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By :
BENSEGHIR Selsabil & HALTALI Amina

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**Modeling and Identification of Dynamical Systems
and Time Series Using Fuzzy Logic Type 3**

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Prof. BENNACER Hamza	University of M'Sila	Chair
Prof. LADJAL Mohamed	University of M'Sila	Examiner
Prof. OUALI Mohamed Assam	University of M'Sila	Supervisor
Dr. HADJAB Moufdi	University of M'Sila	Co- Supervisor

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بِسْمِ اللَّهِ الرَّحْمَنِ الرَّحِيمِ



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giving us the health and patience to complete this modest work.*

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Dedication

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A. Most Commonly Used Abbreviations

DS :	Dynamical System.
TS :	Time Series.
ECG :	Electrocardiogram.
EEG :	Electroencephalogram.
EMG :	Electromyography.
SISO :	Single Input Single Output.
MIMO:	Multiple Input Multiple Output.
LTI :	Linear Time Invariant.
AR :	Auto-Regressive.
TF :	Transfer Function.
SS :	State Space.
BIBO:	Bounded Input Bounded Output.
ARX:	Auto-Regressive with Exogenous Input.
I/O :	Input/Output.
ODE :	Ordinary Differential Equation.
PID:	Proportional Integral Derivative.
FL :	Fuzzy Logic.
T1-FS:	Type-1 Fuzzy Set.
T2-FS:	Type-2 Fuzzy Set.
IT2-FS:	Interval -2 Fuzzy Set.
GT2 FS:	General Type-2 Type Fuzzy Set.
IT3-FS:	Interval Type-3 Fuzzy Set.
FOU:	Footprint of Uncertainty.
TSK:	Takagi–Sugeno– Kang.
FIS:	Fuzzy Inference System.
KM:	Karnik–Mendel.
RMSE:	Root Mean Square Error.
MSE:	Mean Square Error.
MAE:	Mean Absolute Error.
NMSE:	Normalized Mean Square Error.
CPU:	Central Processing Unit.
AI:	Artificial Intelligence.

B. List of Symboles

$x(t)$:	State vector.
$u(T)$:	Input signal.
$y(t)$:	Output signal.
t :	Time.
p :	Plant/System.
$e(t)$:	Error.
A, B, C, D :	State matrices.
$G(s)$:	Transfer function.
$X(s)$:	Laplace transform input.
$Y(s)$:	Laplace transform output.
s :	Complex variable.
θ :	Parameter vector.
$\mu(x)$:	Membership function.
X :	Universe of discourse.
A, B :	Fuzzy sets.
σ :	Standard deviation.
m :	Mean value.
T :	T-norm.
S :	S-norm
$\hat{y}$:	Estimated output.
y :	Actual output.
e :	Error.
N :	Number of samples

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General Introduction

Modeling and prediction of dynamical systems and time series have become important research topics in many scientific and engineering fields due to their wide range of applications, including industrial processes, control systems, economics, energy management, and biomedical engineering. These systems are often characterized by nonlinear behaviors, uncertainties, noise, and incomplete information, which make conventional mathematical modeling approaches difficult to apply effectively.

To address these challenges, intelligent modeling techniques have attracted considerable attention in recent decades. Among these techniques, fuzzy logic systems have proven to be powerful tools for representing uncertain and imprecise information while preserving interpretability and flexibility. Traditional Type-1 fuzzy logic systems have been successfully used in the modeling and control of nonlinear dynamical systems. However, their capability to handle high levels of uncertainty remains limited in complex environments.

To overcome these limitations, advanced fuzzy frameworks such as Type-2 and Type-3 fuzzy logic systems have been introduced. In particular, Type-3 fuzzy systems provide a higher degree of freedom for uncertainty representation, allowing better handling of ambiguities and variations present in real-world dynamical systems and time series data. Their enhanced structure makes them suitable for improving modeling accuracy and robustness in uncertain and nonlinear conditions.

A crucial aspect of fuzzy system design lies in the selection and adjustment of membership function parameters, as these parameters significantly influence the overall system performance. Consequently, optimization techniques are commonly employed to automatically tune the membership functions in order to achieve better prediction accuracy and system adaptability.

This work focuses on the modeling and analysis of dynamical systems and time series using Type-3 fuzzy logic systems. The proposed work aims to develop an efficient fuzzy modeling framework capable of accurately representing nonlinear dynamics and managing uncertainties associated with real-world data. Furthermore, optimization methods are employed to adjust the parameters of the Type-3 membership functions in order to enhance the performance of the proposed model.

The main objectives of this work are:

- To study the fundamental concepts of dynamical systems, time series analysis, and fuzzy logic theory;
- To investigate the structure and properties of Type-3 fuzzy systems;
- To develop a fuzzy modeling approach for nonlinear dynamical systems and time series;
- To optimize the parameters of the membership functions using suitable optimization techniques;
- To evaluate the effectiveness of the proposed approach through simulations

This dissertation is organized as follows. The three first chapters present the theoretical background related to dynamical systems, time series, modeling and identification and fuzzy logic concepts. The fourth chapter describes the proposed modeling methodology and presents the simulation results, performance evaluation, conclusions, and future research perspectives.

Chapter I

Dynamical Systems and Time Series

I.1. Introduction

A dynamical system is a mathematical model that describes how the state of a system evolves over time according to a set of rules, often expressed by differential or difference equations. A time series is an ordered sequence of observations of a system recorded over time like ECG, EEG, EMG. Real-world dynamical systems and times series are often nonlinear, uncertain, and subject to disturbances, making their modeling and prediction challenging. Traditional methods may fail to capture this complexity, especially when data are imprecise or noisy. Type-3 fuzzy logic provides an advanced approach for handling uncertainty and improving the identification and prediction of dynamical systems and time-series behavior.

In this chapter, we will address two main sections of this work: dynamical systems and time series theory. The first section focuses on all aspects related to dynamical systems (State of the art, characterization and identification of systems, the Weierstrass theorem, the Stone-Weierstrum theorem, input-output state Representation of systems, linear systems, and Nonlinear Systems), while the second section is dedicated to time series (definition of time series, characteristics of time series, different Type of time series, component of time series and electrocardiogram).

I.2. State of the Art

Mathematical systems theory has evolved over five decades into a powerful scientific discipline with wide applicability, concerned with the analysis and synthesis of dynamical systems. The most advanced using well-established methods based on linear algebra, complex variable theory, and the theory of ordinary linear differential equations, this part of the theory deals with systems defined by linear operators. Well-known design techniques have been established for dynamical systems because they are closely related to their stability properties, and because over the past century, necessary and sufficient conditions for the stability of linear time-invariant systems have been generated.

On the other hand, since the stability of nonlinear systems can typically only be determined on a system-by-system basis, it is not surprising that there are currently no design techniques for large classes of such systems that simultaneously satisfy the requirements of stability, robustness, and good dynamical response.

Significant progress has been achieved in adaptive identification and control over the last thirty years to identify and manage linear time-invariant plants with unknown constraints.

Based on well-established findings in linear systems theory, the identifier and controller structures were chosen. In these situations, stable adaptive rules for parameter adjustment that guarantee the global stability of the pertinent overall systems are likewise based on linear system characteristics and stability outcomes that are renowned for these kinds of systems [1].

As a result, the proposed models represent a first step in this direction. The second goal is to present a prescriptive approach for the dynamic modification of the parameters. founded on back propagation. In this context, the phrase "dynamic back propagation" is introduced. Clearly stating the numerous theoretical presumptions that must be made in order to have well-posed problems is the third and final goal. Throughout the study, computer simulations and block diagram representations of systems that are frequently employed in systems theory are utilized to demonstrate the different ideas presented.

I.3. Characterization and Identification of Systems

In systems theory, system identification and characterization are basic issues. The mathematical representation of a system is the focus of the characterization problem; an operator is used to convey a model of a system P from a space of input $\mathcal{U}$ into a space of output $\mathcal{Y}$ and the goal is to describe the class $\mathbb{P}$ which P belongs. Considering a class $\mathbb{P}$ and the reality that $P \in \mathbb{P}$ determining a class is the identification problem $\hat{\mathcal{P}} \subset \mathcal{P}$ and a component $\hat{p} \in \hat{\mathcal{P}}$ so that $\hat{p}$ approximations P in a preferred way. The spaces in static systems $\mathcal{U}$ and $\mathcal{Y}$ are subsets of $\mathbb{R}^n$ and $\mathbb{R}^m$ respectively, whereas it is typically assumed that they are bounded Lebesgue integrable functions on the interval in dynamical systems $[0, T]$ or $[0, \infty]$ in all situations, the designated input-output pairs implicitly define the operator P . The selection of the identification model class $\hat{\mathcal{P}}$ as well as the particular technique employed to ascertain $\hat{p}$ depends on a number of variables that are connected to both analytical attractiveness and the required level of accuracy. Among them is the model's suitability $\hat{p}$ to represent the simplicity of P , how easily it can be found, how easily it can be expanded if it doesn't meet requirements, and ultimately whether the $\hat{p}$ decided whether to utilize it online or offline. In real-world applications, a lot of these choices inevitably rely on the prior knowledge of the plant that needs to be identified [1].

I.4. Identification of Static and Dynamic Systems

One common example of identifying static systems is the pattern recognition challenge. Small sets $U_i \subset \mathbb{R}^n$ are divided into elements $\gamma_i \in \mathbb{R}^m$; ($i = 1, 2, \dots$) by a decision function P in the output space. The components of U_i indicate the class-corresponding pattern vectors

γ_i . In dynamical systems, the input-output pairings of time automatically define the operator P defining a particular plant. Operations $u(t), y(t), t \in [0, T]$. In both situations, the goal is to ascertain $\hat{p}$ so that:

$$\|\hat{y} - y\| = \|\hat{p}(u) - p(u)\| \leq \epsilon, \quad u \in \mathcal{U}. \quad (\text{I.1})$$

for some desired $\epsilon > 0$ and a suitably defined norm (de-noted by $\|\cdot\|$) on the output space in (1), $\hat{p}(u) = \hat{y}$ de-notes the identification model's output, and so $\hat{y} - y \triangleq e$ is the error between the output produced by $\hat{p}$ and the output that was noticed [1].

I.5. The Weierstrass Theorem and the Stone-Weierstrum Theorem

Let $C([a, b])$ represent the interval-defined space of continuous real-valued functions $[a, b]$ with the norm of $f \in C([a, b])$ defined by:

$$\|f\| = \sup \{ |f(t)| : t \in [a, b] \}. \quad (\text{I.2})$$

According to Weierstrass's well-known approximation theorem, any function in $C([a, b])$ can be arbitrarily closely approximated by a polynomial. Alternatively, the collection of polynomials is dense in $C([a, b])$. Naturally, the approximation of continuous functions makes extensive use of Weierstrass's theorem and its generalization to several dimensions: $f: \mathbb{R}^n \rightarrow \mathbb{R}^m$ using polynomials. All approximation processes for dynamical systems can begin with the Stone-Weierstrass theorem, a generalization of Weierstrass's theorem ascribed to Stone.

 **Theorem** (Stone-Weierstrass [2]): Let $\mathcal{U}$ be a metric space that is compact. Should $\hat{\mathcal{P}}$ is a subalgebra of $C(\mathcal{U}, \mathbb{R})$ which contain the constant functions and separates points of $\mathcal{U}$ then $\hat{\mathcal{P}}$ is dense in $C(\mathcal{U}, \mathbb{R})$.

In the issues that interest us, we will presume that the plant P to be recognized as belonging to the area $\wp$ of causal, time-invariant, bounded, and continuous operators [3]. The Stone-Weierstrass theorem states that if $\hat{\mathcal{P}}$ meets the requirements of the theorem, a model that is a part of $\hat{\mathcal{P}}$ can be selected to approximate any given operator $P \in \wp$.

The classic works of Volterra, Wiener, Barret, and Urysohn are among the many studies on the characterization of nonlinear functional. The Stone-Weierstrass theorem can be used to demonstrate that, under specific circumstances, a given nonlinear functional can be represented by a corresponding series, such as the Wiener series or the Volterra series. Despite the impressive theoretical few of the work that these represent has been widely used

to identify significant classes of real-world nonlinear systems. Our focus in this study is primarily on representations that allow dynamic systems to be identified and controlled online using finite dimensional nonlinear difference (or differential) equations. These nonlinear models are discussed in the next paragraph and are widely recognized in the systems literature.

I.6. Input-State-Output Representation of Systems

In systems theory, the technique of using vector differential or difference equations to model dynamical systems is currently well-established and applicable to a sizable class of systems. The differential equations can describe as following:

$$\begin{aligned} \frac{dx(t)}{dt} &\triangleq \dot{x}(t) = \phi [x(t), u(t)] & t \in \mathbb{R}^+ \\ y(t) &= \psi [x(t)] \end{aligned} \quad (I.3)$$

where $x(t) \triangleq [x_1(t), x_2(t), \dots, x_n(t)]^T$, $u(t) \triangleq [u_1(t), u_2(t), \dots, u_p(t)]^T$ and, $y(t) \triangleq [y_1(t), y_2(t), \dots, y_m(t)]^T$ depict an order system with p inputs and m outputs n with $u_i(t)$ representing the inputs, $x_i(t)$ the state variables, and $y_i(t)$ the outputs of the system ϕ and ψ are static nonlinear maps defined as $\phi : \mathbb{R}^n \times \mathbb{R}^p \rightarrow \mathbb{R}^n$ and $\psi : \mathbb{R}^n \rightarrow \mathbb{R}^m$. The system's state at time t is represented by the vector $x(t)$, which is based on the state at time $t_0 < t$ and the interval-defined input $u [t_0, t]$. The output $y(t)$ is determined totally by the state of the system at time t . The input-state-output representation of the system is represented by equation (I.2). Discrete-time systems, which can be described by difference equation corresponding to the differential equations stated in (I.3). These take the shape of:

$$\begin{aligned} x(k+1) &= \phi [x(k), u(k)] \\ y(k) &= \psi [x(k)] \end{aligned} \quad (I.4)$$

where $u(\cdot)$, $x(\cdot)$, and $y(\cdot)$ are sequences of discrete time. Nonetheless, the majority of the findings can also be applied to continuous time systems. Assuming that the system described in (I.3) is time invariant and linear, its behavior is governed by the following equations:

$$\begin{aligned} x(k+1) &= Ax(k) + Bu(k) \\ y(k) &= Cx(k) \end{aligned} \quad (I.5)$$

where A , B , and C are $(n \times n)$, $(n \times p)$, and $(m \times n)$ matrices, respectively. The system is then parameterized by the triple $\{C, A, B\}$. The theory of linear time-invariant systems, when C , A , and B are known, is highly developed, and over the past three decades, ideas like controllability, stability, and observability of such systems have been thoroughly investigated. Techniques for figuring out the control input $u(\cdot)$ to maximize a performance criteria are also widely recognized. The tractability of these different issues may be ultimately attributed to the fact that they may be reduced to the solution of n linear equations with n unknowns. On the other hand, the nonlinear equation problems of the form (4), where the functions ϕ and ψ are known, produce nonlinear algebraic equations for which there are no comparable effective solutions. As a result, in order to make the issues analytically tractable, a number of assumptions must be made, as demonstrated in the sections that follow [1].

I.7. Identification

When the functions ϕ and ψ in (I.4), or the matrices A , B , and C in (I.5), are unknown, the identification of the unknown system referred to as the plant in the sections that follow becomes a challenge [4, 5].

The input and output of a time-invariant, causal discrete-time dynamical plant are $u(\cdot)$ and $y(\cdot)$, respectively, where $u(\cdot)$ is a uniformly limited function of time. Although the parameters' values are unknown, it is presumed that the plant is stable given a known parameterization.

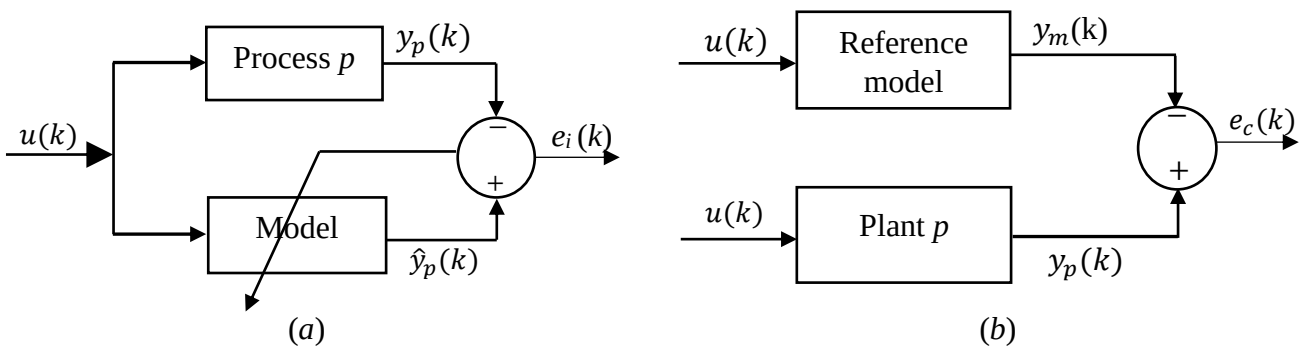


Figure I.1. (a) Identification, (b) Model reference adaptive control [3].

The goal is to create an appropriate identification model (Fig. I.1 (a)) that, given the same input $u(k)$ as the plant, produces an output $\hat{y}_p(k)$ which approximates $y_p(k)$.

I.8. Linear Systems

The creation of identification models for linear time-invariant plants with unknown characteristics is currently widely recognized. For a controllable and observable single-input single-output (SISO) plant, the matrix A and the vectors B and C in (I.5) can be chosen such that the plant equation can be represented as:

$$y_p(k+1) = \sum_{i=0}^{n-1} \alpha_i y_p(k-i) + \sum_{j=0}^{m-1} \beta_j u(k-j) \quad (I.6)$$

where α_i and β_j are continuous unknown parameters. For the multi-input-output (MIMO) system, a comparable form is also feasible. This suggests that the output at time $k+1$ is a linear combination of the input and output's historical values. The selection of the following identification models is motivated by equation (I.5):

➤ **parallel model:**

$$\hat{y}_p(k+1) = \sum_{i=0}^{n-1} \hat{\alpha}_i(k) \hat{y}_p(k-i) + \sum_{j=0}^{m-1} \hat{\beta}_j(k) u(k-j). \quad (I.7)$$

➤ **Series parallel model**

$$\hat{y}_p(k+1) = \sum_{i=0}^{n-1} \hat{\alpha}_i(k) y_p(k-i) + \sum_{j=0}^{m-1} \hat{\beta}_j(k) u(k-j) \quad (I.8)$$

where $\hat{\alpha}_i (i = 0, 1, \dots, n-1)$ and $\hat{\beta}_j (j = 0, 1, \dots, m-1)$ are parameters that can be changed. The parallel identification model's (I.6) output at time $k+1$ is $\hat{y}_p(k+1)$ and is a linear combination of both the input's and its previous values. The series-parallel model $\hat{y}_p(k+1)$ is a linear mix of the plant's historical input and output values. The series-parallel model is proven to be better at producing stable adaptive laws. In such a situation, a typical adaptive algorithm has the form:

$$\hat{\alpha}_i(k+1) = \hat{\alpha}_i(k) - \eta \frac{e(k+1) y_p(k-i)}{1 + \sum_{i=0}^{n-1} Y_p^2(k-i) + \sum_{j=0}^{m-1} u^2(k-j)} \quad (I.9)$$

where $\eta > 0$ determines the step size. The continuous vector of plant properties in the topics that follow $[\alpha_0, \dots, \alpha_{n-1}, \beta_0, \dots, \beta_{m-1}]^T$ will be denoted by p and that of the identification model $[\hat{\alpha}_0, \dots, \hat{\alpha}_{n-1}, \hat{\beta}_0, \dots, \hat{\beta}_{m-1}]^T$ by $\hat{p}$.

It can be demonstrated that linear state feedback stabilizes controllable linear time-invariant plants. Adaptive controllers have been created using this information for such plants. For instance, the control input can be produced as a linear combination of the input and output's historical values if an upper bound on the plant's order is known. If $\theta(k)$ shows

the vector for the control parameter, and it can be demonstrated that a constant vector θ^* exists such that when $\theta(k) = \theta^*$. The input-output characteristics of the plant and controller are identical to those of the reference model. Adaptive algorithms to make modifications $\theta(k)$ are now widely recognized and have the overall form depicted in (I.8) in a steady manner [1].

I.9. Nonlinear Systems

Controllability and observability are crucial when formulating the identification and control problems for linear systems. Other well-known findings from the theory of linear systems are also used to select a reference model, appropriately parameterize the plant, and ensure that the desired controller is present. Controllability, observability, feedback stabilization, and observer design for nonlinear systems are among the topics that several authors have written in recent years [4, 5]. Despite these efforts, nonlinear systems lack constructive processes that are comparable to those for linear systems. Therefore, selecting identification and controller models for nonlinear plants is a challenging task, and effective identification and control must rely on a number of firm presumptions about the plant's input-output behavior. For instance, if a SISO system is given by equation (I.3), we will assume that n measurements of the input and output can be used to reconstruct the system's state. More specifically, $y_p(k) = \psi[x(k), Y_p(k+1) = \psi[\phi[x(k), u(k)], \dots, y_p(k+n-1) = \psi[\phi[\dots \phi[\phi[x(k), u(k)]u(k+1)], \dots, u(k+n-2)]]]$ yield n nonlinear equations in n unknowns $x(k)$ if $u(k), \dots, u(k+n-2), y_p(k), \dots, y_p(k+n-1)$ are given, and we'll suppose that for every set of values of $u(k)$ in a compact region in $\mathcal{U}$ there is a special solution to the aforementioned issue. This makes it possible to suggest identification methods for nonlinear systems along lines that are comparable to those in the linear case.

Even when the function ϕ is available and the state vector is known in (I.3), the calculation of $u(\cdot)$ getting the plant to follow a desired path is an issue that is equally challenging.

Therefore, it is necessary to assume the presence of appropriate inverse operators in order to generate the control input. If the input is thought to be generated by a controller structure $u(\cdot)$, additional presumptions must be made to guarantee the presence of a constant control parameter vector in order to accomplish the intended goal. All of these suggest that

rigorous solutions to the identification and control problems will require significant advancements in nonlinear control theory.

I.10. Time Series

I.10.1. Definition of time series

A time series (also referred to as a chronological or chronic series) consists of a collection of observations of a variable recorded sequentially over successive time intervals or specific points in time. It provides valuable information about how a phenomenon evolves over time. In other words, a time series represents the output obtained through the continuous monitoring and tracking of particular events or processes. The analysis of these observations helps to understand past behavior, forecast future outcomes, and support decision-making across various fields [6,7].

From a mathematical perspective, a time series can be expressed using the following functional relationship:

$$Y_T = f(t) \quad (\text{I.10})$$

where Y_T is the time series value at time t and t , can be depending on the case (minute, hour, day, week, year, etc.).

Time series data are commonly illustrated using graphical representations in which the horizontal axis corresponds to time and the vertical axis represents the values of the variable under investigation [8, 9]. Such visualizations are considered a fundamental step in time series analysis, as they clearly reveal patterns, trends, and fluctuations over time. These graphical insights form the basis for developing reliable models, thereby facilitating the analysis of the phenomenon's behavior, understanding its historical evolution, and predicting its future dynamics.

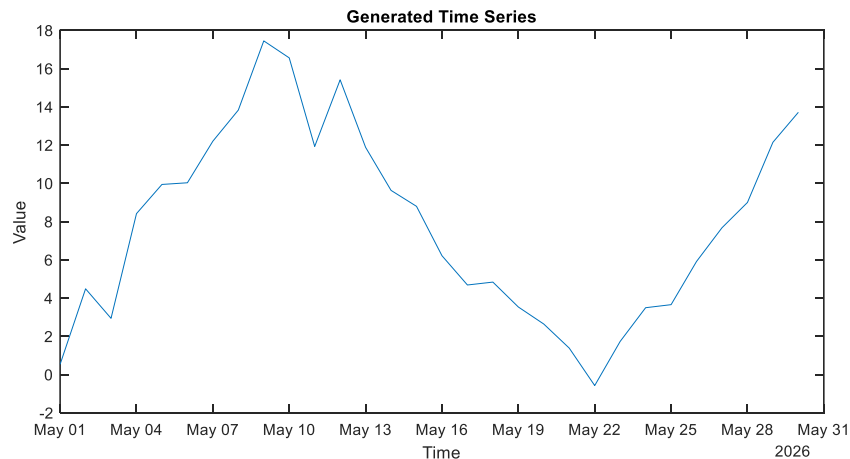


Figure I.2. Example of graphical representation of time series.

I.10.2. Time Series Characteristics

Time series data may display a variety of patterns that significantly influence the choice of analytical techniques and forecasting methods. The figure. I.3 highlights six fundamental characteristics of time series, each describing a distinct form of variation commonly observed in real-world data [10].

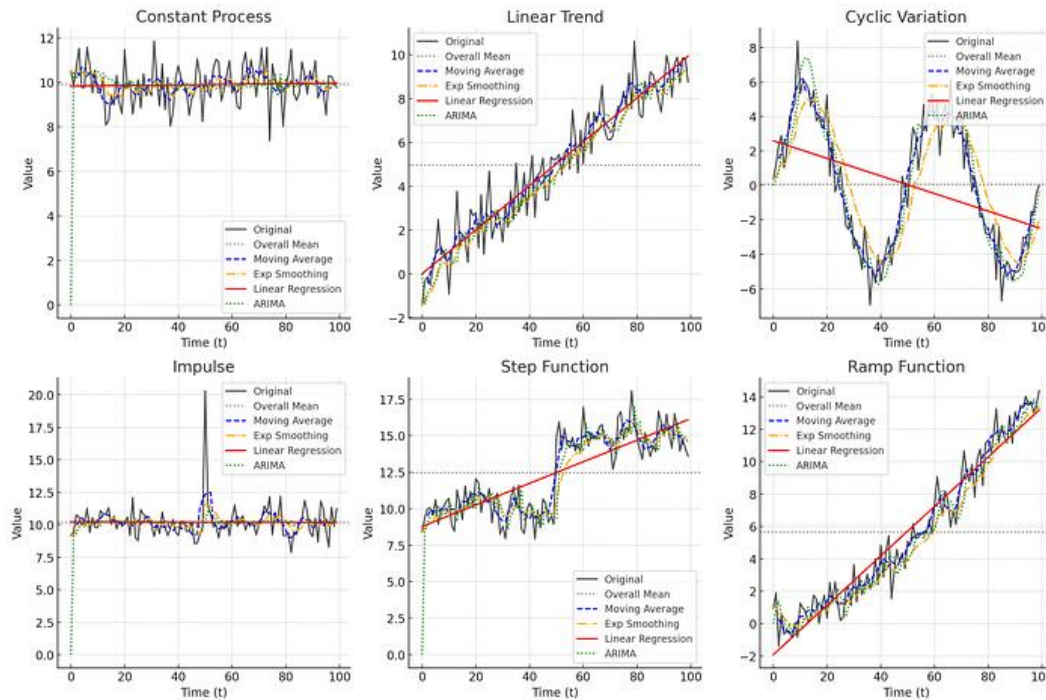


Figure I.3. Characteristics of Time Series Behaviors [10].

I.10.2.1. Constant Process (Stationary Mean)

- **Description:** A constant process oscillates around a fixed mean value, showing no apparent trend or seasonal pattern.
- **Example:** Daily temperature fluctuations measured in a controlled environment.
- **Importance:** Many statistical techniques, such as autoregressive (AR) models, rely on the assumption of stationarity. When the mean of a time series varies over time, transformations or differencing are often required to achieve stationarity.

I.10.2.2. Linear Trend

- **Description:** A time series that exhibits a steady upward or downward movement over time.

- **Example:** Population growth, inflation rates, or long-term trends in stock market indices.
- **Importance:** Identifying trends is crucial for long-term forecasting. Simple approaches like linear regression or moving averages can be applied, while advanced methods such as Holt's exponential smoothing allow for dynamic trend modeling.

I.10.2.3. Cyclic Variation

- **Description:** Fluctuations that repeat over time but do not follow a fixed or regular period, distinguishing them from seasonal patterns.
- **Example:** Economic business cycles characterized by alternating periods of expansion and recession.
- **Importance:** The analysis of cyclic behavior typically requires long historical records. Techniques such as spectral analysis or Fourier transforms are useful for identifying and studying these cycles.

I.10.2.4. Impulse (Transient Shock)

- **Description:** A sudden and isolated increase or decrease in the series that does not conform to a recurring pattern.
- **Example:** A significant political event influencing stock prices or a natural disaster disrupting supply chains.
- **Importance:** Impulsive events can adversely affect models that assume smooth or continuous behavior. Special treatments, including outlier detection or intervention analysis, are necessary to account for such shocks.

I.10.2.5. Step Function

- **Description:** An abrupt and permanent shift in the level of the time series.
- **Example:** The introduction of a new pricing policy by a company that results in a lasting increase in revenue.
- **Importance:** Step changes often signal structural breaks or regime shifts, requiring change-point detection or segmented modeling techniques.

I.10.2.6. Ramp Function (Nonlinear Trend)

- **Description:** A trend that changes its rate over time, either accelerating or decelerating.
- **Example:** The diffusion of new technology, where adoption starts slowly and then increases rapidly as acceptance grows.
- **Importance:** Linear models are inadequate for capturing nonlinear trends. More sophisticated methods, such as polynomial regression, logistic growth models, or machine learning techniques, are better suited for modeling such behavior.

I.10.3. Different Types of Time Series

I.10.3.1. Trend

The trend shows the general tendency of the data to increase or decrease during a long period of time. A trend is a smooth, general, long-term, average tendency. It is not always necessary that the increase or decrease is in the same direction throughout the given period of time. It is observable that the tendencies may increase, decrease or are stable in different sections of time (*E.g:* Population, Agricultural Production, No. of schools) [11].

I.10.3.2. Seasonal Variations

These are the rhythmic forces which operate in a regular and periodic manner over a span of less than a year. They have the same or almost the same pattern during a period of 12 months. This variation will be present in a time series if the data are recorded hourly, daily, weekly, quarterly, or monthly. These variations come into play either because of the natural forces or man-made conventions. The various seasons or climatic conditions play an important role in seasonal variations (*E.g:* production of crops depends on seasons, the sale of umbrella and raincoats in the rainy season, and the sale of electric fans and A.C. shoots up in summer seasons) [11].

I.10.3.3. Cyclic Variations

The variations in a time series which operate themselves over a span of more than one year are the cyclic variations. This oscillatory movement has a period of oscillation of more than a year. One complete period is a cycle. This cyclic movement is sometimes called the 'Business Cycle' It is a four-phase cycle comprising of the phases of prosperity, recession, depression, and recovery. The cyclic variation may be regular or not periodic. The upswings and the downswings in business depend upon the joint nature of the economic forces and the interaction between them [11].

I.10.3.4. Random or Irregular Movements

There is another factor which causes the variation in the variable under study. They are not regular variations and are purely random or irregular. These fluctuations are unforeseen, uncontrollable, unpredictable, and are erratic.

These forces are earthquakes, wars, flood, famines, and any other disasters [11].

I.10.4. Component of Time Series

I.10.4.1. Trend component

The trend represents the long-term movement or general direction of a time series over an extended period. It may be positive or negative, depending on whether the series shows a sustained increase or decrease over time. When no clear upward or downward movement is observed, the series is considered stationary in its mean. Accordingly, trends can be classified as: upward trend, downward trend, or stationary (horizontal) trend [12].

I.10.4.2. Cyclical component

The cyclical component refers to recurring patterns of rise and fall around the long-term trend. These fluctuations are not fixed in duration and usually occur over longer periods. The length and nature of cycles vary depending on the industry or economic activity under consideration.

I.10.4.3. Seasonal component

Seasonality arises when a time series displays regular and predictable variations at specific times within a year, such as the same month or quarter each year. A common example is retail sales, which typically increase during December due to holiday demand.

I.10.4.4. Irregular component

The irregular component captures random, unpredictable variations in a time series. This element reflects unforeseen events or random noise that cannot be systematically explained. In time series forecasting, the goal is to model the trend, cyclical, and seasonal components as accurately as possible so that the remaining unexplained variation corresponds only to this random component.

I.10.5. Electrocardiogram ECG

I.10.5.1. Definition

The electrocardiogram (ECG) is a graphic representation of the electrical potential that controls the heart's muscular activity. The electrodes on the skin's surface collect this

potential. These potentials are located at the boundary between the excited zones and those not myocardial excitation, and are measured between two body surface sites [13,14].

I.10.5.2. ECG Waves and Intervals

The electrocardiogram records the depolarization and repolarization of the auricular and ventricular muscle masses. This process is recorded as a series of deflections or oscillations superposed on a zero-potential line, often known as an isoelectric or base line. If these deflections are situated, they are considered positive. above the isoelectric line, and negative if not.

The ECG has many electrodes and varying intervals between them for each cardiac beat. (cf., Figure 1.6) [15, 16, 17, 18]:

- **P wave:** is associated with auricular depolarization. Rounded, it's a small-amplitude wave. The physiological rhythm is sinusoidal. In this instance, the auricular waves P often come before the ventricular complexes. Positive Sinusoidal waves P are always present.
- **PR interval:** is the time between the beginning of P and the beginning of QRS. It takes time for the electrical flow of Atria to be transmitted to the ventricles.
- **Segment PR:** this is a short, isoelectric segment that divides the ventricle's wave p.

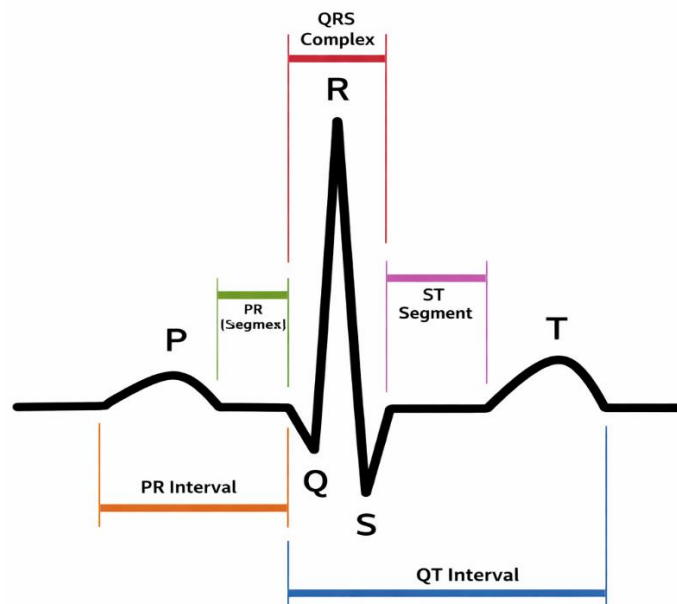


Figure I.4. Example of graphical representation of ECG signal [18].

- **The QRS Complex:** is made up of many fast deflections and is indicative of ventricular depolarization.
 - Q wave: physiologically, it has a small amplitude (less than one rung of the complex QRS).
 - R wave: whether or whether it precedes a Q wave, it is by definition the first positive wave.
 - S wave is a negative wave that follows a R wave.
- **Segment ST:** it corresponds to the onset of ventricular repolarization. It follows the base line horizontally.
- **T wave:** this is the electrical indicator of ventricular repolarization. Because of its gradual end, its duration is precise. Generally speaking, she is guided in the same direction as the QRS complex. Its shape is asymmetrical, with a primary. alternating between a Low slope, a rounded peak, and a second descending steep slope.
- **The QT interval:** is measured from the start of the QRS complex to the end of the T wave, represents the entirety of cardiac depolarization and repolarization.

The usual durations of the various ECG components in a healthy subject are the: wave $P < 0.12s$, complex $QRS = 0.06 \text{ à } 0.10s$, wave $T = 0.2s$ environ, interval $PR = 0.12 \text{ à } 0.20s$, segment $ST = 0 \text{ à } 0.16s$, interval $QT = 0.33 \text{ à } 0.43s$.

The usual amplitudes of waves P, Q, S, and T are around 0.1 mV; wave R may occasionally exceed 1 mV. The measurement of an RR space is used to determine the cardiac frequency.

I.11. Conclusion

This chapter established the theoretical foundation for dynamical systems and time series analysis, which are essential for modeling and understanding complex real-world phenomena. Key concepts of dynamical systems were introduced, including the differences between linear and nonlinear systems, along with challenges related to stability, controllability, observability, and system identification. Classical approximation results, such as the Weierstrass and Stone–Weierstrass theorems, were discussed to justify the use of suitable model structures for complex system behaviors. The importance of system identification was emphasized, particularly when dealing with unknown systems using input–output data, highlighting the limitations of conventional linear methods and motivating the adoption of intelligent techniques such as neural networks and fuzzy logic.

In addition, this chapter introduced time series analysis as a fundamental tool for describing and forecasting temporal dynamics, with particular emphasis on biomedical signals such as the electrocardiogram (ECG), which exemplifies nonlinear and dynamic real-world data. Overall, the chapter establishes the conceptual foundation for the subsequent chapters, preparing the ground for advanced modeling and identification approaches based on intelligent methods, including neural networks and Type-3 fuzzy logic.

Chapter II

Modeling and Identification Theory

II.1. Introduction

The goal of dynamic system modeling is to find a mathematical representation that is as straightforward and compact as feasible while yet accounting for the relationships that exist between the systems' inputs and outputs.

In this chapter, we present the fundamental concepts related to the modeling process, we will address two main sections of this work: Modeling and Identification. The first section focuses on all aspects related to dynamic system modeling and the second section focuses on all aspects related to the identification process.

II.2. The Mathematical Modeling

The word "to model" means to proceed with a modeling. This means the establishment of models, including those used in automated, computer, operational, and economic research. The ability to "model" would involve translating a real-world situation into mathematical language (using equations, functions, sets, geometric configurations, graphs, probability laws, statistical tools, etc.)." using, comprehending, and creating a digital or geometric simulation based on modeling and software. If accepted in its broadest sense, the "modeling" skill refers to the use of a collection of mathematical ideas, methods, and theories that enable the description, comprehension, and prediction of the evolution of phenomena outside of mathematics [19].

After presenting the official and general definition of modeling, we will now clarify the perspective adopted in this research.

The term "modeling" has a connection to the fields in which it operates. For my part, we will use the concept of modeling as a way to "re-present" situations of a particular "reality" in a mathematical model, "re-presenting" in the sense of presenting this situation with a new description related to the model selected. This will be done within the framework of a mathematical education for everyone, and we would add three specificities to this representation process:

- *Representation of "functional" objects of a certain "reality" by "abstract" or "schematized" objects in a model where theoretical treatment can be applied.*
- *"Analogical" or "metaphoric" representation: natural processes are mimicked in settings that encourage observation and study,*
- *"Selective representation" refers to the need to retain some aspects of the scenario while ignoring others in a modeling effort.*

The following diagram illustrates the modeling process:

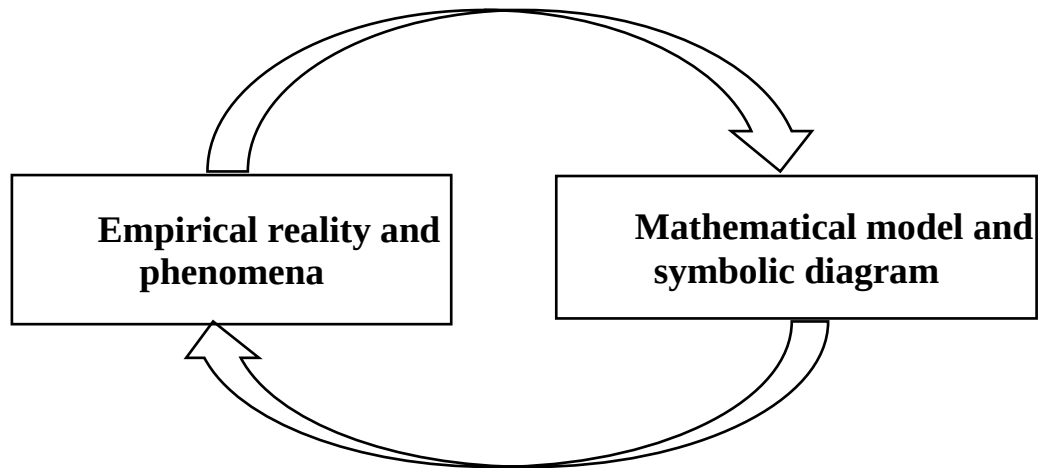


Figure II.1. Modeling process [20].

This scheme operates in two ways:

- **From reality to the model:** descriptive models (such as "transforming" and "interpreting" "information"); this Meaning relates to a heuristic function.
- **From model to reality:** predictive models (i.e., models that "anticipate" an action) correlate to a justifiable function [20].

II.3. Definition of a Mathematical Model

According to the *littré* Dictionary of the French Language, we understand that a model in the literal sense of the word "is for things of a mind the equivalent of models in the arts." Then, we are brought back to a more familiar idea. In fact, everyone knows that, in the arts, a model is a depiction of an existing or intended object. Therefore, we may state that a mathematical model is a mathematical representation of a reality element, whether that part already exists or if it emerges from reality [19].

II.4. Illustration of the Transfer Function

Linear dynamic systems have a very significant characteristic. According to this perspective, if a system's input is sinusoidal with a frequency f_c , then its output will likewise be sinusoidal with a frequency f_c but an amplitude and one that is most likely diérentes. These magnitude and phase differences are functions of frequency and describe what is known as the system's response in frequency, it is feasible to turn a system's representation of its temporal domain into an input/output representation of its frequent domain, known as the function of transfer, by applying Laplace's transformation. In doing so, she also converts the differentially governing equation into an algebraic equation that is frequently easier to analyze.

The following equation defines the Laplace transform of a time-domain function $f(t)$:

$$F(P) = \int_0^{+\infty} f(t) \cdot e^{-pt} \cdot dt \quad (\text{II.1})$$

where the parameter $P = \delta + \omega \cdot j$ is a complicated frequency value in practice, it is extremely uncommon to evaluate a Laplace transformation directly (unless you are certain of how to do it). It is much more common to do research. the transformation of a temporal function in the Laplace tableau (Table1), which replicates the most common functions in the service domain [21].

Table II.1: Laplace and Laplace inverse of the most common functions.

$f(t)$	$F(P)$
Impulsion = $\delta(t)$	1
Echelon = E	$\frac{E}{P}$
t^n (n=1 s'appelle rampe)	$\frac{n!}{p(n+1)}$
e^{-at}	$\frac{1}{p+1}$
$\sin(\omega \cdot t)$	$\frac{\omega}{p^2 + \omega^2}$
$\cos(\omega \cdot t)$	$\frac{p}{p^2 + \omega^2}$

II.5. Properties of the Laplace Transform

II.5.1. Derivative

The Laplace transform of a function's derived name is particularly significant:

$$L\left\{\frac{d^n f}{dt^n}\right\} = P^n \cdot F(P) - P^{(n-1)} \cdot f(0^+) - P^{(n-2)} \cdot f'(0^+) - \dots - f^{(n-1)}(0^+) \quad (\text{II.2})$$

✓ **Example:**

when analyzing SISO (single-input/single-output) systems, such as those controlled by an equation corresponding to constant coefficients, as seen below:

$$a_n \cdot \frac{d^n y(t)}{dt^n} + \dots + a_0 \cdot \frac{dy(t)}{dt} = b_0 \frac{d^n u(t)}{dt^n} + a_0 \frac{du(t)}{dt} \quad (\text{II.3})$$

The Laplace transform of this equation is provided below.

$$a_n \cdot P_n \cdot Y(P) + \dots + a_0 \cdot Y(P) = b_n \cdot P_n \cdot U(P) + b_0 \cdot U(P) \quad (\text{II.4})$$

where $Y(P)$ and $U(P)$ are the corresponding transformations of Laplace's $y(t)$ and $u(t)$.

The transfer function $H(P)$ in a linear system that is the relationship between Laplace's transformation and its output $Y(P)$ and that of its entrance $U(P)$ It is written [21]:

$$H(P) = \frac{Y(p)}{U(P)} = \frac{b_n + P_n + \dots + b_0}{a_n + P_n + \dots + a_0} \quad (\text{II.5})$$

II.5.2. The integral

The Laplace transform of a function's integral is provided by:

$$L[\int f(t).dt] = \frac{F(P)}{P} + \dots + \frac{1}{P}] t = 0 \quad (\text{II.6})$$

II.5.3. Delay

The Laplace transform of a function with a delay is provided by [21]:

$$L[f(t - \tau)] = F(P) \cdot e^{-\tau \cdot P} \quad (\text{II.7})$$

II.6. Representation of the state space

The representation of a real-world behavior by space is shown below:

$$\begin{cases} \dot{X} = A \cdot X + B \cdot U \\ Y = C \cdot X + D \cdot U \end{cases} \quad (\text{II.8})$$

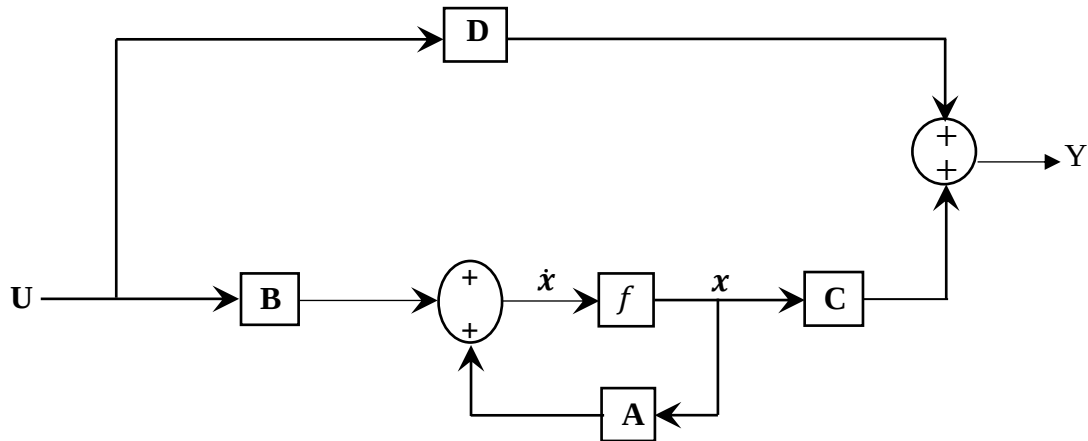


Figure II.2. Matrix state representation [21].

where X is the vector of the dimension's state variables ($n \times 1$), $\dot{X}$ is the temporal derivative of the dimension state vector ($n \times 1$). U is the dimension command vector ($m \times 1$) Y is the dimension's departure vector ($q \times 1$), A is the dimension's evolution matrix ($n \times n$), B is the

dimension's command matrix ($n \times m$), C is the dimension's observation matrix ($(q \times n)$), and D is the dimension's direct-action matrix ($q \times m$).

The departure equation (Equation 8) is required because it frequently contains state variables that are unavailable or do not clearly show interest. The output matrix C is used to calculate the variables of interest (or combinations of variables) that are accessible to be used by the controller. Furthermore, it is frequently assumed that there is no direct action required to force the elements of matrix D by zero [21].

❖ **Observation:**

Using the representation of the space of state as follows, we can also directly determine the function of transfer:

$$G(P) = \frac{Y(P)}{U(P)} = C \cdot (P \cdot I - A)^{-1} \cdot B + D \quad (\text{II.9})$$

where: $I(n \times n)$ is the identity matrix [21].

II.7. Static and dynamic modeling

An output $y(t)$ of a statically defined system is independent of the values of the input $u(T)$ with T to predict the stationary states of an industrial process's departure or to release grandeurs that are time-independent [22].

A dynamic system's output, $y(t)$, is dependent on the values of the incoming $u(T)$ with $T < t$. The inputs and outputs have been related to each other for dynamic modeling, either via differential equations (time-modeling) continuous) either by recurring equations or by differential equations (time-discrete model) [22].

II.8. Model classification

II.8.1. Categorization based on conceptual mode

Three different types of models are distinguished based on the used data:

- ✓ **Knowledge models:** These models are constructed based on a physical, chemical, biological, or other process analysis, using either general laws, based on empirical laws (finance, economy), or principles (laws of mechanics, electromagnetism, thermodynamics, quantitative physics, etc.), that regulate the phenomena that occur inside the processes under study. These models typically lack adjustable parameters or have very few adjustable parameters. Establishing a model of knowledge about the processes being studied is always desirable in practice. However, it frequently happens that the process is too complicated or

if the phenomena which are too little understood to allow for the establishment of a sufficiently specific knowledge model for the intended application. Then, it is acceptable to create entirely empirical models that are based solely on the outcomes of process measurements [23].

- ✓ **The " Black box " models:** The " Black box " models are mostly based on measurements made on the inputs and outputs of the process to be modeled. The modeling then consists of using, to represent the relationships between inputs and outputs, parametrized equations (algebraic, differentials, or recurrence), and estimate the parameters based on available measurements in order to obtain the maximum accuracy while using the fewest number of adjustable parameters possible. The domain of validity of such a model cannot be extended beyond the domain of entries that are represented in the learning measures [23].

II.8.2. Instability-based classification

Two sorts of models can be distinguished based on their intended use, independent of the previous classification.

- **The simulation models (or simulators):** A simulation model is used in a way that is independent of the process it represents. Therefore, it must exhibit behavior that is as similar to that of the procedure. These models are used to validate the concept of a system before it is manufactured (concept assisted by a mechanical or microelectronic computer, etc.), for staff training (voltage simulators), for long-term planning, i.e. From the perspective of the model's structure, previous results, measured on the modeling process, cannot be considered model inputs. The model's use and parameter estimates comprise two distinct and sequential phases (non adaptive learning) [23].
- **The prediction models (or predictors)** A prediction model is used in tandem with the process for which it is intended. It predicts the process's departure at a short time scale in comparison to constant times du processes. The predictors are employed in the command system itself (command with internal model) or in the synthesis of command laws. From the perspective of the model's structure, the measured outputs on the process may serve as model inputs. Estimating the parameters and using the model can be done simultaneously if necessary (adaptive learning, especially if the characteristics of the process change over time) [23].

II.9. The Objective of Modeling

A model can be created in one of two ways: either to simulate a process for educational purposes or to identify irregularities in functioning, default diagnostics, computer-assisted conception, i.e., To synthesize a command law or be integrated into a command device [23].

II.10. System identification

II.10.1. Definition of identification

Identification is the process of determining, based on knowledge about a certain number of system inputs and varieties, a model known as the identification model, which sends out sufficiently close outputs to the same inputs that the system provides.

The identification process involves determining a set of parameters in a model that best describes the process. There are two steps in the identification process: the first is fixing the equations' form, also known as the qualitative phase or characterization; the second is finding the numerical values of the coefficients that appear in these equations, also known as the quantitative phase or parameter estimation. These numerical values are chosen so that the model's behavior is as close to the system's as possible .

II.10.2. Identification Structures

There are two types of identification models since the output of a dynamic system depends on its input and final state.

II.10.2.1. Parallel identification

The model's output in the case of a dynamic system with discrete time is determined by its inputs and outputs:

$$\hat{y}(k+1) = f[\hat{y}(k), \hat{y}(k-1), \dots, \hat{y}(k-n+1); u(k), u(k-1), \dots, u(k-m+1)] \quad (\text{II.10})$$

Certain precautions must be taken throughout the identification process. The main method is to use born-born inputs and born-born outputs; the system is stable and is known as BIBO (Bonded Input Bonded Output).

The drawback of parallel identification is that it does not guarantee that the parameters will converge and that the error tends to zero [24].

The following diagram shows the construction of a parallel identification:

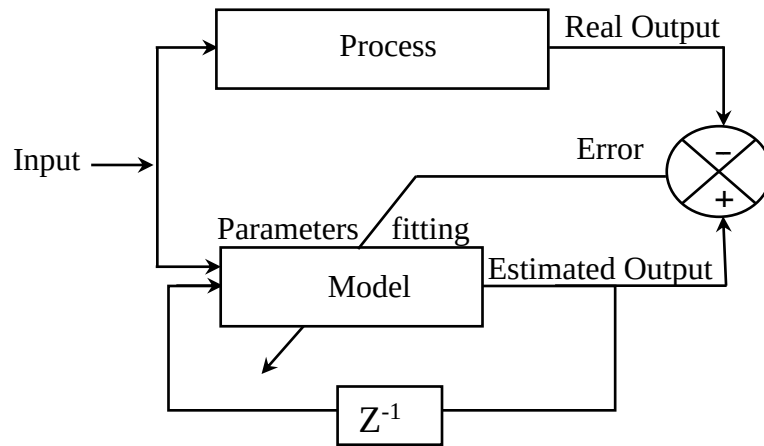


Figure II.3. Parallel identification structure

II.10.3. Identification Procedure

An accurate system model is crucial to automated control. The initial stage in any subsequent process, such as controller design, fault detection or prediction, etc., is always determining a model. A model can be constructed for simple systems using physical and mathematical understanding to characterize the characteristics or behaviors of basic subsystems as building blocks, then put these blocks together to create a model of the entire system. This methodology is difficult to apply to compact and complex systems which cannot be unpacked or analyzed in details. Instead, another methodology, called system identification, is considered. System identification focuses on creating models of dynamical systems using experimental data.

We observe that a mathematical model is merely an approximation of some features regarded as identification goals and is never an accurate representation of a real-life system. The measured data are presumed to be well-generated for these objectives in such a setting.

Table II.2 identification Procedure [25].

1	Select and save an input/output dataset (U_k, Y_k)
2	Choosing class of models or the model structure.
3	Determining the “best” model in the class, guided by the data with a criterion of fit.
4	Validating the obtained model via a test procedure.

II.10.3.1. Linear System Identification

The family of linear ARX models is the model class that is typically described as:

$$y_k = f(x_k, \theta) = \theta^T x_k \quad (\text{II.12})$$

where $\theta \in \mathbb{R}^{n_a+n_b}$ is the parameter vector and x_k is the regression vector. The problem of linear system identification can be stated as follows.

✓ **Problem 1.** Given a data set $D_0 = \{(u_k, y_k)\}_{k=1}^N$ estimate the model orders n_a and n_b and the model parameter vector θ .

For many years, the identification of linear systems has been extensively researched in the literature [26, 25]. There are several ways to estimate the parameter vector θ in the straightforward scenario when the orders n_a and n_b are known.

II.10.3.2. Nonlinear System Identification

Two main model classes for nonlinear systems: parametric models and non-parametric models. In nonlinear settings, model identification can become more difficult because the optimization problem is often non-convex, which may lead to convergence issues. Additionally, there is a higher risk of overfitting or underfitting, depending on model complexity and data quality. This section discusses both parametric and non-parametric approaches to nonlinear modeling. It also presents a widely used class of non-parametric models and introduces regularization techniques as a strategy to control model complexity. These concepts are essential for achieving reliable and robust identification of nonlinear systems

a. Parametric models

Models having a definite structure and a limited number of unknown parameters that must be ascertained from the data are known as parametric models. A decent model is quickly provided by a well-posed parametric model class. However, models in such a narrow class have little flexibility, which can easily result in the underfitting issue: the model derived from the training data may not accurately represent the system's nonlinearity.

b. Non-parametric models

Models that have both the structure and the parameters to be estimated from the data are known as non-parametric models. Current machine learning techniques take into account non-parametric models that may adequately approximate any function. As a the linearly parameterized function class of function expansions is an illustration of a non-parametric model class [27].

$$H = \{f: f = \sum_{i=1}^M \alpha_i g_i(\cdot), \alpha_i \in \mathbb{R}, M \in \mathbb{N}\} \quad (\text{II.13})$$

where α_i are function weights and g_i are referred to as basic functions. The typical ones are Radial Basis Functions (RBF), wavelet functions, kernel functions...

Overfitting and underfitting are two issues that need to be carefully handled. For the model to produce an ideal fit of the data, the standard function classes offer enough flexibility. Therefore,

the model would learn the noise as a result of minimizing the error on a data set. In addition to the target function, which overfits the training set. If the error on an independent test data set is much larger than the error on the training data set, overfitting can be identified. To avoid further biases from extrapolation, both data sets must fall within the same range of the response variables. Underfitting, as opposed to overfitting, occurs when the function classes are not adaptable enough to match the data.

The majority of nonlinear modeling techniques use a regularization procedure to manage the model's complexity (or flexibility) and prevent overfitting, as will be shown later.

II.11. Conclusion

This chapter introduced the fundamental concepts of modeling and identification of dynamical systems and time series. It presented the principles of mathematical modeling, transfer functions, Laplace transforms, and state-space representation, as well as different model classifications and modeling objectives. This chapter also explained the basics of system identification, including identification structures, procedures, and linear and nonlinear modeling approaches. These theoretical foundations provide the basis for the following chapters dedicated to intelligent and fuzzy-logic-based system modeling and identification.

Chapter III

Fuzzy Logic

III.1. Introduction

The term "fuzzy set" was first used in 1965 when professor Lotfi A. Zadeh of the University of Berkeley in the United States published an article titled " fuzzy sets (fuzzy sets) " [28]. Since then, there have been numerous significant theoretical advances, in the field and has been swiftly followed by several researchers conducting theoretical studies. Simultaneously, other researchers are focused on using logic to solve problems that are thought to be challenging. Thus, in 1975, Mamdani, a professor in London, developed a strategy for process control and obtained encouraging results on the operation of a vaporizer. In 1978, F. L. Smith analyzes the control of a cement four. This is also the most notable industrial use of fuzzy logic. In the majority of modern applications, fuzzy logic allows for the consideration of several types of knowledge, quantifiable concepts and operators in the automation of systems, Applications are also being multiplied in the field of continuous production processes and automation. The flow of logic arises because it involves an approach that is fundamentally practical, effective, and efficient. It is sometimes said that it makes it easier to systematize things that fall under the realm of empiricism and are therefore challenging to manage. The theory of fuzzy sets provides a useful and easily implementable method in applications in real time; it enables the transcription and dynamic rendering of conceptual or operational knowledge.

III.2. Type-1 Fuzzy Logic

III.2.1. Type-1 Fuzzy Sets

According to the theory of sets, an entity either belongs to or does not belong to a sets. Many mathematical theories have their roots in the concept of set. However, this fundamental idea is unable to account for circumstances that are basic and frequently encountered. As an example, it is easy to define all apples if one considers the universe of fruits. However, it will be more challenging to define all ripe apples. As a result, the concept of ripe apples is gradual.

The concept of fuzzy sets was developed in order to account for such circumstances. The theory of fuzzy sets is based on the idea of partial belonging: each species gradually or partially belongs to finished fuzzy sets. Each boundaries of a fuzzy set (see Figure III.1) are "fuzzy" or "graduals" rather than "nets" [29].

The " Membership function " $\mu_B(X)$ of an ensemble flou B is defined on a set of values known as the "univers du discours X ," which corresponds to the concept of "function caracteristique" in classical logic. The function of belongingness, which typically ranges from 0 to 1, quantifies the degree to which each x in X belongs to B .

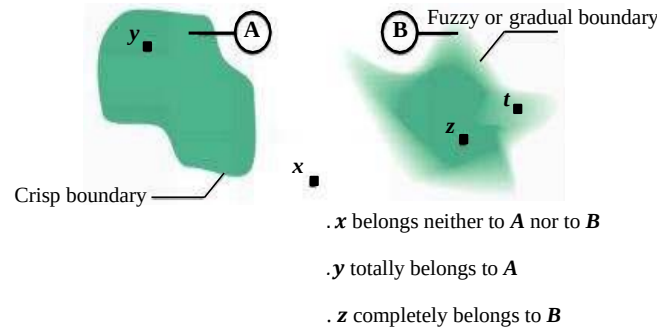


Figure III.1. Comparison between a classical set and a fuzzy set [29].

In the universe of discourse X , a fuzzy sets B may be expressed as follows [28, 30-32]

$$B = \{(x, \mu_B(x)) | x \in X\} \quad (\text{III.1})$$

$\mu_B(x)$ is the degree to which x belongs to B .

Different kinds of fuzzy sets will be generated depending on the type of function of belonging. The functions of belonging could be divided into two categories: those made of straight lines and those with a Gaussian form. The most frequently used Membership functions are shown in (Figures (III.2)):

- ✓ **The triangular function:** is defined by a lower value (a), a higher value (b), and a modular value (m) such that $a < m < b$ (see Figure III.2(a)).
- ✓ **The Gaussian function:** is defined by its mean m and its standard deviation σ (see Figure III.2(d)).
- ✓ **Singleton function:** this function takes the value 0 throughout the whole discourse universe, with the exception of the point at which it takes the value 1 (see Figure III.2(c)).

III.2.2. Base operations on fuzzy sets

Let A and B be two fuzzy sets, and let $\mu_A(x)$ and $\mu_B(x)$ be their respective functions of approximation on the universe of discourse X . Similar to the theory of classical ensembles, intersection is defined as the union of fuzzy sets and the complementary part of a fuzzy sets. These relationships are represented by the operators "and" or "and" non." New membership functions associated with these operators are established:

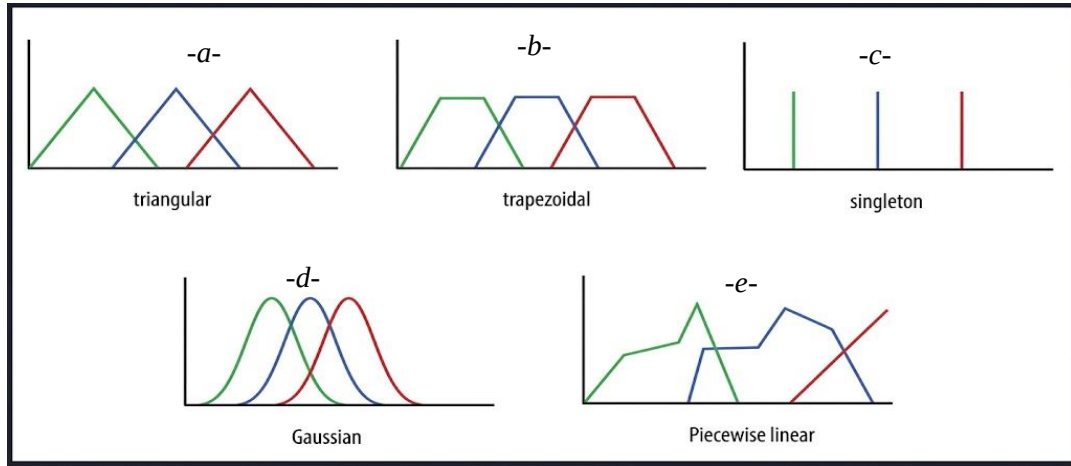


Figure III.2. Membership function: (a) Triangular, (b) Trapezoidal, (c) Singleton, (d) Gaussian, (e) Piecewise linear [29].

- x belongs to A and $B \Leftrightarrow x \in A \cap B \Leftrightarrow \mu_{A \cap B}(x)$.
- x belongs to A or $B \Leftrightarrow x \in A \cup B \Leftrightarrow \mu_{A \cup B}(x)$.
- x belongs to the complement of $A \Leftrightarrow x \in \bar{A} \Leftrightarrow \mu_{\bar{A}}(x)$.

The operator "et" is defined by a triangular norm (t – norm) T :

$$T : [0,1] \times [0,1] \rightarrow [0,1]$$

$$(x, y) \mapsto z = xTy \quad (\text{III.2})$$

Additionally, the operator "or" is defined by a triangular co-norm (T^*), often known as the norm(S):

$$S : [0,1] \times [0,1] \rightarrow [0,1]$$

$$(x, y) \mapsto z = xSy \quad (\text{III.3})$$

The most often used operators in fuzzy logic are:

1) The operator "and" for the normal t, which is the intersection of two entities A and B , can be expressed as:

- ✓ The "Min" function: $\mu_{A \cap B}(x) = \min(\mu_A(x), \mu_B(x))$.
- ✓ The arithmetic function "Sum «: $\mu_{A \cup B}(x) = \mu_A(x) + \mu_B(x)$.

2) The operator "or" for the standard system, which corresponds to the union of two ensembles A and B . It can be expressed as:

- ✓ The "Max" function: $\mu_{A \cup B}(x) = \max(\mu_A(x), \mu_B(x))$.
- ✓ The "sum" function: $\mu_{A \cup B}(x) = \mu_A(x) + \mu_B(x)$.

3) The "non" operator is defined as: $\mu_{\bar{A}}(x) = 1 - \mu_A(x)$. [29]

III.2.3. Linguistic variables

Concepts of systems are typically represented by linguistic variables in fuzzy logic. A linguistic variable is one whose values include words or phrases that are often used in natural or artificial languages [33]. A linguistic variable is defined by [34]:

$$(X, U, T(X), \mu_x) \quad (\text{III.4})$$

"X" denotes the variable's name, "U" is the universe of discourse associated with the variable X (also referred to as "sentential" [30]), $T(x) = \{T1, T2, \dots, Tn\}$ is the collection of linear values of the variable X (also referred to as linguistic terms or linguistic etiquettes), and finally, " μ_x " are the functions of belonging associated with the collection of linguistic terms.

III.2.4. Fuzzy Inference System

The purpose of a fuzzy input system (FIS) is to convert input data to output data based on the assessment of a set of data. The fuzzification process is the source of the entries, and the expert's knowledge is typically used to define all of the rules. Three steps make up a FIS (see Figure III.3): a) fuzzification, b) inference, and c) defuzzification.

The first step is fuzzification, which entails characterizing the linguistic variables used in the system. Therefore, it involves a transformation of real-world inputs into a defined fluid portion on a presentation space associated with the input. This re-presentation space is typically a fuzzy subset, every input and output variable is linked to fuzzy subset during the fuzzification process.

The "Aggregation" is a mechanism that allows information from a system to be condensed through a set of defined rules for the presentation of each given problem. Every rule yields a partial conclusion, which is then agreed upon with the other rules to provide a conclusion (aggregation).

The next step is defuzzification, which is the opposite of fuzzification and enables the conversion of fuzzy inferences into a non-fuzzy value [29].

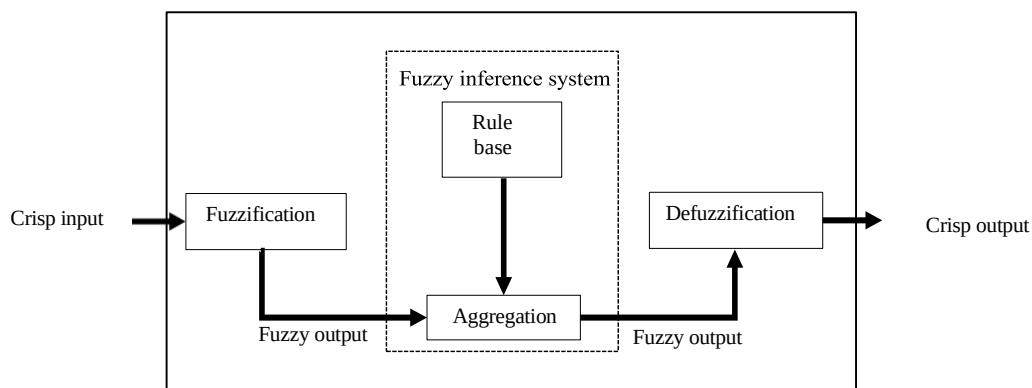


Figure III.3. Fuzzy System [29].

III.2.5. Type-1 Fuzzy Logic Example

The "Takagi-Sugeno Kang (TSK)" fuzzy model is a universal approximation of real functions that are defined in a closed subset with a real number of n dimensions. This powerful feature of the TSK model has various applications in the modeling of dynamic systems [35]. A TSK fuzzy is defined by a collection of IF-THEN rules that represent the entered relations departure from a region. The most commonly used type of Takagi–Sugeno–Kang (TSK) fuzzy model is the first-order model. It consists of a set of fuzzy rules, each having p antecedents. The rule structure is generally expressed as follows:

$$\begin{aligned} R^l : & \text{IF } x_1 \text{ is } F_1^l \text{ and } x_2 \text{ is } F_2^l \text{ and } , \dots , \text{ and } x_p \text{ is } F_p^l , \\ & \text{THEN } y^l = c_0^l + c_1^l x_1 + c_2^l x_2 + , \dots , + c_p^l x_p , \end{aligned} \quad (\text{III.5})$$

where $l = 1, 2, \dots, M$; c_j^l are the consequent parameters, for $j = 0, 1, \dots, p$; x_j is the input of the fuzzy system; y^l is the output of the l^{th} rule; and F_j^l is the fuzzy set $j = 0, 1, \dots, p$. The final output of the incomplete first-order TSK model is derived as follows [36]:

$$r = \sum_{l=1}^M f^l y^l \quad (\text{III.6})$$

where f^l are the firing strengths of the rules defined as follows:

$$f^l = T_{j=1}^p \mu_{F_j^l}(x_j) \quad (\text{III.7})$$

where T denotes the t -norm.

when the membership functions are Gaussian (see Figure III.2(d)):

$$\mu_{F_j^l}(x_j) = \exp \left[-\frac{1}{2} \left(\frac{x_j - m_j^l}{\sigma_j^l} \right)^2 \right] \quad (\text{III.8})$$

and when the t -norm is used, Equation (III.6) can be expressed as follows:

$$r = \sum_{l=1}^M y^l \prod_{j=1}^p \exp \left[-\frac{1}{2} \left(\frac{x_j - m_j^l}{\sigma_j^l} \right)^2 \right] \quad (\text{III.9})$$

where m_j^l and σ_j^l are the centers (means) and the standard deviations of the l^{th} Fuzzy set, respectively.

III.3. Type-2 Fuzzy Logic

In 1975, Zadeh, the founder of fuzzy logic, introduced type-2 fuzzy logic as an extension of type-1 fuzzy logic [33]. A type-2 ensemble is characterized by a fuzzy membership function, where the degree of membership of each element in the ensemble is itself a fuzzy set in $[0, 1]$. The main issue with type 1 fuzzy systems is that they may produce unsatisfactory results when uncertainty and imprecision are present. The uncertainty may exist at the level of the function of belonging's form or in one of its parameters. The advent of the type 2 fuzzy logic somewhat resolved this issue with the type-1 logic.

III.3.1. Presentation of a Type-2 Fuzzy Sets

One way to think about a type-2 membership function is as a function with two variables. We have a secondary affiliation, denoted by the ordinary number μ_2 , for every x in the universe of discourse and primary affiliation μ_1 . Therefore, a type-2 function of affiliation can be expressed as follows:

$$\mu_2(x, \mu_1): X \times [0,1] \rightarrow [0,1] \quad (\text{III.10})$$

where X Is the input space x [29] .

➤ **Example:**

We consider a type-1fuzzy set that is characterized by a Gaussian function of average M and standard deviation σ_x . For each element x , the degree of membership is determined by:

$$m(x) = \exp\left(-\frac{1}{2} \left(\frac{x-M}{\sigma_x}\right)^2\right) \quad (\text{III.11})$$

within the concept of type-1 fuzzy sets, an ordinary number that falls within the interval $[0, 1]$ is the degree of belonging. We assume that this appearance is not a number but rather a type-1 set. The elements of this sets domain are indicated by μ_1 , and the primary membership of x is called. It is noted that these primary memberships are $\mu_2(x, \mu_1)$, and it is referred to as the secondary belonging of x .

In the event that the secondary members are Gaussians with average $m(x)$ and standard deviation σ_m :

$$\mu_2(x, \mu_1) = \exp\left(-\frac{1}{2} \left(\frac{\mu_1 - m(x)}{\sigma_m}\right)^2\right) \quad (\text{III.12})$$

with $\mu_1 \in [0, 1]$ and $m(x)$ Is given by Equation (III.11) , hence:

$$\mu_2(x, \mu_1) = \exp\left(-\frac{1}{2} \left(\frac{\mu_1 - \exp\left(-\frac{1}{2} \left(\frac{x-M}{\sigma_x}\right)^2\right)}{\sigma_m}\right)^2\right) \quad (\text{III.13})$$

for the case of a type-2 fuzzy set, the degree of affiliation of each group comprises all primary and corresponding secondary affiliations, and it can be expressed as follows:

$$\mu(x) = \frac{\int_{\mu_1 \in [0,1]} \mu_2(x, \mu_1)}{\mu_1}, x \in X \quad (\text{III.14})$$

Figure III.4 (a) shows a three-dimensional representation of a type-2 Gaussian set with a primary Gaussian membership function, whereas Figure III.4 (b) depicts the distribution of Gaussian fuzzy type corresponding affiliation to $x = 4.25$ [29].

III.3.2. Types of Type-2 Fuzzy Sets

According to the shape of the primary membership, three main types of Type-2 fuzzy sets can be distinguished: Interval, Gaussian, and Triangular [29].

III.3.2.1. Interval Type-2 Fuzzy Set

In these kinds of sets, each point's degree of belonging is an ordinary set whose domain of definition is inside the interval $[0, 1]$ [37, 38]. All secondary belongings in this instance are equal to 1. It should be noted that although if every element of a type-2 interval set is an ordinary set, the set itself is type-2 since the degrees of affiliation are sets rather than ordinal numbers.

III.3.2.2. Gaussian Type-2 Fuzzy Set

Each point in this type of set has a set type-1 Gaussian whose domain of definition is included in the interval $[0, 1]$. We see that it is not necessary for the primary membership function to be Gaussian [37].

III.3.2.3. Triangular Type-2 Fuzzy Set

In this kind of set, each point's degree of membership is a type-1 triangular set whose domain of definition is included in the interval $[0, 1]$ [37].

III.3.3. Operations on Type-2 Fuzzy Sets

Let's say we have two type-1 fuzzy sets, $A \in X$ and $B \in X$, with membership functions ϕ_1 and ϕ_2 [29].

$$A = \sum_i \phi_1(x_i)/x_i \quad (\text{III.15})$$

$$B = \sum_i \phi_2(x_i)/x_i \quad (\text{III.16})$$

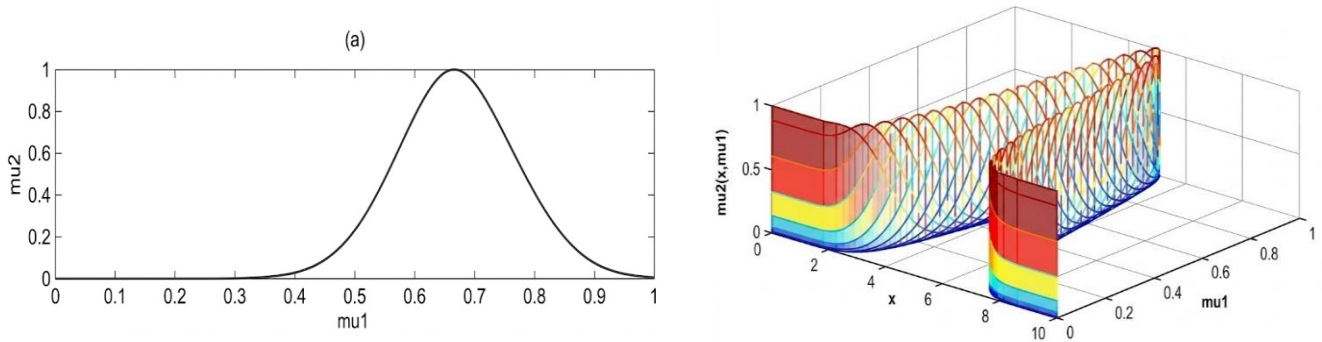


Figure III.4 (a) Three-dimensional representation of a type-2 Gaussian set (b) The degree of corresponding fuzzy membership to $x = 4.25$ [29].

The membership functions of the union, intersection, and complement will be determined by using the T -conorm max and the norm T -norm [39, 40]:

$$\mu_{A \cup B} = \max\{\theta_1(x_i, \theta_2(x_i))\} \forall_i \quad (\text{III.17})$$

$$\mu_{A \cap B} = \min\{\theta_1(x_i, \theta_2(x_i))\} \forall_i \quad (\text{III.18})$$

$$\mu_{\bar{A}}(x_i) = 1 - \theta_1(x_i) \forall_i \quad (\text{III.19})$$

$$\mu_{\bar{B}}(x_i) = 1 - \theta_2(x_i) \forall_i \quad (\text{III.20})$$

Since A and B are type-1 fuzzy sets, their membership degree values $\theta_1(x_i)$ and $\theta_2(x_i)$ are ordinal numbers. Therefore, for each x_i , the operations (III.15) – (III.20) can be directly evaluated in a single step.

Continuing, let's assume that $\tilde{A} \in X$ and $\tilde{B} \in X$ are fuzzy sets type 2. In this case, the degrees of similarity $\theta_1(x_i)$ and $\theta_2(x_i)$ are fuzzy sets type 1. Therefore, we will need to understand (generalize) the operations *min*, *max*, and negation towards fuzzy sets in order to calculate the union, intersection, and complement of $\tilde{A}$ and $\tilde{B}$. We will use Zadeh's well-known principle of extension [41] for this.

Let $\mu_{\tilde{A}}(x)$ and $\mu_{\tilde{B}}(x)$ be two fuzzy membership functions (fuzzy sets in $J \subseteq [0,1]$).

➤ **Type-2 Fuzzy Set $\tilde{A}$ and $\tilde{B}$:**

$$\mu_{\tilde{A}}(x) = \sum_i f_x(u_i) / u_i ; u_i \in J \quad (\text{III.21})$$

$$\mu_{\tilde{B}}(x) = \sum_i g_x(\omega_i) / \omega_i ; \omega_i \in J \quad (\text{III.22})$$

It should be noted that in equations (III.21) and (III.22), u_i and w_i are factual variables that are only used to distinguish between x 's primary belongings in $\tilde{A}$ and $\tilde{B}$, respectively. By applying the zadeh principal extension, a binary operation $*$ can be extended to $\mu_{\tilde{A}}(x)$ and $\mu_{\tilde{B}}(x)$:

$$\mu_{\tilde{A}}(x) * \mu_{\tilde{B}}(x) = \sum_{ij} (f_x(u_i) \wedge g_x(\omega_j)) / u_i * \omega_j \quad (\text{III.23})$$

When this principal (for fuzzy sets) is applied to fuzzy sets $\tilde{A}$ and $\tilde{B}$, the membership functions for union, intersection, and complement are defined as follows:

• **For the union**

$$\begin{aligned} \tilde{A} \cup \tilde{B} &\Leftrightarrow \mu_{\tilde{A} \cup \tilde{B}}(x) = \mu_{\tilde{A}}(x) \cup \mu_{\tilde{B}}(x) ; x \in X \\ &= (\sum_i f_x(u_i / u_i) \cup (\sum_j g_x(\omega_i / \omega_i)) \\ &= \sum_{ij} (f_x(u_i) \wedge g_x(\omega_j)) / u_i \vee \omega_j \end{aligned} \quad (\text{III.24})$$

• **For the junction**

$$\begin{aligned} \tilde{A} \cap \tilde{B} &\Leftrightarrow \mu_{\tilde{A} \cap \tilde{B}}(x) = \mu_{\tilde{A}}(x) \cap \mu_{\tilde{B}}(x) ; x \in X \\ &= (\sum_i f_x(u_i / u_i) \cap (\sum_j g_x(\omega_i / \omega_i)) \\ &= \sum_{ij} (f_x(u_i) \wedge g_x(\omega_j)) / u_i \vee \omega_j \end{aligned} \quad (\text{III.25})$$

• **For the completion**

$$\begin{aligned} \bar{\tilde{A}} &\Leftrightarrow \mu_{\bar{\tilde{A}}}(x) = \neg \mu_{\tilde{A}}(x) ; x \in X \\ &= \sum_i f_x(u_i / (1 - u_i)) \end{aligned} \quad (\text{III.26})$$

$$\begin{aligned}\bar{\bar{B}} &\Leftrightarrow \mu_{\bar{\bar{B}}}(x) = \neg\mu_{\bar{B}}(x); x \in X \\ &= \sum_j g_x(\omega_i/(1 - \omega_j))\end{aligned}\quad (\text{III.27})$$

where $\wedge$ and $\vee$ represent the max and min operations, respectively. In this case (Type-2), and as in [37, 38], we will refer to the operations $\cup, \cap$, and $\neg$ as *join*, *meet*, and *negation*, respectively. The same notations will be used in the continuous case.

III.4. Structure of a Type-2 Fuzzy System

A type-2 fuzzy system is similar to the type-1 system (see section III.2), with the exception that type-2 fuzzy sets are converted into type-1 sets prior to the defuzzification step by using a type reducer. Figure III.5 depicts a type-2 flow [42, 43, 44].

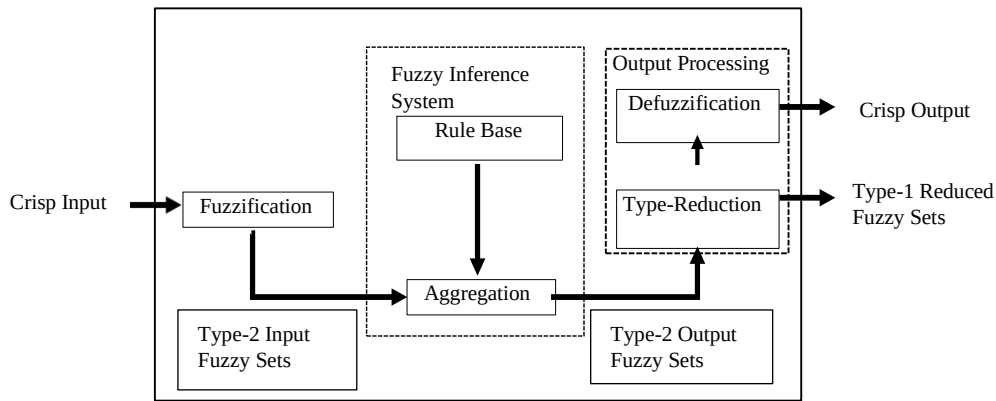


Figure III.5. shows the structure of a fuzzy type 2 [42].

III.4.1. Fuzzification

In contrast to the type-1 function, the type-2 function provides several degrees of membership (or dimensions) for each input. Consequently, the uncertainty will be reduced. This presentation will enable us to consider what was overlooked by the type-1 [29].

III.4.2. Rule base

In the type-1 case, we often have rules of the form *IF – THEN*, and the rule link has the following form:

$$\begin{aligned}R^l: & \text{IF } x_1 \text{ is } F_1^l \text{ and } x_2 \text{ is } F_2^l \text{ and, } \dots, \text{ and } x_p \text{ is } F_p^l, \\ & \text{THEN } y \text{ is } G^l\end{aligned}\quad (\text{III.28})$$

where $x_1 \in X_1, x_2 \in X_2, \dots, x_p \in X_p$ are the inputs, the F_i^l s are fuzzy sets, such as $i = 1, 2, \dots, p, y \in Y$ is the output and G^l are the fuzzy sets of consequences.

The only difference between type-1 and type-2 is the nature of membership functions; therefore, in the case of type-2, the structure of members will remain exactly the same, the only difference being that some (or all) of the membership functions will be of type-2. Consequently, the relationship between a type-2 group has the following form [44, 45]:

$$\begin{aligned}
 R^l: & \text{IF } x_1 \text{ is } \tilde{f}_1^l \text{ and } x_2 \text{ is } \tilde{f}_2^l \text{ and } , \dots , \text{ and } x_p \text{ is } \tilde{f}_p^l , \\
 & \text{THEN } y \text{ is } \tilde{G}^l
 \end{aligned} \tag{III.29}$$

III.4.3. Inference

Interval Type-2 Fuzzy Sets are used in this theory and for reasons of calculation simplification [46]. The base of fuzzy (III.28), which establishes a fuzzy type-2 relation between the space of input $(X_1 \times X_2 \times \dots \times X_p)$ and the space of output (Y) of a type-2 fuzzy, is used in this method. We take note of this type 2 fuzzy membership function by $\mu_{\tilde{F}_1^l} \times \dots \times \tilde{F}_1^l \tilde{G}^{l(x,y)}$ where $\tilde{F}_1^l \times \tilde{F}_p^l$ indicate the cartesian product of $\tilde{F}_1^l, \tilde{F}_2^l, \dots, \tilde{F}_p^l$ and $\underline{x} = (x_1, x_2, \dots, x_p)^T$ when an input $\underline{x}'$ is applied, the composition of the fuzzy set $\underline{x}'$ to which $\underline{x}'$ belongs and the ratio R^l are determined using the following composition sup-Star [46, 47]:

$$\mu_{\underline{x}'} \circ \tilde{F}_1^l \times \dots \times \tilde{F}_p^l \rightarrow \tilde{G}^{l(y)} = \bigcup_{\underline{x} \in \underline{x}'} [\mu_{\underline{x}'} \cap \mu_{\tilde{F}_1^l} \times \dots \times \tilde{F}_p^l \rightarrow \tilde{G}^{l(\underline{x}, y)}] \tag{III.30}$$

In a singleton fuzzification, each crisp input x_j is represented as a fuzzy set that has full membership at that point and zero elsewhere. So the membership function becomes:

$$\mu_{\underline{x}'} \circ \tilde{F}_1^l \times \dots \times \tilde{F}_p^l \rightarrow \tilde{G}^{l(y)} = \mu_{\underline{x}'} \times \tilde{F}_p^l \rightarrow \tilde{G}^{l(\underline{x}', y)} \tag{III.31}$$

Take note of the entire output that corresponds to the same rule since the second member of (III.31) is calculated using the implication minimum or product [46, 47] (which corresponds to the operation met with T—norm minimum or product in the case of type-2), (III.31) can be written as follows:

$$\mu_{\tilde{B}^l} = \mu_{\tilde{F}_1^l} \times \dots \times \tilde{F}_p^l(\underline{x}') \cap \mu_{\tilde{G}^l}(y) \tag{III.32}$$

which is equivalent to:

$$\mu_{\tilde{B}^l} = \mu_{\tilde{F}_1^l}(x_1) \cap \mu_{\tilde{F}_2^l}(x_2) \cap \dots \cap \mu_{\tilde{F}_p^l}(x_p) \cap \mu_{\tilde{G}^l}(y) = \mu_{\tilde{G}^l}(y) \cap \left[\prod_{i=1}^p \mu_{\tilde{F}_i^l}(x_i) \right] \tag{III.33}$$

Note that the operation is based on the selected T – norm. Since only type-2 fuzzy interval are used and the normal production operation is carried out, the activation interval linked to the j^{th} set, the input type-1 fuzzy interval of the output is determined by:

$$F^l(\underline{x}) = [\underline{f}^l(\underline{x}), \bar{f}^l(\underline{x})] \tag{III.34}$$

where $\underline{f}^l(x) = \underline{\mu}_{\tilde{F}_1^l}(x_1) * \underline{\mu}_{\tilde{F}_2^l}(x_2) * \dots * \underline{\mu}_{\tilde{F}_p^l}(x_p)$ et $\bar{f}^l(x) = \bar{\mu}_{\tilde{F}_1^l}(x_1) * \bar{\mu}_{\tilde{F}_2^l}(x_2) * \dots * \bar{\mu}_{\tilde{F}_p^l}(x_p)$

The terms $\underline{\mu}_{\tilde{F}_p^l}(x_p)$ and $\bar{\mu}_{\tilde{F}_p^l}(x_p)$ are respectively lower and higher degrees of membership relative to $\mu_{\tilde{F}_p^l}(x_p)$.

III.4.4. Type Reduction

In a type-1 fuzzy system, the output sets are type-1 fuzzy sets, we perform the defuzzification in order to obtain a crisp output by combining the output sets.

Since the output sets in the type-2 case are type-2, we must use extended versions of the type-1 defuzzification methods (by applying the extension principle). Because type-1 defuzzification provides us with an ordinary number upon the output of the defuzzification process carried out in the type-2 case results in an entire type-1 fuzzy at the end. Because this operation converts a type-2 set into a type-1 set, it is called "reduction of type" and the resulting type-1 set is called "set of type reduced." The reduced type set is then refuzzed to obtain an ordinary number.

Various methods have been proposed to calculate the output of a type-2 fuzzy [48]. The method proposed by Karnik and Mendel (KM) [49], which is well-known for the method of centers of sets (COS: Center of Sets). The expression of the total type reduced by this method is provided by [50]:

$$(Y_{cos}, Y^1, \dots, Y^M, W^1, \dots, W^M) = \int_{y^1} \dots \int_{y^M} \dots \int_{W^1} \dots \int_{W^M} \frac{1}{\frac{\sum_{i=1}^M w^i y^i}{\sum_{i=1}^M w^i}} \quad (III.35)$$

Given that each set in Equation (III.34) is a type-1 interval, Y_{cos} is also a type-1 interval with a domain on the real axis.

$$(Y_{cos}, Y^1, \dots, Y^M, W^1, \dots, W^M) = [y_l, y_r] \quad (III.36)$$

where y^l and y_r are two points, left and right, characterizing the reduced set y_{cos} .

Note by $f^l(\underline{x})$ the l^{th} element activation interval (equation III.34) and y^j an element of the interval type 2 $Y^j = [y_l^j, y_r^j]$. Karanik and Mandel [51] have developed an algorithm to calculate the extreme points y^l and y_r as follows:

1. Discretize the output space Y into a sufficient number of points by choosing as segments the centroids of the fuzzy sets involved in the activation of the rules.
2. Sort y^j in ascending order: $y^1 \leq y^2 \leq \dots \leq y^M$.
3. Initialize f^j by taking as the starting point: $f^j = \frac{f^j + \bar{f}^j}{2}, j=1, 2, \dots, M$.
4. Calculate: $y_l = \frac{\sum_{j=1}^M y^j f^j}{\sum_{j=1}^M f^j}$ and to set down $y_l' = y_l$.
5. Find a switching point N ($1 \leq N \leq M - 1$) which satisfies: $Y^N \leq y_l \leq Y^{N+1}$.
6. To set down: $f^j = \begin{cases} \bar{f}^j, & j < N \\ f^j, & j \geq N \end{cases}$.

7. Calculate: $y_l = \frac{\sum_{j=1}^M y^j f^j}{\sum_{j=1}^M f^j}$.
8. To set down: $y_l'' = y_l$.
9. If $y_l'' \neq y_l$ go to the step 6, if $y_l'' = y_l$ stop and put down $y_l = y_l''$.
10. To set down $y_l \neq y_l''$ and go to step 3.

The algorithm for calculating $y_l, y_l', y_l'', y^N, y^{N+1}$ is similar to that of the algorithm y_l except that all instances of y and N are replaced by $y_r, y_r', y_r'', y^L, y^{L+1}$ and L respectively.

III.4.5. Defuzzification

To obtain an ordinary output from a type-2 floating-point system, we must defuzzify the entire reduced type. Several methods have been proposed to achieve this goal, see [46].

III.5. Fuzzy logic system type 3

III.5.1. Overall View

Figure 1 provides an illustration of how IT2, GT2, and IT3 fuzzy sets differ from one another. GT2-FS is a type-1 FS, IT3-FS is a type-2 FS, and the secondary membership for an IT2-FS is a crisp value. Then, compared to the IT2-FS and GT2-FSs, the IT3-FSs are able to model a greater degree of fuzziness and uncertainty. Additionally, when GT2-FSs and IT2-FSs are compared, we find that the secondary membership in GT2-FSs is a fuzzy number while it is a set number in IT2-FSs. Then, compared to IT2-FSs, GT2-FSs are more capable of modelling uncertainty [53].

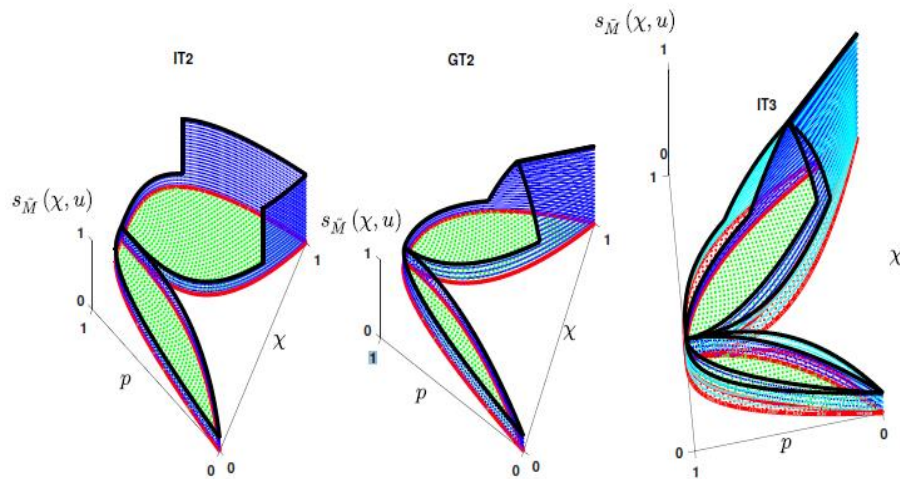


Figure III.6. An overview of the fuzzy sets IT2, GT2, and IT3 (some horizontal and vertical slices) [53].

The UB and LB of the footprint-of-uncertainty (FOU) in IT3-FSs are fuzzy numbers, but they are crisp values in IT2-FSs and GT2-FSs. This is the other primary distinction. Figure III. 7 shows the FOU for three different kinds of FS. In many cases, determining the upper and lower

ranges of uncertainties is difficult. Then, when defining the uncertainty range is a crucial issue, the IT3-FSs can assist us.

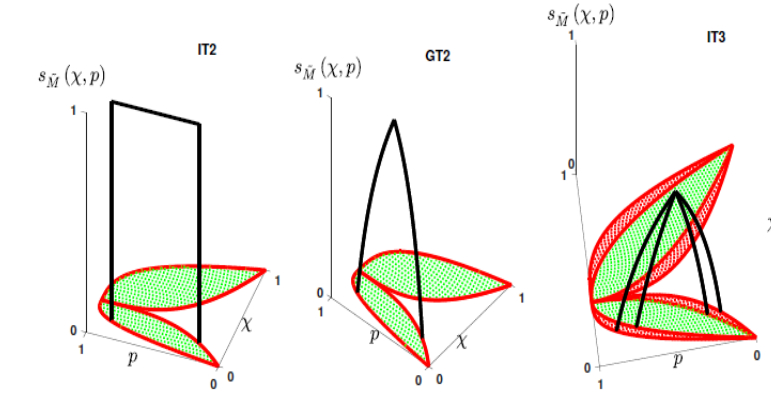


Figure III.7. FOU for fuzzy sets IT2, GT2, and IT3 [53].

Examine the membership function (MF) $\times M$. Table 1 lists the different forms of $\tilde{M}$ in the context of IT2, GT2, and IT3.

Table III.1. Mathematical comparison of different MF types.

	$\tilde{M}$
GT2	$\sum_{x \in X} \sum_{p \in J_x} \frac{S_M(x, p)}{(x, p)}, J_x \subseteq [0, 1]$
IT2	$\sum_{x \in X} \sum_{p \in J_x} \frac{1}{(x, p)}, J_x \subseteq [0, 1]$
IT3	$\sum_{x \in X} \sum_{p \in J_x} \frac{1}{\frac{S_{\tilde{M}}(x, p)}{(x, p)}}, J_x \subseteq [0, 1]$

In Table 1, $\sum \sum$ represents the union of x and p . Table 1 illustrates how the IT2-FS and GT2-FS differ in that the secondary MF is constant. The consistency of the secondary membership in the interval counterpart distinguishes interval MFs from general type-2 MFs. Put differently, in the IT2-FS, we have $S_{\tilde{M}}(x, p) = 1$ when compared to the type-3 FS, the secondary membership $S_{\tilde{M}}(x, p)$ is written as an interval type-2 FS $\frac{1}{S_{\tilde{M}}(x, p)}$ [53].

An improved version of the traditional Type-1 and Type-2 fuzzy logic systems, Type-3 fuzzy logic is intended to simulate greater degrees of uncertainty in tasks including inference, modelling, and decision-making. The goal of each level in the fuzzy logic hierarchy is to address increasingly complicated types of imprecision:

- ✓ **Type-1 fuzzy logic** deals with precise membership functions;
- ✓ **Type-2 fuzzy logic** permits the membership degree to be fuzzy, typically represented as a fuzzy set of values (such as an interval);

- ✓ **Type-3 fuzzy logic** goes one step further by permitting the secondary membership functions in Type-2 fuzzy sets to be fuzzy sets, thereby producing "fuzzy membership of fuzzy memberships."

Figure III.8 shows the output of the type-3 Fuzzy Logic model, which incorporates higher-order uncertainty. The provided MFs with some vertical and horizontal cuts are shown in Figures III.9 and III.8. It is evident that the upper and lower bounds of the foot of uncertainty (FOU) in the used MF are interval type-1 MFs rather than known values when comparing the designed MF shown in Figure III.9 to the traditional general type-2 MF presented in Figure III.10.

The proposed FLS can withstand a greater degree of uncertainty thanks to this extra degree of freedom. Because of its ability to handle non-symmetric distributions and high-order uncertainty, the designated membership function was employed. The used MF enables flexible FOU modelling, improving robustness under noisy or faulty data, in contrast to Gaussian membership functions that assume symmetric uncertainty.

The initial step in the process involves obtaining the inputs (data collection step). The inputs are represented by $x_1, x_2, \dots, x_n$, where each x_i denotes a specific feature relevant to the system. Each x_i input is associated with a set of membership functions $\tilde{M}_i^j(x_i)$, where j is the j^{th} input's membership function and i is the i^{th} input itself. The secondary membership α_k is represented by horizontal slices in the type-3 fuzzy logic system (T3FLS). Four primary membership values are taken into account for each slice: the upper and lower boundaries on the right side, represented as $\bar{P}_{i,r,\alpha_k}$ and $\underline{P}_{i,r,\alpha_k}$, With the matching boundaries on the left, indicated as $\bar{P}_{i,l,\alpha_k}$ and $\underline{P}_{i,l,\alpha_k}$. These limits show how unpredictable the membership grade and its relative dependability are. For the membership function, the firing degrees are computed as follows [54]:

where: $c_{\tilde{M}_i^j}$ is the center of $\tilde{M}_i^j$, $\vartheta_{\tilde{M}_i^j}$ and $\bar{\vartheta}_{\tilde{M}_i^j}$: are the left and right-side widths of $\tilde{M}_i^j$; α_k is the horizontal slice's value; the membership functions' widths are represented by Δ_r and Δ_l .

The upper and lower boundary memberships of the membership functions are used to determine the rule firings $\tilde{M}_i^j$, as follows:[52]

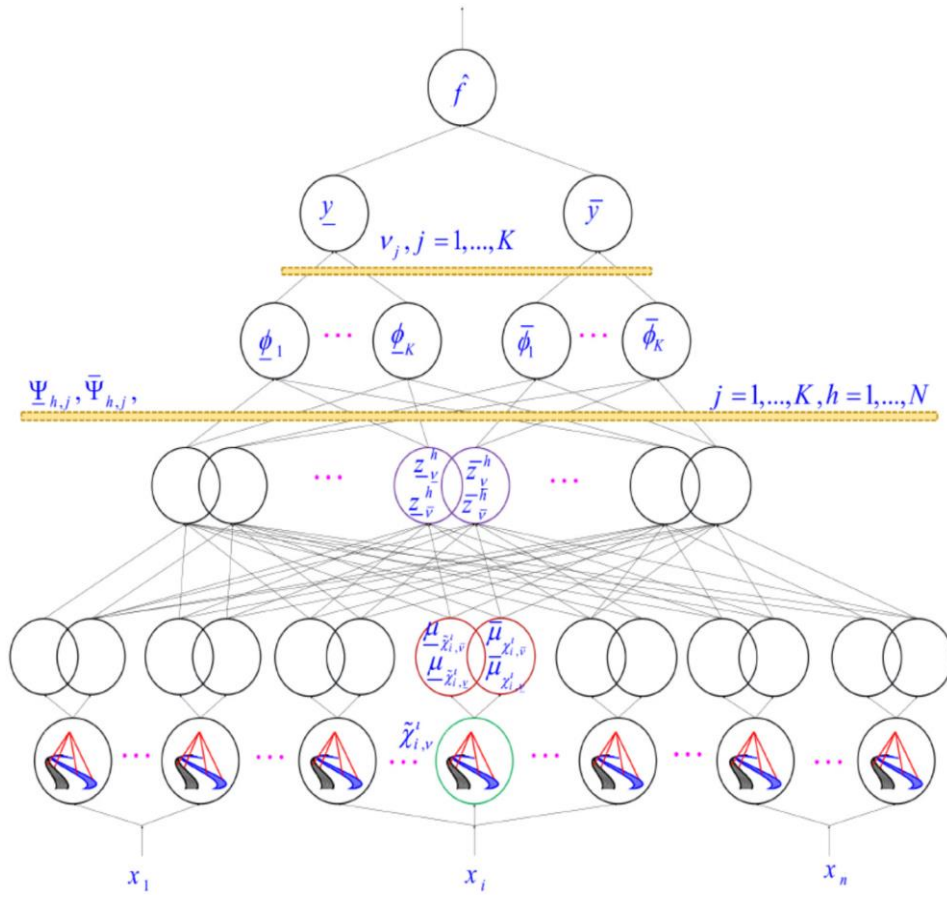


Figure III.8. Fuzzy logic system type 3 [52].

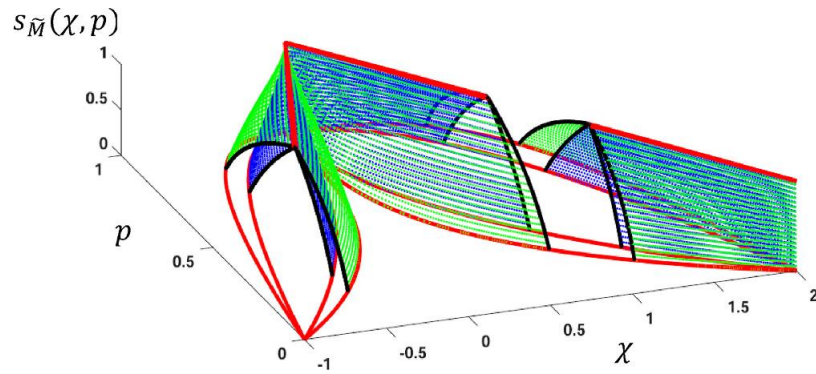


Figure III.9. Fuzzy membership function of type 3 [52].

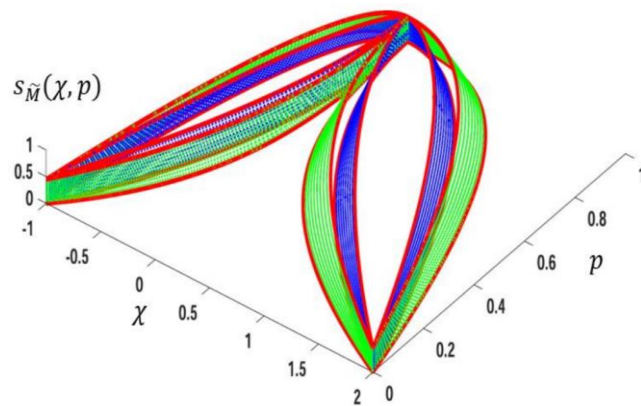


Figure III.10. The utilized membership function's horizontal slice [52].

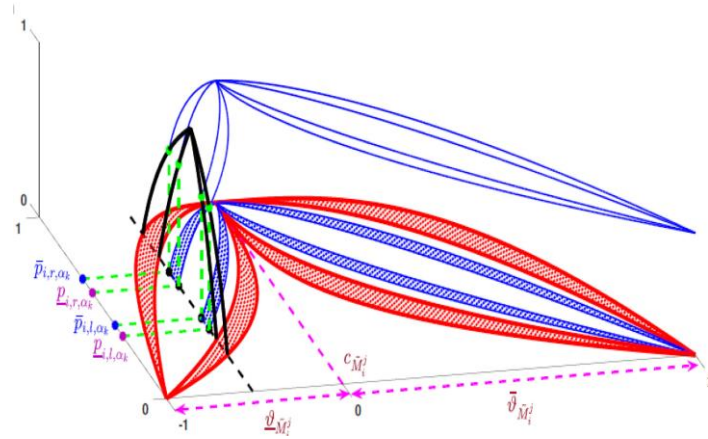


Figure III.11. The type 3 membership function's FOU [52].

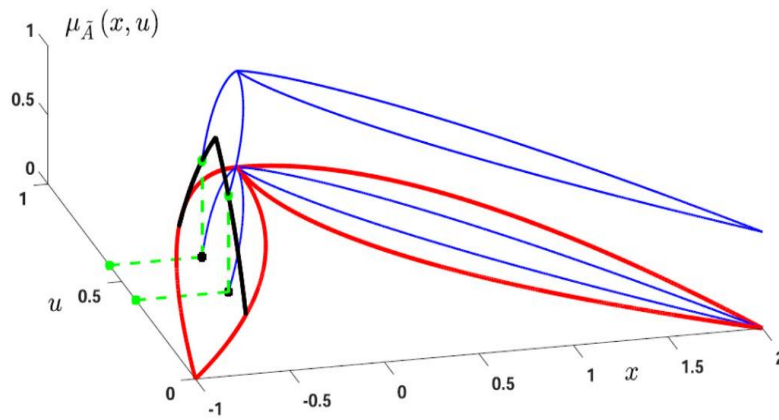


Figure III.12. The general type 2 membership function's FOU [52]

III.6. Conclusion

This chapter presented the theoretical foundations of fuzzy logic systems, with particular emphasis on Type-3 fuzzy logic. It first introduced the principles of Type-1 fuzzy logic, including fuzzy sets, membership functions, linguistic variables, and fuzzy inference systems. Then, Type-2 fuzzy logic was discussed as an extension capable of handling higher levels of uncertainty through fuzzy membership degrees and type-reduction mechanisms. Finally, the chapter focused on Type-3 fuzzy logic, highlighting its advanced ability to model complex and high-order uncertainties compared to Type-1 and Type-2 systems. These concepts provide a strong basis for the development of intelligent modeling and control methods for dynamical systems and time series applications.

Chapter IV

Method and Results

IV.1. Introduction

Modeling complicated systems is a challenging task. Many researchers have considered using artificial intelligence in order to deal with this issue. This chapter will introduce a method of modelling dynamic systems and time series based on the fuzzy logic type 3 and the Kalman filter as an optimization algorithm. This method involves the parallel connection of two fuzzy sub-models. The primary model, which represents an ordinary model with low resolution. In order to overcome the resolution problem and obtain a model with a higher resolution, we have introduced a second fuzzy sub-model known as the model of error, which represents the model of error between the primary model's output and the real data of the dynamic system or the time series under consideration. The error model represents the uncertainty in the primary model; this uncertainty is reduced by simply subtracting the output of the error model from the output of the primary model, which results in a parallel connection between the two models. This connection leads to a single final model that is complete, unique, and has a higher resolution.

In this chapter, we will discuss the proposed method based on the type 3 fuzzy logic and the optimization algorithm based on the Kalman filter. This method incorporates the ability to learn of the type 3 fuzzy logic with the Kalman filter to approximate dynamic systems and time series.

IV.2. Proposed Method

Figure IV.1 shows the overall layout. In type 3 membership functions (MFs), the UB and LB for the FOU are interval fuzzy sets rather than explicit values.

Moreover, the SM is a type-2 MF. Below is a summary of the computation:

Get the inputs in the initial stage. X_i denotes the inputs, $i = 1, \dots, n$. Every input X_i has Q_i MFs, as seen in Figure IV.1. A stand for the j^{th} MF input X_i . The memberships (firing degrees) are then calculated for $\bar{M}_i^j$. For every horizontal slice α_k (secondary membership), we have four main memberships. The top and lower boundaries of the right side are shown by two values, $\bar{P}_{r \alpha_{ik}}$ and $\underline{P}_{r \alpha_{ik}}$, whereas the upper and lower bounds of the left side are indicated by two values, $\bar{P}_{l \alpha_{ik}}$ and $\underline{P}_{l \alpha_{ik}}$. The memberships for the proposed MF are then calculated. Figure IV.2 shows the MF memberships, which are calculated as follows:

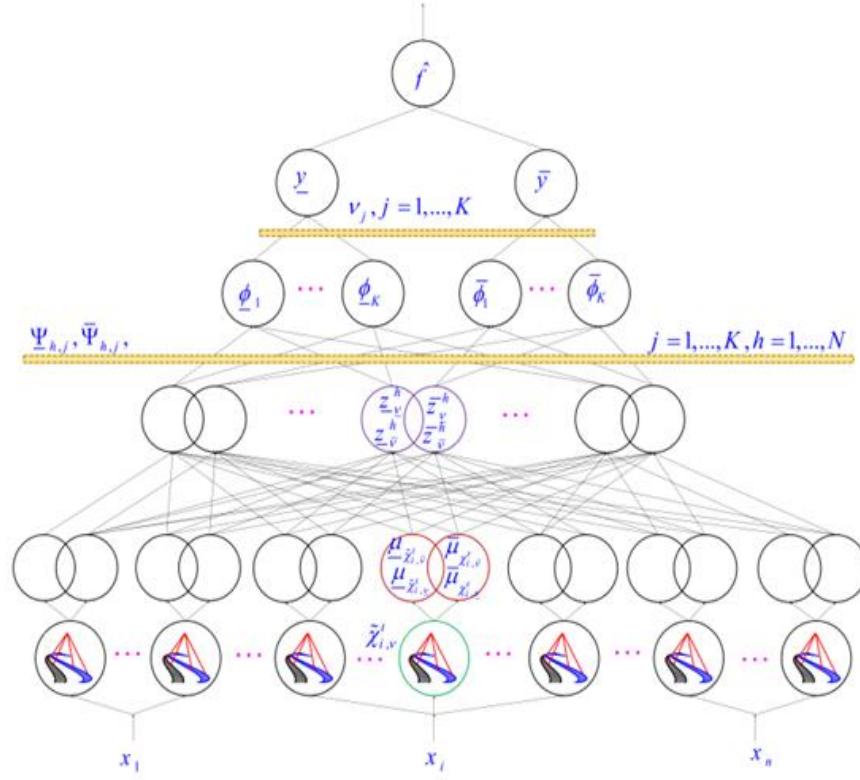


Figure IV.1. Suggested T3-FLS.

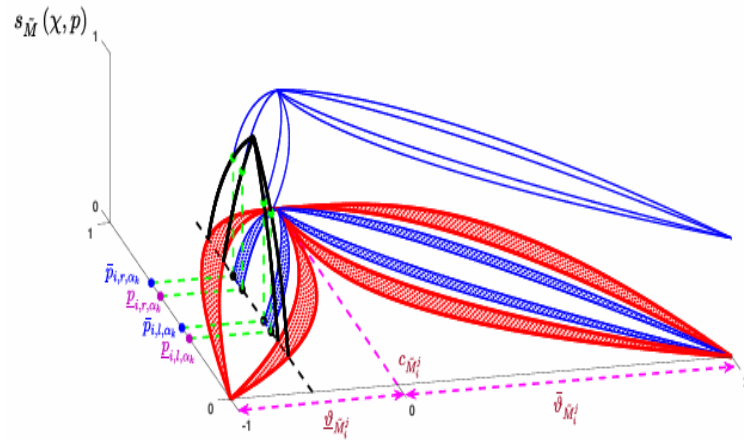


Figure IV.2. Type-3MF.

$$\bar{P}_{i,r,\alpha_k} = \begin{cases} 1 - \left(\frac{|x_i - c_{\bar{M}_i^j}|}{\vartheta_{\bar{M}_i^j}} \right)^{\Delta_r(1-\alpha_k)+1} & \text{if } c_{\bar{M}_i^j} - \vartheta_{\bar{M}_i^j} < X_i \leq c_{\bar{M}_i^j} \\ 1 - \left(\frac{|x_i - c_{\bar{M}_i^j}|}{\bar{\vartheta}_{\bar{M}_i^j}} \right)^{\Delta_r(1-\alpha_k)+1} & \text{if } \bar{M}_i^j < X_i \leq c_{\bar{M}_i^j} + \bar{\vartheta}_{\bar{M}_i^j} \\ 0 & \text{if } X_i > c_{\bar{M}_i^j} + \bar{\vartheta}_{\bar{M}_i^j} \text{ or } X_i \leq c_{\bar{M}_i^j} - \vartheta_{\bar{M}_i^j} \end{cases} \quad (\text{IV.1})$$

$$\underline{P}_{i,r,\alpha_k} = \begin{cases} 1 - \left(\frac{|x_i - c_{\bar{M}_i^j}|}{\underline{\vartheta}_{\bar{M}_i^j}} \right)^{1/(\Delta_r(1-\alpha_k)+1)} & \text{if } c_{\bar{M}_i^j} - \underline{\vartheta}_{\bar{M}_i^j} < X_i \leq c_{\bar{M}_i^j} \\ 1 - \left(\frac{|x_i - c_{\bar{M}_i^j}|}{\bar{\vartheta}_{\bar{M}_i^j}} \right)^{1/(\Delta_r(1-\alpha_k)+1)} & \text{if } \bar{M}_i^j < X_i \leq c_{\bar{M}_i^j} + \bar{\vartheta}_{\bar{M}_i^j} \\ 0 & \text{if } X_i > c_{\bar{M}_i^j} + \bar{\vartheta}_{\bar{M}_i^j} \text{ or } X_i \leq c_{\bar{M}_i^j} - \underline{\vartheta}_{\bar{M}_i^j} \end{cases} \quad (\text{IV.2})$$

$$\bar{P}_{i,l,\alpha_k} = \begin{cases} 1 - \left(\frac{|x_i - c_{\bar{M}_i^j}|}{\underline{\vartheta}_{\bar{M}_i^j}} \right)^{\Delta_l(1-\alpha_k)+1} & \text{if } c_{\bar{M}_i^j} - \underline{\vartheta}_{\bar{M}_i^j} < X_i \leq c_{\bar{M}_i^j} \\ 1 - \left(\frac{|x_i - c_{\bar{M}_i^j}|}{\bar{\vartheta}_{\bar{M}_i^j}} \right)^{\Delta_l(1-\alpha_k)+1} & \text{if } \bar{M}_i^j < X_i \leq c_{\bar{M}_i^j} + \bar{\vartheta}_{\bar{M}_i^j} \\ 0 & \text{if } X_i > c_{\bar{M}_i^j} + \bar{\vartheta}_{\bar{M}_i^j} \text{ or } X_i \leq c_{\bar{M}_i^j} - \underline{\vartheta}_{\bar{M}_i^j} \end{cases} \quad (\text{IV.3})$$

$$\underline{P}_{i,l,\alpha_k} = \begin{cases} 1 - \left(\frac{|x_i - c_{\bar{M}_i^j}|}{\underline{\vartheta}_{\bar{M}_i^j}} \right)^{1/(\Delta_l(1-\alpha_k)+1)} & \text{if } c_{\bar{M}_i^j} - \underline{\vartheta}_{\bar{M}_i^j} < X_i \leq c_{\bar{M}_i^j} \\ 1 - \left(\frac{|x_i - c_{\bar{M}_i^j}|}{\bar{\vartheta}_{\bar{M}_i^j}} \right)^{1/(\Delta_l(1-\alpha_k)+1)} & \text{if } \bar{M}_i^j < X_i \leq c_{\bar{M}_i^j} + \bar{\vartheta}_{\bar{M}_i^j} \\ 0 & \text{if } X_i > c_{\bar{M}_i^j} + \bar{\vartheta}_{\bar{M}_i^j} \text{ or } X_i \leq c_{\bar{M}_i^j} - \underline{\vartheta}_{\bar{M}_i^j} \end{cases} \quad (\text{IV.4})$$

where $\underline{\vartheta}_{\bar{M}_i^j}$ and $\bar{\vartheta}_{\bar{M}_i^j}$ are the widths of the left and right sides of $\bar{M}_i^j$, and $c_{\bar{M}_i^j}$ is the center of $\bar{M}_i^j$ (see Figure .IV.2). The horizontal slice's value is represented by α_k $k=1,\dots,k$. The MFs' breadth is represented by Δ_r and Δ_l . The MFs have narrow forms when Δ_r and Δ_l are small. Taking into account the UB and LB memberships of MFs $\bar{M}_i^j$, the following rule firings are obtained as:

$$\bar{z}_{r,\alpha_k}^h = \bar{s}_{\bar{M}_1^{h_1}|r,\alpha_k}^{h_1}(X_1) \bar{s}_{\bar{M}_2^{h_2}|r,\alpha_k}^{h_2}(X_2) \dots \bar{s}_{\bar{M}_n^{h_n}|r,\alpha_k}^{h_n}(X_n) \quad (\text{IV.5})$$

$$\underline{z}_{r,\alpha_k}^h = \underline{s}_{\bar{M}_1^{h_1}|r,\alpha_k}^{h_1}(X_1) \underline{s}_{\bar{M}_2^{h_2}|r,\alpha_k}^{h_2}(X_2) \dots \underline{s}_{\bar{M}_n^{h_n}|r,\alpha_k}^{h_n}(X_n) \quad (\text{IV.6})$$

$$\bar{z}_{l,\alpha_k}^h = \bar{s}_{\bar{M}_1^{h_1}|l,\alpha_k}^{h_1}(X_1) \bar{s}_{\bar{M}_2^{h_2}|l,\alpha_k}^{h_2}(X_2) \dots \bar{s}_{\bar{M}_n^{h_n}|l,\alpha_k}^{h_n}(X_n) \quad (\text{IV.7})$$

$$\underline{z}_{l,a_k}^h = \underline{s}_{\bar{M}_1^{h_1}|l,\alpha_k}(X_1) \underline{s}_{\bar{M}_2^{h_2}|l,\alpha_k}(X_2) \dots \underline{s}_{\bar{M}_n^{h_n}|l,\alpha_k}(X_n) \quad (\text{IV.8})$$

where $\bar{M}_i^{h_i}$ is the h_i -th MF for input X_i and $k=1,\dots,K$. The following is the h -th rule:

$$\begin{aligned} &\text{If } x_1 \text{ is } \bar{M}_1^{h_1}, \dots, \text{ and } \dots x_n \text{ is } \bar{M}_n^{h_n}, \text{ then} \\ &y_h \in \left[\left[\underline{\psi}_{l,h}, \underline{\psi}_{r,h} \right] \quad \left[\bar{\psi}_{l,h}, \bar{\psi}_{r,h} \right] \right] \end{aligned} \quad (\text{IV.9})$$

where the parameters of the rule are $\underline{\psi}_{l,h}$, $\underline{\psi}_{r,h}$, $\bar{\psi}_{l,h}$, and $\bar{\psi}_{r,h}$. The result of each rule is an interval type-3 fuzzy number, with fuzzy LB and UB. To put it another way, the UB belongs to $[\bar{\psi}_{l,h}, \bar{\psi}_{r,h}]$, while the LB belongs to $[\underline{\psi}_{l,h}, \underline{\psi}_{r,h}]$.

Finally, The output's simplest estimate is recommended as:

$$y = \frac{\sum_{k=1}^K (\bar{\phi}_k + \underline{\phi}_k) \alpha_k}{\sum_{k=1}^K \alpha_k} \quad (\text{IV.10})$$

where:

$$\bar{\phi}_k = \frac{\sum_{h=1}^N (\bar{z}_{r,a_k}^h \bar{\psi}_{r,k}^h + \bar{z}_{l,a_k}^h \bar{\psi}_{l,k}^h)}{\sum_{h=1}^N \bar{z}_{r,a_k}^h + \bar{z}_{l,a_k}^h} \quad (\text{IV.11})$$

$$\underline{\phi}_k = \frac{\sum_{h=1}^N (\underline{z}_{r,a_k}^h \underline{\psi}_{r,k}^h + \underline{z}_{l,a_k}^h \underline{\psi}_{l,k}^h)}{\sum_{h=1}^N \underline{z}_{r,a_k}^h + \underline{z}_{l,a_k}^h} \quad (\text{IV.12})$$

IV.3. The suggested modeling technique

Our goal is to provide a flexible, reliable, and effective model to represent dynamic systems and time series. Initially, a primary model is created with the help of the real data of the dynamic system's input-output and as well as the temporal data under consideration. The error between the output of the dynamic system or the time series under consideration and the output of the primary model is then modelled in order to create the error model. This latter model accounts for the errors in the primary model that can be easily eliminated by simply subtracting the output of the error model from the output of the primary model. The details of this method will be discussed in the following sections.

Any unknown function can be represented or modelled by a fuzzy model $y = f(x)$ via making use of the input-output data. The idea is to find a relationship between the input and output of the fuzzy model by adjusting its parameters over time and using optimization algorithms so that an objective function reaches its minimum. Our method will consist of three steps in order to get the desired output:

- **Step 1:** involves the identification of the primary model.

- **Step 2:** involves the identification of the error process.
- **Step 3:** involves the conception of the final model.

IV.3.1. The parameters of the fuzzy model to be modified

The parameters adjusted during the optimization process are those of the employed Type-3 membership functions, namely:

- ✓ **Center $c_{\bar{M}_i}$:** Center (mean) the membership function;
- ✓ **Left Spread / Width $\underline{\vartheta}_{\bar{M}_i}$:** Left-side width (lower bound);
- ✓ **Right Spread / Width $\bar{\vartheta}_{\bar{M}_i}$:** Right-side width (upper bound);
- ✓ **Right Foot of Uncertainty Points $\bar{P}_{i,r,\alpha_k}$, $\underline{P}_{i,r,\alpha_k}$:** Right lower and upper bounds at slice α_k ;
- ✓ **Slice Level α_k :** Horizontal slice (level of uncertainty);

IV.3.2. Cost function

In mathematical optimization, the term "cost function" refers to a function that serves as a criterion for identifying the optimal solution to an optimization problem. The objective function used is the mean square error criterion (MSE: Mean Square Error), which combines the actual and estimated values as follows:

$$MSE = \frac{\sum_{k=1}^N (y_k - \hat{y}_k)^2}{N} = \frac{\sum_{k=1}^N e^2}{N} \quad (IV.13)$$

N is the size of the measurements, and y_k and $\hat{y}_k$ are the real measurement and its estimate, respectively.

IV.3.3. Identification of the Primary Model

In this step, the entire set of input/output data is used to determine the primary fuzzy model f_p for the process that is the dynamic system or time series under consideration (see Figure IV.3).

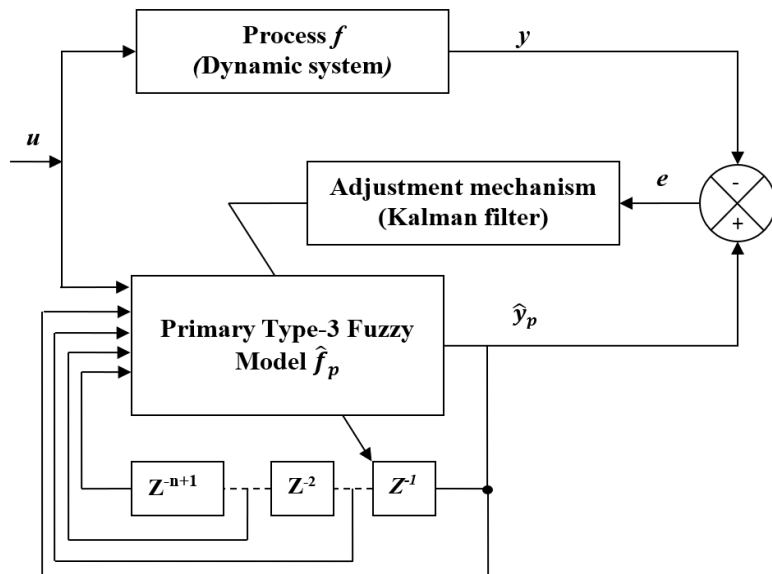


Figure IV.3. Primary model identification.

with:

f : function that has y as its output and the temporal sequence that we wish to identify;

u : The system's entrance;

$\hat{y}_p$: the primary model's output;

$e = \hat{y}_p - y$: error of modelling, used in the optimization algorithm's cost function (to activate the adjustment mechanism).

The fuzzy identification principle is based on a real-time modification of $\hat{f}_p$. The adjustment mechanism shown in Figure 2 is the Kalman filter that will modify the fuzzy model's parameters $\hat{f}_p$ in such a way that the error e between the process's output y and the primary model's output $\hat{y}_p$ reaches the smallest value.

To identify $\hat{f}_p$, The autoregressive moving average (ARMA) fuzzy model is a modeling technique that is part of a group of prediction formulas that attempt to predict a system's output by relying on previous results and the system's input (a previous output becomes a new input).

IV.3.4. Identification of the Error Process

The Error process (Ep) is defined as a parallel connection between the process (dynamical system or time series under consideration) and the primary fuzzy model, as seen in the following figure:

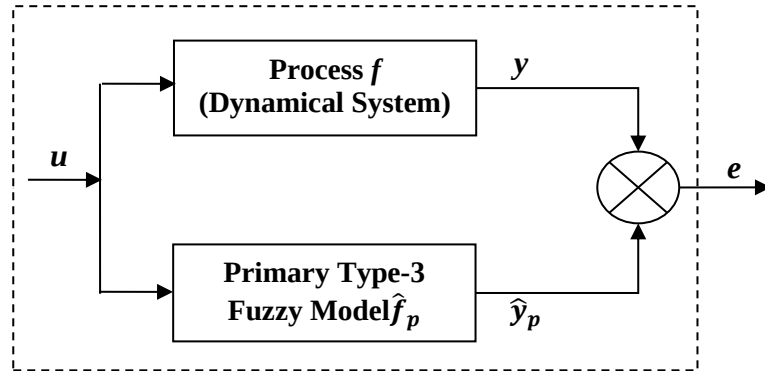


Figure IV.4. Error process.

The output of the error process (Ep) is defined as follows:

$$e = \hat{y}_p - y. \quad (\text{IV.14})$$

Once the output of the error process (Ep) is obtained (Equation 2), a fuzzy model can be used to model the error e . This model will be known as the fuzzy error model. Since the error is thought of as a temporal phenomenon, it is convenient to model it using a fuzzy autoregressive (AR) model, which attempts to predict the new output based on previous results.

Figure IV.5 illustrates the second phase's structure, where $\hat{e}$ is the output of the fuzzy error model and e_1 is the difference between the output of the error process and the output of the error fuzzy model.

The current task is to adjust the error fuzzy model's parameters until the error e_1 reaches its minimum.

IV.3.5. Final model conception

In this particular stage, the primary model $\hat{f}_p$ and the error model $\hat{E}_p$ are connected in a parallel pattern (see image). IV.5) in order to obtain the final model $\hat{f}_F$. This arrangement will allow us to compensate for the residual error (modeling error) obtained in the first step (primary model).

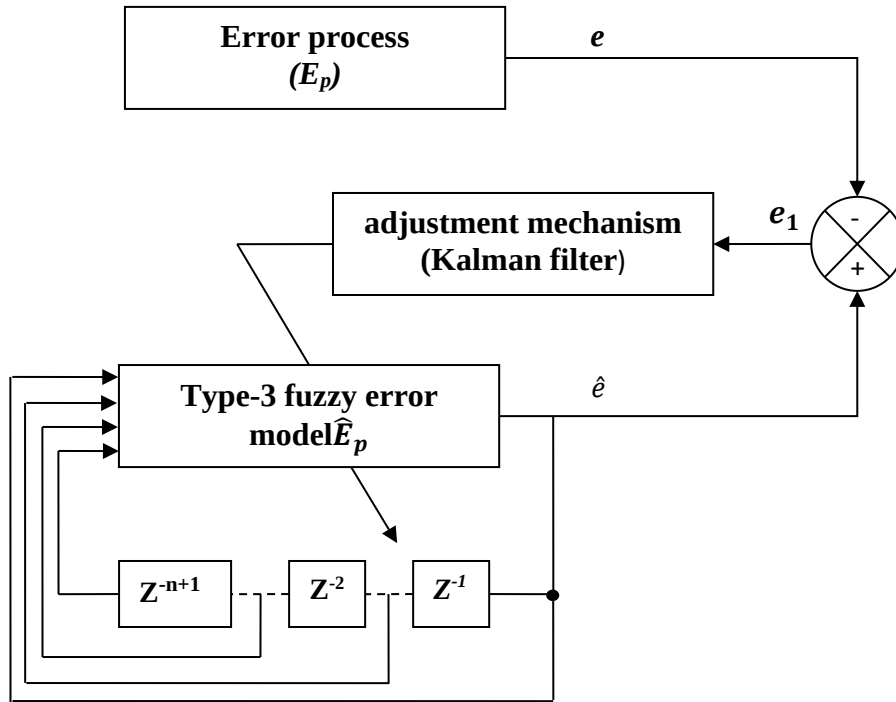


Figure IV.5. Identification of the error process.

Consequently, the global modeling error is minimized. The finished model's output can be expressed as follows:

$$\hat{y} = \hat{y}_p - \hat{e}. \quad (\text{IV.15})$$

where $\hat{y}_p$ and $\hat{e}$ are the outputs of the primary fuzzy model and the output of the fuzzy error model, respectively.

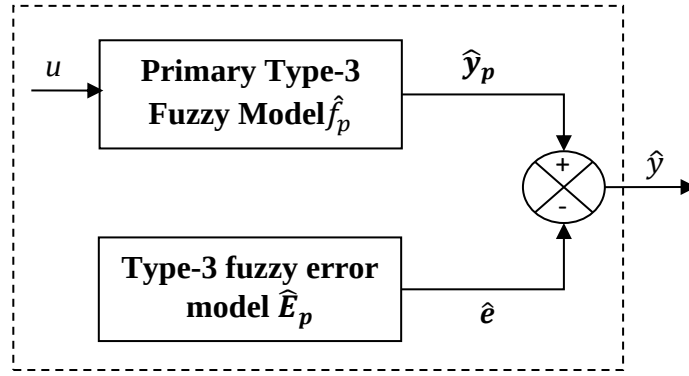


Figure. IV.6. Final model $\hat{f}_F$.

IV.4. Simulation Results

IV.4.1. Modeling of dynamic systems

In this section, we will present and discuss the simulation results of our method used for the modeling of dynamical systems. The following three dynamical systems will be taken into consideration for modeling:

1. Model I:

$$y_p(k+1) = f[y_p(k), y_p(k-1), \dots, y_p(k-n+1)] + \sum_{i=0}^{m-1} \beta_i u(k-1) \quad (\text{IV.16})$$

2. Model II

$$y_p(k+1) = \sum_{i=0}^{n-1} \alpha_i y_p(k-1) + g[u(k), u(k-1), \dots, u(k-m+1)] \quad (\text{IV.17})$$

3. Model III

$$y_p(k+1) = f[y_p(k), y_p(k-1), \dots, y_p(k-n+1)] + g[u(k), u(k-1), \dots, u(k-m+1)] \quad (\text{IV.18})$$

IV.4.1.1. Modeling of System I

In this subsection, we will investigate the effectiveness of the fuzzy modeling technique on a complex system (Equation. IV.16), or the nonlinearity is only related to the output, taking into account the specific example given by the following equations:

$$y_p(k+1) = f[y_p(k), y_p(k-1)] + u(k) \quad (\text{IV.19})$$

with:

$$f[y_p(k), y_p(k-1)] = \frac{y_p(k)y_p(k-1)[y_p(k)+2.5]}{1+y_p^2(k)+y_p^2(k-1)} \quad (\text{IV.20})$$

and:

$$u(k) = \sin \frac{2\pi k}{25} \quad \text{for } 1 \leq k \leq 5 \text{ \& } 150 \leq k \leq 200 \quad (\text{IV.21})$$

$$u(k) = 0.5 \sin \frac{2\pi k}{10} + 0.5 \sin \frac{2\pi k}{5} \quad \text{for } 50 \leq k \leq 150 \quad (\text{IV.22})$$

Figures IV.7-a, IV.7-b, and IV.7-c, respectively, illustrate the simulation results as follows:

- ✓ **Figure IV.7-a:** shows a superposition of the dynamic system's output and the primary fuzzy model's output.
- ✓ **Figure IV.7-b:** shows a superposition of the dynamic system output and the final model output.
- ✓ **Figure IV.7-c:** shows a superposition of the modeling error and the output of the model of the modeling error.

By a visual examination of figures IV.7-a, IV.7-b, and IV.7-c reveals that the final model is significantly better than the primary model, and that the final model's error has been reduced, which enables the final model to be effectively improved.

IV.4.1.2. Modeling of System II

In the following, we examine the effectiveness of our method on a more complex system (Equation IV.17), where the nonlinearity is just related to the input by taking into account the following equations:

$$y_p(k+1) = 0.3y_p(k) + 0.6y_p(k-1) + f[u(k)] \quad (\text{IV.23})$$

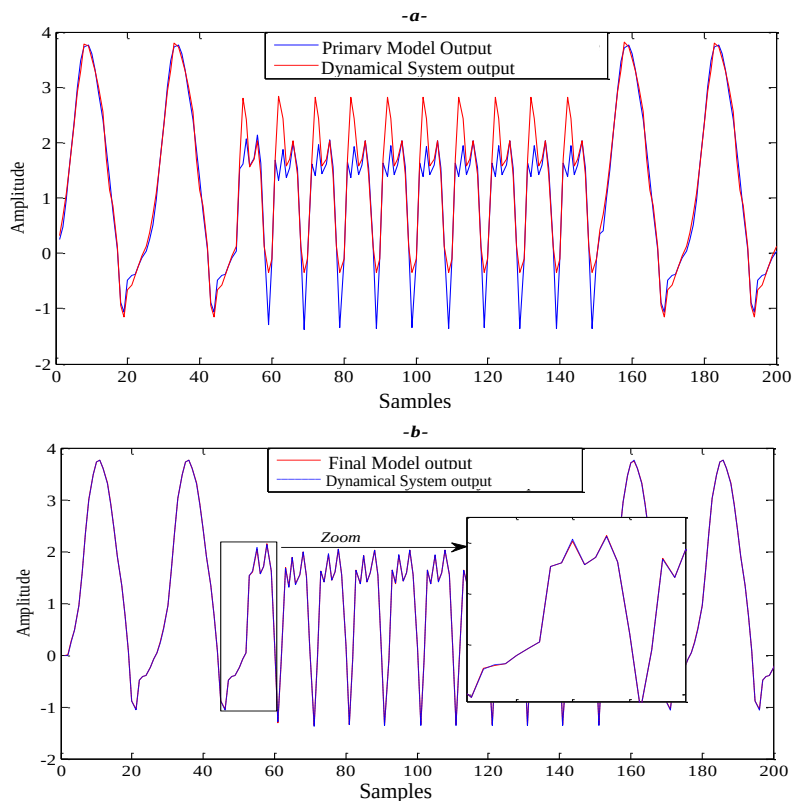
with:

$$f(u) = 0.6 \sin(\pi u) + 0.3 \sin(3\pi u) + 0.1 \sin(5\pi u) \quad (\text{IV.24})$$

and:

$$u(k) = \sin \frac{2\pi k}{250}. \quad (\text{IV.25})$$

The final results of the fuzzy identifier are shown in Figure IV.8. By examining the zooms, we confirm the effectiveness of our method for this kind of issue.



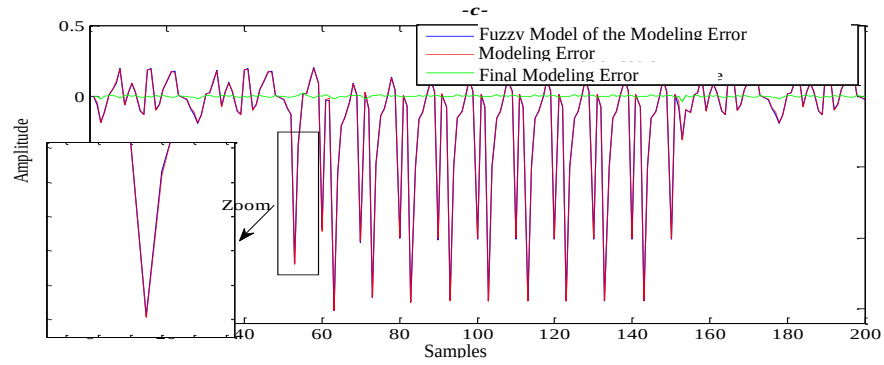
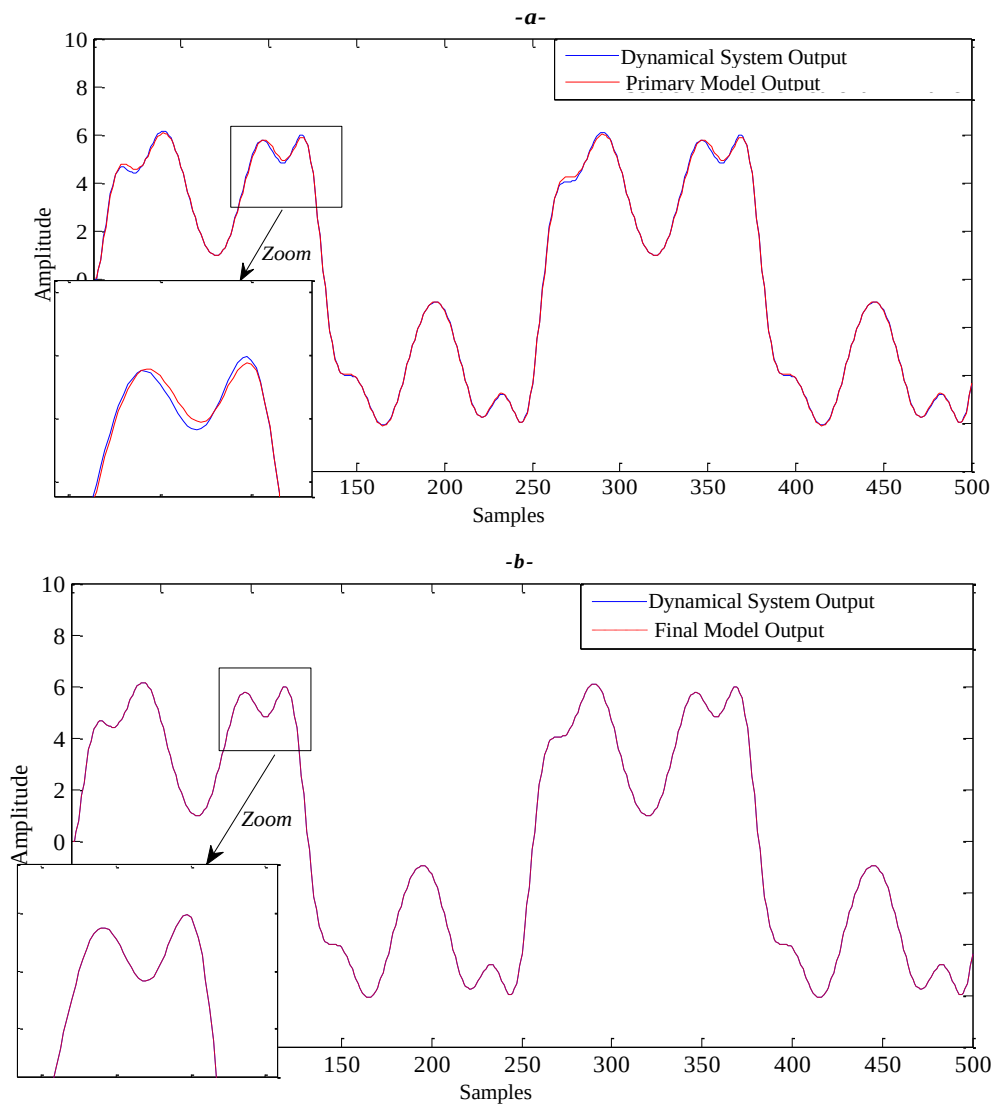


Figure IV.7. Simulation results of the dynamic system Model I: (a) Primary Model, (b) Final Model, and (c) Modeling Errors.



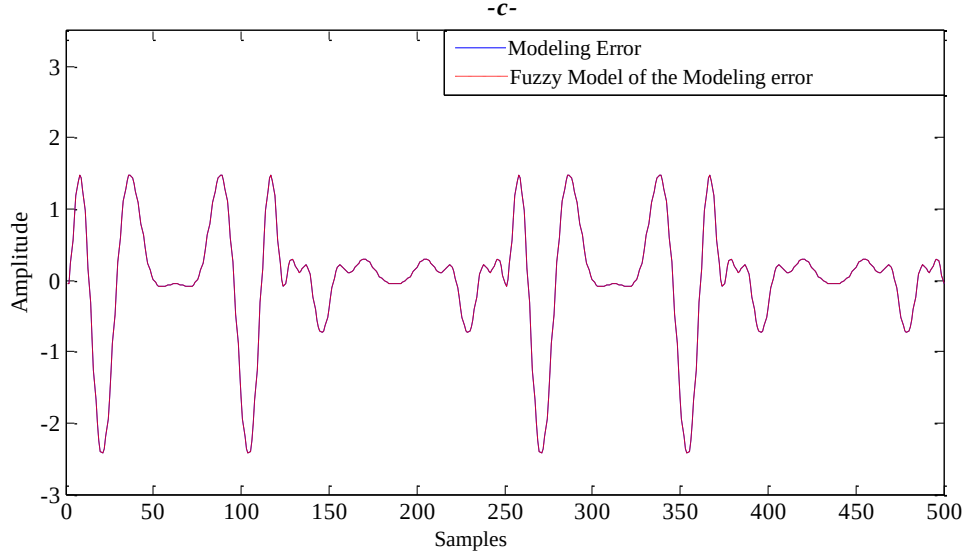


Figure IV.8. Simulation results of the dynamic system Model II: (a) the Primary model, (b) the final model, and (c) the modeling errors.

IV.4.1.3. Modeling of System III

The model III (Equation IV.18) will be used to verify the effectiveness of our method for more complicated system, taking into account the specific case described by the following equations:

$$y_p(k+1) = f(y_p(k), u(k)) = \frac{y_p(k)}{1+y_p(k)^2} + u^3(k) \quad (\text{IV.26})$$

The unknown function has a more complex structure since its independent variables are the input and output signals. The entry signal is selected as follows:

$$u(k) = \sin\left(\frac{2\pi k}{25}\right) + \sin\left(\frac{2\pi k}{10}\right) \quad (\text{IV.27})$$

The final results of the fuzzy identifier are shown in Figure IV.9. By examining the zooms, we consistently verify the effectiveness of our approach for this kind of issue.

IV.4.2. Modeling of time series

In this section, we will present and discuss the simulation results of our method used to model time series. Two categories of time series have been taken into consideration:

- 1- **Synthetic time series:** developed with the use of mathematical models to understand:
 - ✓ A synthetic ECG Signal.
 - ✓ The Mackey-Glass time series is recognized as a benchmark function problem.
- 2- **real time series:** real ECG signal.

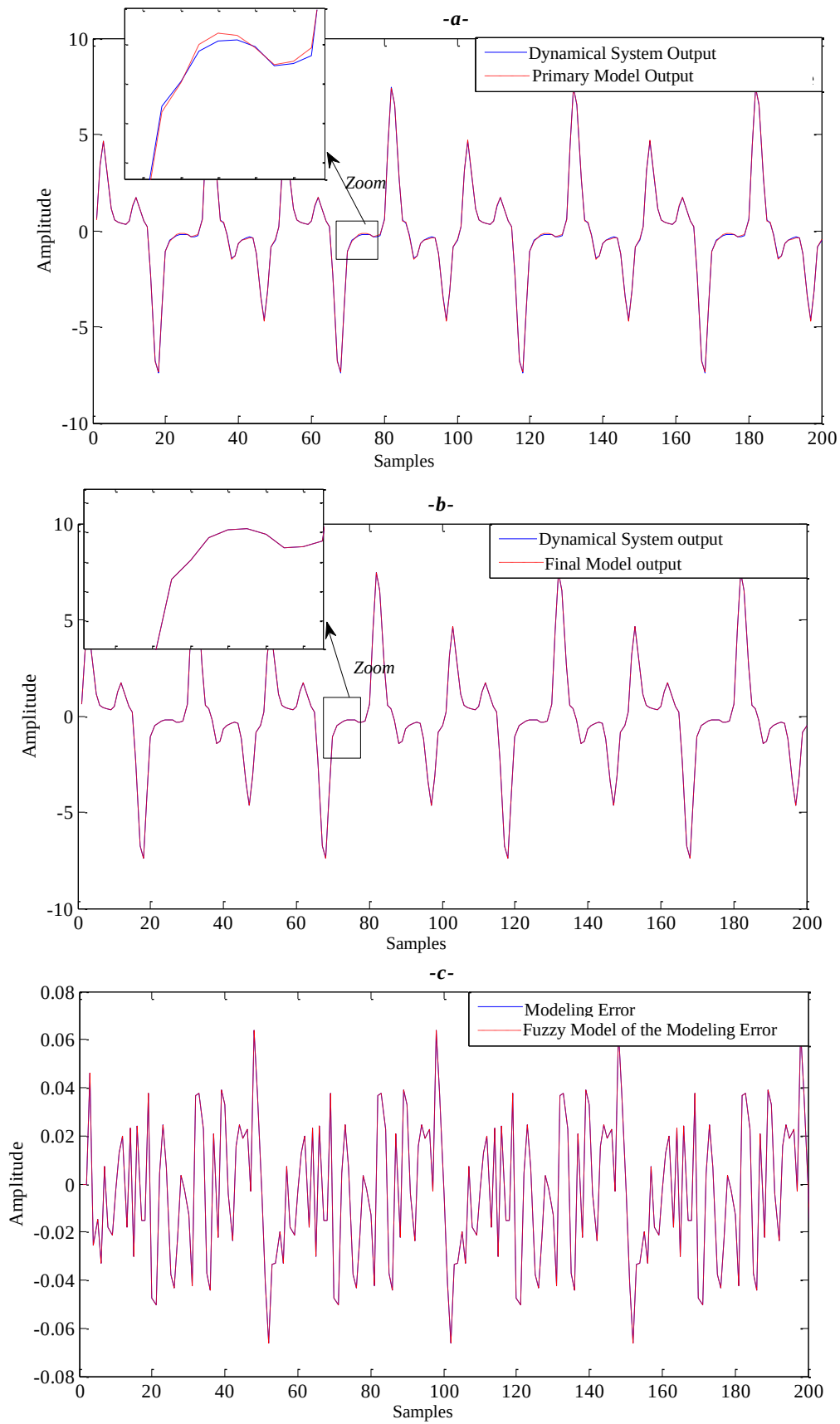


Figure IV.9. Simulation results of the dynamic system Model III: (a) the Primary model, (b) the final model, and (c) the modeling errors.

IV.4.2.1. Modeling of synthetic ECG signal

In this section, a mathematical model that enables the creation of an ECG signal will be used. This model can be expressed as the sum of the three subsequent equations:

$$\begin{cases} \dot{x}_1 = \alpha x_1 + \omega x_2 \\ \dot{x}_2 = \alpha x_2 + \omega x_1 \\ \dot{x}_3 = -\sum_{i \in \{P, Q, R, S, T\}} a_i \Delta \theta_i \exp\left(\frac{-\Delta \theta_i^2}{2b_i^2}\right) - (x_3 - z_0) \end{cases} \quad (\text{IV.28})$$

with $\Delta \theta_i = (\theta - \theta_i)$, $\theta = \arctan2(x_2, x_1)$, a_i are the parameters that determine the Gaussian amplitudes, the b_i represent the Gaussian's' larger and z_0 represents the ECG signal's base line, which is written as follows:

$$z_0 = A \sin(2\pi f_r t) \quad (\text{IV.29})$$

with : $a = 0.15mv$ and $f_r = 0.25Hz$.

Table 1 represents the parameters of model (IV.28) used to generate a synthetic electrocardiogram signal.

Table 1: ECG model parameters.

Index (i)	P	Q	R	S	T
<i>Time (s)</i>	-0.2	-0.05	0	0.05	0.3
θ_i (radians)	$-\pi/3$	$-\pi/12$	0	$\pi/12$	$\pi/2$
a_i	1.2	-0.5	30.0	-7.5	0.75
b_i	0.25	0.1	0.1	0.1	0.4

The previously discussed method of modeling time series will be used to model the synthetic ECG signal. The simulation results are shown in Figures IV.10 -a, IV.10-b, and IV.10-c, respectively.

- ✓ **Figure IV.10-a:** shows a superposition of the synthetic ECG signal with the output of the primary fuzzy model.
- ✓ **Figure IV.10-b:** shows a superposition of the synthetic ECG signal and its final fuzzy model output.
- ✓ **Figure IV.10-c:** shows a superposition of the modeling error, the model of the modeling error, and the final modeling error.

A cursory visual examination of Figures IV.10-a, IV.10-b, and IV.10-c reveals that the final model is far better than the initial model, and that the final model's error has been reduced, allowing for effective improvement.

IV.4.2.2. Modeling of a real ECG signal

The application of the fuzzy modeling technique for modeling of a real ECG signal will be discussed in this section. For this purpose, the ECG signal 16539.dat from the MIT-BIH Physio Bank data base will be regarded as an unknown time series that we wish to model. Figures IV.11-a, IV.11-b, and IV.11-c, respectively, show the simulation results as follows:

- ✓ **Figure IV.11-a:** shows a superposition of the ECG signal 16539.dat and the output of the primary fuzzy model.
- ✓ **Figure IV.11-b:** shows a superposition of the ECG signal 16539.dat and the output of the final fuzzy model of this signal.
- ✓ **Figure IV.11-c:** shows a superposition of the modeling error and the model of the modeling error.

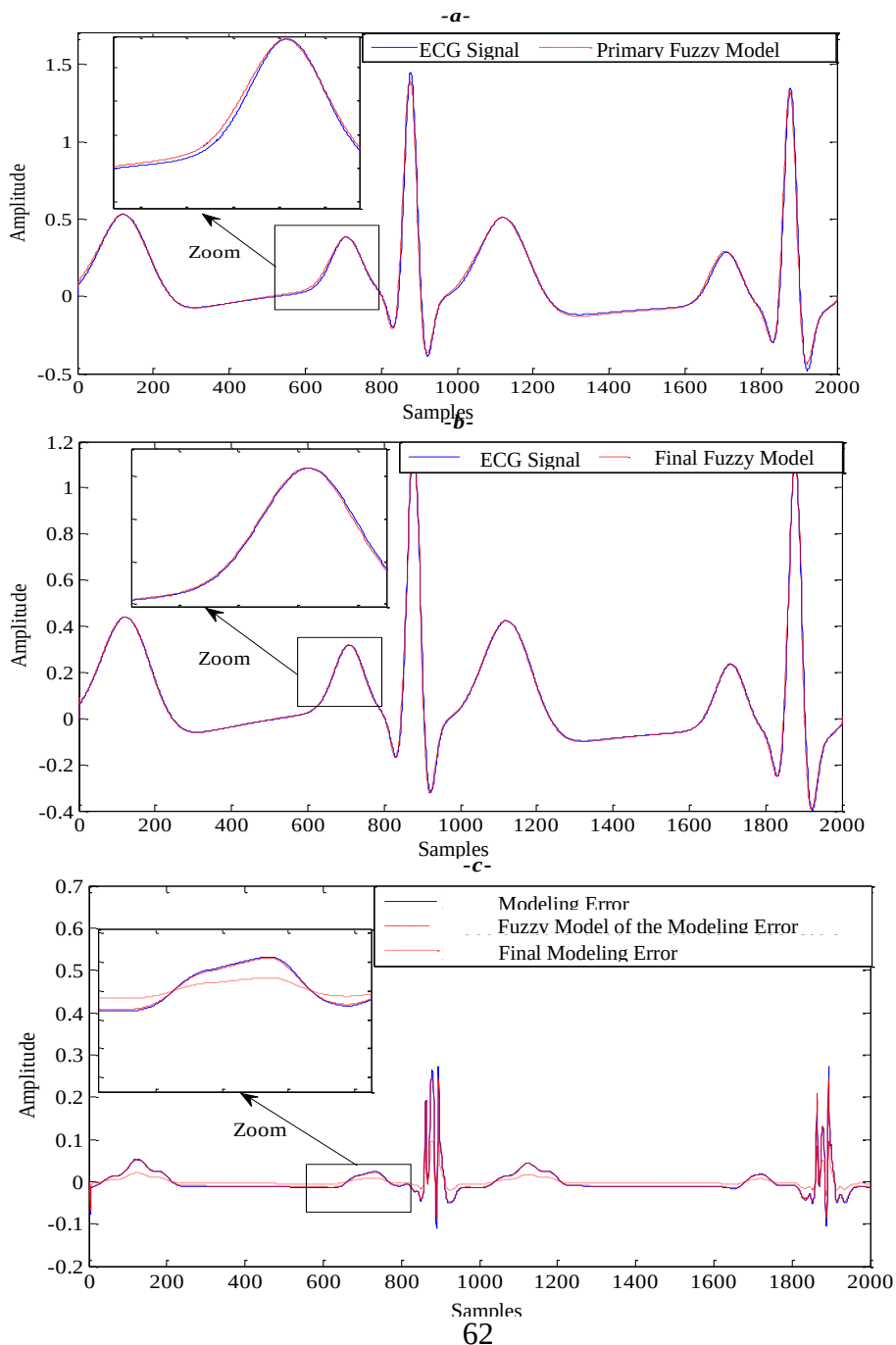


Figure IV.10. Simulation results of the synthetic ECG signal: (a) Primary Model, (b) Final Model, and (c) Modeling Errors.

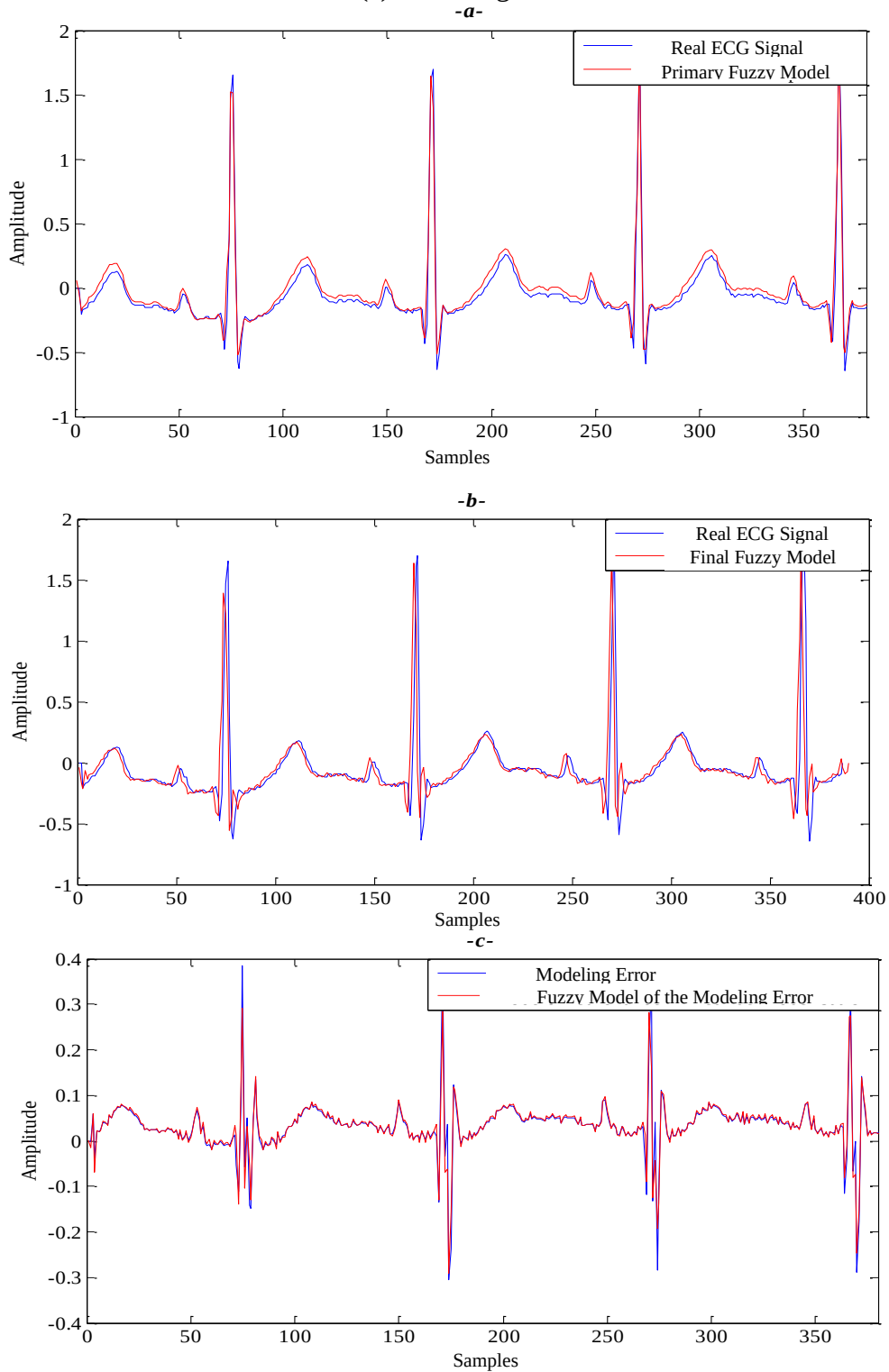


Figure IV.11. Simulation results of the real ECG signal: (a) the Primary model, (b) the Final model, and (c) the modeling errors.

IV.4.2.3 Mackey-Glass function modeling

The Mackey-Glass time series is recognized as a time series problem, which is defined by the following fundamental equation:

$$\frac{dx(t)}{dt} = \frac{0.2 * x(t-\tau)}{(1+x(t-\tau)^{10})} - 0.1 * x(t) \quad (\text{IV.30})$$

Equation IV.30 was used to create 1200 samples of Mackey-Glass function, and our modeling method was then used to model these data. The modeling result is shown in Figure IV.13.

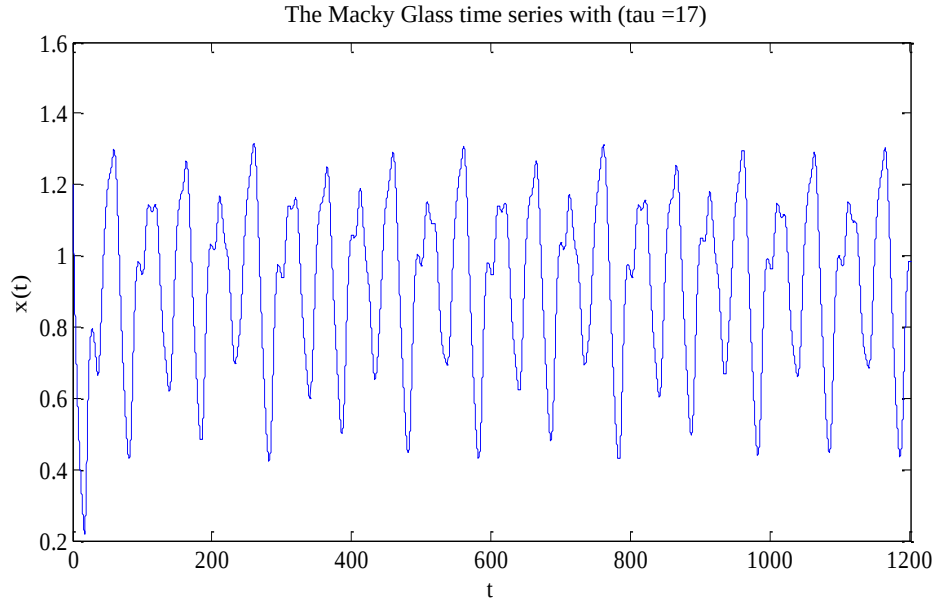


Figure IV.12. Macky Glass function.

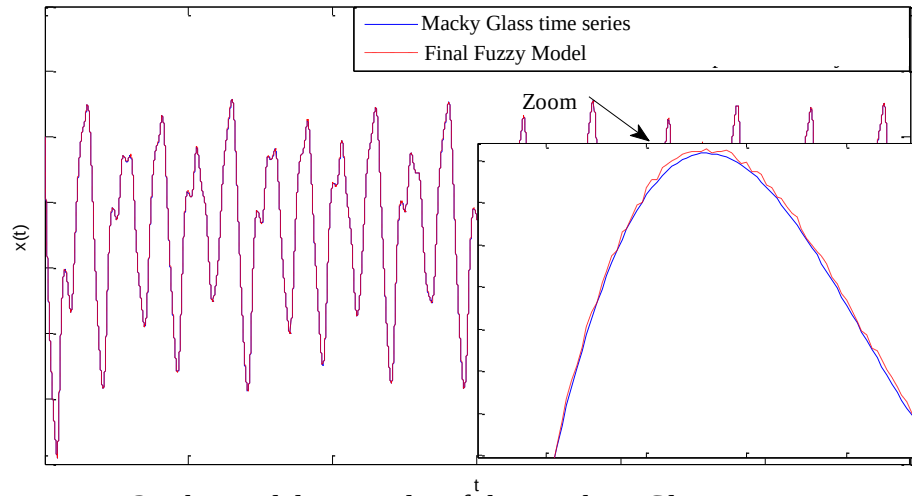


Figure. IV.13. The modeling results of the Mackey–Glass time series.

IV.5. Conclusion

In this chapter, we have developed a method for modeling dynamical systems based on fuzzy logic type 3 and an optimization algorithm based on the Kalman filter. This method allows us to realize the concept of identification by adding a new identification module called the fuzzy error model, which was introduced as a complement of the primary model to improve the model's quality, which has provided a good accuracy. The optimization of the proposed structure's parameters has been ensured by using the optimization algorithms based on the Kalman filter. Several simulation results have been presented to assess the effectiveness of our method.

General conclusion

In this work, a modeling approach based on Type-3 fuzzy logic systems has been proposed for the analysis and prediction of dynamical systems and time series. The objective of this work was to develop an intelligent modeling framework capable of handling nonlinearities and uncertainties commonly encountered in real-world systems.

The study began with a review of the fundamental concepts related to dynamical systems, time series analysis, and fuzzy logic theory. Particular attention was given to Type-3 fuzzy systems due to their enhanced ability to represent and manage uncertainty compared with conventional fuzzy approaches. Subsequently, an optimization process was employed to adjust the parameters of the membership functions in order to improve the performance and accuracy of the proposed model.

The obtained simulation results demonstrate that the proposed model provides satisfactory performance in terms of approximation accuracy. The optimized Type-3 fuzzy system was able to efficiently capture the nonlinear behavior of the studied dynamical systems and accurately model the considered time series data. Furthermore, the results confirm the effectiveness of the optimization procedure in enhancing the quality of the fuzzy model.

Overall, this work highlights the potential of Type-3 fuzzy logic systems as powerful tools for dynamical system modeling and time series prediction under uncertain conditions. The proposed methodology can be extended and applied to various engineering and scientific applications involving complex nonlinear processes.

As future perspectives, further investigations may focus on the integration of advanced optimization algorithms, hybrid intelligent techniques, or deep learning methods in order to further improve modeling performance and computational efficiency. Additionally, the proposed approach may be validated using larger real-world datasets and real-time applications.

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Abstract - This dissertation develops an innovative approach for modeling and identifying complex non-linear dynamical systems and time series (such as real ECG signals and the Mackey-Glass chaotic time series) using Type-3 Fuzzy Logic Systems optimized by the Kalman Filter. The research is structured into four chapters: Chapter I covers the theoretical foundations of dynamical systems and temporal dynamics. Chapter II reviews mathematical modeling and system identification procedures. Chapter III focuses on fuzzy logic evolution, showcasing Type-3 fuzzy logic's superior ability to handle high-order uncertainties. Chapter IV details the proposed methodology, which connects a low-resolution primary fuzzy model in parallel with a fuzzy error model to reduce uncertainty. Parameters are optimized via the Kalman filter, achieving outstanding high-resolution tracking and minimal modeling error.

Keywords: Dynamical Systems, Time Series, System Identification, Type-3 Fuzzy Logic, Kalman Filter, ECG Signal.

Résumé – Ce mémoire présente le développement d'une méthode innovante pour la modélisation et l'identification des systèmes dynamiques complexes et des séries temporelles (telles que les signaux ECG et la série de Mackey-Glass) en utilisant la logique floue de Type-3 optimisée par le filtre de Kalman. Le travail s'articule autour de quatre chapitres : le premier établit les fondements théoriques des systèmes dynamiques et des séries temporelles. Le deuxième est dédié aux théories de modélisation et d'identification des systèmes (structures linéaires et non linéaires). Le troisième explore l'évolution de la logique floue du Type-1 au Type-3, mettant en évidence sa capacité supérieure à gérer des niveaux élevés d'incertitude et de bruit. Enfin, le quatrième chapitre introduit l'approche proposée basée sur la connexion en parallèle d'un modèle primaire et d'un modèle d'erreur floue pour corriger les incertitudes, avec une optimisation par le filtre de Kalman, validée par des simulations hautement précises.

Mots-clés: Systèmes Dynamiques, Séries Temporelles, Identification des Systèmes, Logique Floue de Type-3, Filtre de Kalman, ECG.

الملخص - تناول هذه المذكرة تطوير طريقة مبتكرة لنمذجة وتحديد الأنظمة الديناميكية والسلاسل الزمنية المعقدة مثل إشارات تخطيط القلب ECG وسلسلة ماكي-غلاس (باستخدام المنطق الضبابي من النوع الثالث (Type-3 Fuzzy Logic) المحسن بواسطة مرشح كالمان (Kalman Filter) يمتد العمل عبر أربعة فصول متكاملة؛ حيث يضع الفصل الأول الأساس النظري للأنظمة الديناميكية غير الخطية والسلاسل الزمنية، بينما يُخصّص الفصل الثاني لدراسة نظريات النمذجة والتعرف على الأنظمة وهياكلها الخطية وغير الخطية. ويقدم الفصل الثالث تعمقاً تدريجياً في المنطق الضبابي من النوع الأول والثاني وصولاً إلى النوع الثالث المتقدم لتمييزه بقابليته العالية على معالجة المستويات المرتفعة من عدم اليقين والتشويش في البيانات. أخيراً، يقدم الفصل الرابع الطريقة المقترحة المعتمدة على ربط نموذجين فرعيين على التوازي: "النموذج الأولي" منخفض الدقة، و"نموذج الخطأ الضبابي" لتصحيح الفروقات، وتم تحسين معاملات هذه البنية المدمجة بواسطة مرشح كالمان، مما أظهر نتائج محاكاة دقيقة جداً ومطابقة للبيانات الحقيقية.

الكلمات المفتاحية: الأنظمة الديناميكية، السلاسل الزمنية، تحديد الأنظمة، المنطق الضبابي من النوع الثالث، مرشح كالمان، إشارة تخطيط القلب (ECG).