left (right) K-Cauchy sequence in X is for every $\varepsilon > 0$ there exists$n_0 \epsilon \mathbb{N}$ such that $p(x_n + x_k) < \varepsilon \quad (resp: p(x_k - x_n) < \varepsilon)$whenever $n \geq k \geq n_0$.
- (x,p) is called left(right) K-complete if every left(right) K-couchy sequence in X is convergent with respect to τ_p
- (x,p) is called bicomplete(or bi-Banach) if the normed space (X, p^s) is complete.

Example 1 .2.4

As an important example, let u the asymmetric norm on the real vector space $\mathbb{R}$defined be

$u(x) := x^+ = max\{x, 0\}$

In this case $\bar{u}(x) = max\{-x, 0\} = x^- and\ u^s(x) = max\{-x, x\} = |x|$.

Obviously ($\mathbb{R}$,u) is a bi-Banach space.

The asymmetric norm u is called usual asymmetric norm .

1.3 Continuous linear operators.

Let$(X, p)$and(Y, q)two asymmetric normed spaces. We denote by $LC(X, y)$ the set of all continuous linear mappings form (X, p) into (Y, q) and by $LC^s(X, Y)$ the set of all continuous linear mappings form normed space (X, p^s) to normed space (Y, q^s) .

Definition 1.3.1

A linear map$T: (X, p) \rightarrow (Y, q)$ is called bounded if there exist a positive constant Ksuch that

$q(T(x)) \leq Kp(x)$

For all $x \epsilon X$.

The next result and its consequences can be found in [5]or[6] and will be used in the sequel .

Theorem 1.3.2

$T \epsilon LC(X, Y)$ if and only if T is bounded.

The following example show that the set $LC(X, Y)$ is not necessarily a vector space but it is a cone (or normed semi-vector space) . that is,

$T + S \epsilon LC(X, Y) and\ \alpha T \epsilon LC(X, Y)$for all $T, S \epsilon LC(X, Y) and\ \alpha \geq 0$.

Example 1.3.3 see[11]

let id be the identity function form $(\mathbb{R}, u)$into itself. Clearly id is a continuous linear function but $-id$ is not continuous, because if $x < 0, u(-x) = -x$,so

$sup\{u(-x) : u(x) \leq 1\} = \infty$

Thus we conclude that $LC(X, Y)$ is not a vector space in general.

Following(23 ,theorem1), we can consider an asymmetric norm on the cone $LC(X, Y)$ of all linear continuous mappings T from (X,p) into (Y,q) defined by the formula

$p_q^*(T) := sup\{q(T(x)) : p(x) \leq 1\}$

And also

$p_q^*(T) = inf\{K > 0 : q(T(x)) \leq Kp(x)\}$

Proposition 1.3.4

If the linear map $T : (X, p) \to (Y, q)$ is continuous, then$T : (X, \bar{p}) \to (Y, \bar{q})$ is continuous .Hence

$LC(X, Y) \subseteq LC^s(X, Y)$.

Proof.

let$T \in LC(x, y)$.than there is $K > 0$ *such that*

$\bar{q}(T(x)) = q(T(-x)) \leq K\bar{p}(x)$.

Therefore T is continuous from $(X, \bar{p}) to (X, \overline{q})$,hence

$$q^s(T(x)) \leq max\{kp(x), k\bar{p}(x)\} = kp^s(x)$$

We conclude that $LC(X, Y) \subseteq LC^S(X, Y)$.

Dual of an asymmetric normed space

Arelevant special case of continuous linear operator between asymmetric normed spaces arises when we take $(Y, q) = (\mathbb{R}, u)$.In this case p_u^* will be simply denoted by $p^*$and we put

$X^* = \{T : (X, p) \to (\mathbb{R}, u), T \text{is linear and continuous}\}$.

This cone is referred to as the dual space of (X,p).Observe that$((X^*, P^*)$ is a bi-Banach cone where $p^* := p_u^*$ (see[6],theorem1).

1.4 Example of bi-Banach space

In what follows, we study and detail an important example given in [13]

For $1 \leq p \leq \infty$,we will denote by ℓ_p the set of infinite sequences $x = (x_n)_n \subset \mathbb{R}$ such that

$$\ell_p = \left\{ (x_n)_n \subset \mathbb{R}: \sum_{n=0}^{\infty} |x_n|^p < \infty \right\}$$

It is well known that $(\ell_p, \|.\|_p)$ is a Banach space , where $\|.\|_p$ is the norm on ℓ_p defined by

$$\|x\|_p = \left(\sum_{n=0}^{\infty} |x_n|^p \right)^{\frac{1}{p}}, x = (x_n)_n \in \ell_p$$

Fix $p \geq 1$, for each $x = (x_n)_n \in \ell_p$ define

$$x^+ := (x_n^+)_n \; and \; \|x\|_{+p} = \|x^+\|_p := \left(\sum_{n=0}^{\infty} (x_n^+)^p \right)^{\frac{1}{p}}$$

We will show that $\|.\|_{+p}$ is an asymmetric norm on ℓ_p,such that the norm $(\|.\|_{+p})^s$ is equivalent to $\|.\|_p$. We need the following lemma.

Lemma 1.4.1

For $x = (x_n)_n \in \ell_p, y = (y_n)_n \in \ell_p \; and \; a \in \mathbb{R}^+$ the following statements hold

a) $x = x^+ - (-x)^+$.

b) $(ax)^+ = ax^+$.

c) $(x_n + y_n)^+ \leq x_n^+ + y_n^+ \; for \; all \; n \in \mathbb{N}$.

Proposition 1.4.2

For each $p \geq 1$, $\|.\|_{+p}$ is an asymmetric norm on ℓ_p .

Proof .

Let $x = (x_n)_n \in \ell_p$ such that $\|x\|_{+p} = \|-x\|_{+p} = 0$. Then $x^+ = (-x)^+$ and by (a) in the previous lemma, $x = 0$.On the other hand ,it is clear that $\|0\|_{+p} = 0$.

Now let $a \in \mathbb{R}^+$, $x = (x_n)_n \in \ell_p$ and $y = (y_n)_n \in \ell_p$, then

$\|(ax)\|_{+p} = \|(ax)^+\|_{+p} = a\|x\|_{+p}, by \; (b)$ in the previous lemma.

Finally by (c) in the previous lemma we have

$$\|x+y\|_{+p} = \|(x+y)^+\|_p \le \left(\sum_{n=0}^{\infty}(x_n^+ + y_n^+)^p\right)^{\frac{1}{p}}$$

And so

$$\|x+y\|_{+p} = \|x^+ + y^+\|_p \le \|x^+\|_{+p} + \|y^+\|_{+p} = \|x\|_{+p} + \|y\|_{+p}$$

Corollary 1.4.3

$(\ell_{\mathrm{p}}, \|\,.\,\|_{+\mathrm{p}})$is an asymmetric normed linear space, for each $p \ge 1$.

Proposition 1.4.4

For each $p \ge 1$ we have

$$(\|\,x\|_{+p})^s \le \|\,x\|_p \le \|x\|_{+p} + \|-x\|_{+p}$$

When ever$x \in \ell_p$.

Proof.

Let $x = (x_n)_n \in \ell_p$. Then ,it is clear that

$$(\|\,x\|_{+p})^s = max\{\|x\|_{+p}, \|-x\|_{+p}\} \le \|x\|_p$$

Finally , by (a) in the previous lemma , we obtain

$$\|x\|_p = \|x^+ - (-x)^+\|_p \le \|x^+\|_p + \|(-x)^+\|_p = \|x\|_{+p} + \|-x\|_{+p}$$

Corollary 1.4.5

For each $p \ge 1$we have

$$(\|\,.\,\|_{+p})^s \le \|\,.\,\|_p \le 2(\|\,.\,\|_{+p})^s$$

Therefore $(\|\,.\,\|_{+p})^s and\ \|\,.\,\|_p$are equivalent norms in ℓ_p.

Corollary 1.4.6

For each $p \ge 1$, $(\ell_{\mathrm{p}}, \|\,.\,\|_{+\mathrm{p}})$ is a bi-Banach space.

1.5 Multilinear operators between normed spaces .

Let $m\epsilon\mathbb{N}$ and consider $X_j (j = 1, \dots, m), Y$ the normed spaces over $\mathbb{K}, (either \mathbb{R} or \mathbb{C})$.

A mappings $T\colon X_1 \times \dots \times X_m \to Y$ is called multilinear (or m-linear) if the mappings

$$T_j\colon\ X_j \ \to\ \ Y$$

$$x^j \longmapsto T(x^1, \dots, x^j, \dots, x^m)$$

Are linear for each set of fixed $x^{\kappa}\epsilon X_{\kappa}, \kappa \neq j, i.e$

$$T\left(x^1, \ldots, \lambda x^j + y^j, \ldots, x^m\right) = \lambda T\left(x^1, \ldots, x^j, \ldots, x^m\right) + T\left(x^1, \ldots, x^j, \ldots, x^m\right)$$

For all $\lambda \epsilon \mathrm{K}\ and\ x^j, y^j \epsilon X_j (j = 1, \ldots, m)$

The vector space of such mappings is dented by $L(X_1, \ldots, X_m; Y)$.If$Y = \Bbbk$, we write $L(X_1, \ldots, X_m)$.

Remark 1.5.1

The set **S** of all vectors in Y of the form $T(x^1, \ldots, x^m), x^j \epsilon X_j (j = 1, \ldots, m)$ is not in general a vector subspace of Y.In order to see this, let$X_1 = X_2$ and Y be vector spaces such that $dim(X_1) = dim(X_2) = 2$ and dim $(Y) = 4$ .Select a basis $\{a_1, a_2\}$ in X_1 and a basis $(e_i)_{i=1}^4$ in Y and define the bilinear mapping T by $T(x^1, x^2) = \xi_1 \eta_1 e_1 + \xi_1 \eta_2 e_2 + \xi_2 \eta_1 e_3 + \xi_2 \eta_2 e_4$

Where $x^1 = \xi_1 a_1 + \xi_2 a_2\ and\ x^2 = \eta_1 a_1 + \eta_2 a_2$.It is easy to see that

$$\boldsymbol{S} = \left\{z = \sum_{i=1}^4 \lambda_i e_i \epsilon Y : \lambda_1 \lambda_4 - \lambda_2 \lambda_3 = 0\right\}$$

Let $z_1 = 2e_1 + 2e_2 + e_3 + e_4\ and\ z_2 = e_1 + e_3$.It is clear that $z_1, z_2 \epsilon$**S** but $\mathrm{z}_1 - \mathrm{z}_2 \notin$ **S**, it follows that **S** is not a subspace of Y.(see [7] ,section1.1)

Let us consider the space $X_1 \times \ldots \times X_m$ endowed with the norms $\|\,.\,\|_\infty$and *s* defined by

$$\|x\|_\infty = \max_{1 \le j \le m} \|x^j\| and\ s(x) = \sum_{j=1}^m \|x^j\|,$$

For all $x = (x^1, \ldots, x^m) \epsilon X_1 \times \ldots \times X_m$.

Definition 1.5.2

An m-linear mapping $T: X_1 \times \ldots \times X_m \to Y$ is continuous if it is continuous as a function between two normed spaces .

As a consequence of this definition, similar to the linear case ,we have a result that gives the characterization of the continuous m-linear mapping .

Theorem 1.5.3

Let $X_1 \times \ldots \times X_m, Y$ be normed spaces. For $T \epsilon L(X_1, \ldots, X_m; Y)$the following asserrions are equivalent .

1. T is continuous.
2. T is continuous in (0,...,0).

3. There is a constant $\mathrm{K} \geq 0$ with

$$\|T(x^1, \dots, x^m)\| \leq \mathrm{K}\|x^1\| \dots \|x^m\| \qquad (1.2)$$

For all $x^j \epsilon X_j (j = 1, \dots, m)$

4. $\|T\| = \sup_{\|x^j\| \leq 1, j=1,\dots,m} \|T(x^1, \dots, x^m)\| < \infty.$

We will write $\mathcal{L}(X_1, \dots, X_m; Y)$ for the vector space of all continuous m-linear mappings .
If $Y = \mathbb{K}, we\ write\ \mathcal{L}(X_1, \dots, X_m)$.
It is easy to see that
$\|T\| = inf\{\mathrm{K} \geq 0, verifying\ the\ inequality (1.2)\}$
Defines a norm on $\mathcal{L}(X_1, \dots, X_m; Y)$ which is complete norm when $\| . \|_Y$is complete .For the general theory of multilinear mappings we refer to [8]or [9].

Chapter 2

Continuous multilinear operators between asymmetric normed spaces

2.1 Characterization of continuous multilinear mapping between asymmetric normed spaces.

We give a characterization of continuous multilinear mappings in a way analogous to that used to characterize linear mappings between asymmetric normed spaces .For studying the continuity norms instead of asymmetric norms. We will see why this in the remark 2.1.5after characterize the continuity by means of an inequality .

Definition 2.1.1
An N-asymmetric norm is an asymmetric norm p on the real vector space X, for which p(x)=0 implies x=0 . We say that the pair (x,p)is a N-asymmetric normed space .

Example 2.1.2
It is easy to see that the function
$p: \mathbb{R} \to \mathbb{R}^+, p(x) = |x| + max\{x, 0\}$ (2.1)
Is an N-asymmetric (called the usual N-asymmetric norm) . more generally, we can define an N-asymmetric norm p on the Banach lattice X by the formula
$p(x) = \|x\| + \|max\{x, 0\}\|.$
Throughout this chapter ,$(X_1, p_1), \dots, (X_m, p_m)$ will be real N-asymmetric normed spaces and (Y,q) be an asymmetric normed space .let us consider the space $X_1 \times \dots \times X_m$ endowed with the N-asymmetric norms p_∞ and s defined by

$$p_\infty(x) = \max_{1 \le j \le m} p_j(x^j) and\ s(x) = \sum_{j=1}^{m} p_j(x^j)$$

For all $x = (x^1, \dots, x^m) \epsilon X_1 \times \dots \times X_m$.we know that s and $p_\infty$are equivalent asymmetric norm on $X_1 \times \dots \times X_m$whose induced topology coincides with the product topology (see[10],lemma 6)

Definition 2.1.3
An m-linear mapping $T: X_1 \times \dots \times X_m \to Y$ is continuous if is continuous as a function between two asymmetric normed spaces .
By$LC_{(p_1,\dots,p_m;)}(X_1, \dots, X_m; Y)$we denote the set of all continuous multilinear mappings between the N-asymmetric normed space $X_1 \times \dots \times X_m$and the

asymmetric normed spaceY and by $LC^{s}_{(p^{s}_{1},\dots,p^{s}_{m};q^{s})}(X_1,\dots,X_m;Y)$ the normed vector space of all continuous multilinear operators between the normed vector spaces $(X_1,p^s_1)\times\dots\times(X_m,p^s_m) and\ (Y,q^s)$.

Theorem 2.1.4

let $(X_1,p_1),\dots,(X_m,p_m)$be N-asymmetric normed spaces,(Y,q)be an asymmetric normed space and $T\colon X_1\times\dots\times X_m\to Y$be a multilinear mapping .the following statements are equivalent :

(i) T is continuous

(ii) T is continuous in (0,...,0)

(iii) There is a constant $M\geq 0$such that

$$q\big(T(x^1,\dots,x^m)\big)\leq Mp_1(x^1)\dots p_m(x^m) \qquad (2.2)$$

For every $x^j\epsilon X_j, j=1,\dots,m$

(iv)

$$\| T|_{(p_1,\dots,p_m;q)} = \sup_{p_j(x^j)\leq 1, j=1,\dots,m} q\big(T(x^1,\dots,x^m)\big)<\infty \quad (2.3)$$

Proof :

(i) implies (ii)is obvious

(ii)⇒(iii). Assume that T is continuous at (0,...,0). Then we can choose r>0 such that $T\left(B_{p_\infty}(0,r)\right)\subset B_q(0,r). since$

$$p_\infty\left(\frac{rx^1}{2p_1(x^1)},\dots,\frac{rx^m}{2p_m(x^m)}\right)=\frac{r}{2}<r$$

For all$(x^1,\dots,x^m)\in X_1\times\dots\times X_m$with $x^j\neq 0, j=1,\dots,m,$one has

$$q\left(T\left(\frac{rx^1}{2p_1(x^1)},\dots,\frac{rx^m}{2p_m(x^m)}\right)\right)<1$$

By multilinearity of T we obtain(2.2) with$M=\frac{2^m}{r^m}.If\ x^j=0$for some j=1,...,m we have $T(x^1,\dots,x^m)=0$and the inequality(2.2) Remains valid .

$(iii)\Rightarrow(i)$ .Let us consider the space $X_1\times\dots\times X_m$endowed with the asymmetric norm $\mathcal{S}$.Let us fix $a=(a^1,\dots,a^m)\epsilon X^1\times\dots\times X^m$and we prove that $T\big(B_s(a,r)\big) \subset B_q(T(a),\epsilon)$,for every $\epsilon > 0$,where $r < min\left\{1,\frac{\epsilon}{\kappa M}\right\}$ $and\ \kappa =$ $\max_{1\le j\le m}\left\{p_\infty(a)^{j-1}\big(1+p_\infty(a)\big)^{m-j}\right\}$.

Let $y = T(z) \in T\big(B_s(a,r)\big) with\ z = (z^1,\dots,z^m)$. than ,using the inequality (2.2) and taking into account that

$$T(z)-T(a)=\sum_{j=1}^{m} T(a^1,\dots,a^{j-1},z^j-a^j,z^{j+1},\dots,z^m),$$

We obtain

$$q(y-T(a)) \le \sum_{j=1}^{m} Mp_1(a^1)\dots p_{j-1}(a^{j-1})p_j(z^j - a^j)p_{j+1}(z^{j+1})\dots p_m(z^m)$$

$$\le \sum_{j=1}^{m} Mp_j(z^j - a^j)p_\infty(a)^{j-1}\ p_\infty z^{m-j}\ .$$

On the other hand ,since $p_\infty \le s$, we get that

$p_\infty(z) \le s(z-a)+p_\infty(a) < r+p_\infty(a) < 1+p_\infty(a)$

Which yields

$$q\big(y-T(a)\big) < \sum_{j=1}^{m} Mp_j(a^j - a^j)\big(1+p_\infty(a)\big)^{m-j}p_\infty(a)^{j-1}$$

$$< \sum_{j=1}^{m} Mp_j(z^j-a^j)\kappa$$

$$=\kappa Ms(z-a)$$

$$<\kappa Mr < \kappa M\frac{\epsilon}{\kappa M}=\epsilon$$

$(iii)\Rightarrow(iv)$.Starting form (2.2)we get

$$\| T|_{(p_1,\dots,p_m)} = \sup_{p_j(x^j)\le 1, j=1,\dots,m} q\big(T(x^1,\dots,x^m)\big)$$

$$\leq \sup_{p_j(x^j)\leq 1, j=1,\dots,m} M p_1(x^1)\dots p_m(x^m) = M < \infty$$

$(iv) \Rightarrow (iii)$. If $p_j(x^j) = 0$ for some j=1 ,...,m, the inequality (2.2) is evident.Suppose that $\| T|_{(p_1,\dots,p_m;q)} < \infty$,than there exists a constant $M \geq 0$ such that

$$q\left(T\left(\frac{x^1}{p_1(x^1)}, \dots, \frac{x^m}{p_m(x^m)}\right)\right) \leq M$$

For all $x^j \in X_j$ with $p_j(x^j) \neq 0, j = 1, \dots, m$ and we obtain (2.2) .

Remark 2.1.5

For many asymmetric norms, there are no continuous multilinear mappings. those are ones that has may non-zero elements $x^1 \epsilon X_1 such\ that p_1(x^1) = 0$ (i.e p₁ is not an N-asymmetric norm) . Indeed, if the m-linear mapping T is continuous ,than $T(x^1, \dots, x^m) = 0\ for\ all\ x^j \epsilon X_j\ with\ j > 1$.this is obvious, since if $T(x^1, \dots, x^m) \neq 0$ we have

$$q(T(x^1, x^2, \dots, x^m)) > 0\ or\ q(T(x^1, -x^2, \dots, x^m)) > 0$$

What contradicts that

$$0 < q(T(x^1, x^2, \dots, x^m)) \leq M p_1(x^1) p_2(x^2) \dots p_m(x^m) = 0$$

Or

$$0 < q(T(x^1, -x^2, \dots, x^m)) \leq M p_1(x^1) p_2(-x^2) \dots p_m(x^m) = 0$$

Now we give an easy example of a continuous bilinear mapping .

Example 2.1.6

We can define the bilinear map $T: (\mathbb{R}, p) \times (\mathbb{R}, p) \to (\mathbb{R}, u) by T(x, y)$ =xy,where u is the usual asymmetric norm on $\mathbb{R}$ defined by(1.1) and p is the usual N-asymmetric norm defined by (2.1).It is easy to see that

$$u(T(x, y)) = \max\{xy, 0\} \leq |x||y| \leq p(x)p(y),$$

For all $x, y \epsilon \mathbb{R}$.

Proposition 2.1.7

let $\mathrm{T} \epsilon \mathrm{LC}_{(\mathrm{p}_1,\dots,\mathrm{p}_{2m+1};\mathrm{q})}(\mathrm{X}_1, \dots, \mathrm{X}_{2m+1}; \mathrm{Y})$. Than

$$\mathrm{q}(\mathrm{T}(\mathrm{x}^1, \dots, \mathrm{x}^m)) \leq \| \mathrm{T}|_{(\mathrm{p}_1,\dots,\mathrm{p}_m;\mathrm{q})} \mathrm{p}_1(\mathrm{x}^1) \dots \mathrm{p}_m(\mathrm{x}^m) \quad (2.4)$$

For all $x^j \in X_j, j = 1, \dots, m$.Moreover$\| \mathrm{T}|_{(\mathrm{p}_1,\dots,\mathrm{p}_m;\mathrm{q})}$ can becalculated also by the formula

$\| \mathrm{T}|_{(\mathrm{p}_1,\dots,\mathrm{p}_m;\mathrm{q})} = \inf\{\mathrm{M} \geq 0 : \mathrm{M}\ \text{satisfies}(2.2)\} \quad (2.5)$

Proof .

For every $x^j \in X_j$ such that $x^j \neq 0$,j=1,...,m , the inequality is obvious . on the other hand, if λ is the right side member of the equality (2.5) then it is clear that $\lambda \leq \| T|_{(p_1,\dots,p_m;q)}$ .For the reverse inequality ,$M \geq 0$ satisfies (2.2) ,follows that

$$\| T|_{(p_1,\dots,p_m;q)} = \sup_{p_j(x^j)\leq 1, j=1,\dots,m} q(T(x^1,\dots,x^m)) \leq M$$

And so $\| T|_{(p_1,\dots,p_m;q)} \leq \lambda$

An immediate consequence of the theorem 2.1.4 is the following corollary.

Corollary 2.1.8

Let $m \in \mathbb{N}$.the following are equivalent for the multilinear mapping $T: X_1 \times \dots \times X_{2m+1} \to Y$

- T is continuous from$(X_1, p_1) \times \dots \times (X_{2m+1}, p_{2m+1}) to\ (Y, q)$.
- T is continuous form$(X_1, \bar{p}_1) \times \dots \times (X_{2m+1}, \bar{p}_{2m+1}) to\ (Y, \bar{p})$

Consequently

$$\mathrm{LC}_{(\mathrm{p}_1,\dots,\mathrm{p}_{2m+1};\mathrm{q})}(\mathrm{X}_1, \dots, \mathrm{X}_{2m+1}; \mathrm{Y}) \subset \mathrm{LC}^{s}_{(\mathrm{p}^s_1,\dots,\mathrm{p}^s_{2m+1};\mathrm{q}^s)}(\mathrm{X}_1, \dots, \mathrm{X}_{2m+1}; \mathrm{Y})$$

And

$\|T\| := \|T\|_{(p^s_1,\dots,p^s_{2m+1};q^s)} \leq \| T|_{(p_1,\dots,p_{2m+1};q)}$

For all $T \in LC_{(p_1,\dots,p_{2m+1};)}(X_1, \dots, X_{2m+1}; Y)$.

Proof.

The equivalence $(i) \Leftrightarrow (ii)$ it follow from

$$\sup_{\substack{\bar{p}_j(x^j)\leq 1 \\ j=1,\dots,2m+1}} \bar{q}\big(T(x^1,\dots,x^{2m+1})\big) = \sup_{\substack{p_j(x^j)\leq 1 \\ j=1,\dots,2m+1}} q(T(-x^1,\dots,-x^{2m+1}))$$

$$= \sup_{\substack{p_j(x^j)\le 1\\ j=1,\dots,2m+1}} q(T(x^1,\dots,x^{2m+1}))$$

With this we have $\| T|_{(p_1,\dots,p_{2m+1};q)} = \| T|_{(\bar{p}_1,\dots,\bar{p}_{2m+1};\bar{q})}$

Now let $T \in LC_{(p_1,\dots,p_{2m+1};)}(X_1,\dots,X_{2m+1};Y)$. For all$x^j \in X_j$ such that $p_j^s(x^j) \le 1$, j=1,...,2m+1,we have

$q\left(T(x^1,\dots,x^{2m+1})\right) \le \| T|_{(p_1,\dots,p_{2m+1};q)}$

And

$\bar{q}\left(T(x^1,\dots,x^{2m+1})\right) \le \| T|_{(\bar{p}_1,\dots,\bar{p}_{2m+1};\bar{q})}$

This implies that

$\|T\| = \sup_{p_j^s(x^j)\le 1, j=1,\dots,2m+1} q^s\left(T(x^1,\dots,x^m)\right) \le \| T|_{(p_1,\dots,p_{2m+1};q)} < \infty$

And the proof follow.

Remark 2.1.9

Asin the linear case, the$set\,LC_{(p_1,\dots,p_m;q)}(X_1,\dots,X_m;Y)$ is a cone(or normed semi-vector space), that is T+S ,$\alpha T \in LC_{(p_1,\dots,p_m;q)}(X_1,\dots,X_m;Y)$for all T,S belongs to $LC_{(p_1,\dots,p_m;q)}(X_1,\dots,X_m;Y)$ and all $\alpha > 0$ .we have no proof for the fact that this set is a vector space or not .

Now we introduce an asymmetric semi-norm on the cone $LC_{(p_1,\dots,p_m;q)}(X_1,\dots,X_m;Y)$. In the following proposition, the properties of the asymmetric semi-norm are easy to verify.

Proposition 2.1.10

The number $\|.|_{(p_1,\dots,p_m;q)}$is an asymmetric semi-norm on the cone of all continuous multilinear mappings between N-asymmetric normed spaces $X_1 \times \dots \times X_m and\ Y.$

As the linear case ,we consider an extended asymmetric norm on the space $LC^s_{(p_1^s,\dots,p_m^s;q^s)}(X_1,\dots,X_m;Y)$ by the same formula

$$\| T|_{(p_1,\dots,p_m;q)} = \sup_{p_j(x^j)\le 1} q\left(T(x^1,\dots,x^m)\right)$$

With the possibility that$\| T|_{(p_1,\dots,p_m;q)} = +\infty$

With the asymmetric norm $\|.|_{(p_1,\dots,p_m;q)}$we associates an extended norm on

$LC^s_{(p_1^s,\dots,p_m^s;q^s)}(X_1,\dots,X_m;Y)$defined by

$\| T|^s_{(p_1,\dots,p_m;q)} = max\{\| T|_{(p_1,\dots,p_m;q)}, \| -T|_{(p_1,\dots,p_m;q)}\}$

Corollary 2.1.11

For we haveall $T \in LC^s_{(p_1^s,\dots,p_m^s;q^s)}(X_1,\dots,X_m;Y)$

$$\|T\| \ \leq \| T|^s_{(p_1,\dots,p_m;q)} \qquad (2.6)$$

Proof .

In order to establish (2.6), we may suppose $\| T|^s_{(p_1,\dots,p_m;q)} < \infty$. Than we obtain$\| T|_{(p_1,\dots,p_m;q)} < \infty$ and $\| -T|_{(p_1,\dots,p_m;q)} < \infty$.For every$x^j \in X_j$,j=1,...,m, we get

$$\begin{aligned} q\left(T(x^1,\dots,x^m)\right) &\leq \| T|_{(p_1,\dots,p_m;q)} p_1(x^1)\dots p_m(x^m) \\ &\leq \| T|^s_{(p_1,\dots,p_m;q)}\, p_1^s(x^1)\dots p_m^s(x^m) \end{aligned}$$

And

$$\begin{aligned} \bar{q}\left(T(x^1,\dots,x^m)\right) &\leq \| -T|_{(p_1,\dots,p_m;q)} p_1(x^1)\dots p_m(x^m) \\ &\leq \| T|^s_{(p_1,\dots,p_m;q)}\, p_1^s(x^1)\dots p_m^s(x^m) \end{aligned}$$

Consequently

$$q^s\left(T(x^1,\dots,x^m)\right) \leq \| T|^s_{(p_1,\dots,p_m;q)}\, p_1^s(x^1)\dots p_m^s(x^m)$$

And thus $\|T\| \ \leq \| T|^s_{(p_1,\dots,p_m;q)}$.

2.2 Completeness properties

For studing the completeness properties of the cone $LC_{(p_1,\dots,p_m;q)}(X_1,\dots,X_m;Y)$ we need the following .

Lemma 2.2.1

The extended norm $\|.\,|^s_{(p_1,\dots,p_m;q)}$con be calculated by the following formula

$$\| T|^s_{(p_1,\dots,p_m;q)} = \sup\nolimits_{p_j(x^j)\leq 1, j=1,\dots,m} q^s\left(T(x^1,\dots,x^m)\right) \qquad (2.7)$$

For all $T \in LC^s_{(p_1^s,\dots,p_m^s;q^s)}(X_1,\dots,X_m;Y)$.

Proof .

Let α is the right side member of the equality (2.7) .Than

$$\begin{aligned}\| -T|_{(p_1,\dots,p_m;q)} &= \sup_{p_j(x^j)\leq 1, j=1,\dots,m} q\left(-T(x^1,\dots,x^m)\right)\\ &\leq \sup_{p_j(x^j)\leq 1, j=1,\dots,m} q^s\left(T(x^1,\dots,x^m)\right)\end{aligned}$$

And

$$\begin{aligned}\| T|_{(p_1,\dots,p_m;q)} &= \sup_{p_j(x^j)\leq 1, j=1,\dots,m} q\left(T(x^1,\dots,x^m)\right)\\ &\leq \sup_{p_j(x^j)\leq 1, j=1,\dots,m} q^s\left(T(x^1,\dots,x^m)\right)\end{aligned}$$

This implies$\| T|^s_{(p_1,\dots,p_m;q)} \leq \alpha$.Also, for all $x^j \in X_j$ such that $p_j\left(x^j\right) \leq 1, j = 1,\dots,m, we\ have$

$q\left(T(x^1,\dots,x^m)\right) \leq \| T|_{(p_1,\dots,p_m;q)} \leq \| T|^s_{(p_1,\dots,p_m;q)}$

And

$q\left(-T(x^1,\dots,x^m)\right) \leq \| -T|_{(p_1,\dots,p_m;q)} \leq \| T|^s_{(p_1,\dots,p_m;q)}$

Than $\alpha \leq \| T|^s_{(p_1,\dots,p_m;q)}$.

Proposition 2.2.2

Let m an odd natural number. The set$LC_{(p_1,\dots,p_m;q)}(X_1,\dots,X_m;Y)$ is closed in the space

$\left(LC^s_{(p_1^s,\dots,p_m^s;q^s)}(X_1,\dots,X_m;Y), \|.|^s_{(p_1,\dots,p_m;q)}\right)$

Proof.

Let $(T_n)_n$ be a sequence in $LC_{(p_1,\dots,p_m;q)}(X_1,\dots,X_m;Y)$ that converges to $T \in LC^s_{(p_1^s,\dots,p_m^s;q^s)}(X_1,\dots,X_m;Y)$with respect to the extended norm $\|.|^s_{(p_1,\dots,p_m;q)}$. we will show that $T \in LC_{(p_1,\dots,p_m;q)}(X_1,\dots,X_m;Y)$. Indeed, there exist $n_0 \in \mathbb{N}$ such that

$$\begin{aligned}\| T|_{(p_1,\dots,p_m;q)} &\leq \| T|^s_{(p_1,\dots,p_m;q)}\\ &\leq \| T_{n_0} - T|^s_{(p_1,\dots,p_m;q)} + \| T_{n_0}|^s_{(p_1,\dots,p_m;q)}\\ &\leq 1 + \| T_{n_0}|^s_{(p_1,\dots,p_m;q)} < \infty\end{aligned}$$

In the following theorem we show that the cone $LC_{(p_1,\dots,p_m;q)}(X_1,\dots,X_m;Y)$ is bi-Banach if Y is a bi-Banach asymmetric space .

Theorem 2.2.3

Let $(X_1,p_1),\dots,(X_m,p_m)$be N-asymmetric normed spaces,(Y,q) be an asymmetric normed space and assume that (Y,q) is bi-Banach .then $LC^{s}_{(p_1^s,\dots,p_m^s;q^s)}(X_1,\dots,X_m;Y)$ is a bi-Banach space with respect to the asymmetric norm$\|.|_{(p_1,\dots,p_m;q)}$ .Consequently, if m is odd, $\left(LC_{(p_1,\dots,p_m;q)}(X_1,\dots,X_m;Y),\|.|_{(p_1,\dots,p_m;q)}\right)$is a bi-Banach cone .

Proof .

consider a Cauchy sequence$(T_n)_n \subset LC^{s}_{(p_1^s,\dots,p_m^s;q^s)}(X_1,\dots,X_m;Y)$with respect to the extended norm$\|.|^{s}_{(p_1,\dots,p_m;q)}$ .Hence for all $\varepsilon > 0$,there exists $n_0 \in \mathbb{N}$ such that

$\|T_n - T_k\| \le \| T_n - T_k|^{s}_{(p_1,\dots,p_m;q)} < \varepsilon$

For all $n,k \ge n_0$,which means that $(T_n)_n$ is a Cauchy sequence in the Banach space $\left(LC^{s}_{(p_1^s,\dots,p_m^s;q^s)}(X_1,\dots,X_m;Y),\|.\|_{(p_1^s,\dots,p_m^s;q^s)}\right)$

Thus, there exists $T \in LC^{s}_{(p_1^s,\dots,p_m^s;q^s)}(X_1,\dots,X_m;Y)$ such that $\|T_n - T\| \longrightarrow 0. As\ T_n - T$ is continuous we get that

$q^s\left(T_n(x^1,\dots,x^m) - T(x^1,\dots,x^m)\right) \le \|T_n - T\| p_1^s(x^1),\dots,p_m^s(x^m)$

And $\left(T_n(x^1,\dots,x^m)\right)_n$ is convergent to $T(x^1,\dots,x^m)$ in the Banach space (Y,q^s) for all$x^j \in X_j, j = 1,\dots,m$ .then there is $k \ge n_0$ such that $q^s\left((T - T_k)(x^1,\dots,x^m)\right) < \varepsilon$.By using the formula (2.7),for every $n \ge n_0$ and $x^j \in X_j$ with $p_j(x^j) \le 1, j = 1,\dots,m$, we have

$$q^s\left((T - T_n)(x^1,\dots,x^m)\right) = q^s\left(T(x^1,\dots,x^m) - T_n(x^1,\dots,x^m)\right)$$

$$\le$$

$$q^s\left(T(x^1,\dots,x^m) - T_k(x^1,\dots,x^m)\right) + $$

$$q^s\left(T(x^1,\dots,x^m) - T_n(x^1,\dots,x^m)\right)$$

$$< \varepsilon + \| T_n - T_k|^{s}_{(p_1,\dots,p_m;q)}$$

$$< 2\varepsilon$$

By taking the supremum over all $x^j \in X_j\ with\ p_j(x^j) \le 1, j = 1,\dots,m$, we

obtain
$\| T - T_n |_{(p_1,\ldots,p_m;q)}^{s} < 2\varepsilon \ , for\ every\ n \geq n_0$

The second claim is a consequence of the first one and of the above proposition .

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Abstract :

in this memory we introduce and characterize the continuity of multilinear mappings between asymmetric normed spaces .in particular, we study the completeness properties of the asymmetric normed semi-vector space of these mappings . the second purpose of the memory is to present the concept of linearization of continuous multilinear operators, on asymmetric normed spaces .

Résumé:

Dans cette mémoire ,nous introduisons et caractérisons la continuité des opérateurs multilinéaires entre espaces asymétriques normés .nous étudions en particulier les propriétés de complétude de semi-espace vectoriel asymétrique normé de cesopérateurs. Le deuxiéme objectif de la mémoire est de présenter le concept de linearization d’opérateurs multilinéaires continus sur des espaces asymétriques normés.

ملخص:

في هذه المذكرة قدمنا و عرفنا استمرار المؤثرات متعددة الخطية المعرف على فضاءات نظيمية لا متماثلة . ومنها توصلنا الى خصائص الفضاءات الناتجة لهذه التطبيقات

الهدف الثاني لهذه المذكرة هو عرض مفهوم خطية للمؤثرات متعددة الخطية المستمرة المعرفة على فضاءات نظيمية لا متماثلة .