PEOPLE'S DEMOCRATIC REPUBLIC OF ALGERIA

MINISTRY OF HIGHER EDUCATION AND SCIENTIFIC RESEARCH

UNIVERSITY OF MOHAMED BOUDIAF-MSILA

FACULTY OF TECHNOLOGY

COMMON BASE SCIENCE AND TECHNOLOGY DEPARTMENT

Fundamental Electrical Engineering 1

(Courses and applications)

For: 2nd Year level Engineer in Electrical Engineering

Directed by

Dr. HELLALI LALLOUANI

Academic Year: 2024/2025

الجـمـهـوريـة الـجـزائريـة الديمـقـراطية الشـعـبيـة
PEOPLE'S DEMOCRATIC REPUBLIC OF ALGERIA

وزارة الـتـعـلـيم الـعـالي و البحث العلمي
جامـعــة محمد بوضياف بالمسيلـة
كــلـيــة التكنولوجيا
قسم الهندسة الكهربائية

MINISTRY OF HIGHER EDUCATION AND SCIENTIFIC RESEARCH
MOHAMED BOUDIAF UNIVERSITY OF M'SILA
FACULTY OF TECHNOLOGY
DEPARTMENT OF ELECTRICAL ENGINEERING

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شهادة إدارية

بخصوص مطبوعة الدروس الخاصة بالأستاذ
هلالي العلواني

بناءً على محضر اللجنة العلمية لقسم الهندسة الكهربائية تحت رقم: 365/ق.هـ.ك/2024 المنعقد بتاريخ 06 نوفمبر 2024 والمتضمن تعين الخبراء: الأستاذ بلحمدي ساعد أستاذ بجامعة المسيلة الأستاذ بن يطو لطفي أستاذ بجامعة المسيلة، والأستاذ عكة علي أستاذ محاضر –أ بالمدرسة العليا للأساتذة ببوسعادة2 وذلك لتقييم مطبوعة الدروس الخاصة بالأستاذ هلالي العلواني أستاذ محاضر –ب- بقسم الهندسة الكهربائية لجامعة المسيلة تحت عنوان :

" Electrotechnique Fondamentale 01 "

مطبوعة دروس مكتوبة باللغة الإنجليزية تحت عنوان:

" Fundamental Electrical Engineering 01"

وبعد إطلاع رئيس اللجنة العلمية ورئيس القسم على التقارير الواردة والتي كانت كلها ايجابية، وعليه فإن اللجنة لا ترى مانعا أن تتخذه سندا في تدريس طلبة السنة الثانية مهندس دولة تخصص كهروتقني، شعبة كهروتقني، ميدان علوم وتكنولوجيا وأن تعتمد في أي تقييم للمسار العلمي للأستاذ المعني.

رئيس اللجنة العلمية
أ. د بوقرة عبد الرحمان

رئيس القسم
دفداف البروك

ملاحظة: سلمت هذه الشهادة للمعني(ة) لاستعمالها في حدود ما يسمح به القانون.

Foreword

This document is intended for students in the second year of the State Electrical Engineering program. The content is in line with the program for the Fundamental Electrical Engineering I module. The aim of Fundamental Electrical Engineering I is to introduce students to the basic concepts of mathematics (complex numbers) and the fundamental laws of electricity. This is followed by the analysis of DC, single-phase AC and three-phase AC circuits. Magnetic circuits and electrical transformers are then presented in a highly educational manner. We conclude with technological descriptions of a number of electrical machines. The aim of this module is to familiarize students with concepts specific to electrical engineering, enabling them to continue their university studies in electrical engineering, electronics and automation.

Course syllabus

Subject: Fundamental Electrical Engineering 1

Stream: Electrical Engineering

Level: 2nd year Engineering

Hours: (Class: 1h30, Practical: 1h30, Practical: 1h30)

Credits: 4

Coefficient: 2

Evaluation: Continuous assessment: 40%; Exam: 60%.

General course objective:

Know the basic principles of electrical engineering.

Understand the operating principle of transformers and electrical machines.

Recommended prerequisites :

Fundamental electricity.

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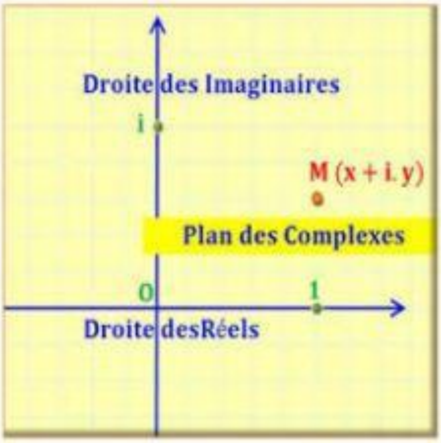
Chapitre I

Mathematical review of complex numbers

I.1 Introduction

Complex numbers are a mathematical tool that will enable us to treat and study electrical circuits as complex as possible in a purely algebraic way.

After a reminder of the mathematical laws of complex numbers. In this chapter, we'll look at the main electrical dipoles and the fundamental laws governing them.



I.2 Complex numbers (CN)

The equation $x^2+1=0$ has no solution in the space of real numbers **R**. It must be solved using the square roots of a negative number, which led to the invention of complex numbers. In $\mathbb{C}$, the set of complex numbers, it has two solutions: *i* and - *i* with $i^2=-1$.

The notation *i* was introduced by Euler, the great Swiss mathematician. In this document, we'll use *j* instead of *i*. In electricity, *i* am a notation used for current intensity [1].

I.3 Algebraic representation of a NC

Let be a complex number, $z=a+jb$ with a and b real.

Writing a + jb is called the algebraic form of z, with:

a = Re (z) : which represents the real part of z

and b = Im(z) , is the imaginary part of z.

We can take these two values (a and b) as coordinates in the Cartesian plane

Plane (see figure 1.1).

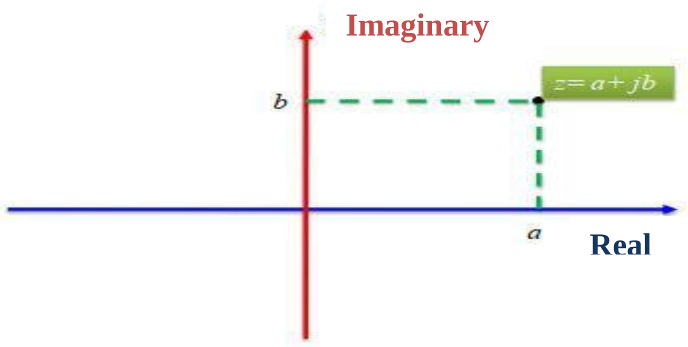


Figure 1.1 Cartesian representation of a complex number (z).

I.4 Geometric representation of a NC

In a plane with an orthonormal reference frame (O, u, v), any complex number $z = a + jb$ is associated with the point P (a, b) and, conversely, any point in the plane can be associated with a complex number.

P (a, b) is the image of $z = a + jb$ and z is the affix of the vector $\vec{OP} = a\vec{u} + b\vec{v}$.

I.5 Trigonometric representation of a NC

Let P be the image of $z = a + jb$ in the reference plane (O; u, v).

- Modulus of z is the positive real : $r = |z| = \sqrt{a^2 + b^2}$

- Argument of z ($z \neq 0$) is the number θ defined as $2k\pi$ ($k \in \mathbb{Z}$) with :

$$\sin(\theta) = \frac{b}{r}$$
$$\cos(\theta) = \frac{a}{r}$$
$$\theta = \arctan(b/a)$$

Geometrically θ is, to within $2k\pi$ the measure of angle (u, OP). The trigonometric form of z is then:

$$z = \rho(\cos(\theta) + \sin(\theta))$$

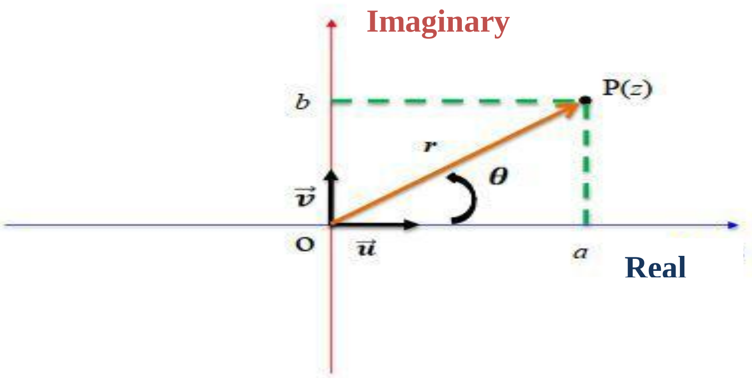


Figure 1.2 Trigonometric representation of a NC (Z).

I.5.1 Polar representation of a NC

Let be a complex number $z\,(z \neq 0)$, with r, θ the modulus and argument respectively. The polar representation is given in the following form:

$$z=[r,\theta]$$

I.5.2 Exponential representation of a NC

Euler's formula links the complex exponential with cosine and sine in the complex plane:

$$\forall\theta\in\mathbb{R}: e^{j\theta}=\cos(\theta)+\sin(\theta)$$

Theorem

Let θ be a real number, and Euler's formula is expressed as :

$$\cos(\theta)=\frac{e^{j\theta}}{2}+\frac{e^{-j\theta}}{2}$$
$$\sin(\theta)=\frac{e^{j\theta}}{2j}-\frac{e^{-j\theta}}{2j}$$

We have $e^{j\theta}=\cos(\theta)+j.\sin(\theta)$ and $e^{-j\theta}=\cos(-\theta)+j.\sin(-\theta)=\cos(\theta)-j.\sin(\theta)$

And So we have :

$$e^{j\theta}=\cos(\theta)+j.\sin(\theta)$$
$$e^{-j\theta}=\cos(\theta)-j.\sin(\theta)$$

By adding and subtracting the two equalities member by member, we obtain Euler's formulas.

A complex number can then be written in exponential form, as shown in Figure 1.3 :

$z = e^{j\theta}$

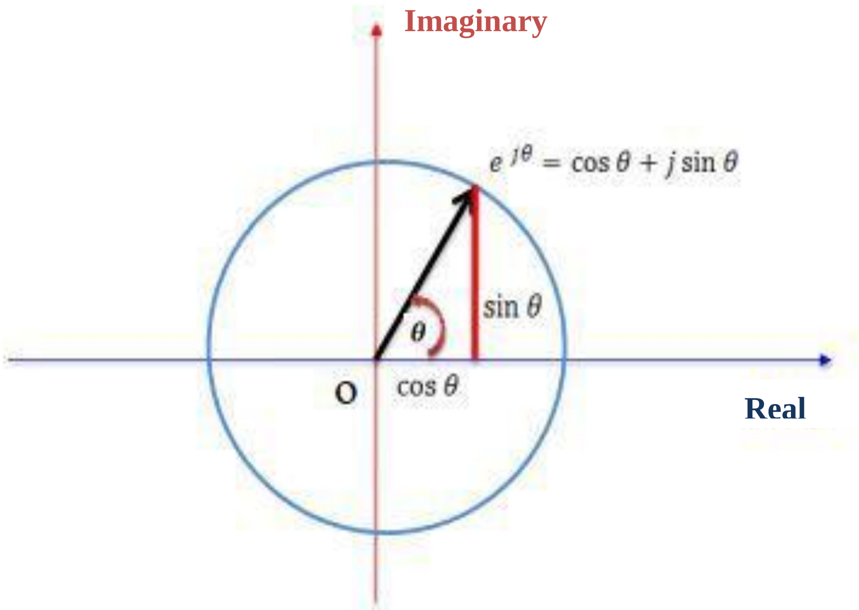


Figure 1.3 Exponential form of a NC *Z*

I.6 Moivre's formula

The n-th power of a complex number z , is given by Moivre's formula [2]:

$$(\cos(\theta) + j.\sin(\theta))^n = \cos(n\theta) + j.\sin(n\theta)$$

Demonstration

For a complex number of modulus r = 1, z can be written as :

$$z = e^{j\theta}$$

For a natural number n : $Z^n = (\cos(\theta) + j.\sin(\theta))^n = (e^{j\theta})^n = e^{jn\theta} = \cos(n\theta) + j.\sin(n\theta)$

Thus $(\cos(\theta) + j.\sin(\theta))^n = \cos(n\theta) + j.\sin(n\theta)$

I.6.1. Operations on NCs

I.6.1.1 Conjugate complex

Let be a complex number $z = a + jb$ .The conjugate complex of z is denoted z. It is the same number but with an opposite imaginary part $\bar{z} = a - jb$

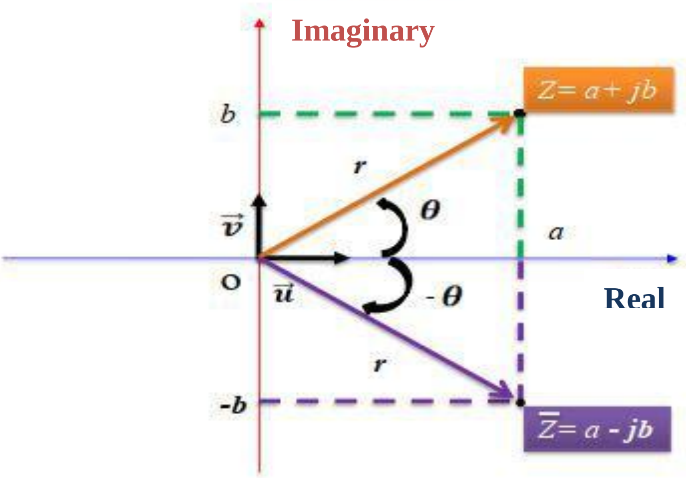


Figure 1.4 Conjugate complex.

I.6.1.2 Addition

The addition of two complex numbers in algebraic form $z=a+jb$ and $\grave{z}=\grave{a}+j\grave{b}$

consists in adding the real part to the real part, and the imaginary parts to each other. $z+\grave{z}=(a+\grave{a})+j(b+\grave{b})$

Applying the reasoning, the subtraction of complex numbers is given by :

$$z-\grave{z}=(a-\grave{a})+j(b-\grave{b})$$

I.6.1.3 Product

The product of these two numbers is:

$$z*\grave{z}=(a*\grave{a})-(b*\grave{b})+j(a*\grave{b}-b*\grave{a})$$

For the product, it is often more interesting to use the polar form.

$$z=[r_1,\theta_1]$$
$$\grave{z}=[r_2,\theta_2]$$

The product of these two numbers in polar form is then :

$$z*\grave{z}=[(r_1*r_2),(\theta_1*\theta_2)]$$

I.6.1.4 Division

For division,

$$\frac{z}{\grave{z}}=\frac{[r_1,\theta_1]}{[r_2,\theta_2]}=[\frac{r_1}{r_2},(\theta_1-\theta_2)]$$

I.7 Application of NC to electricity

Let us consider a sinusoidal alternating current $i\ t$ whose expression as a function of time is given by:

$$i(t)=I\sqrt{2}(\sin(\omega t+\varphi))$$

To this sinusoidal quantity, we associate a Fresnel vector. We can then associate $i\ t$ with a complex representation

$$\underline{I}=\lfloor I,\varphi\rfloor=I.(\cos(\varphi)+j\sin(\varphi))$$

Similarly for the applied voltage, we can write

$$\underline{U}=\lfloor U,\varphi\rfloor=U.(\cos(\varphi)+j\sin(\varphi))$$

Note that the complex representation is very efficient for calculating electrical circuit parameters [3].

Exercises for the first chapter

Exercise 01:

1. Give the algebraic form of the complex numbers below:

a. $z_1 = \frac{1+j}{j}$ **b.** $z_2 = \frac{1}{1-j}$ **c.** $z_3 = \frac{-2+j}{2+j}$

d. $z_4 = \frac{5+2j}{1-2j}$ **e.** $z_5 = \left(\frac{1+j}{2-j}\right)^2$ **f.** $z_6 = \frac{2+5j}{1-j} + \frac{2-5j}{1+j}$

2. Consider the two complex numbers defined by: $z_1 = 1 + j$; $z_2 = 5 - 2j$

- Determine the algebraic form of the following numbers:

a. $z_1 + z_2$ **b.** $z_1 - z_2$ **c.** $z_1 - 2\,z_2$ **d.** $z_1 + \bar{z}_2$

e. $z_1 . \bar{z}_2$ **f.** z_1^2 **g.** $\frac{z_1}{z_2}$ **h.** $\frac{z_2}{z_1 - z_2}$

Exercise 02:

1. Give the polar form (modulus, argument) of:

$1 + j\,;\ 1 - j\,;\ 3 + 4j\,;\ \sqrt{3} + j\,;-1 + 7j\,;1 + 3j\,;-5 - 3j\,;$

2. Determine the modulus, an argument of each of the given numbers of: $z_1 = \sqrt{6} - \sqrt{2}j\,;\ z_2 = -\frac{1}{2} - j\frac{1}{2}\,;\ z_3 = -\frac{1}{2} + j\frac{\sqrt{3}}{2}$. Deduce the module and argument of $z_1 z_2\,;\ z_1 z_3;\ z_2^2$

Exercise 3:

1. Calculate the modulus, argument and conjugate of the following complex numbers:

$z_1 = 2 + 3j\,;\ z_2 = 2 - 3j\ ;z_3 = -2 + 3j\ ;z_4 = -2 - 3j$

2. Give the polar (trigonometric) and exponential forms

3. Perform the following operations: $z_1 + z_2\,;\ z_3 - z_4;\ z_4 z_2;\ z_3/z_4$.

Exercise 04:

Write the following complex numbers in algebraic form:

$$z_1 = 2e^{j\frac{2\pi}{3}} ; z_2 = \sqrt{2}e^{j\frac{\pi}{8}} ; z_1 = 3e^{-j\frac{7\pi}{3}} ; z_4 = \left(2e^{j\frac{\pi}{4}}\right) \left(e^{-j\frac{3\pi}{4}}\right) ;$$

$z_5 = \dfrac{2e^{j\frac{\pi}{4}}}{e^{-j\frac{3\pi}{4}}}$; z_6: the number with modulus 2 and argument $\frac{\pi}{4}$; z_7: $\left(3 , \frac{-\pi}{8}\right)$

Exercise 5:

The total impedance of an electrical circuit is given by: $z_T = \frac{z_1 z_2}{z_1 + z_2} + z_3$

With $z_1 = 5 - 3j$; $z_2 = 4 + 7j$; $z_3 = 4 - 7j$

1. Determine z_T in Cartesian, exponential and trigonometric form.
2. Calculate the modulus and argument of z_T

Exercise 6:

Given: $z_1 = 1 + 2j$; $z_2 = -3 - \frac{j}{2}$; $z_3 = \sqrt{7} + j$; $z_4 = 4j$. Put the following numbers into algebraic form:

$$z_1 + z_2 ; z_1 - z_2 ; z_1 - z_3 ; z_1 + z_4 ; z_1^2 ; z_1 z_2 ; z_1 z_3 ; z_1 z_4 ; z_3 \bar{z}_3 ; \frac{1}{z_4} ; \frac{z_1}{z_4}.$$

Chapter II

Fundamental Laws of Electricity in Continuous, Harmonic and Transient Regimes

II.1 Continuous Regimes

Continuous regimes refer to the use of DC voltage or current generators, such as batteries, accumulators, DC generators and dynamos.

In continuous regimes, voltages and currents do not depend on time; the only thing that characterizes them is their average value.

Continuous regimes are defined as an operation in which the intensities of the electric currents through the various branches of the circuit have a constant value.

II.1.1 Passive and active dipoles

A ***passive dipole*** is a dipole that consumes electrical energy and converts all this energy into *heat*. Examples include a resistor, a light bulb, etc. The voltage-current characteristic U(I) passes through the origin: U = 0 V I = 0 A.

Otherwise, we speak of an ***active dipole*.** The U(I) characteristic does not pass through the origin. An active is not symmetrical and its two terminals must be distinguished: there is a *polarity*. Examples: battery, solar cell, dynamo (generating dipoles).

II.1.1.1 Resistive dipole

a) Resistance:

A resistive dipole is a passive dipole. The energy it absorbs is entirely consumed by the Joule effect, i.e. dissipated in the form of heat [4]. A resistive dipole is represented by a rectangle

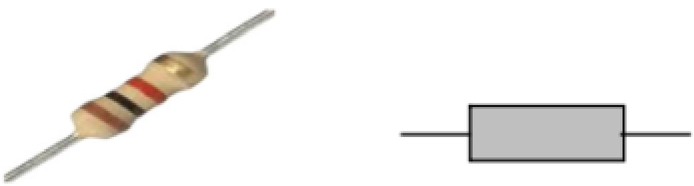


Figure 2.1 Resistive dipole

On the other hand, the resistance of a conducting wire depends on the nature of the conductor and its dimensions, and is calculated from the relationship

$$R = \frac{\rho . l}{S}$$

R : The value of the conductor resistance in ohms [Ω].

ρ : The value of the conductor's resistivity in ohms. Meters

l : The length of the conductor in meters [m].

S : La section du conducteur en mètres ² [m²]

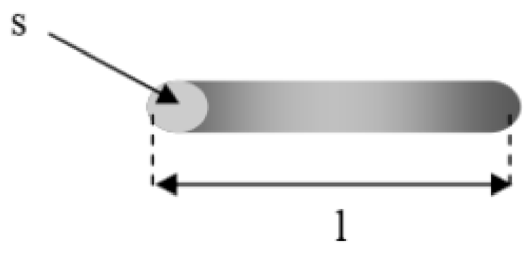
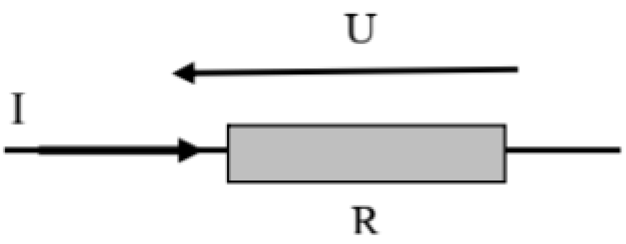


Figure 2.2 Straight conductor

b) Ohm's law:

The electric voltage U across a resistive element through which a current of intensity I flows is shown below [11].



Voltage U is expressed as a function of current I by Ohm's law.

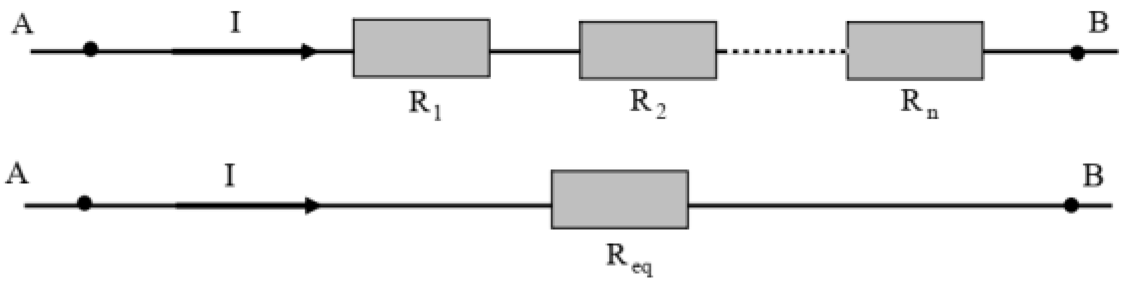
$U = R.I$

U : The electrical voltage across the resistive dipole in volts [V].

R : The resistance value in ohms [Ω].

I : Current intensity in amperes [A].

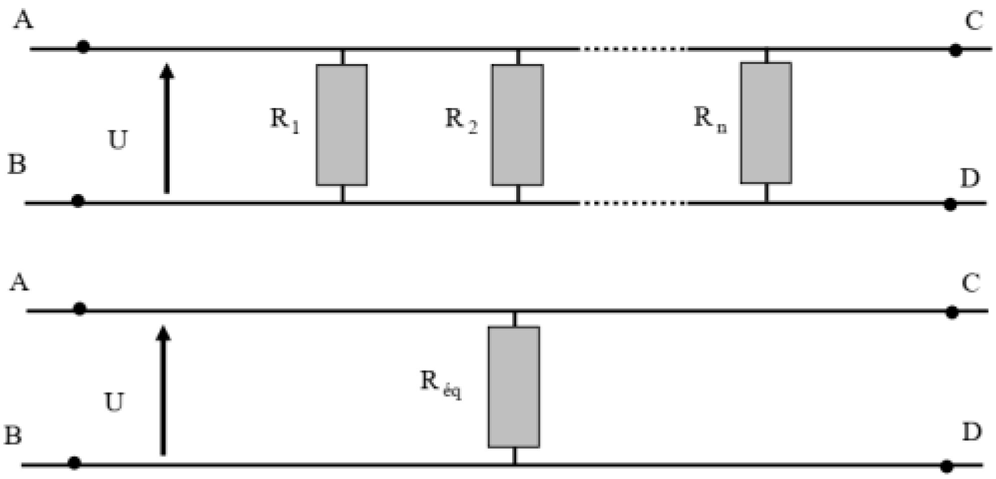
c) Association of resistive elements in series:



The equivalent resistance is calculated using the following equation:

$$R_{eq} = \sum_{i=1}^{n} R_i$$

d) Association of resistive elements in parallel:



The equivalent resistance is calculated using the following equation:

$$\frac{1}{R_{eq}} = \sum_{i=1}^{n} \frac{1}{R_i}$$

II.1.1.2 Capacitive dipole:

The characteristic relationship of an ideal capacitor is [5]:

$$i = C\frac{di}{dt}$$

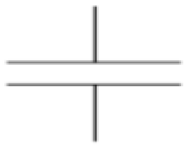


Figure 2.3 Capacitive dipole

U : is the voltage across the component, expressed in volts (symbol: V) ;

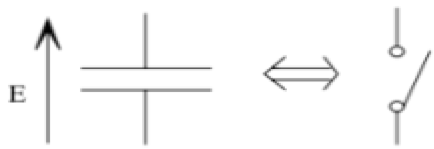
C is the electrical capacitance of the capacitor, expressed in farads (symbol: F) ;

I is the current flowing through the component, expressed in amperes (symbol: A) ;

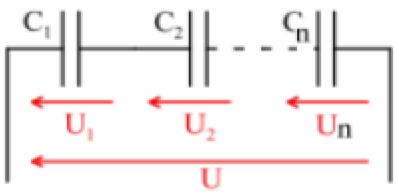
$\frac{di}{dt}$ is the derivative of voltage with respect to time

a. Continuous capacitance

In continuous regimes, the current through the capacitor is zero. This is because the capacitor behaves like an open switch.

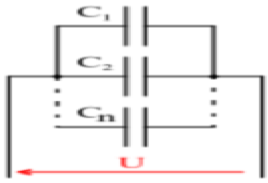


b. **Association of capacitors in series**



$$\frac{1}{C_{eq}}=\sum_{i=1}^{n}\frac{1}{C_i}$$

c. **Association of capacitors in parallel:**



$$C_{eq}=\sum_{i=1}^{n}C_i$$

II.1.1.3 Inductive dipole:

The voltage across a coil is related to the current by the following relationship [5]:

$$u(t)=L\frac{di}{dt}$$

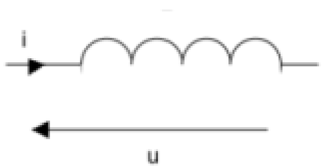
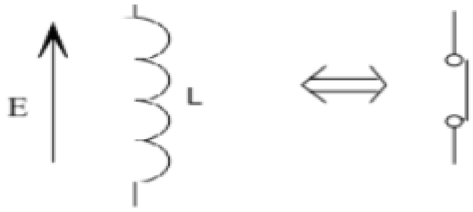


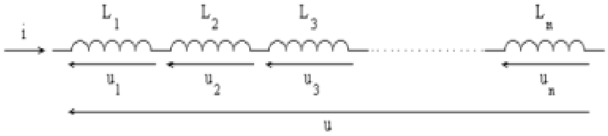
Figure 2.4 inductive dipole

a) Continuous inductance

In continuous regimes, the voltage across the inductance is zero. In effect, the inductor behaves like a closed switch.



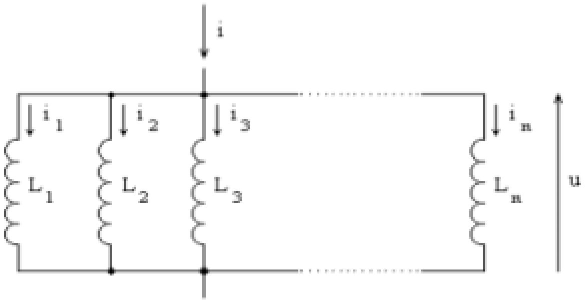
b) Association of series inductors



$$L_{eq}=\sum_{i=1}^{n}L_i$$

c) Association of parallel inductors

$$\frac{1}{L_{eq}}=\sum_{i=1}^{n}\frac{1}{L_i}$$



II.2 Harmonic regime (variable, sinusoidal)

The state of a system in which the variation over time of the quantities characterizing it is sinusoidal is called the sinusoidal (or harmonic) regime. In this case, the electrical circuit is supplied with a sinusoidal AC voltage V(t) and flows with a sinusoidal AC current i(t) [6].

II.2.1 Characterization of a periodic signal

a) Period:

The period, T, of a periodic signal is the shortest time after which the signal repeats itself identically. Its unit in the international unit system (S.I.) is the second, noted s.

b) Frequency:

Frequency is the number of periods per second. It is linked to the period by the following relationship following relationship: Its SI unit is the Hertz, symbol Hz.

$f = \frac{1}{T}$ with f in Hertz and T in sec.

c) Maximum and minimum voltage

The maximum voltage U_{max} is the highest value assumed by u(t) over time.

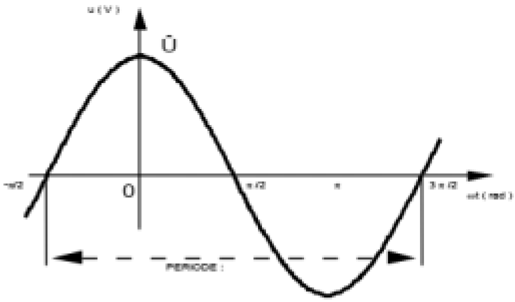
The minimum voltage U_{min} is the lowest value assumed by u(t) over time.

The unit of U_{max} and U_{min} in the I.S. is the volt.

d) Instantaneous voltage

$$u(t) = \hat{u}.cos(\omega t + \phi)$$

with:



- u : instantaneous voltage value.
- Û : maximum voltage value, in volts.
- ω : voltage pulsation, in radians per second.
- ϕ : phase of voltage at initial instant, in radians.
- $\omega t + \phi$: voltage phase at instant t, in radians.
- $\omega = 2\pi f$. With *f*: signal frequency in Hertz.
- $T = 1/f$ with *T:* signal period in seconds.

e) Mean value

The mean value u_{moy}, of any periodic quantity *u*(*t*) , is calculated from the relation :

$$u_{moy} = \frac{1}{T} \int_{(T)} u(t).dt$$

Note: The mean value of a sinusoidal alternating signal is zero.

f) RMS value

The RMS value u_{eff} , of a periodic quantity *u*(*t*) is calculated from the relation :

$$u_{eff} = U = \sqrt{\frac{1}{T} \int_{(T)} u(t)^2.dt}$$

The RMS value of a voltage, *u*(*t*) , sinusoidal and only in this case, can be deduced from the relation :

$$U = \frac{\hat{u}}{\sqrt{2}}$$

II.2.2 Representation of a sinusoidal signal:

II.2.2.1 Cartesian representation:

Cartesian representation uses sinusoidal functions of time.

The following two quantities are defined in this way [7]:

- $u(t) = \hat{u}.cos(\omega t + \phi_u)$
- $v(t) = \hat{v}.cos(\omega t + \phi_v)$

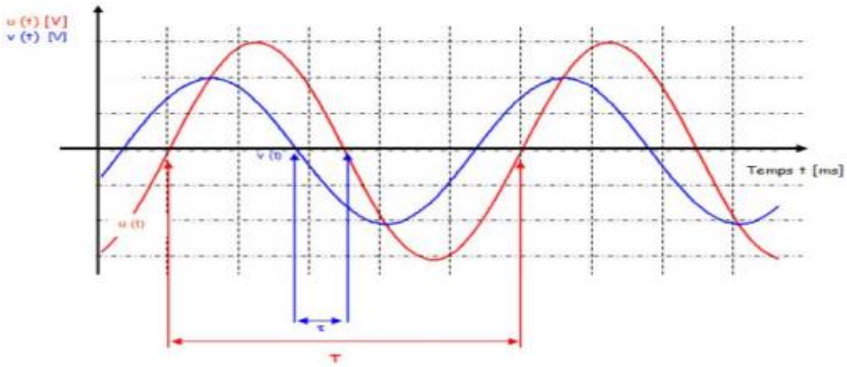


Figure 2.5: Cartesian representation of sinusoidal quantities.

The two quantities u (t) and v (t) do not intersect the x-axis at the same instant. They are shifted in time. This time shift, noted τ, always has the same value, and the quantities u (t) and v (t) do not have the same phase at the initial instant. They are out of phase.

a. Calculating the phase shift angle

The phase shift angle, ϕ, indicates the angle by which the two quantities u (t) and v(t) are out of phase. It corresponds to the difference between the initial phases ϕ_u and ϕ_v, and is defined as follows:

$$\phi = \phi_u - \phi_v$$

Φ : Phase shift angle between u (t) and v (t) in radians [rad].

Φ_u : The phase of u (t) at the initial instant in radians [rad].

Φ_v : The phase of v (t) at the initial instant in radians [rad].

b. Graphical determination of the phase shift angle

Phase shift can also be determined from the Cartesian representation of the two curves curves u(t) and v(t). We proceed as follows:

- Measure the quantity representing the period T.
- Measure the quantity representing the time difference τ.
- Evaluate the relationship between the angle ϕ in radians and the time difference τ in seconds.

$$T \rightarrow 2\pi = 360^{\circ}$$
$$\tau \rightarrow \phi$$

The phase shift can be deduced using the relationship

$$\phi = \frac{2\pi\tau}{T}$$

Φ : The phase shift angle between u (t) and v (t) in radians [rad].

Φ_u : The time difference between u (t) and v (t) in seconds [s].

Φ_v : The period of u (t) and v (t) in seconds [s].

c. Current-voltage phase shift:

If the two quantities to be studied are a voltage u(t) and a current i(t), they are defined as follows:

- $u(t) = \hat{u}.cos(\omega t + \phi_u)$
- $i(t) = \hat{i}.cos(\omega t + \phi_i)$

If voltage u is chosen as the phase origin, i.e. (t) $\phi_u = 0$, voltage u(t) and current i(t) are defined as follows:

- $u(t) = \hat{u}.cos(\omega t)$
- $i(t) = \hat{i}.cos(\omega t + \phi_i)$

$\phi_i \succ 0$: Current i(t) lags voltage u(t).

$\phi_i \prec 0$:Current i(t) leads voltage u(t).

$\phi_i = 0$:Current i(t) and voltage u(t) are in phase.

II.2.2.2 Fresnel representation (vector):

When two sinusoidal voltages u(t) and v(t) are under study, they can be represented using represented using vectors based on their characteristics [8]:

- $u(t) = \hat{u}.cos(\omega t)$
- $v(t) = \hat{v}.cos(\omega t + \phi_v)$

In this case we have :

- The voltage u(t) has zero phase at the initial instant.
- Voltage v(t) is phase-shifted by the angle ϕ_v with respect to u(t).
- The RMS values of u(t) and v(t) are equal to U and V in succession.

The vector characterizing the voltage u (t) :

- Is carried by the axis taken as the phase origin.
- Its length is proportional to U, the rms value of u(t).
- The vector characterizing the voltage v (t) :
- Is offset by an angle ϕ_v in the positive direction with respect to the phase origin axis.
- Its length is proportional to V, which is the RMS value of v(t).

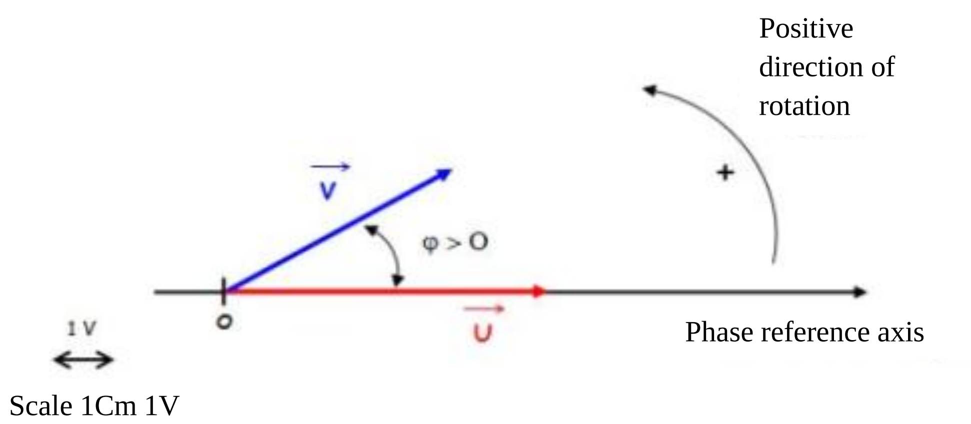


Figure 2.6: Fresnel representation, v (t) ahead of u (t)

Under the same conditions as above, but if the angle ϕu_{is} negative, then the voltage v (t) lags behind (t) u and the representation becomes:

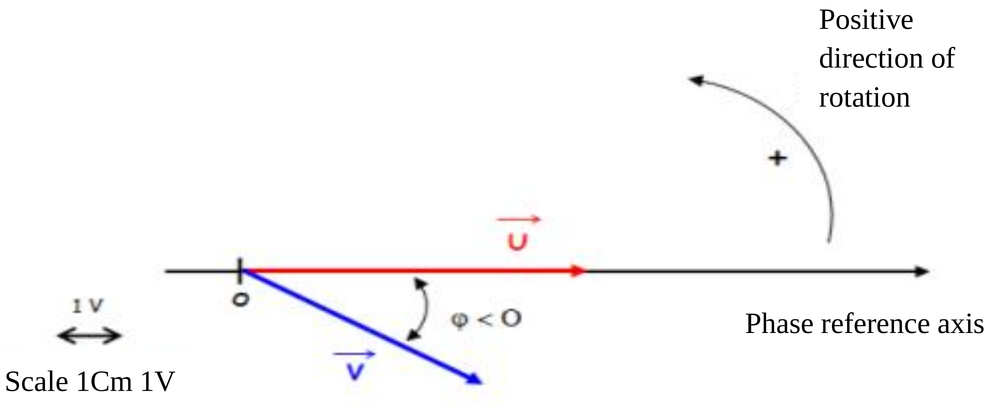


Figure 2.7: Fresnel representation, v (t) lagging behind u (t)

II.2.3 Complex impedance notation

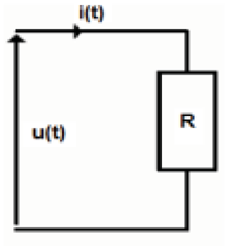
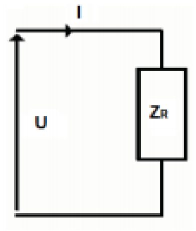
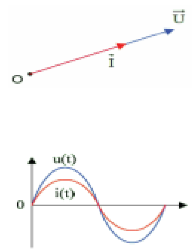
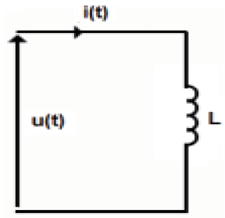
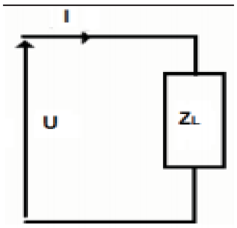
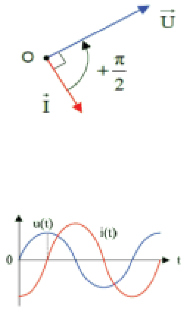
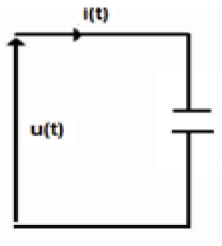
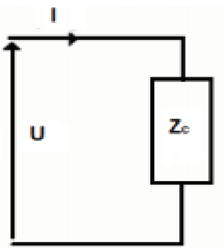
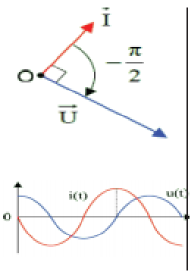
The complex impedance Z of a dipole in sinusoidal steady state is the quotient of the complex voltage $u(t) = \hat{u}.e^{j\phi_u}$ and the complex current. $i(t) = \hat{i}.e^{j\phi_i}$

In electrical engineering, we find the following three main dipoles [9]:

Resistive dipole

Inductive dipole

Capacitive dipole

	Circuit diagram	Complex number modeling	Voltage	Modulus and phase	Graphical representation
Resistive dipole			$U = R\,I(t)$	$Z_R = R$ $\varphi_R = 0°$	
Inductive dipole			$U_L = L\dfrac{dI(t)}{dt}$	$Z_L = j\,L\omega$ $\varphi_L = +\dfrac{\pi}{2}$	
Capacitive dipole			$U_c = \dfrac{1}{c}\int I(t)dt$	$Z_c = \dfrac{1}{jC\omega} = -\dfrac{j}{C\omega}$ $\varphi_c = -\dfrac{\pi}{2}$	

NOTE

$Z = |Z|.e^{j\phi}$ =R+Jx

- If X =0 **impedance is resistive** and $\phi = 0$
- If R=0 **Impedance is purely inductive** and $\phi = \frac{\pi}{2}$
- If R=0 **Impedance is purely capacitive** and $\phi = -\frac{\pi}{2}$

II.2.4 Sinusoidal power (instantaneous, active, reactive, apparent):

In physics, a power represents a quantity of energy per unit of time. Its unit is the Watt (1W = 1J/s). The concept of power is an indispensable tool in electrical engineering, often providing a global view of systems and enabling certain problems to be easily solved using the power balance technique [11].

a. Instantaneous power

The instantaneous value of power is by definition the product of the instantaneous values of voltage and current, giving :

$$P(t) = U(t).\ I(t) \text{ (expressed in Watts)}$$

Because of the phase shift between U and I on the dipole, we're going to identify several notions of Power

b. Active power:

As a general rule, the power that drives energy conversion systems is the average power of the systems, also known as active power. Active power is the average value of instantaneous power. Noted P, it is expressed in Watts (W), and depends on the RMS values of U and I and the phase shift φ between the two quantities.

$P = U\ I \cos\varphi$ *(Watts)*

The active power absorbed by a receiver is always positive. The theoretical definition of active power brings us directly to the notion of power factor.

c. Reactive power

Reactive power arises when the installation contains both inductive and capacitive loads. Power supplies to the magnetic circuits of electrical machines (motors, transformers, fluorescent ballasts, etc.) consume reactive power. **By** analogy**,** reactive power Q is given by the relation :

$Q = U\ I \sin\varphi$ ***(expressed in reactive volt-amperes (var)).***

The sign of reactive power depends on the angle of phase shift produced by the receiver in question:

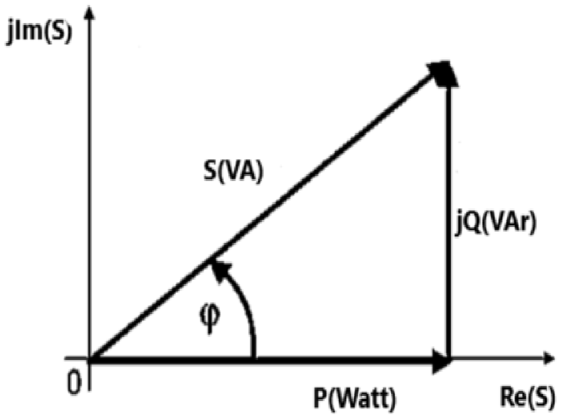
- for an inductive receiver ($\varphi > 0$), reactive power is positive,
- for a capacitive receiver ($\varphi < 0$), reactive power is negative.

d. Apparent *power*

This power is often referred to as “sizing power”, and is a characteristic of the insulation and conductor cross-section, i.e. the dimensions of the equipment. Apparent power is a design feature of electrical machines. The apparent power rating is :

$S_n = U_n I_n$ **(Volts Amperes)**

This produces a right-angled triangle from which we derive the relationship between P, Q and S

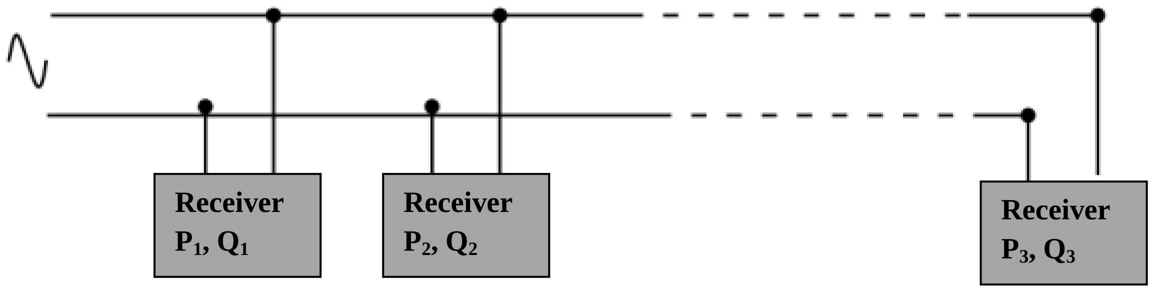


$$S^2 = P^2 + Q^2 \qquad Cos\,\varphi = \frac{P}{S}$$

Active power	**P=U.I.Cosφ**	**U---Volts** **I---Amperes** **P---Watts W**
Reactive power	**Q=U.I.Sinφ**	**U---Volts** **I---Amperes** **Q--- Voltamperes reactive VAR**
Apparent power	**S=U.I**	**U---Volts** **I---Amperes** **S---voltamperes VA**

II.2.5 Boucherot's theorem:

Boucherot's theorem states the conservation of active and reactive power. In any electrical circuit [11]:



The total active power consumed by the installation is equal to the arithmetic sum of the active powers consumed by each receiver.

$$P_T = P_1 + P_2 + \ldots\ldots\ldots + P_n = \sum_{i=1}^{n} P_i$$

The total reactive power consumed by the installation is the algebraic sum of the reactive powers consumed by each load.

$$Q_T = Q_1 + Q_2 + \ldots\ldots\ldots + Q_n = \sum_{i=1}^{n} Q_i$$

On the other hand, apparent power is not conserved. S is not equal to

$$S_T \neq S_1 + S_2 + \ldots\ldots\ldots + S_n$$

To apply Boucherot's method to a circuit or installation, we need to draw up a balance sheet of active and reactive powers. This balance can be presented in the form of a table.

Table 1: Power balance.

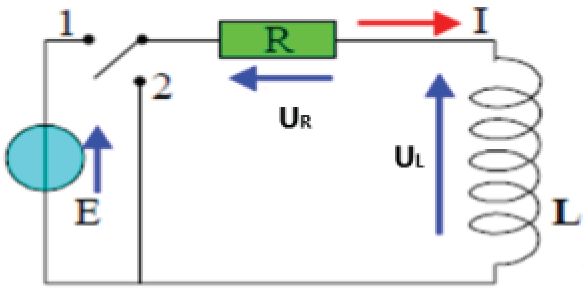
Dipole	**active power**	**reactive power**
Receiver 1	P_1	Q_1
Receiver 2	P_2	Q_2
Receiver 3	P_3	Q_3
installation	$P = P_1 + P_2 + P_3$	$Q = Q_1 + Q_2 + Q_3$

II.3 Transient regime

These are the particular changes in electrical quantities that occur when the characteristics of an electrical circuit change abruptly. In general, they do not occur repeatedly, otherwise they are referred to as periodic steady state [6] [9].

II.3.1 RL circuit

The RL circuit consists of a generator connected in series with a coil.



Depending on the position of the reverser, either free-running (position 2) or forced-running (position 2) is available.

Forced operation: $E = U_R + U_L = R.I + L\dfrac{dI}{dt}$

The differential equation is $\dfrac{E}{L} = \dfrac{I}{\tau} + \dfrac{dI}{dt}$ a $\tau = \dfrac{L}{R}$ first-order differential equation. The solution of this equation is the sum of the two solutions, the general solution without second member and the particular solution.

General SSM solution:

$$\frac{dI}{dt} + \frac{I}{\tau} = 0 \quad \rightarrow \frac{dI}{I} + \frac{1}{\tau}dt = 0 \quad \rightarrow \int \frac{dI}{I} = -\frac{1}{\tau}\int dt \quad \rightarrow I = A\,e^{-\frac{t}{\tau}}$$

Special solution SP:

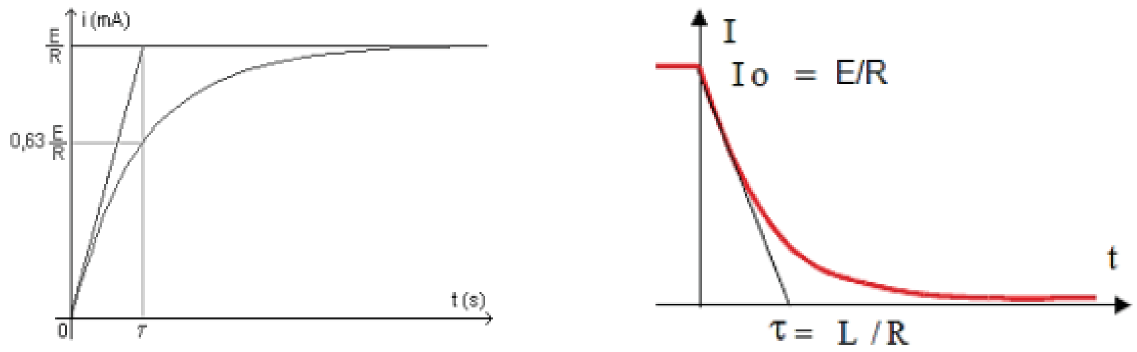
The particular solution I = Constant we replace in the equation $\dfrac{E}{L} = \dfrac{I}{\tau} + \dfrac{dI}{dt}$ we obtain $\dfrac{E}{R} = I$

The solution of the differential equation is: $I_L(t) = A\,e^{-\frac{t}{\tau}} + \frac{E}{R}$

Initial conditions, if t=0,I_L=0 then : $-\dfrac{E}{R} = A$

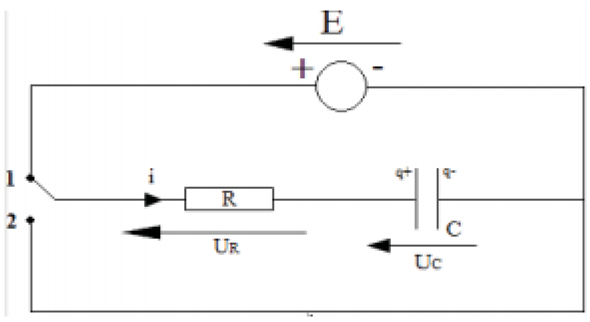
Then $I_L = -\frac{E}{R}(1-e^{-\frac{t}{\tau}})$

- $U_L = L\frac{dI}{dt} = -\frac{L}{\tau}(I_0 e^{-\frac{t}{\tau}}) = -RI_0 e^{-\frac{t}{\tau}}$



II.3.2 RC circuit - Charging and discharging a capacitor.

The RC circuit consists of a generator, a resistor and a capacitor. In their series configuration, RC circuits can be used to create low-pass or high-pass electronic filters. The time constant τ of an RC circuit is given by the product of the value of the two elements making up the circuit τ = RC.



Capacitor charging

Flip the switch to position 1, and apply a voltage E across the capacitor.

$$E = RI + U_C \text{ , } I = C\frac{dU_C}{dt}$$

$E = RC\frac{dU_C}{dt} + U_C$ So $\frac{E}{RC} = \frac{dU_C}{dt} + \frac{1}{RC}U_C$

The solution to this equation is: $U_C(t) = Ae^{-\frac{t}{\tau}} + E$

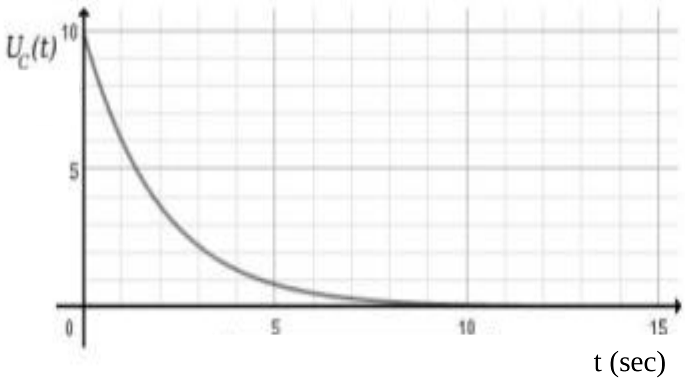
Initial conditions, si t=0, U_C=0 alors: $-E = A$, $U_C(t) = E(1 - e^{-\frac{t}{\tau}})$

Capacitor discharge

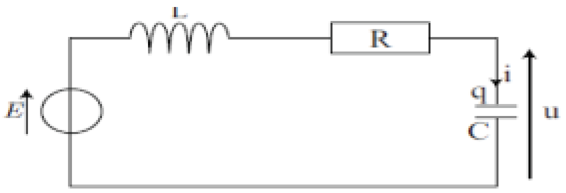
Flip the switch to position 2 and apply a voltage E across the capacitor.

$0 = RI - U_C$, $I = C\frac{dU_C}{dt}$, $0 = \tau\frac{dU_C}{dt} + U_C$

The solution to this equation is : $U_C(t) = Ee^{-\frac{t}{\tau}}$



II.3.3 RLC circuit



According to the law of meshes, we have : $-E + RI + L\frac{dI}{dt} + \frac{1}{C}\int Idt = 0$

Let's find the differential equation, and consider the voltage and current across the capacitor,

$\frac{dq}{dt} = I$, $q = CU_C$ Then $I = C\frac{dU_C}{dt}$

$RC\frac{dU_C}{dt} + LC\frac{d^2U_C}{dt^2} + U_C = E$, $\ddot{U}_C + \frac{R}{L}\dot{U}_C + \frac{1}{LC}U_C = \frac{1}{LC}E$

The natural pulsation is $\omega_0^2 = \frac{1}{LC}$ the frequency at which this system oscillates when in free evolution

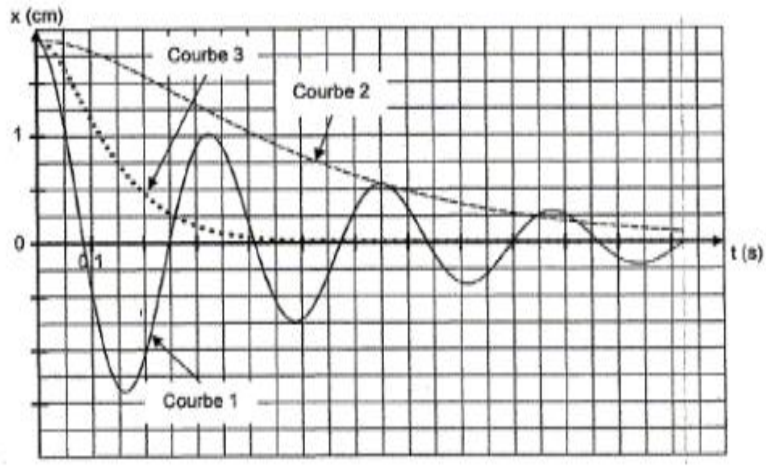
$2\lambda = \frac{\omega_0}{Q} = \frac{R}{L}$ so $Q = \sqrt{\frac{L}{C}} \times \frac{1}{R}$ the quality factor is a unitless measure of an oscillator's damping rate

We define $\Omega = \sqrt{\lambda^2 - \omega_0^2}$

If $\lambda \prec \omega_0$ pseudo-periodic - curve 1

If $\lambda \succ \omega_0$ aperiodic - curve 2

If $\lambda = \omega_0$ critical - curve 3

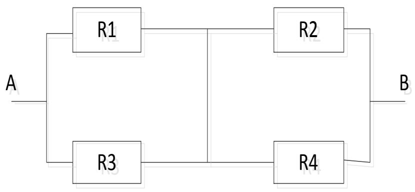


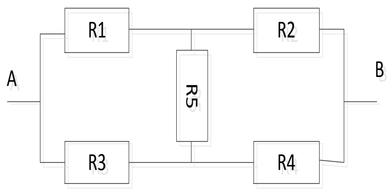
Exercises for the second chapter

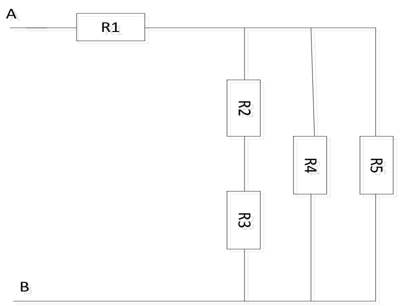
Exercise 01:

Calculate the equivalent resistance seen from points A and B for the different circuits:

We give: $R_1 = 100\Omega$; $R_2 = 150\Omega$ $R_3 = 200\Omega$; $R_4 = 500\Omega$; $R_5 = 500\Omega$

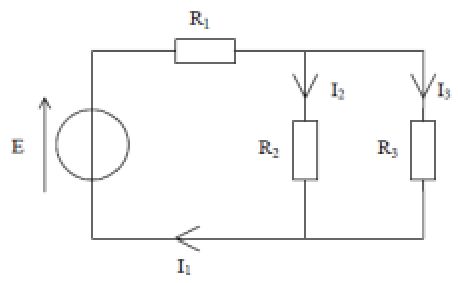






Exercise 02:

Consider the electrical circuit shown opposite. Calculate:



1. Equivalent resistance R_{eq}

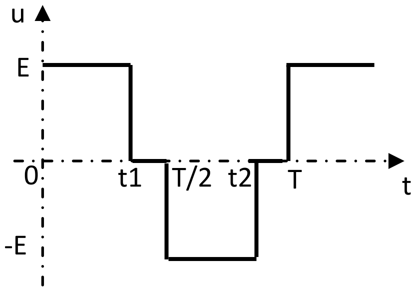
2. Current intensity I_2 and I_3

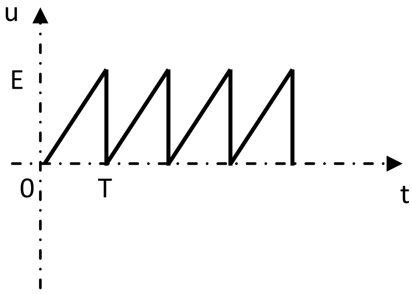
3. Voltage U_2 across R_2

On donne E= $6V$; $R_1 = 270\Omega$; $R_2 = 470\Omega$; $R_3 = 220\Omega$

Exercise 3:

Find the mean and RMS values of the periodic signals given in the figures below:





Exercise 04:

Find the phase shift between voltage and current given by the expressions below and plot their waveform and vector (Fresnel):

1.$v(t) = 15\sin(wt + 30°)\ i(t) = 5\sin(wt + 60°)$

2.$v(t) = 20\sin(wt + 30°); i(t) = 10\sin(wt - 70°)$

3. $v(t) = 20\sin(wt + 20°)\ \ i(t) = 5\sin(wt - 20°)$

4. $v(t) = 8\cos(wt - 60°); i(t) = 4\sin(wt - 70°)$

Exercise 5:

Consider the following sinusoidal signals:

1. $v_1(t) = 40\sin(wt)\ ;\ v_2(t) = 30\sin(wt + 30°);\ v_3(t) = 20\cos(wt - 45°)$
2. $I_1(t) = 4\sqrt{2}\sin(wt - 30°)\ ;\ I_2(t) = 2\sqrt{2}\sin(wt - 150°)$

We give: $v(t) = v_1(t) + v_2(t) +; v_3(t)\ ; I_3(t) = I_1(t) - I_2(t)$

Find the instantaneous expression of $v(t)\ ; I_3(t)$ using:

Complex notation, vector representation and the analytical method

Chapter III

Electrical circuits and power

III.1 Single-phase circuits and electrical power

Consider an electrical power transmission system where the generator supplies a fixed voltage (sinusoidal AC). The energy is transported to the receiver (consumer shown in figure 3.1) via a single-phase distribution line [11].

The system (source and receiver) is said to be single-phase.

Single-phase circuits consist of a single phase and neutral. Single-phase current can be generated from a three-phase network, by connecting one of the three phases and the neutral. Receivers represent electrical installations and blocks, buildings. Generally speaking, they can be classified into three types:

Inductive receiver (motor).

Resistive receiver (oven, radiator).

Capacitive receiver (capacitor).

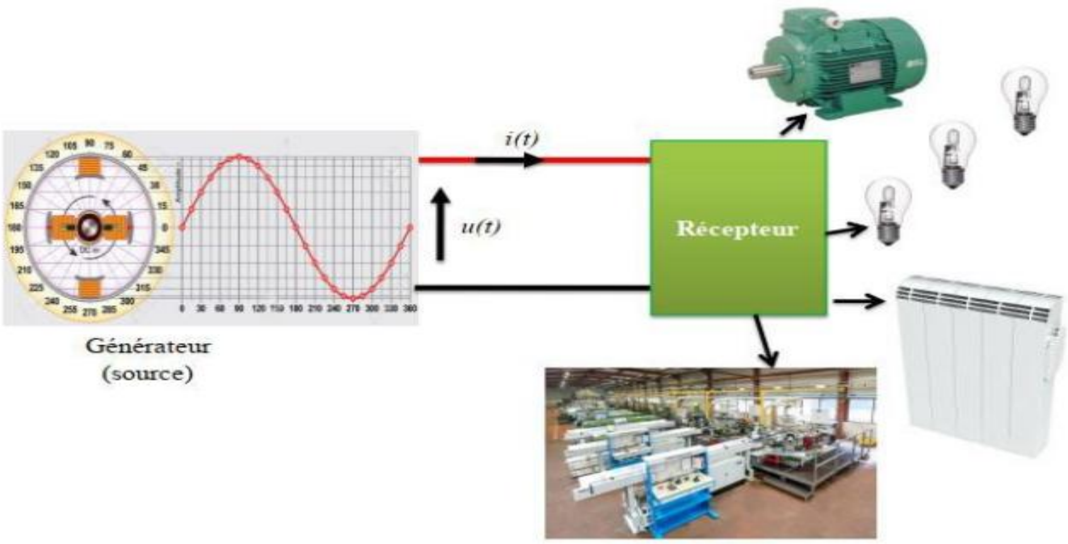


Figure 3.1 Single-phase installation

III.1.1 Single-phase electrical powers

The electrical power equations established in the previous chapter, corresponding to average, active, reactive and apparent power, can be represented geometrically by means of a triangle called the “power triangle”.

The electrical installation is characterized by a power triangle.

Consider the case of an inductive receiver (motor): the current I lags behind the voltage V, which is taken as the phase origin. Vectors V and I are replaced by cos and sin components.

To sum up, we have:

Mean (active) power $P = VI \cos \varphi$

Reactive power $Q = VI \sin \varphi$

Apparent power $S = VI$

Power factor $\cos \varphi = \mathrm{p}/ S$

A capacitive receiver can be processed in the same way.

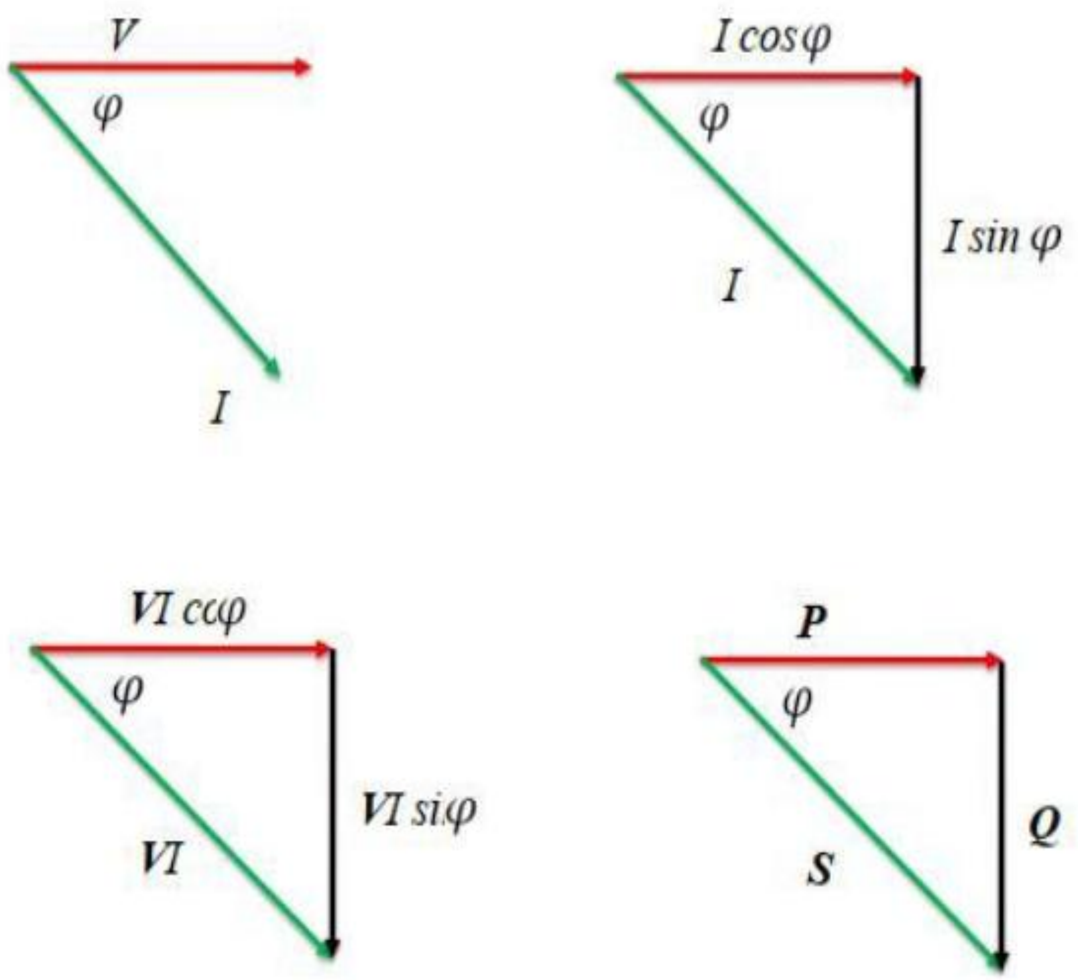
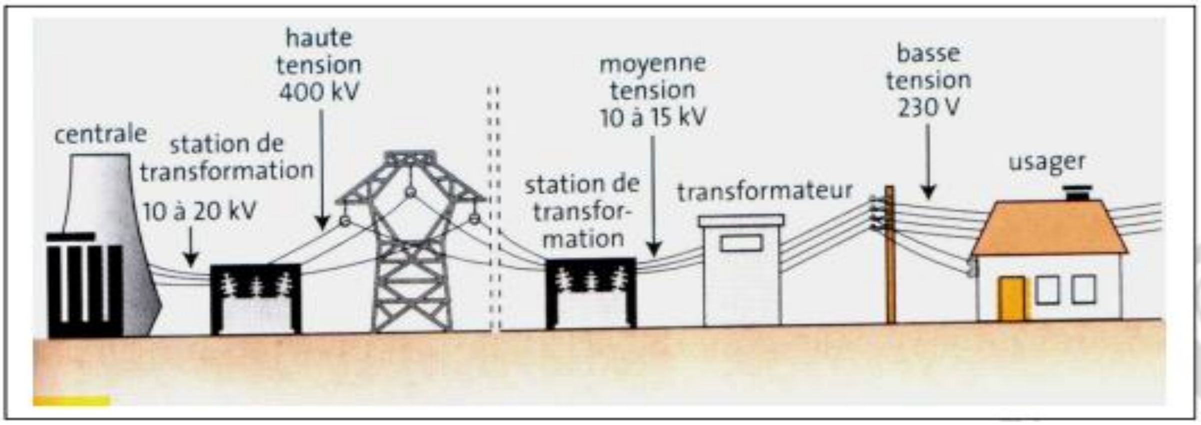


Figure 3.2 Power triangle for an inductive load

III.2 balanced three-phase system

Electricity is generated from a variety of sources, including fossil fuels (coal, natural gas or oil), hydropower, solar energy, wind power and nuclear energy [10].



- **Advantages of three-phase over single-phase**

1. Three-phase machines are 50% more powerful than single-phase machines of the same size, so they cost less.

2. When transporting electrical energy, losses are lower in three-phase. For these reasons, electricity distribution (from the power station to consumers) is based on four terminals. Three phase terminals and one neutral terminal.

III.2.1 Definitions

A three-phase system is a set of 3 sinusoidal quantities (voltages or currents) of the same frequency, out of phase with each other.

The three-phase system is balanced if the voltages (currents) are $2\pi/3$ out of phase with each other and have **the same RMS value** [10]**.**

III.2.2 Three-phase systems

A three-phase installation contains at least:

- **Three-phase network:** This is a source of 3 voltages forming a balanced three-phase voltage system.

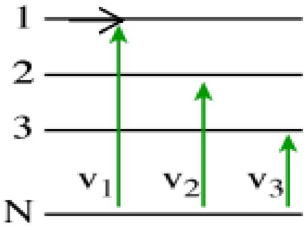
- **Receiver:** a charge made up of 3 identical impedances (if the system is balanced).

- **Connecting lines.**

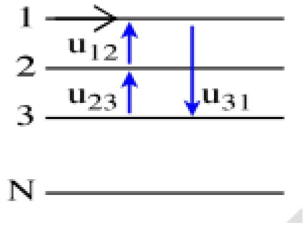
Power is distributed via four terminals:

Three **phase** terminals marked 1, 2, 3 or A, B, C or R, S, T and a **neutral** terminal N.

Simple voltage



Compound voltages



The simple voltages v_1, v_2, v_3 *represent* the potential differences between each line wire and the neutral conductor. They are also called *phase-to-neutral voltages*. Their RMS value is denoted V.

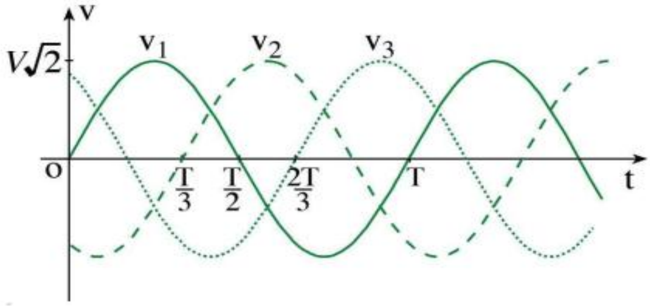
Compound voltages u_{12}, u_{23}, u_{31} *represent* the potential differences between two line wires (phases). They are also known as *phase-to-phase voltages*. Their rms value is U.

Line currents: i_1, i_2, i_3

The^{4th} wire, called neutral N. The neutral current $i_N = i_1 + i_2 + i_3 = 0$ if the system is balanced.

Simple voltages

Three-phase balanced voltage system:

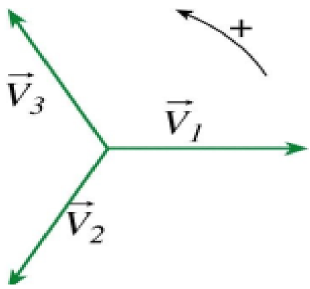


$$V_1(t) = V\sqrt{2}\,sin(\omega t)$$

$$V_2(t) = V\sqrt{2}\,sin(\omega t - \frac{2\pi}{3})$$

$$V_3(t) = V\sqrt{2}\,sin(\omega t - \frac{4\pi}{3})$$

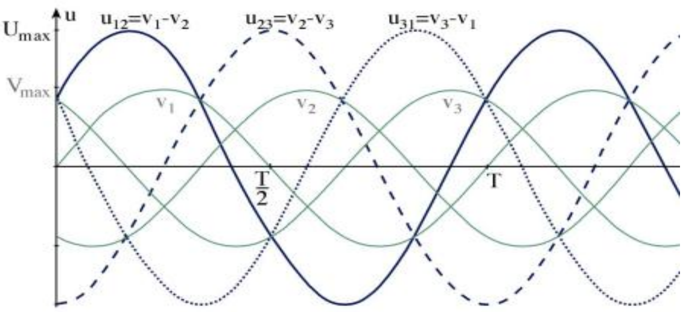
Fresnel representation:



$$V_1(t) = Ve^{j0};\; V_2(t) = Ve^{j(-\frac{2\pi}{3})};\; V_3(t) = Ve^{j(-\frac{4\pi}{3})}$$

Balanced system: $\vec{V}_1 + \vec{V}_2 + \vec{V}_3 = \vec{0}$

Compound voltages

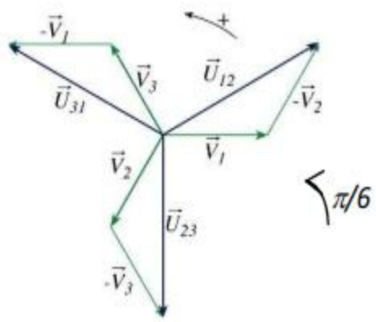


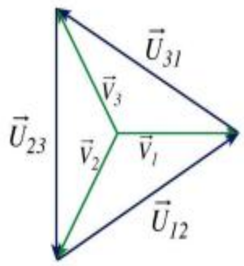
$$V_1 - V_2 = U_{12} \Rightarrow \vec{V}_1 - \vec{V}_2 = \vec{U}_{12}$$

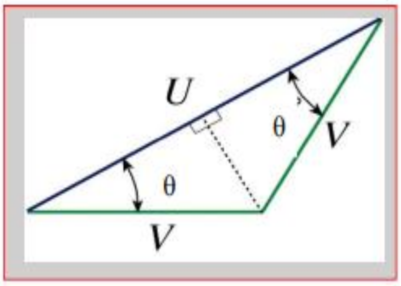
$$V_2 - V_3 = U_{23} \Rightarrow \vec{V}_2 - \vec{V}_3 = \vec{U}_{23}$$

$V_3 - V_1 = U_{31} \Longrightarrow \vec{V}_3 - \vec{V}_1 = \vec{U}_{31}$

Fresnel representation:







According to the triangle we have:

$$V_1 - V_2 = V\sqrt{2}\left[cos\,0 + j\,sin\,0 - cos(\frac{-2\pi}{3}) - sin(\frac{-2\pi}{3})\right] = V\sqrt{2}\left[1 + 0 + \frac{1}{2} + j\frac{\sqrt{3}}{2})\right] = V\sqrt{2}\sqrt{3}\left[\frac{\sqrt{3}}{2} + j\frac{1}{2}\right]$$

$$U_{12} = Ue^{j\frac{\pi}{6}}\ ;\ U_{23} = Ue^{j(-\frac{\pi}{2})}\ ;\ U_{31} = Ue^{j(-\frac{7\pi}{6})}$$

Therefore

$$U_{12}(t) = U\sqrt{2}\,sin(\omega t + \frac{\pi}{6})$$

$$U_{23}(t) = U\sqrt{2}\,sin(\omega t - \frac{\pi}{2})$$

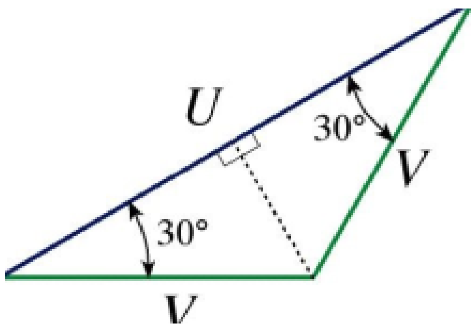
$$U_{31}(t) = U\sqrt{2}\,sin(\omega t - \frac{7\pi}{6})$$

Relationship between V and U

We deduce the relationship between the RMS value of simple voltages and that of compound voltages:

$U/2 = V\,cos(30)$

Hence : $U = V\sqrt{3}$



Note: The network is characterized by its two voltages: simple voltage and compound voltage, which is always present. Simple voltage exists only if the neutral is present on the network. These two voltages are in the ratio of 3. The network is characterized by its voltage

III.2.3 Generator and receiver couplings

III.2.3.1 Definition of a three-phase receiver

Balanced three-phase receivers consist of three identical dipoles of impedance Z.

The currents flowing through the Z elements of the receiver are called receiver **phase currents** and are denoted **J.**

The currents flowing in the wires of the three-phase network are called **line currents** and are denoted I.

A. Star coupling: Y

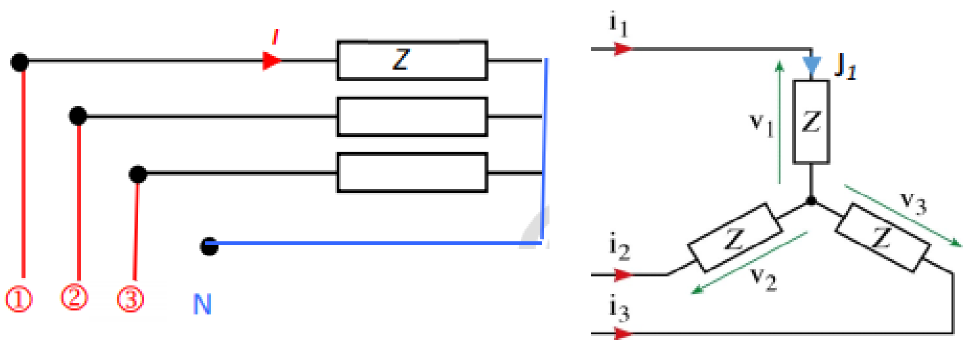


Figure 3.3 Star coupling

Note

- Receiver phase current J = line current I
- The terminal voltage of an element Z is the simple network voltage V.

Power ratings

For one receiver phase: $P_1 = V.I.\cos\varphi$ with $\varphi(\vec{I},\vec{V})$

For the complete receiver: $P = 3V.I.\cos\varphi$

Finally for star coupling:

$$P = \sqrt{3}.U.I.\cos\varphi$$

$$Q = \sqrt{3}.U.I.\sin\varphi$$

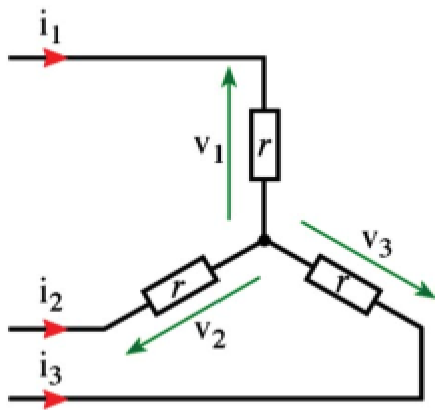
$$S = U.I$$

Power factor: $K = \cos\varphi$

Joule effect losses

Let's consider only the resistive part of the receiver.

For one phase of the receiver: $P_{J1} = r.I^2$



For the complete receiver: $P = 3.P_{J1} = 3r.I^2$

Resistance seen between two terminals: $R = 2.r$.

Finally, for star coupling: $P = \frac{3}{2}.R.I^2$

B. Delta coupling: ΔΔ

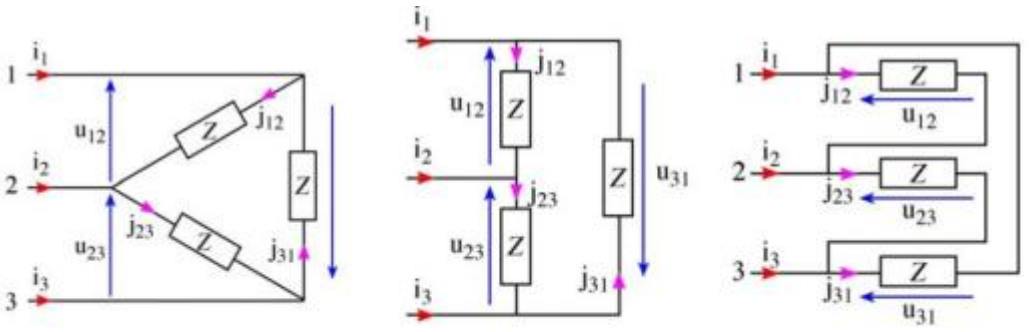


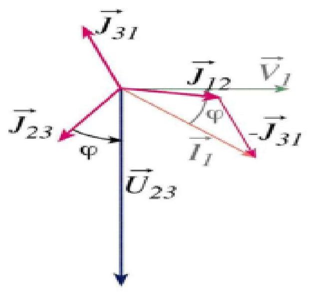
Figure 3.4 Triangle coupling

Each terminal of each of the 3 components is connected to a phase of the network.

The voltage at the terminals of an element is the composite voltage of the network: U

The law of nodes imposes: $i_3 = j_{31} - j_{23}$; $i_2 = j_{23} - j_{12}$; $i_1 = j_{12} - j_{31}$;

Using the same calculation as for voltages, we find $I = \sqrt{3}.J$



Note

- Line current $I = \sqrt{3} \times$ Receiver phase current J
- The voltage across an element Z is the compound voltage U

Powers

For a receiver phase: $P_1 = U.J.\cos\varphi$ with $\varphi(\vec{U},\vec{J})$

For the complete receiver: $P = 3.P_1 = 3U.J.\cos\varphi$

Finally for the delta coupling:

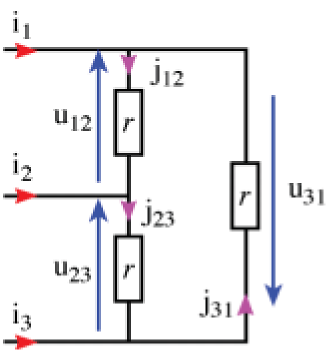
$P = \sqrt{3}.U.I.\cos\varphi$

$Q = \sqrt{3}.U.I.\sin\varphi$

$S = U.I$

Power factor: $K = \cos\varphi$

Joule effect losses



Let's consider only the resistive part of the receiver.

For one phase of the receiver: $P_{J1} = r.I^2$

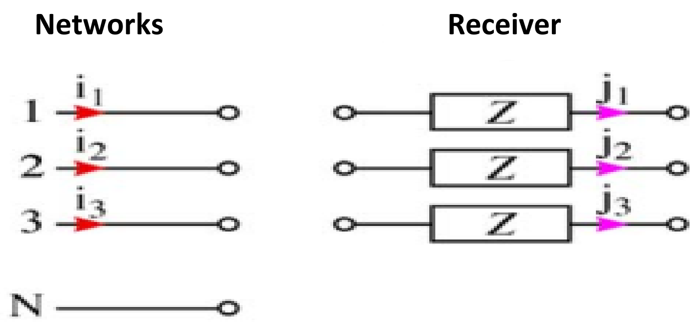
For the complete receiver: $P = 3.P_{J1} = 3r.J^2$

Resistance seen between two terminals: $R = \dfrac{2.r.r}{2.r + r} = \dfrac{2}{3}r$

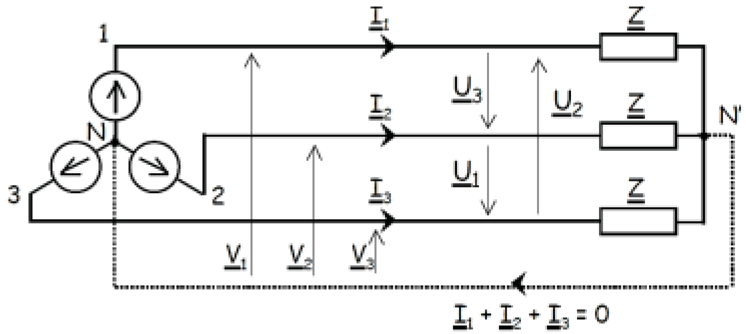
Finally, for delta coupling: $P = \dfrac{3}{2}.R.I^2$

III.2.4 Different types of generator-receiver coupling

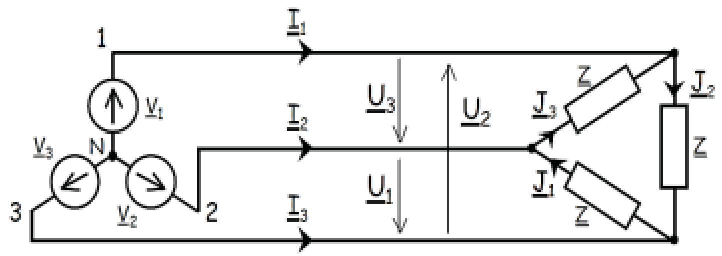
The network and the receiver can be connected in two different ways: star or delta.



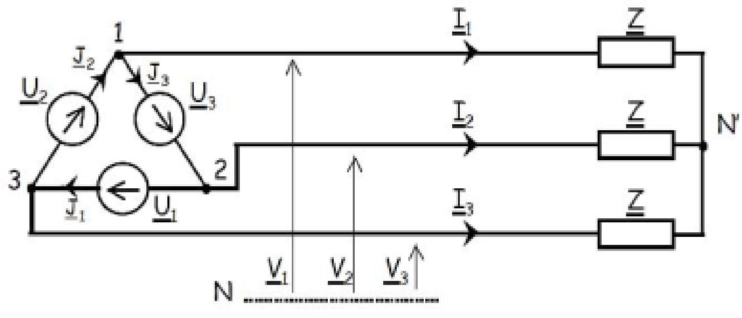
III.2.4.1 Star - star coupling Y-Y



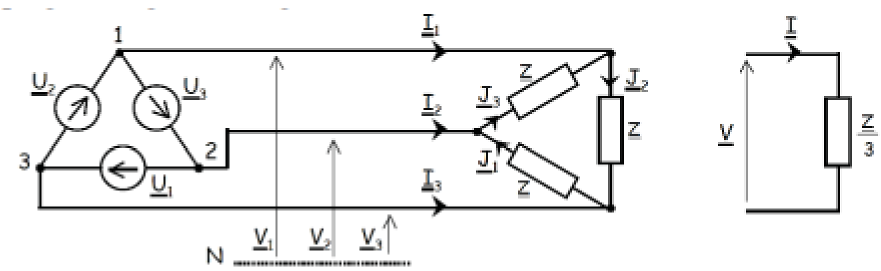
III.2.4.2 Star-delta coupling Y-D



III.2.4.3 Delta-star coupling D-Y



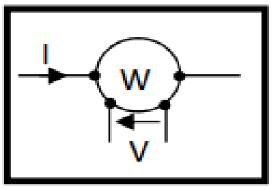
III.2.4.4 Delta - Delta coupling D -D



III.2.5 Power measurement

III.2.5.1 The wattmeter:

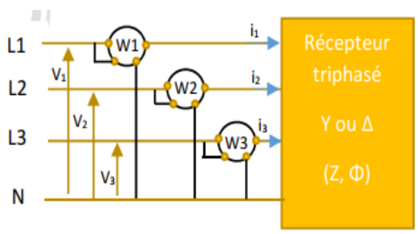
measures the active power consumed by a dipole. It consists of two circuits, one carrying the same current **i(t)** as the dipole, the other subject to the same voltage **V(t)** as the dipole. It shows **<v(t).i(t)>.**



In sinusoidal operation, it indicates $W = V.I.cos(\vec{V},\vec{I}) = \vec{V}.\vec{I}$

therefore : The indication is **algebraic** [12].

III.2.5.2 Measuring active power with three wattmeters.

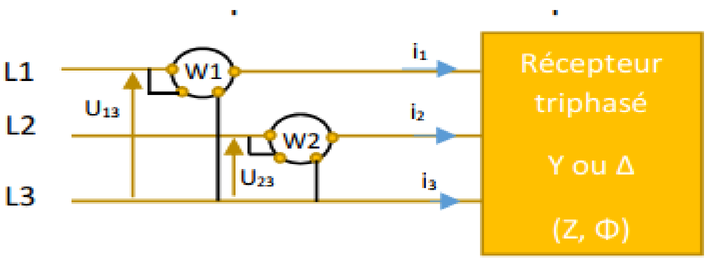


Measuring active power with three wattmeters gives :

$$P = \vec{V}_1.\vec{I}_1 + \vec{V}_2.\vec{I}_2 + \vec{V}_3.\vec{I}_3 = W_1 + W_2 + W_3$$

III.2.5.3 Two wattmeter method for measuring P and Q

If the neutral is not connected, or if the current through it is zero, only two wattmeters are used to measure power.



This means that :

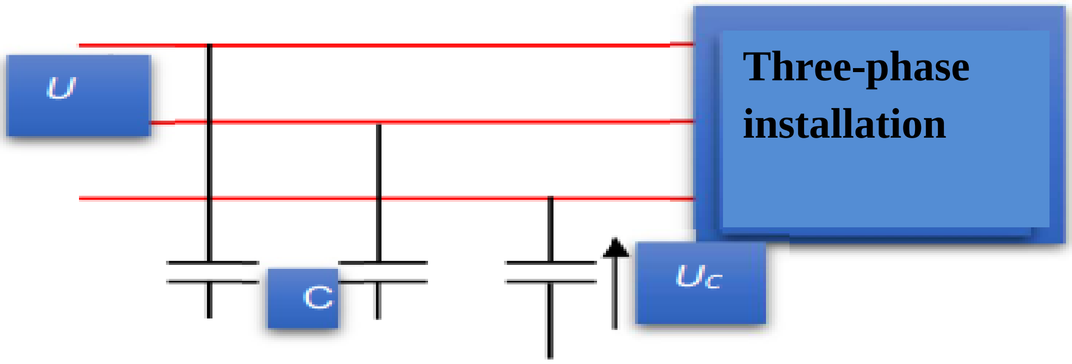
$P = W_1 + W_2 = \sqrt{3}.U.I.cos\,\varphi$ and

$$Q = \sqrt{3}.(W_1 - W_2) = \sqrt{3}.U.I.sin\,\varphi$$

III.2.6 Recovery the power factor:

If an electrical installation consumes too much reactive energy, its line currents will increase considerably, even though its active power remains unchanged. Industrial customers are required to have a power factor greater than *0.93*.

Note: To determine the value of the power factor, 3 capacitor banks must be placed at the input of the installation, which can be connected in star or delta configuration.



III.2.6.1 Star coupling of capacitors

When the power factor increases from $\cos\varphi_1$ to $\cos\varphi_2$. The phase shift decreases and reactive power falls from Q_1 = P.tan φ_1 à Q_2 = P.tan φ_2. This difference $Q_c = Q_2 - Q_1$ is provided by three capacitors of unit capacitance C, supplied at voltage V, so total reactive power :

$$Q_C = -3V^2.C.w$$

We deduce: $C_\gamma = \dfrac{P(\tan\varphi_2 - \tan\varphi_1)}{3V^2.w} = \dfrac{P(\tan\varphi_2 - \tan\varphi_1)}{U^2.w}$

III.2.6.2 Delta-coupling of capacitors

The same applies to the delta circuit, the only difference being that the capacitors are supplied with voltage U, i.e. total reactive power: $Q_C = -3U^2.C.w$

We deduce: $C_\Delta = \dfrac{P(\tan\varphi_2 - \tan\varphi_1)}{3U^2.w}$

Note: Delta-coupling of capacitors is more advantageous, as their capacitance is three times smaller than in star-coupling. $C_\Delta = \dfrac{C_\gamma}{3}$

	Star coupling	Delta coupling
Relation between U and V	$U = V\sqrt{3}$	$U = V\sqrt{3}$
Relation between I and J	$I = J$	$I = J\sqrt{3}$
Phase shifting	φ *angle entre* $(\vec{V}_1$ *et* $\vec{I}_1)$ Ou bien $(\vec{V}_2, \vec{I}_2)$ ou $(\vec{V}_3, \vec{I}_3)$	φ *angle entre* $(\vec{U}_{12}$ *et* $\vec{J}_{12})$ Ou bien $(\vec{U}_{23}, \vec{J}_{23})$ ou $(\vec{U}_{31}, \vec{J}_{31})$
Active power	$P = 3.P_1 = 3.V.I.cos\varphi$ $P = \sqrt{3}.U.I.cos\varphi$	$P = 3.P_1 = 3.U.J.cos\varphi$ $P = \sqrt{3}.U.I.cos\varphi$
Reactive power	$Q = \sqrt{3}.U.I.sin\varphi$	$Q = \sqrt{3}.U.I.sin\varphi$
Apparent power	$S = \sqrt{3}.U.I$	$S = \sqrt{3}.U.I$
Power factor	$f_p = cos\varphi$	$f_p = cos\varphi$
Two wattmeter method	$P = W_1 + W_2 = \sqrt{3}.U.I.cos\varphi$ et $Q = \sqrt{3}.(W_1 - W_2) = \sqrt{3}.U.I.sin\varphi$	$P = W_1 + W_2 = \sqrt{3}.U.I.cos\varphi$ et $Q = \sqrt{3}.(W_1 - W_2) = \sqrt{3}.U.I.sin\varphi$

NATURE OF THE CHARGE

If Q=0 then the charge is resistive and is in the form $Z = R.....avec....\varphi = 0$

If P≠0 and Q>0 then the charge is inductive and is of the form $Z = R + jL\omega avec 0 \le \varphi \le \frac{\pi}{2}$

If P≠0 and Q<0 then the charge is capacitive and has the form $Z = R - \frac{j}{C\omega} avec -\frac{\pi}{2} \le \varphi \le 0$

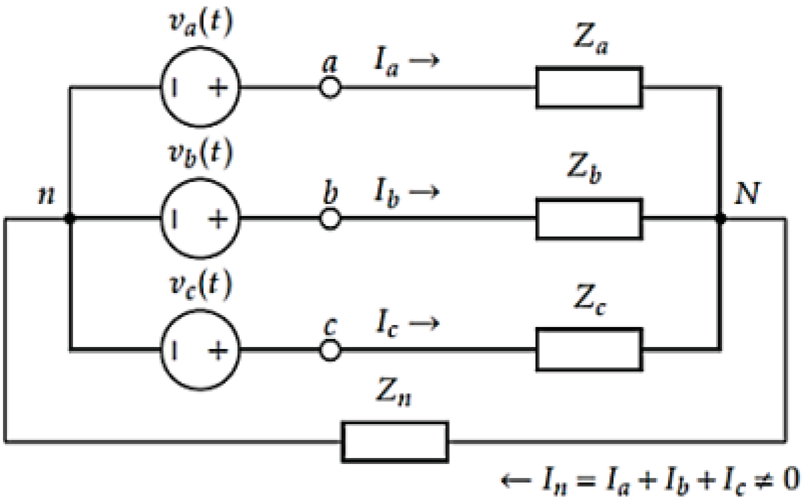
If P=0 and Q>0 then the charge is purely inductive and is of the form $Z = jL\omega avec \varphi = \frac{\pi}{2}$

If P=0 and Q<0 then the charge is purely capacitive and is of the form $Z = -\frac{j}{C\omega} avec \varphi = -\frac{\pi}{2}$

III.3 Unbalanced three-phase systems (symmetrical components):

A three-phase system is said to be unbalanced if all analogous electrical quantities are not equal from one phase to the other. There are three types of unbalanced three-phase circuits:

- Unbalanced source: Short-circuit at the source or in a transformer.
- Unbalanced charge: There may be a short-circuit in the charge, or a poor distribution of single-phase loads on the 3φ network.
- Combination of unbalanced source and unbalanced charge [10].



In practice, unbalanced charges are more common than unbalanced sources. Sources are designed to be as balanced as possible.

One of two study methods can be used to solve these circuits:

1. Use of electrical circuit laws (meshes, nodes, etc.).
2. Symmetrical component method.

The symmetrical component method.

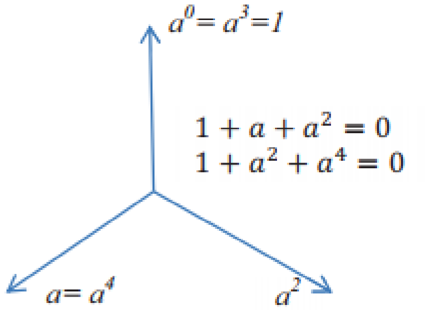
General concepts

Let three quantities be represented by three vectors $\vec{V}_1, \vec{V}_2, \vec{V}_3$. From these three vectors, we define three other vectors according to the formulas:

$$\vec{V}_0 = \frac{\vec{V}_1 + \vec{V}_2 + \vec{V}_3}{3}$$

$$\vec{V}_d = \frac{\vec{V}_1 + a.\vec{V}_2 + a^2.\vec{V}_3}{3}$$

$$\vec{V}_i = \frac{a^2.\vec{V}_1 + \vec{V}_2 + a.\vec{V}_3}{3}$$



$a = e^{j\frac{2\pi}{3}}$ Rotation operator of the three-phase system where $a^2 = e^{j\frac{4\pi}{3}}, a^0 = a^3 = 1$

From $\vec{V}_O, \vec{V}_d, \vec{V}_i$ we can reproduce $\vec{V}_1, \vec{V}_2, \vec{V}_3$

$$\vec{V}_O + \vec{V}_d + \vec{V}_i = \vec{V}_1 + \frac{\vec{V}_2}{3}(1 + a + a^2) + \vec{V}_3(1 + a + a^2)$$

$$\vec{V}_O + a^2.\vec{V}_d + a.\vec{V}_i = \frac{\vec{V}_1}{3}(1 + a + a^2) + \frac{\vec{V}_2}{3}(1 + a^3 + a^3) + \frac{\vec{V}_3}{3}(1 + a^2 + a^4)$$

$$\vec{V}_O + a.\vec{V}_d + a^2.\vec{V}_i = \frac{\vec{V}_1}{3}(1 + a + a^2) + \frac{\vec{V}_3}{3}(1 + a^3 + a^3) + \frac{\vec{V}_2}{3}(1 + a^2 + a^4)$$

$1 + a + a^2 = 1 + a^2 + a^4 = 0$ et $(1 + a^3 + a^3) = 3$

Then
$$\vec{V}_1 = \vec{V}_O + \vec{V}_d + \vec{V}_i$$
$$\vec{V}_2 = \vec{V}_O + a^2.\vec{V}_d + a.\vec{V}_i$$
$$\vec{V}_3 = \vec{V}_O + a.\vec{V}_d + a^2.\vec{V}_i$$

In the system of equation 2:

The first vectors form a three-phase ***homopolar*** system $\vec{V}_O$

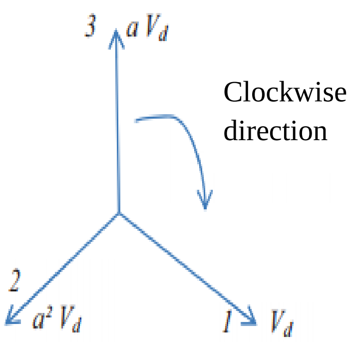
The three vectors $\vec{V}_O + \vec{V}_d + \vec{V}_i$, whose composition is shown below $\vec{V}_1, \vec{V}_2, \vec{V}_3$, are called symmetrical components of the system $\vec{V}_1, \vec{V}_2, \vec{V}_3$.

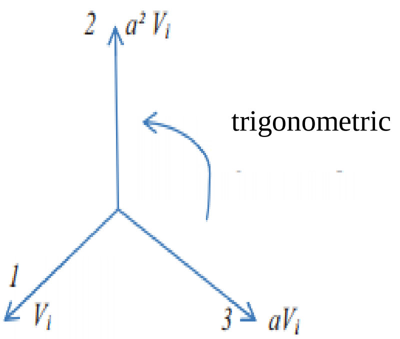
Any three-phase system can be broken down into 3 systems:

A homopolar three-phase system $\vec{V}_O$

A direct symmetrical three-phase system $\vec{V}_d$

An inverse symmetrical three-phase system $\vec{V}_i$





Exercise for the third chapter

Exercise 01:

A balanced three-phase receiver is connected **in star** and supplied by the 50Hz mains with neutral of composite voltage U=380V. Each receiver branch consists of a resistor R=12Ω in series with an inductance L=28.5mH.

1. Give the equivalent schematic.
2. Calculate the simple voltage, the impedance of each branch, the line current, the phase shift between each simple voltage and the corresponding current.
3. Calculate power factor, active and reactive power.
4. Draw the Fresnel representation of the voltages and currents U and I.

Exercise 02:

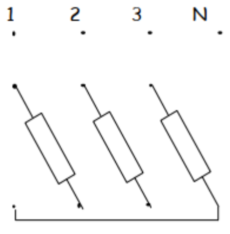
On the three-phase network 220/380V, three (03) identical inductive impedances Z=55Ω and power factor are ($Cos\varphi = 0{,}866$) connected in **delta (Triangle).**

1. Give the equivalent circuit diagram.
2. What do 220V and 380V mean?
3. Determine receiver and line currents.
4. Calculate active and reactive power.
5. Plot receptor currents, line currents and voltages on a vector diagram.

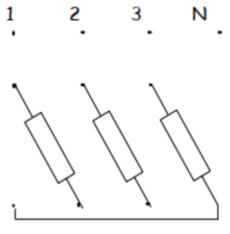
Exercise 03:

Three identical single-phase inductive receivers (coils) with impedance Z=50Ω and power factor 0.8 are connected to the 220/380V 50Hz mains supply.

1. The impedances are **delta (Triangle)-coupled** with neutral.
   - **1.1.** Complete the wiring diagram opposite.
   - **1.2.** Calculate line currents active and reactive power.



2. The impedances are **star-coupled** to the mains.
   - **2.1.** Complete the wiring diagram opposite.
   - **2.2.** Calculate line curent active and reactive power.
   - **2.3.** Calculate the active power ratio P_{Δ}/P_{Y} and conclude.



Exercise 4:

A workshop powered by a 220V/380V, 50Hz balanced three-phase network has the following components:

- A three-phase inductive motor absorbing an active power P_1=2.5kW with$Cos\varphi = 0,7$.

- Three (03) identical single-phase inductive motors operating at 380V, each absorbing 0.8kW with a power factor of 0.72.

- Six (06) Lamps of 100W /220V.

- Three (03) capacitive impedances Z=250Ω, delta (Triangle)-connected, power factor 0.9.

- A three-phase load identified by the two-wattmeter method with: W_1=4,98KW, W_2=1,62KW

1. Propose a diagram for the installation.
2. Calculate the active and reactive power of each element and of the whole system.
3. Deduce the power factor of the workshop and the RMS value of the absorbed current.
4. Determine the capacitance of the 3 delta-connected capacitors used to measure the $Cos\varphi' = 0,98$.
5. Calculate the new RMS value of the line current.

Chapter IV

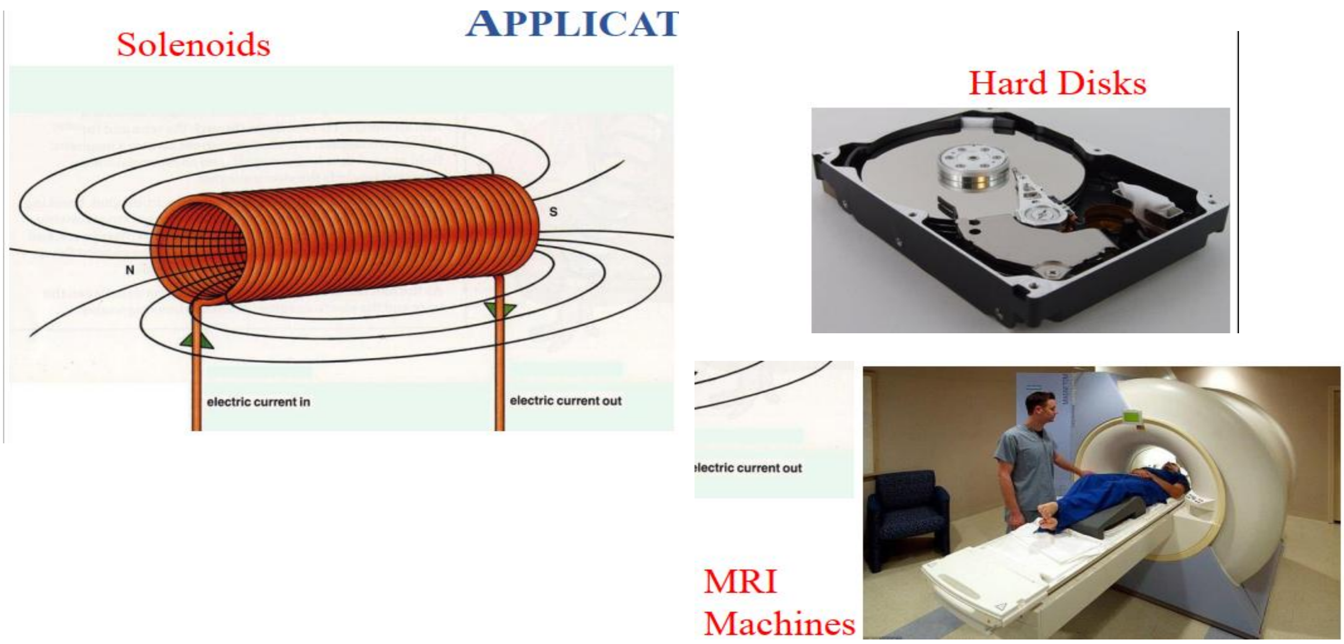
Magnetic circuits

IV.1 Introduction

A magnetic circuit is made up of one or more closed loop paths containing a magnetic flux ϕ (= magnetic field/flux density B × cross-sectional area A).

The flux is usually generated by permanent magnets or electromagnets and confined to a path by magnetic cores consisting of ferromagnetic materials like iron, although there may be air gaps or other materials in the path.

Magnetic circuits are employed to efficiently channel magnetic fields in many devices such as electric motors, generators, transformers, relays, solenoids, loudspeakers, hard disks, MRI machines [11].



IV. 2 Notion of magnetic quantities

IV.2.1 Fundamental laws of electromagnetism

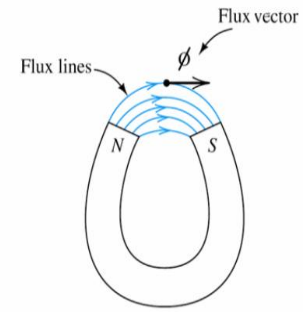
All the phenomena involved in electrical engineering and machines are based on two simple laws, namely[12]:

Biot and Savard's law or Ampere's theorem *(magnetic circuits-transformers)*

Laplace's or Lorentz's force expression *(electrical machines)*

IV.2.2 Principles of Magnetism

Magnetic flux lines form closed paths that are close together where the field is strong and farther apart where the field is weak[12].



Flux lines leave the north-seeking end of a magnet and enter the south-seeking end.

When placed in a magnetic field, a compass indicates north in the direction of the flux lines.

IV.2.3. Magneto-motive Force (M.M.F):

Flux is produced round any current – carrying coil. In order to produce the required flux density, the coil should have the correct number of turns. The product of the current and the number of turns is defined as the coil magneto motive force (m.m.f).

If I = Current through the coil (A) and N = Number of turns in the coil.

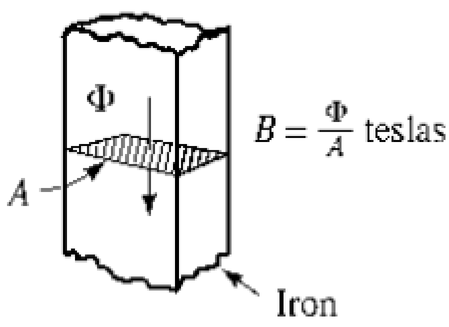
Magneto-motive force = Current x turns.

So $F = N.I$

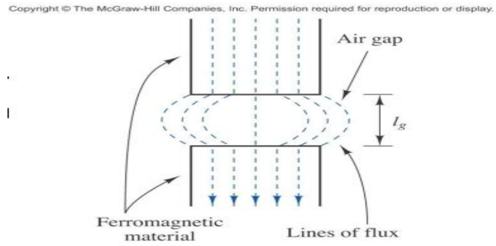
The unit of M.M.F. is ampere–turn (AT) but it is taken as Ampere(A) since N has no dimensions.

IV.2.4 Flux and Flux Density:

The magnetic flux is represented by the symbolΦ. In the SI system, the unit of flux is the weber (Wb). However, we are often more interested in flux density B (i.e., flux per unit area) than in total fluxΦ. Since flux Φ is measured in Wb and area A in m^2, flux density is measured as Wb/m^2, the unit of flux density is called the tesla (T) where $1T = 1Wb/m^2$. Flux density is found by dividing the total flux passing perpendicularly through an area by the size of the area is,



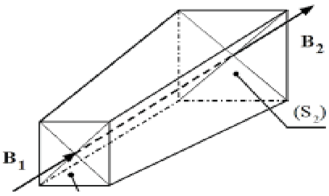
$$B = \frac{\Phi}{A}(Tesla,T)$$



The magnetic flux circulating in area A is the set of lines of force passing through the surface:

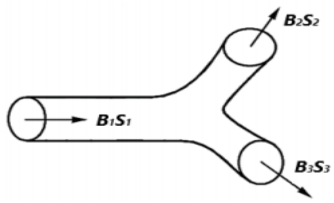
$$\Phi = \int_A B.dA = B.A$$

Magnetic flux is conserved within a field tube.



The same flow that crosses area A_1 crosses surface A_2

$$\Phi = B_1.S_1 = B_2.S_2$$



Case of bifurcation or node (where the flow meets two different paths)

$$\Phi_{TOTAL} = \Phi_1 + \Phi_2$$
$$B_1.S_1 = B_2.S_2 + B_3.S_3$$

IV.2.5 Magnetic Field Intensity (strength):

Magnetic Field Intensity is defined as the magneto-motive force per unit length of the magnetic flux path. Its symbol is H.

$$Magnetic\_field\_int\,ensity = \frac{Magnetomotive\_force}{Mean\_length\_of\_the\_magnetic\_path}$$

$$H = \frac{F}{l} = \frac{N.I}{l} \text{ amperes per metre} (A/m)$$

Where l is the mean length of the magnetic circuit in meters. Magnetic field intensity is also called magnetic field strength or magnetizing force.

IV.2.6 Permeability of free space μ_0 (magnetic constant) :

The permeability of free space or non-magnetic materials is

$\mu_0 = 4.\pi.10^{-7}$ (The units of μ_0 are H/m (Henry per meter))

IV.2.7 Relative Permeability μ_r:

➢The **ratio of the flux density** ***B*** produced in a **material to** the flux density produced in **vacuum** (or in a non-magnetic core) for a particular applied magnetic field strength *H*.

➢ For air and **non-magnetic materials**, μ_r = **1**

➢For **ferromagnetic materials**, e.g. some forms of nickel–iron alloys, the relative permeability can be as large as ~100000 i.e. **~10^6**.

➢ For a material having a relative permeability μ_r,

$B = \mu_0.\mu_r.H = \mu.H$ (The units of μ_0 are H/m (Henry per meter))

Where, $\mu = \mu_0 . \mu_r$ is the absolute permeability

IV.2.8 Reluctance S:

Reluctance (s) is akin to resistance (which limits the electric Current).

Flux in a magnetic circuit is limited by reluctance. Thus reluctance(s) is a measure of the opposition offered by a magnetic circuit to the setting up of the flux.

Reluctance is the ratio of magneto motive force to the flux. Thus

$$\Phi = B.S = \mu \frac{N.I}{l} S = \frac{\mu.S}{l} N.I$$

$$\Phi = B.A.......(1)$$

$$mmf; F = Hl.......(2)$$

Dividing (1) by (2),

$$\frac{\Phi}{F} = \frac{B.A}{H.l} = \mu_0 . \mu_r \frac{A}{l}$$

The formula obtained is called Hopkinson's relation, and it's possible to include in it a quantity that depends only on the fixed characteristics of the circuit:

$\frac{F}{\Phi} = S = \frac{l}{\mu.A}$ called **the reluctance**.

To summarize:

$$\Phi.\Re = N.I$$

The **inverse of reluctance** is known as **permeance** (ease of flux passage)

Important notes

- The equivalent reluctance is the sum of the different reluctances $S_{eq} = S_N + S_e$

- Magnetic excitation is uniform $H_2 = H_1$.

Magnetic inductions are different $B_1 \neq B_2$

IV.3 Ampere's theorem

Ampere's theorem makes it possible to determine the *value of the magnetic field* by means of *electric currents*. Let's take the example of a coil through which a current I flows. The coil then creates a *magnetomotive force* (mmf) that *causes a magnetic flux to flow through* the medium.

This is similar to the phenomenon of electrical circuits: an electromotive force moves electrons that circulate in the medium [11].

The circulation of the **H** vector along any closed curve (C) is equal to the *algebraic sum of the currents* flowing through the surface resting on the contour (C).

Simplifying Ampere's theorem gives the relationship between magnetic field and exciter current.

$$FMM = \oint_C H.dl = \sum_{i=1}^{n} N.I$$

Ii : Currents flowing through (S). *Ni:* Number of turns of conductor *i*.

To create a uniform field, we use a coil to concentrate the field lines in a single in one place. Inside the coil, the magnetic fields ***add up*** to create a ***stronger***, more uniform ***field***.

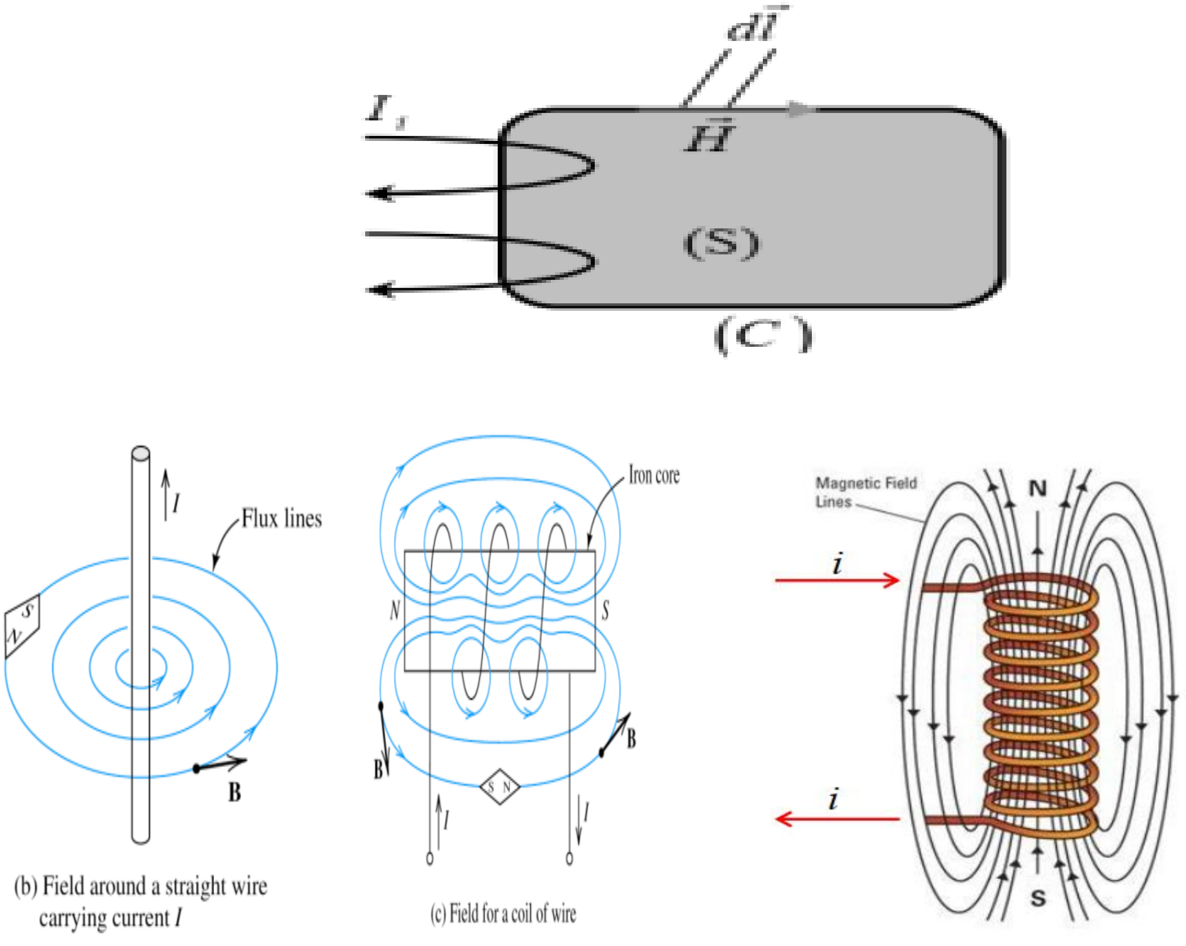


Figure 4.1 Magnetic field created by an electric current.

IV.4 Magnetic Circuit Analogy with Electrical Circuit

What analogies can be drawn between magnetic and electrical circuits?

We can draw analogies between electrical and magnetic circuits.

The aim is to facilitate the analysis of complex, heterogeneous magnetic circuits.

The analogy allows us to calculate the various magnetic parameters in the same way as for an electrical circuit. The table above summarizes the main parameters of this analogy [13]:

Magnetic circuit	**Electrical circuit**
Magnetic flux density ***B*** (T=Wb/m^2)	Current density ***J*** (A/m^2)
Magnetic flux **Φ** (Wb)	Current **I** (A)
Reluctance S (A/Wb)	Resistance R(Ω)
Magnetic flux intensity H(A/m)	Electric field intensity E (V/m)
Magnetomotive force $\mathfrak{F}$: mmf (At)	Electromotive force: fem (V)
Ampere's law NI= Φ $\mathfrak{R}$	Ohm's law V= R.I
Permeance H(Wb/A)	Conductance S(A/V)
Permeability μ (H/m)	Conductivity σ (S/m)

Table 4.1: Magnetic circuit / electrical circuit Analogies

IV.4.1 Heterogeneous series circuit (magnetic circuit with air gap)

In this case, the magnetic circuit is made up of the same circuit as above, except that in this case we have created a vacuum (air gap).

we've created a *vacuum (air gap).* So we have two materials: *ferromagnetic (µ) and air (µ0)*

$$H.(L-\delta)+H_0.\delta = N.I$$

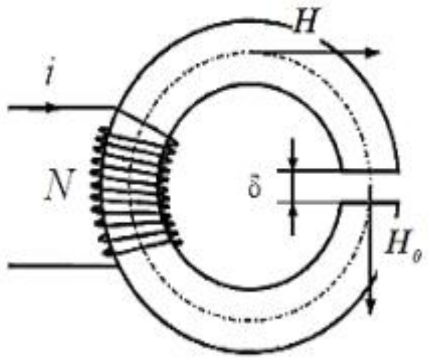
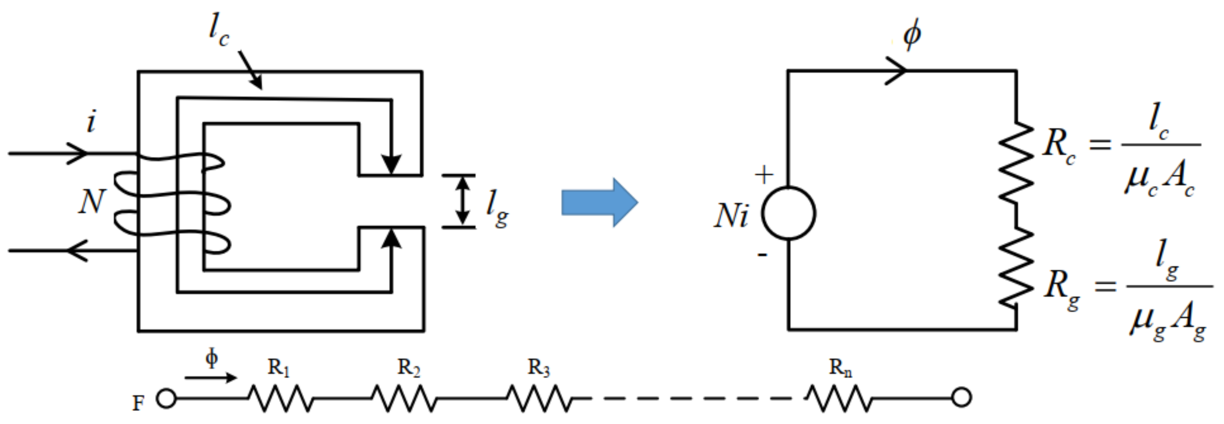


Figure 4.2: Toroid magnetic circuit

IV.4.2 Series connection in magnetic circuit

The applied *mmf* is equal to the sum of the *mmf*s dropped across each series element, while the flux through each series element is the same.



Series reluctance behaves in the same way as series resistors.

$S_{eq} = S_1 + S_2 + S_3 + ... + S_N$

IV.4.3 Parallel circuit (magnetic circuit with bifurcations)

Consider the magnetic circuit shown in figure 4.9. There are 03 lengths, i.e. three reluctances.

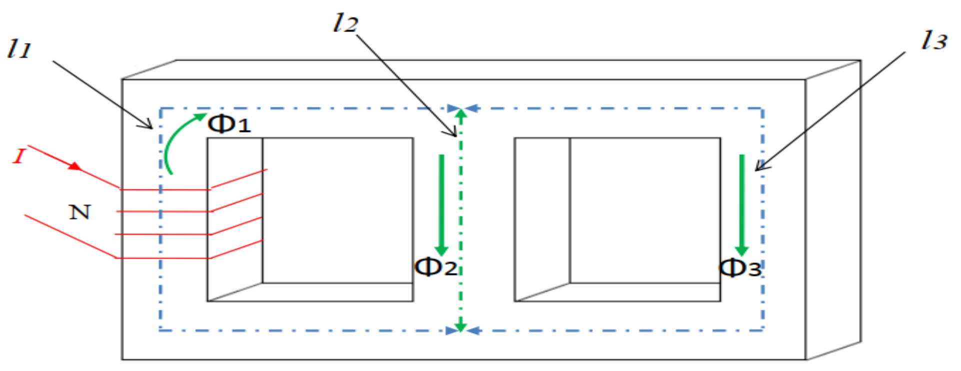


Figure 4.3: Magnetic circuit (rectangular)

By analogy, the equivalent electrical circuit is:

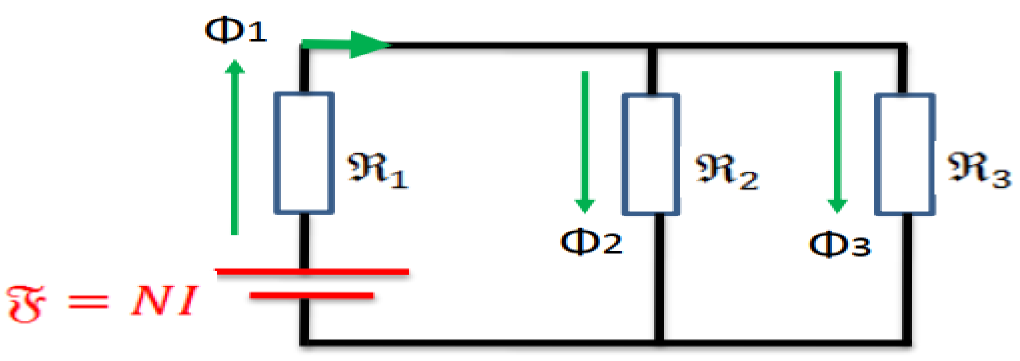


Figure 4.4: equivalent electrical circuit.

Parallel reluctance behaves in the same way as parallel resistors.

$$\frac{1}{S_{eq}} = \frac{1}{S_1} + \frac{1}{S_2} + \frac{1}{S_3} + \ldots\ldots + \frac{1}{S_N}$$

IV.5 B-H Curve [14]

➢The ***B-H*** or **magnetization curve** gives the **relation** between **flux density *B*** and **field intensity *H***.

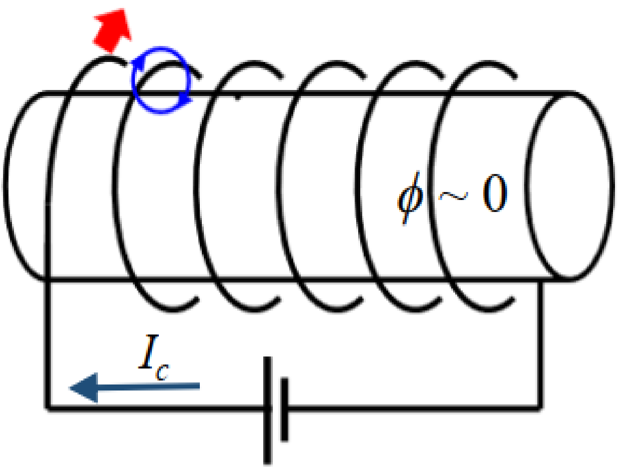
➢ It is **not a straight line** (as naively expected from the relation $B = \mu H$) and is actually **non-linear** as the permeability ***μ*** typically **depends** on the applied field strength ***H***.

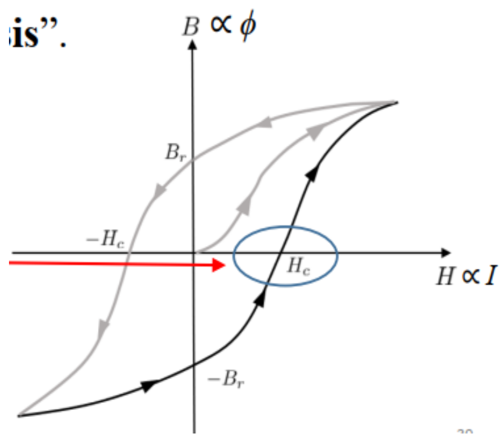
➢ The complete ***B-H* curve** is usually described as a **hysteresis loop**.

The **area** contained **within** a hysteresis **loop** indicates the **energy required** to perform the **'magnetize - demagnetize'** process.

Now as ***H*** is **increased** again **towards positive** values, a **residual magnetism –*B*r** remains at ***H* = 0**. Then ***B* = 0** is again reached at ***H* = +*H*c**, and finally positive saturation occurs, but arrived via a **different path**.

This phenomenon of **non-conformity** (i.e. non-overlapping) of '**increase**' and '**decrease**' curves, is called as "**hysteresis**".

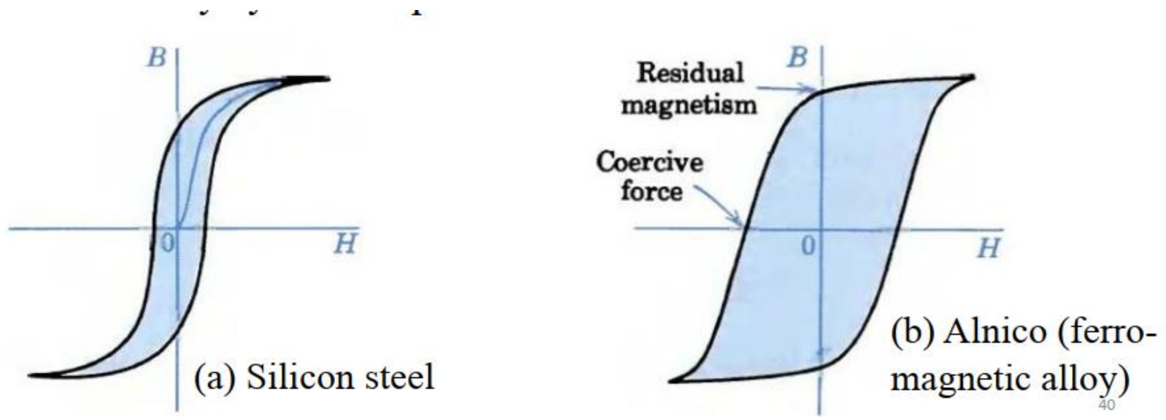




IV.5.1 Hysteresis Loop

➢ Thus, when a magnetic material is magnetized (flux induced), the original state is not returned to when applied magnetizing force/field is removed.

➢ If the magnetizing force is due to an applied AC current, a hysteresis loop results in every cycle/ time period.



IV.5.2 Eddy current losses

➢ If the magnetic core is solid, there can also be Eddy current loss.

➢ Localized Eddy currents excited by AC magnetic flux induced voltages (Lenz's law) result in I^2R losses.

➢ Loss can be reduced by using laminated core designs.

➢ Core in the form of sheets separated by insulating material.

➢ Loss: $P = K_e.B_{max}^2.f^2$

(*Ke* lower for laminated as compared to bulk material)

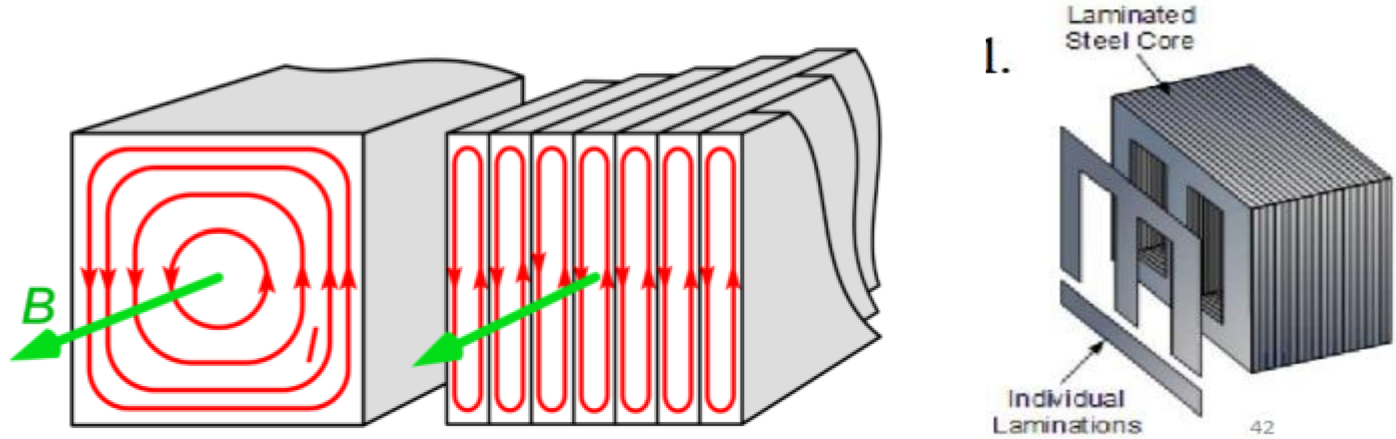


Figure 4.5 Eddy currents in a block of solid material (left), and in laminated material (right).

IV.5.2 Hysteresis losses

The **hysteresis** caused by **cyclic magnetization-demagnetization** leads to some magnetizing **energy** to be **lost**.

➢ The **area** of the ***B-H* loop** gives the amount of **power lost** as heat in the cyclic magnetization-demagnetization process (less area => less loss)

➢ Empirical formula for power loss due to hysteresis: $P_H = K_H.B_m^2.f^2$

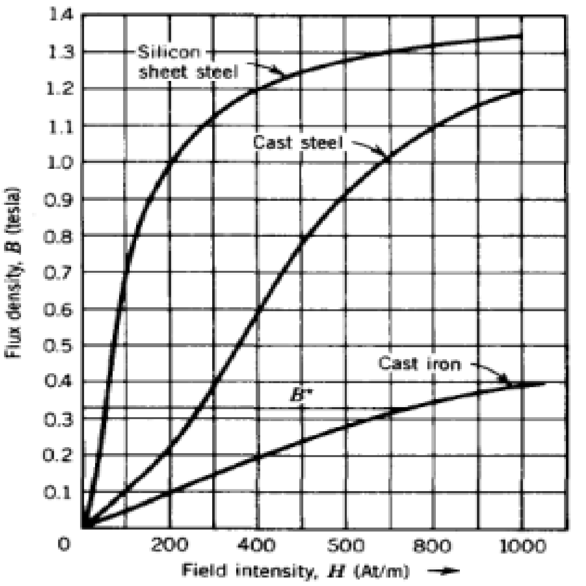
The constants K_H and n **depend** on core **material**.

f is the **frequency** of **AC** magnetizing current.

Typically $n \sim 1.6 - 2$ (called the "**Steinmetz Exponent**")

B_m is the **maximum**/saturated induced **flux density** per cycle

The magnetic circuit differs from the electric circuit in this respect; resistance is normally independent of current in an electric circuit, whereas reluctance depends on the flux density in the magnetic circuit.



The B-H characteristics of three different types of magnetic corescast iron, cast steel, and silicon sheet steel- are shown in Figure.

Note that to establish a certain level of flux density B* in the various magnetic materials the values of current required are different.

IV.6 Self-inductance and self-flux

Consider a closed circuit through which current i flows [15].

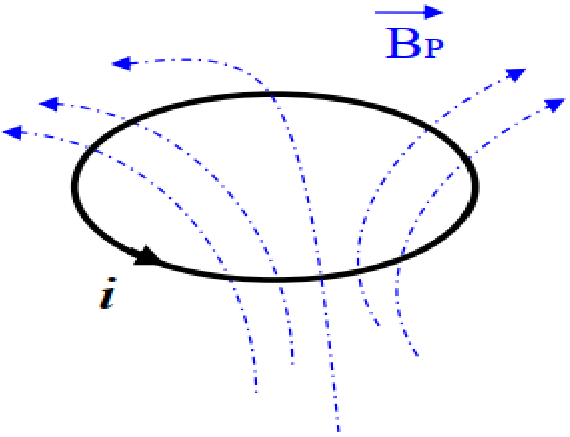


Figure 4.6 Clean flow.

Le champ magnétique B_P créé, est proportionnel à l'intensité, et génère un flux propre :

$$\Phi_P = \iint \overrightarrow{B_p} \cdot d\vec{S}$$

Le flux propre peut donc s'écrire

$$\Phi_P = L . i$$

L : L'inductance propre de la spire, qui s'exprime en Henry (H).

IV.6.1 Inductance propre d'un solénoïde (bobine)

Un champ magnétique uniforme est produit au sein du solénoïde de N spires lorsque celle ci est traversée par un courant i.

The magnetic field B_p created, is proportional to the intensity, and generates a proper flux:

$$\Phi_P = \iint \overrightarrow{B_P}.d\vec{A}$$

The proper flux can therefore be written as

$$\Phi_P = L.i$$

L : The self-inductance of the coil, expressed in Henry (H).

IV.6.2 Self-inductance of a solenoid (coil)

A uniform magnetic field is produced within a solenoid of N turns when a current i is passed through it. is crossed by a current i.

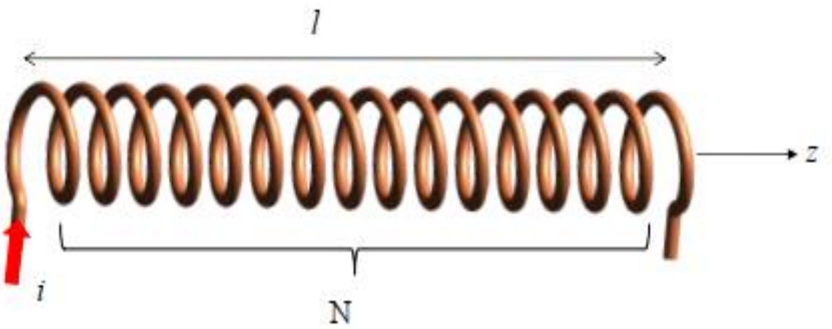


Figure 4.7 Self-inductance.

The field produced is given by:

$$\overrightarrow{B_P} = \mu_0 \frac{N}{l} i.\overrightarrow{\mu_Z}$$

which generates a clean flux ΦP.

$$\Phi_P = \mu_0 \frac{N}{l} i.N.A = \frac{\mu_0 N^2.A}{l} i$$

$$\frac{\mu_0 N^2.A}{l} i = L.i$$

$$L = \frac{\mu_0.A}{l} N^2$$

$$L = \frac{N^2}{S}$$

In reality, not all the field lines created by the winding appear in the magnetic circuit. For essentially manufacturing reasons, some of them loop back into the air near the turns.

We distinguish between flux in the material and leakage flux, which corresponds to the magnetizing and leakage inductances.

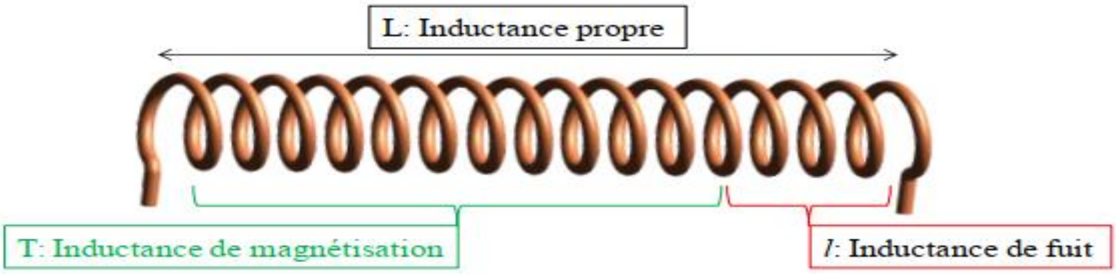


Figure 4.8 Inductance in reality.

IV.6.3 Mutual inductance and coupling coefficient

Consider two closed, wire-shaped, mutually coupled circuits, each carrying currents i_1 and i_2.

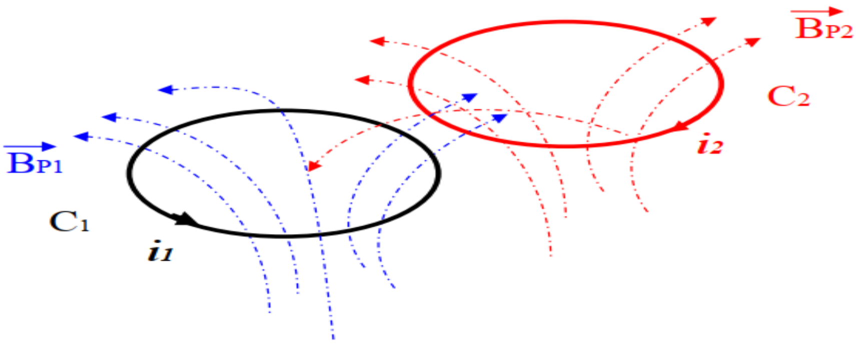


Figure 4.9 Mutual flux.

We have:

$$\vec{B}_{P.TOTAL} = \vec{B}_{p1} + \vec{B}_{p2}$$

Mutual inductance is the coefficient of proportionality between the current in C_2 and the magnetic field flux sent by C_2 into C_1. It depends only on the geometrical characteristics of the two circuits and their relative positions.

Then:

$$\begin{cases} \Phi_1 = \Phi_{11} + \Phi_{21} \\ \Phi_2 = \Phi_{22} + \Phi_{12} \end{cases}$$

$$\begin{cases} \Phi_1 = L_1 I_1 + M_{1,2} I_2 \\ \Phi_2 = L_2 I_2 + M_{2,1} I_1 \end{cases}$$

Where L_1 is the self-inductance of C_1 and $M_{1,2}$ the mutual inductance between C_1 and C_2.

Exercise for the fourth chapter

Exercise 01:

The coil in Figure has *250* turns and is wound on a silicon sheet steel. The inner and outer radii are *20* and *25* cm, respectively, and the toroidal core has a circular cross section. For a coil current of *2.5A*

Find:

1. The magnetic flux density at the mean radius of the toroid.
2. The inductance of the coil, assuming that the flux density within the core is uniform and equal to that at the mean radius.

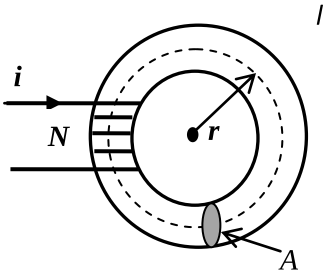


Figure 01

Exercise 02:

For the magnetic circuit shown in **figure 2**

- Find the current I in the coil needed to produce a flux of *0.007Wb*?

1. When the relative permeability of the core is assumed to be constant $\mu_r = 5000$
2. When the core's magnetization curve is as follows:

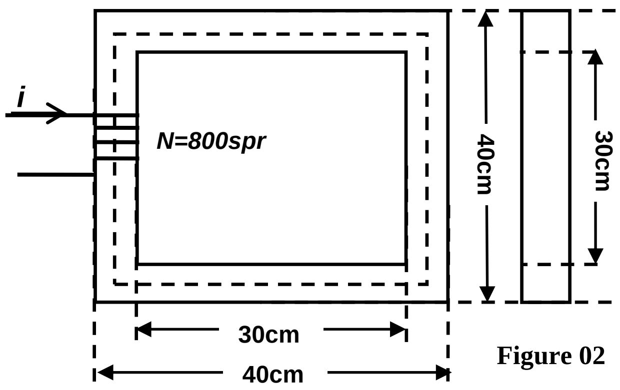


Figure 02

B(T)	*0.3*	*0.4*	*0.5*	*0.6*	*0.7*	*0.8*	*0.9*	*1.25*	*1.3*
H(At/m)	*52*	*58*	*65*	*76*	*90*	*110*	*132*	*380*	*600*

Exercise 03:

The magnetic circuit shown in **figure 3** is made of iron. Flux *is 0.0005Wb*. The coil has 300 turns.

1. Determine the current and the reluctance of the core.
2. Make a slot in the magnetic circuit so as to obtain an air gap *e=1mm* thick (dotted line on figure 3).

- In this case, what is the value of the current required to maintain the same flux as when there was no air gap?
- Determine the *mmf* necessary to and the total reluctance.

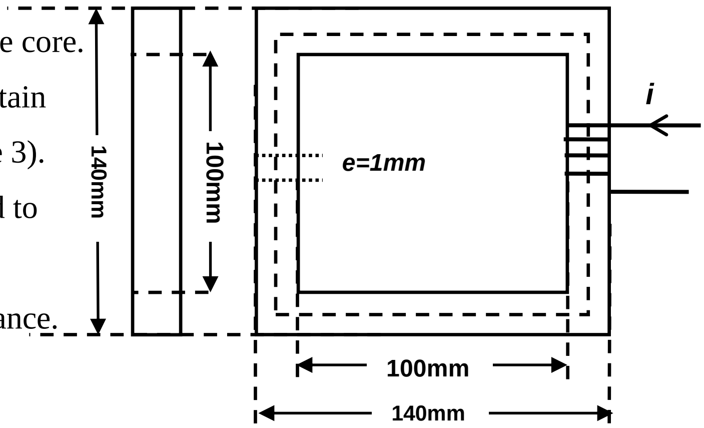


Figure 03

3. Plot the equivalent electrical diagram for the two cases studied.

B(T)	*0*	*0.8*	*1.0*	*1.2*	*1.25*
H(At/m)	*0*	*300*	*600*	*900*	*1200*

Chapter V

Transformers

V.1 Introduction

Electrical power transformer is a static device which transforms electrical energy from one circuit to another without any direct electrical connection and with the help of mutual induction between two windings. It transforms power from one circuit to another without changing its frequency but may be in different voltage level.

A single-phase transformer is a type of power transformer that utilizes single-phase alternating current, meaning the transformer relies on a voltage cycle that operates in a unified time phase [16].

Symboles :

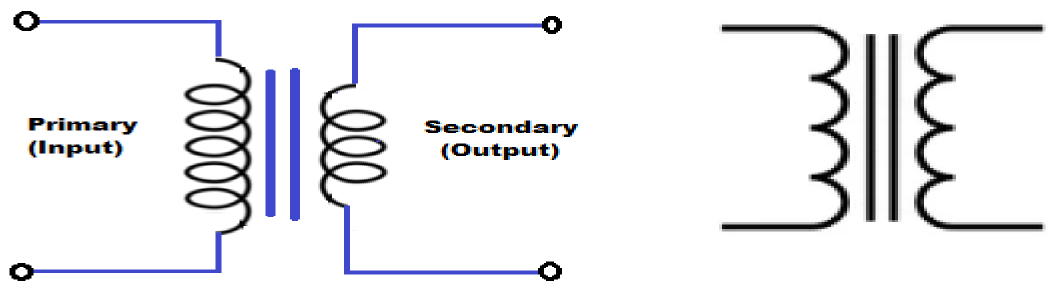


Figure 5.1 Transformer symbols.

V.2 Principle of operation

The working principle of the single phase transformer is based on the Faraday's law of electromagnetic induction 17].

Basically, mutual induction between two or more windings is responsible for transformation action in an electrical transformer.

Faraday's law of electromagnetic induction

According to Faraday’s law, “Rate of change of flux linkage with respect to time is directly proportional to the induced EMF in a conductor or coil”.

$$e = -\frac{d\Phi}{dt}$$

The principle of operation of a transformer has been explained in the following simple steps:

• As soon as the primary winding is connected to a single–phase supply, an AC current starts flowing through it.

• An alternating flux is produced in the core by the AC primary current.

• The alternating flux gets linked with the secondary winding through the core.

• Now, according to Faraday's laws of electromagnetic induction this varying flux will induce voltage into the secondary winding.

V.3 Construction

The three main parts of a transformer are [18]:

• **Primary Winding**: The winding that takes electrical power and produces magnetic flux when it is connected to an electrical source.

• **Magnetic Core**: This refers to the magnetic flux produced by the primary winding. The flux passes through a low reluctance path linked with secondary winding creating a closed magnetic circuit.

• **Secondary Winding**: The winding that provides the desired output voltage due to mutual induction in the transformer.

The primary winding is supplied an alternating electrical source. The alternating current through the primary winding produces an alternating flux that surrounds the winding. Another winding, also known as the secondary winding, is brought close to the primary winding. Eventually, some portion of the flux in the primary will link with the secondary. As this flux is continually changing in amplitude and direction, there is a change in flux linkage in the second winding as well. According to Faraday's law of electromagnetic induction, an electromotive force (emf) is induced in the secondary winding which is called as induced emf. If the circuit of the secondary winding is closed an induced current will flow through it. This is the simplest form of electrical power transformation; this is the most basic working principle of a transformer.

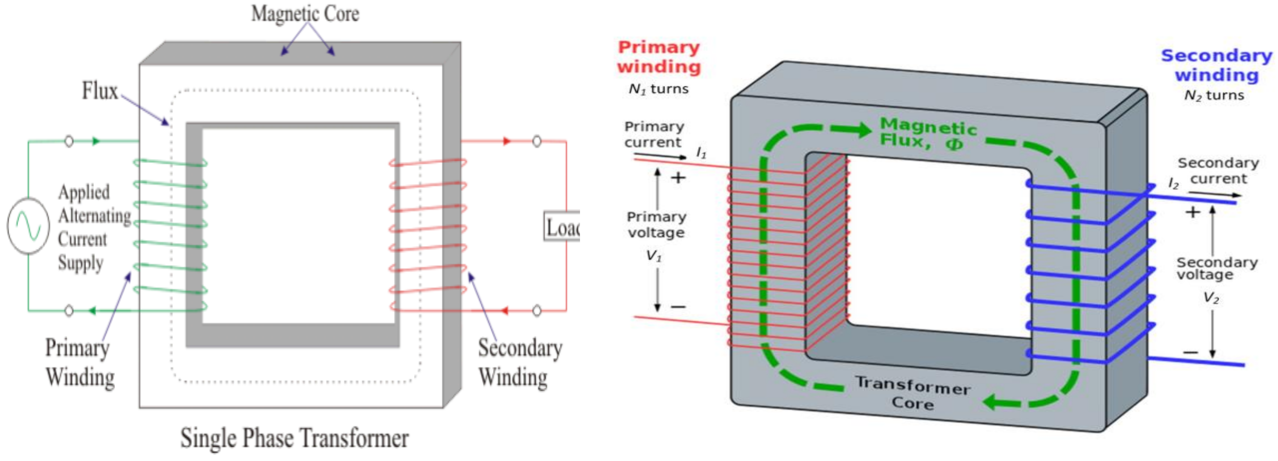


Figure 5.2 Single phases Transformer.

It consists of two coils of electrical wire called inner and outer windings. The primary is usually known to have the higher amount of voltage. Both coils are wrapped around a common closed magnetic iron circuit which is referred to as the core. The core is made up of several layers of iron, laminated together to decrease losses. Being linked at the common core allows power to be transferred from one coil to the other without an electrical connection. When current passes through the primary coil, a magnetic field is created which induces a voltage in the secondary coil. Usually, the primary coil is where the high voltage comes in and then is transformed to create a magnetic field. The job of the secondary coil is to transform the alternating magnetic field into electric power, supplying the required voltage output.

V.4 Nameplate

Like any electrical device, each transformer is spatially calculated to operate:

- Under primary voltage U_{1n}
- Absorbing a current close to a value I_{1n}

U_{1n} and I_{1n} are called nominal values, their product S= $U_{1n} * I_{1n}$ is the device's nominal apparent power.

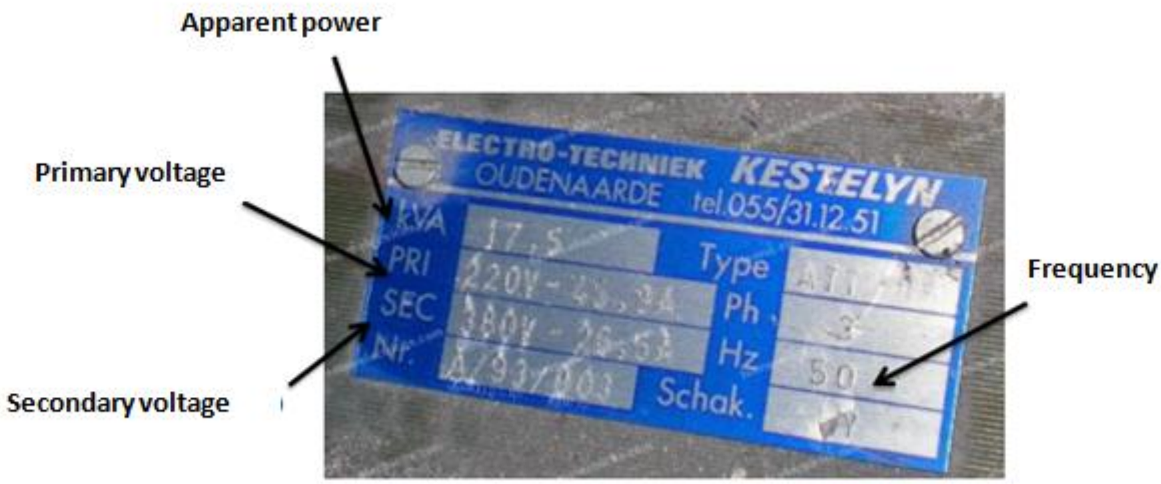


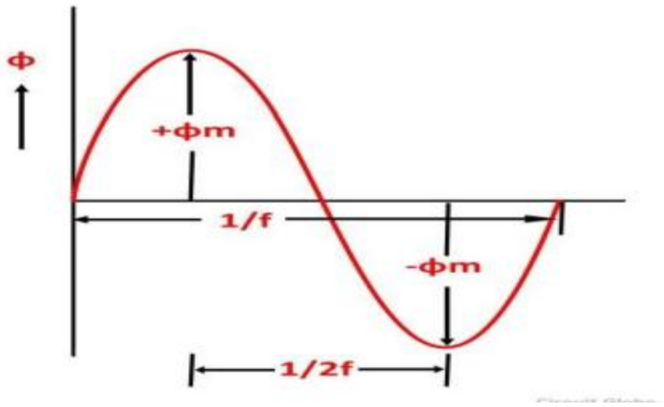
Figure 5.3 Transformer nameplates.

V.5 EMF equation for 1ø transformer:

When a sinusoidal voltage is applied to the primary winding of a transformer, alternating flux Φ_m sets up in the iron core of the transformer. This sinusoidal flux links with both primary and secondary winding. The function of flux is a sine function. The rate of change of flux with respect to time is derived mathematically [16].

The derivation of **EMF Equation** of the transformer is shown below. Let

- Φ_m be the maximum value of flux in Weber
- f be the supply frequency in Hz
- N_1 is the number of turns in the primary winding
- N_2 is the number of turns in the secondary winding
- Φ is the flux per turn in Weber



As shown in the above figure that the flux changes from $+\Phi_m$ to $-\Phi_m$ in half a cycle of 1/2f seconds.

By Faraday's Law

Let E_p is the emf induced in the primary winding

$$E_p = -\frac{d\Phi}{dt} \tag{1}$$

$$E_p = -N_p \frac{d\Phi}{dt} \tag{2}$$

Since Φ is due to AC supply $\Phi = \Phi_m Sin(\omega t)$

$$E_p = -N_p \frac{d}{dt}(\Phi_m Sin(\omega t))$$

$$E_p = -N_p \omega.\Phi_m Cos(\omega t)$$

$$E_p = -N_p \omega.\Phi_m Sin(\omega t - \pi/2) \tag{3}$$

So, the induced emf lags flux by 90 degrees.

Maximum valve of emf

$$E_{p\,max} = N_p \omega.\Phi_m \tag{4}$$

But $\omega = 2.\pi.f$

$$E_{p\,max} = 2.\pi.f.N_p.\Phi_m \tag{5}$$

Root mean square RMS value is

$$E_p = \frac{E_{p\,max}}{\sqrt{2}} \tag{6}$$

Putting the value of E1max in equation (6) we get

$$E_p = \sqrt{2}\pi.f.N_p.\Phi_m \tag{7}$$

Putting the value of π = 3.14 in the equation (7) we will get the value of E1 as

$$E_p = 4.44.f.N_p.\Phi_m \tag{8}$$

Similarly,

$$E_s = \sqrt{2}\pi.f.N_s.\Phi_m \text{ Or } E_s = 4.44.f.N_s.\Phi_m \tag{9}$$

Now, equating the equation (8) and (9) we get

$$\frac{E_s}{E_p} = \frac{4.44.f.N_s.\Phi_m}{4.44.f.N_p.\Phi_m}$$

$$\frac{E_s}{E_p} = \frac{N_s}{N_p} = a$$

The above equation is called the turn ratio where a is known as transformation ratio.

Turn Ratio of the transformer is defined as the ratio of the number of turns of the secondary winding to that of the primary.

The equation (8) and (9) can also be written as shown below using the relation

$\Phi_m = B_m.A_i$ where Ai is the iron area and B_m is the maximum value of flux density.

$E_p = 4.44.f.N_p.B_m.A_i$ Volts and $E_s = 4.44.f.N_s.B_m.A_i$ Volts

For a sinusoidal wave

$$\frac{R.M.S\_value}{Average\_value} == Form\_factor = 1.11$$

V.6 Ideal transformer

An ideal transformer is a lossless device with an input and output winding [17].

- An ideal transformer is a lossless device with an input and output winding.
- The relationship between $V_p(t)$ and $V_s(t)$ of on transformer is:

$$\frac{V_s(t)}{V_p(t)} = \frac{N_s}{N_p} = a$$

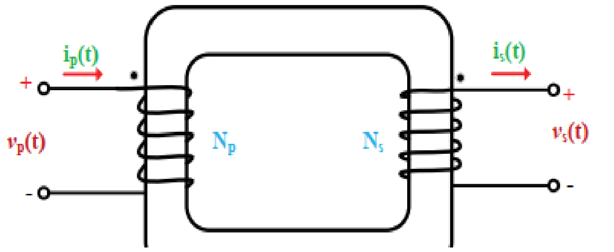


Figure 5.4 Equivalent circuit of an ideal transformer.

Where a -turns ratio of the transformer.

$$N_s . i_s(t) = N_p .. i_p(t)$$

The relationship between $i_p(t)$ and $V_s(t)$ of on transformer is:

$$\frac{i_p(t)}{i_s(t)} = \frac{N_p}{N_s} = \frac{1}{a}$$

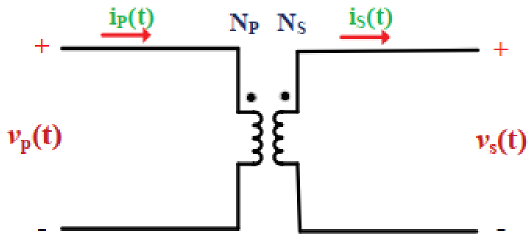


Figure 5.5 Parameters of an ideal transformer.

Where, a = constant

This constant a is known as **voltage transformation ratio**.

▪ If $N_S > N_P$, i.e. a>1, then the transformer is called step-up transformer.

▪ If $N_S < N_P$, i.e. a<1, then the transformer is called step-down transformer.

V.7 Single-phase real transformer:

The basis of transformer operation can be derived from faraday's law [16]

$$e = -\frac{d\Phi}{dt}$$

The Φ in the primary coil can be divided into two components:

- Mutual flux---which remains in the core and links both winding
- Small leakage flux.

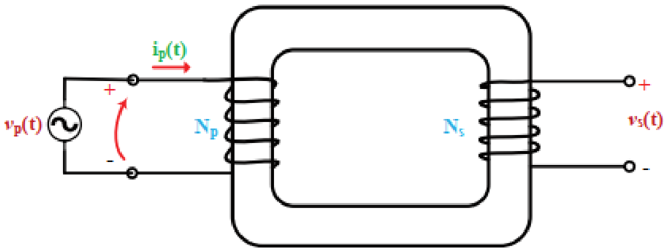


Figure 5.6 A real transformer secondary open circuited.

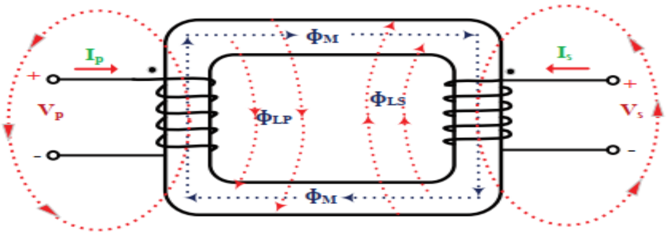


Figure 5.7 Mutual and leakage fluxes in transformer core.

$$\Phi_p = \Phi_M + \Phi_{Lp}$$

Where

- Φ_p - Total average primary flux.

- Φ_M - flux component linking both primary and secondary coils.

- Φ_{Lp} - Primary leakage flux.

Similary the flux in the secondary winding is

$$\Phi_s = \Phi_M + \Phi_{Ls}$$

$V_p(t) = N_p.\frac{d\Phi_p}{dt}$ $V_s(t) = N_s.\frac{d\Phi_s}{dt}$

$V_p(t) = N_p.\frac{d\Phi_M}{dt} + N_s.\frac{d\Phi_{LP}}{dt}$ $V_s(t) = N_s.\frac{d\Phi_M}{dt} + N_s.\frac{d\Phi_{Ls}}{dt}$

$V_p(t) = e_p(t). + e_{Lp}(t)$ $V_s(t) = e_s(t). + e_{Ls}(t)$

$$\frac{e_p(t)}{e_s(t)} = \frac{N_p}{N_s} = a$$

The ratio of $e_p(t)$ to $e_p(t)$ caused by the mutual flux=turns ration of the transformer

In a well-designed transformer $\Phi_M \succ\succ \Phi_{Lp}$ and $\Phi_M \succ\succ \Phi_{Ls}$ ------the ratio of the total voltage on the primary to secondary of a transformer is approximately

$$\frac{V_s(t)}{V_p(t)} = \frac{N_s}{N_p} = a$$

The leakage flux in the primary windings Φ_{LP} and secondary winding Φ_{Ls} produces a voltage e_{LP} and e_{LP} respectively.

$e_{Lp}(t) = N_p.\frac{d\Phi_{LP}}{dt}$ $e_{Ls}(t) = N_s.\frac{d\Phi_{Ls}}{dt}$

$\Phi_{LP} = (\rho.N_p).i_p$ $\Phi_{Ls} = (\rho.N_s).i_s$

Where

ρ - permeance of flux path.

$e_{Lp}(t) = N_p^2.\rho.\frac{di_P}{dt}$ $e_{Ls}(t) = N_s^2.\rho.\frac{di_s}{dt}$

$e_{Lp}(t) = L_P\frac{di_P}{dt}$ $e_{Ls}(t) = L_s\frac{di_s}{dt}$

Where

$L_P = N_p^2.\rho$ is the self inductance of the primary coil

$L_s = N_s^2.\rho$ is the self inductance of the secondary coil

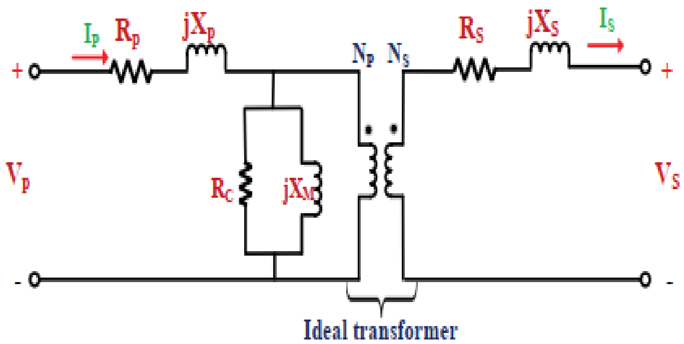


Figure 5.8 Equivalent circuit of a real transformer.

V.7.1. Approximate equivalent circuit of transformer

If only voltage regulation is to be calculated, then even the whole excitation branch (parallel combination of R_C and X_M) can be neglected. Then the equivalent circuit becomes as shown in the figure below [16].

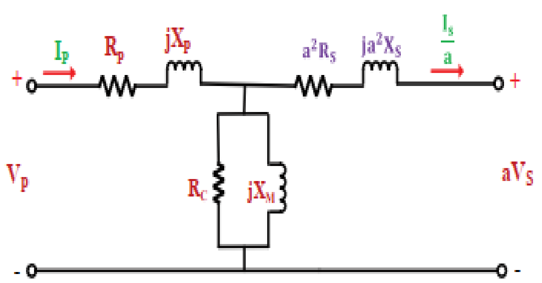


Figure 5.9 The transformer model referred to its primary voltage level.

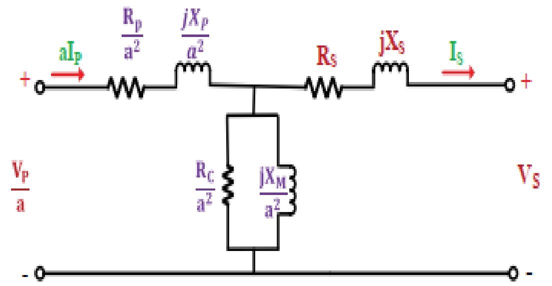


Figure 5.10 The transformer model referred to its secondary voltage level.

$$R_{eqp} = R_p + a^2.R_s$$

$$R_{eqs} = \frac{R_p}{a^2.} + R_s$$

$$X_{eqp} = X_p + a^2.X_s$$

$$X_{eqs} = \frac{X_p}{a^2.} + R_s$$

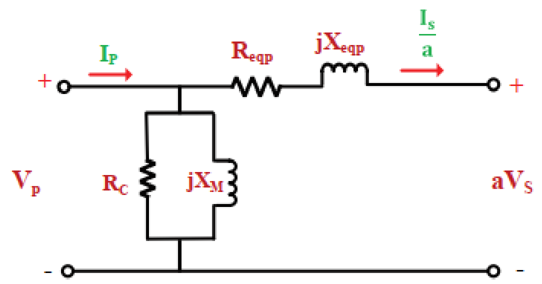


Figure 5.11 Approximate transformer model referred to the primary side

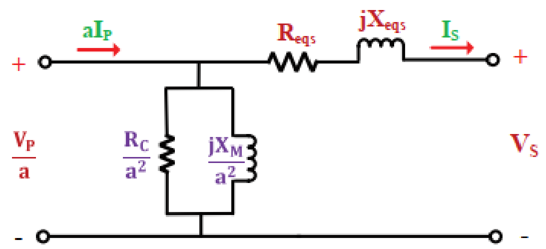


Figure 5.12 Approximate transformer model referred to the secondary side

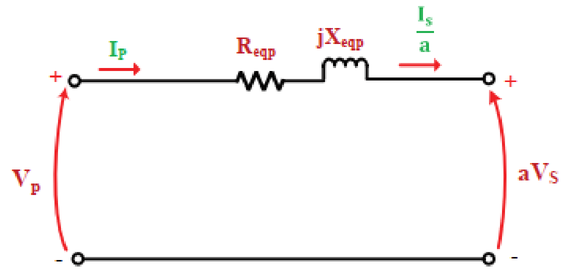


Figure 5.13 Approximate transformer model with no excitation branch, referred to the primary side

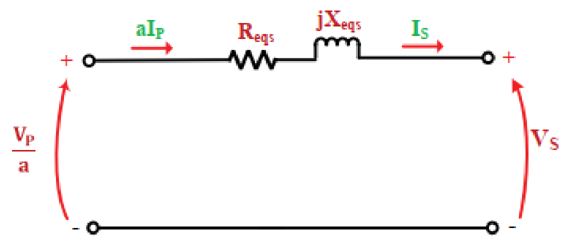


Figure 5.14 Approximate transformer model with no excitation branch, referred to the secondary side

V.8 Losses in Transformer

In any electrical machine, 'loss' can be defined as the difference between input power and output power. An electrical transformer is an static device, hence mechanical losses (like windage or friction losses) are absent in it. A transformer only consists of electrical losses (iron losses and copper losses). Transformer losses are similar to losses Losses in a DC machine except that transformers do not have mechanical losses [17].

Losses in transformer are explained below

V.8.1 Core Losses or Iron Losses

Eddy current loss and hysteresis loss depend upon the magnetic properties of the material used for the construction of core. Hence these losses are also known as **core losses** or **iron losses**.

V.8.1.1 Hysteresis loss in transformer: Hysteresis loss is due to reversal of magnetization in the transformer core. This loss depends upon the volume and grade of the iron, frequency of magnetic reversals and value of flux density. It can be given by, Steinmetz formula:

$W_h = \eta B_{max} 1.6fV$ (watts)

Where,

η = Steinmetz hysteresis constant

V = volume of the core in m^3

V.8.1.2 Eddy current loss in transformer: In transformer, AC current is supplied to the primary winding which sets up alternating magnetizing flux. When this flux links with secondary winding, it produces induced Emf in it. But some part of this flux also gets linked with other conducting parts like steel core or iron body or the transformer, which will result in induced emf in those parts, causing small circulating current in them.

This current is called as eddy current. Due to these eddy currents, some energy will be dissipated in the form of heat.

V.8.2 Copper Loss in Transformer

Copper loss is due to ohmic resistance of the transformer windings. Copper loss for the primary winding is $I_1^2R_1$ and for secondary winding is $I_2^2R_2$. Where, I_1 and I_2 are current in primary and secondary winding respectively, R_1 and R_2 are the resistances of primary and secondary winding respectively. Cu loss is proportional to square of the current, and current depends on the load. Hence copper loss in transformer varies with the load.

V.9 Efficiency of Transformer

Just like any other electrical machine, **efficiency of a transformer** can be defined as the output power divided by the input power. That is efficiency = output / input.

Transformers are the most highly efficient electrical devices. Most of the transformers have full load efficiency between 95% to 98.5%. As a transformer being highly efficient, output and input are having nearly same value, and hence it is impractical to measure the efficiency of transformer by using output / input. A better method to find efficiency of a transformer is using, efficiency = (input - losses) / input = 1 - (losses / input).

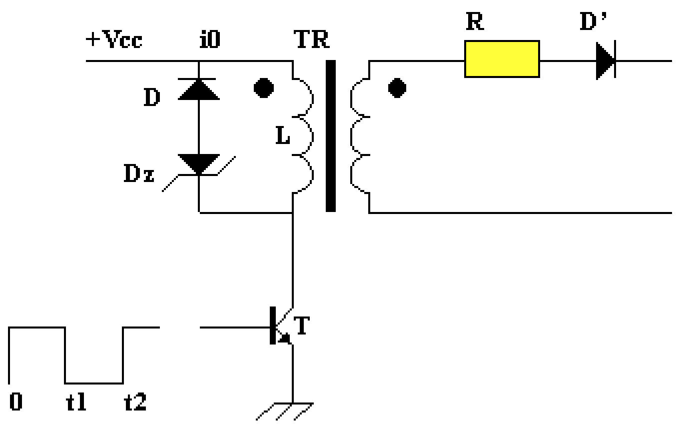
V.10 Other transformers [17]

V.10 .1. Isolation Transformer

An isolation transformer is an electrical device used to transfer electrical energy from a primary circuit to a secondary circuit, while ensuring galvanic isolation between the two circuits. Galvanic isolation prevents the direct passage of electrical current between the circuits, providing a higher level of safety for users and protecting equipment from damage caused by overloads or short circuits.

V.10 .2. Pulse Transformers

Pulse transformers are used to control thyristors, triacs and transistors. Compared with opto-couplers, pulse transformers offer the following advantages: high-frequency operation, simplified assembly, ability to supply high currents, good voltage withstand capability, etc.



V.10.3 Auto Transformer

An A**uto Transformer** is a transformer with only one winding wound on a laminated core. An auto transformer is like a two winding transformer but differ in the way the primary and secondary winding are interrelated. A part of the winding is common to both primary and secondary sides. On load condition, a part of the load current is obtained directly from the supply and the remaining part is obtained by transformer action. An Auto transformer works as a **voltage regulator.**

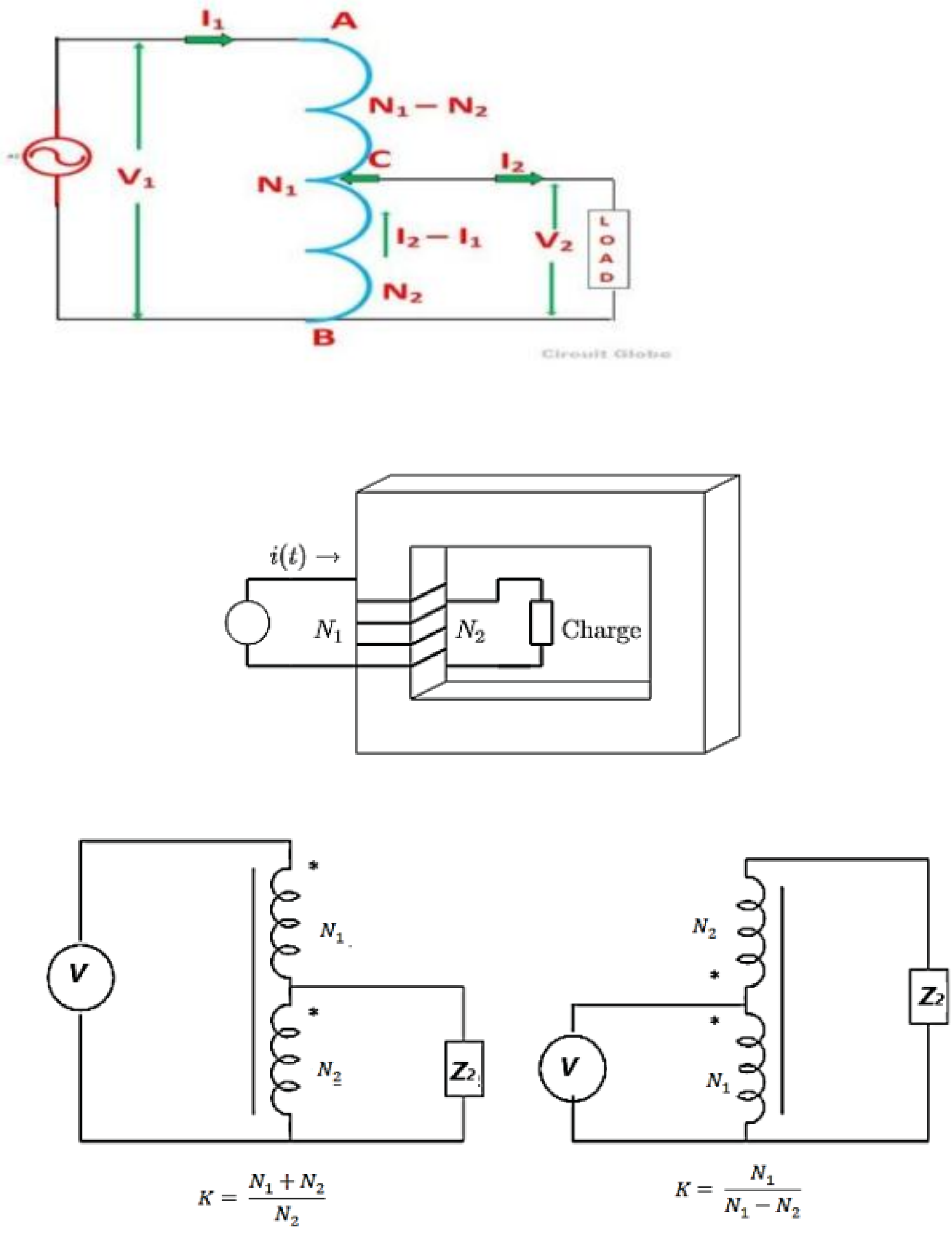


Figure 5.15 Auto – Transformer.

V.10.4 Three-phase transformer

A three-phase transformer consists of three transformers, either separate or combined on one core. The primaries and secondary's of any three-phase transformer can be independently connected in either a wye (Y) or a delta (Δ).

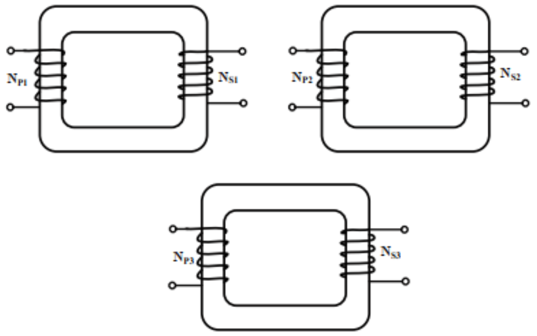


Figure 5.16 A three-phase transformer bank composed of independent transformers

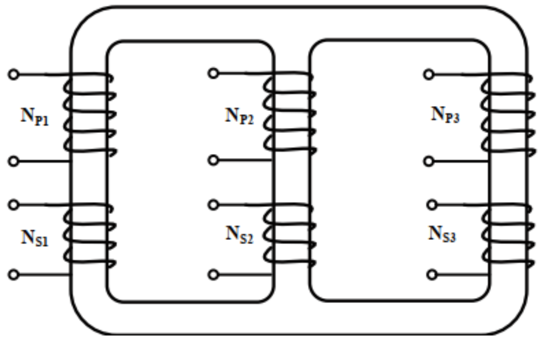


Figure 5.17 A three-phase transformer wound on a single three-legged core.

Chapter VI

Introduction to Electrical Machines

VI.1 Introduction

The **electric mach**ine is an electromechanical converter for bidirectional energy conversion. Depending on the method used to create the magnetic field, a distinction is made between electromagnetic excitations[19].

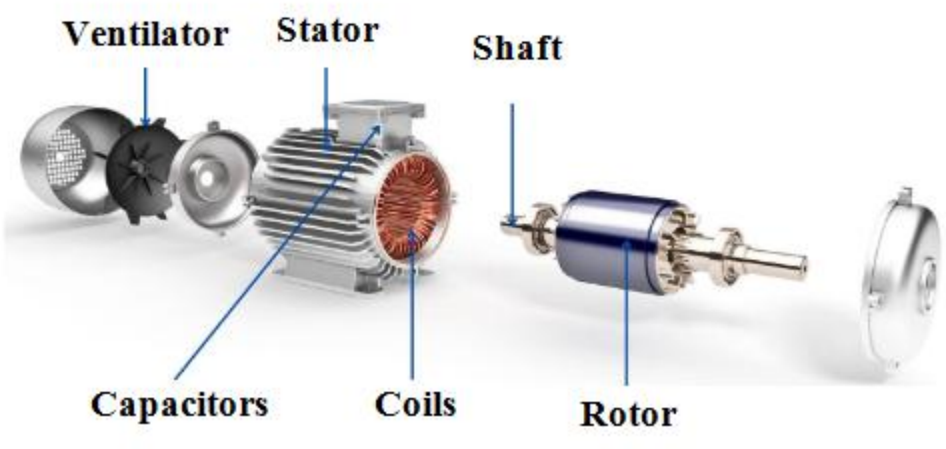


Figure 6.1 Electrical machines.

VI. 2 Construction of an electrical machine

From a mechanical point of view, rotating machines can be broken down into four distinct parts [20]: (fig1.1)

- **The stator**; fixed part of the machine, where the power supply is connected.
- **The rotor;** the rotating part that sets the mechanical load in rotation.

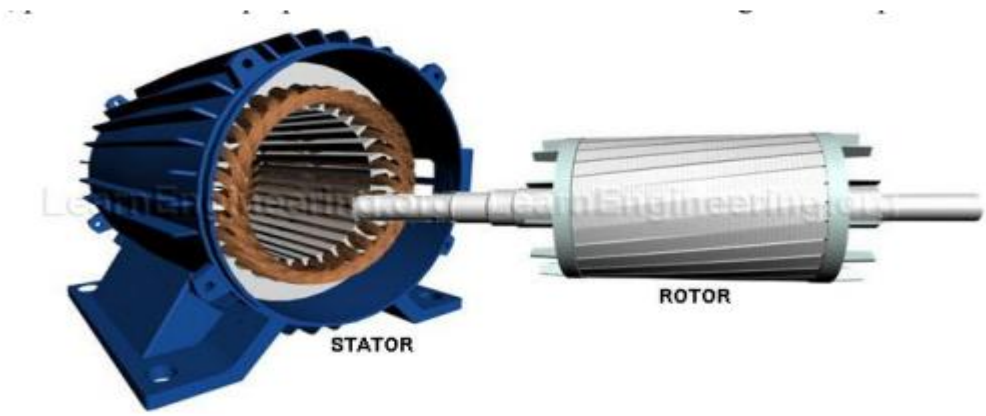


Figure 6.2 Motor constitution

- **Bearings:** the mechanical part that allows the motor shaft to rotate, in the case of bearings, they are precision parts manufactured with great care, so they must be handled with care (fig1.2).

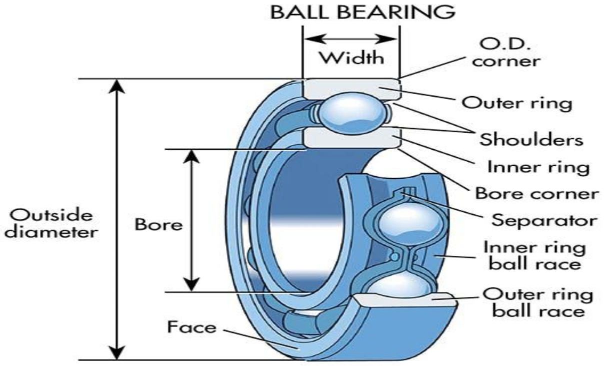


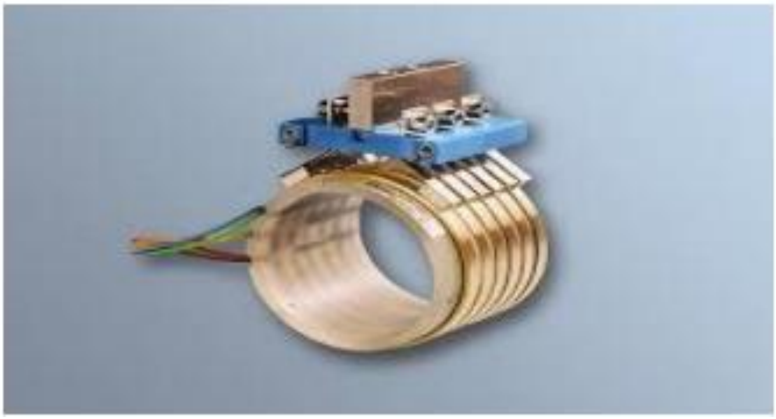
Figure 6.3 Exploded view of a ball bearing.

- **Shaft:** transmission of rotary motion. It comprises :

- A central part which supports the rotor body, the magnetic circuit and the rotating windings ;

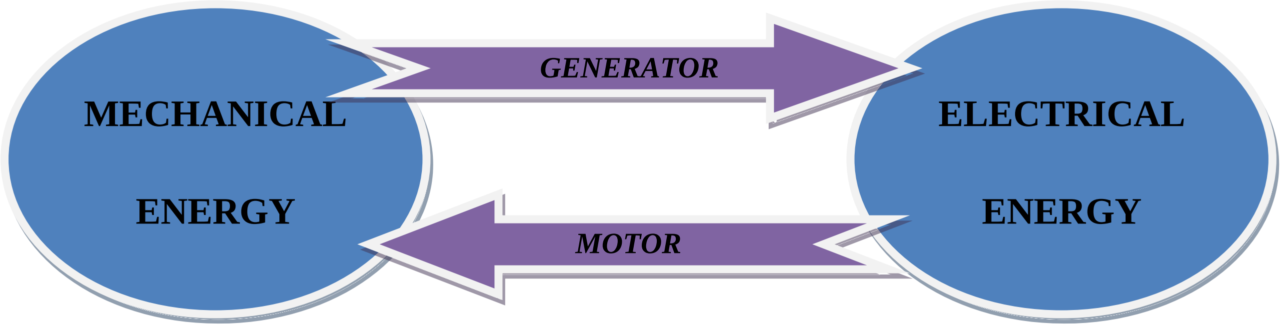
- In general, a shaft extension to which the coupling half connecting the electrical machine to the mechanical machine is attached.

- **The collector**

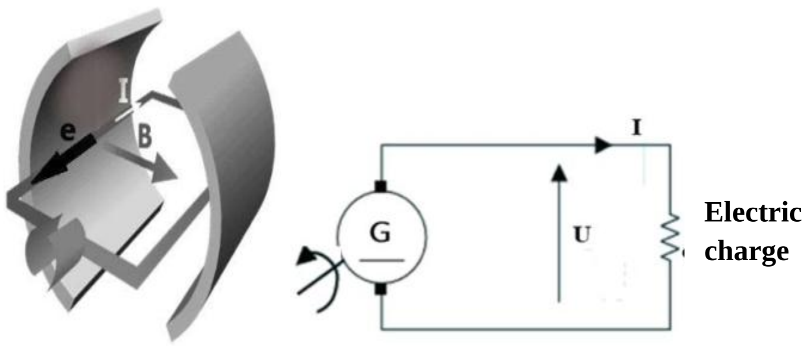


VI.3 Principle of operation

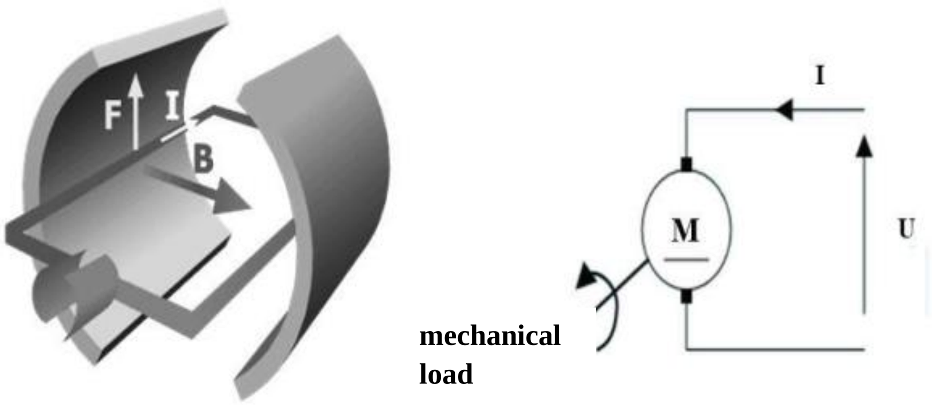
Electric machines involve electric currents and magnetic fields as fundamental elements. Operation is based on the laws of electromagnetism. Electrical machines fall into two main categories [19]:



Generating machines: convert mechanical energy into electrical energy. Their operation is based on the induction of an electric current in a conductive circuit by relative displacement of the circuit and a magnetic field, using a mechanical drive. The electric current induced is either direct or alternating, and the generating machine is called a dynamo or alternator.



Electric motors: their operation is based on obtaining a mechanical force by the action of a magnetic field on an electric circuit through which a current flows, supplied by an external source, which may also produce the magnetic field. The electric current supplied by the external source may be direct or alternating, and the machine is referred to as a direct-current motor or alternating-current motor (synchronous or asynchronous).



VI.4 Power balance

The power balance shows all powers, from mechanical power input to electrical power output[19].

VI.4.1 Generators case

The balance can be summarized using the following diagram:

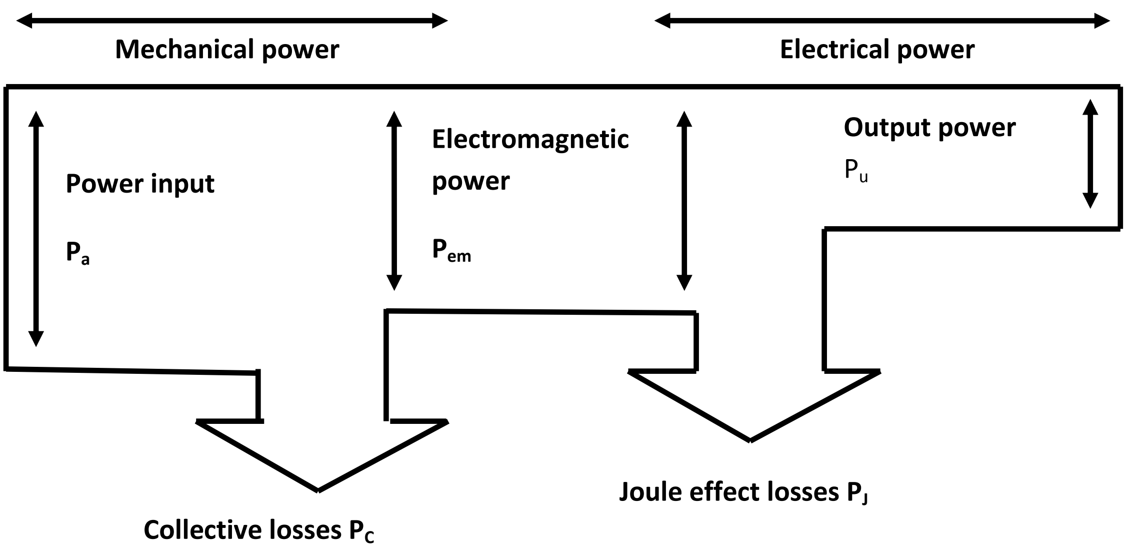


Figure 6.4 Power balance of a generator.

The generator receives a power Pa, the product of the mechanical torque T □ □ from an auxiliary system and the angular velocity All the powers involved in this balance can be calculated from the following relationships

$\mathbf{P_a = T.\,\Omega}$

Pa: Power input in watts [W];

T: Mechanical torque in Newton-meters [N.m];

Ω : Angular speed in radians per second [rad/s].

$\mathbf{P_c = T.\Omega}$

Pc: Collective losses in watts [W];

Tp: Loss torque in Newton-meters [N.m];

Ω : Angular velocity in radians per second [rad/s].

$\mathbf{P_{em} = T_{em}.\,\Omega}$

Pem: Electromagnetic power in watts [W];

Tem: Electromagnetic torque in newton-meters [Nm];

Ω : Angular velocity in radians per second [rad/s].

$$\mathbf{P_{em} = E.\ I}$$

Pem: Electromagnetic power in watts [W];

E: Fem of the generator in volts [V];

I: Armature current in amperes [A].

$$\mathbf{P_j = R.\ I^2}$$

Pj: Joule effect losses in watts [W];

R: Armature resistance in ohms.

I: Armature current in amperes [A].

$$\mathbf{P_u = U.\ I}$$

Pu: Output power in watts [W];

U: Voltage delivered by the generator armature in volts [V];

I: Current in the armature in amperes [A].

Efficiency is the ratio between the useful mechanical power and the electrical power absorbed by the armature, hence:

$$\boldsymbol{\eta} = \mathbf{P_u / P_a}$$

η: Generator armature efficiency

Pu: Output power in watts [W];

Pa: Absorbed power in watts [W].

VI.4.2 Motor case

The balance can be summarized using the following diagram:

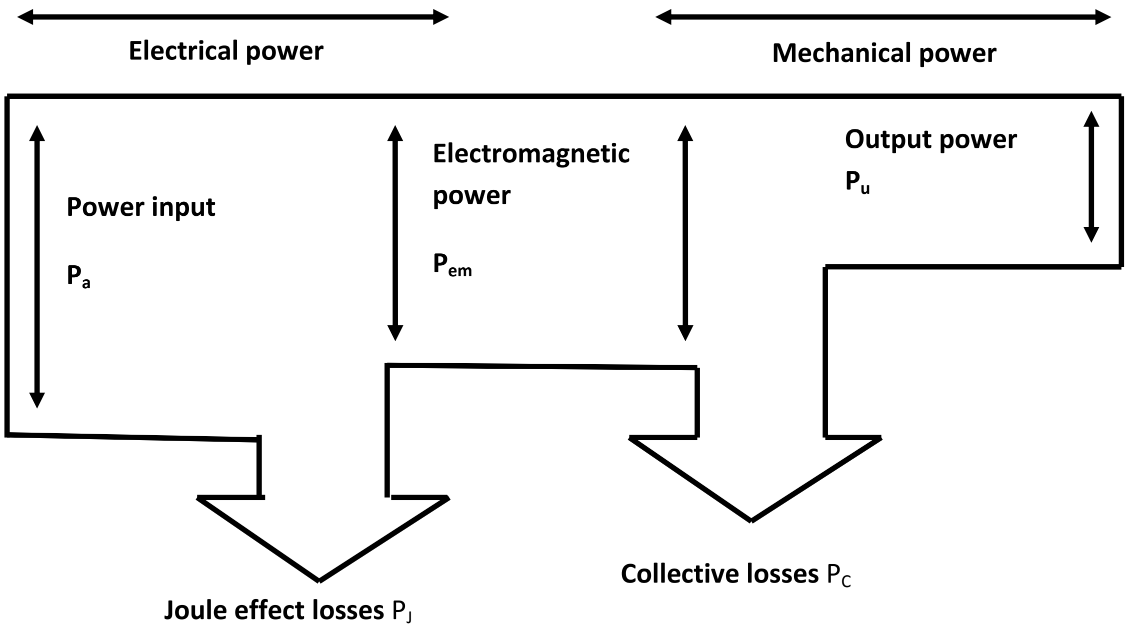


Figure 6.5 motor power balances.

Pa = U. I

Pa: Absorbed power in watts [W];

U: Armature terminal voltage in volts [V];

I: Armature current in amperes [A].

$\mathbf{P_u = T.\ \Omega}$

Pu: Power output in watts [W];

T: Mechanical torque in Newton-meters [N.m];

Ω : Angular speed in radians per second [rad/s].

Efficiency is the ratio between useful mechanical power and electrical power absorbed by the armature, hence:

η =Pu/ Pa

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