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Theme

Control for the Fuel and Oxygen Flows of the Solid Oxide Fuel Cell by two Types of Fuzzy Adaptive PID

Presented on **June 2025**, Under Board of Examiners composed of:

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بسم الله الرحمن الرحيم

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With our sincerest gratitude and utmost respect.

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I dedicate this work to my dear mother and memory of my late dad:
May she find and may he sense, in these words my heartfelt thanks and deep respect for them.
To my brothers and sisters, my family,
and to my wife and my son, my pride and joy.

To everyone who's helped me out whether upfront or behind scenes
and to those who've stood by me through highs and lows of this journey,
and ones who've cheered me on and had my back all way.
To my closest mates, ones who lift my spirits and share my dreams,
especially Saleh bennaoui, smail ossifer
wishing them all best of luck and success.
Cheers from bottom of my heart!

Mostafa Babah

Dedication

This work is wholeheartedly dedicated to:

My dearest mother,
Kind, noble and loving you are for me embodiment of compassion, source of endless tenderness and model of selfless devotion. Your constant support, encouragement and prayers have been my guiding light throughout this journey.

My father,
No words can truly express depth of my love, respect and admiration for you. This thesis is result of sacrifices you made for my education and growth. You have given far more than any parent could ever be asked to give and your guidance has shaped person I am today. May Allah grant your health, long life and endless happiness.

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*And to **my late brother Riad,** whom I lost in tragic road accident—how I wish you could be here beside me. Your absence is deeply felt and your memory lives on in my heart.*

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ملخص

تهدف هذه الدراسة إلى تحسين أداء خلايا الوقود الصلبة (SOFC) من خلال التحكم الدقيق في تدفق الوقود والأوكسجين باستخدام نوعين مختلفين من المتحكمات التكيفية الضبابية: من النوع الأول والنوع الثاني. تم بناء نموذج ديناميكي دقيق لخلية الوقود، وتم دمج متحكم PID تقليدي مع منطق ضبابي من كلا النوعين. المتحكم من النوع الثاني تميز بقدرته الأعلى على التعامل مع عدم اليقين والتشويش مقارنةً بالنوع الأول، رغم تعقيده الحسابي الأكبر. أظهرت النتائج أن النظام الذي يعتمد على التحكم الضبابي التكيفي، خصوصاً من النوع الثاني، يقدم أداءً فائقاً من حيث سرعة الاستجابة، غياب التجاوز، والثبات الديناميكي. كما تم تطبيق تقنيات تحسين مثل آلية Anti-Windup وخوارزمية تفاضلية لتحسين الاستقرار والفعالية. تساهم هذه النتائج في تطوير أنظمة خلايا الوقود المستقبلية وزيادة موثوقيتها في التطبيقات الواقعية.

الكلمات المفتاحية:

خلية الوقود الصلبة، التحكمPID ، المنطق الضبابي، نوع-1، نوع-2، Anti-Windup، التحكم التكيفي، الهيدروجين.

Abstract

This study aims to enhance performance of Solid Oxide Fuel Cells (SOFCs) by precisely controlling fuel and oxygen flow using two types of fuzzy adaptive PID controllers: Type-1 and Type-2. dynamic model of SOFC was developed and traditional PID controller was combined with both fuzzy logic types. Type-2 fuzzy controller showed superior capability in handling uncertainty and noise compared to Type-1, albeit at cost of higher computational complexity. Simulation results revealed that fuzzy adaptive control especially using Type-2 logic greatly improves system response time, eliminates overshoot and enhances dynamic stability. Optimization techniques like Anti-Windup and differential forward algorithms were also applied to improve robustness. These findings contribute to advancing reliable and efficient SOFC systems for real-world applications.

Keywords:

Solid Oxide Fuel Cell, PID control, Fuzzy Logic, Type-1, Type-2, Anti-Windup, Adaptive Control, Hydrogen.

Résumé

Cette étude vise à améliorer la performance des piles à combustible à oxyde solide (SOFC) en contrôlant avec précision les flux de carburant et d'oxygène à l'aide de deux types de régulateurs PID adaptatifs flous : de type-1 et de type-2. Un modèle dynamique précis de la pile été élaboré, et un contrôleur PID classique été enrichi avec une logique floue de chaque type. Le contrôleur flou de type-2 s'est révélé plus performant face aux incertitudes et aux perturbations, bien que plus complexe à implémenter. Les résultats de simulation ont montré que le contrôle adaptatif flou, en particulier le type-2, améliore significativement le temps de réponse, élimine le dépassement et assure une meilleure stabilité. Des techniques comme l'Anti-Windup et l'algorithme différentiel ont également été intégrées pour renforcer la robustesse du système. Ces résultats ouvrent la voie à des systèmes SOFC plus fiables et performants.

Mots-clés :

Pile à combustible à oxyde solide, Contrôle PID, Logique floue, Type-1, Type-2, Anti-Windup, Contrôle adaptatif, Hydrogène.

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Nomenclature

SOFC	Solid Oxide Fuel Cell
PEMFC	Proton Exchange Membrane Fuel Cell
AFC	Alkaline Fuel Cell
MCFC	Molten Carbonate Fuel Cell
PAFC	Phosphoric Acid Fuel Cell
DMFC	Direct Methanol Fuel Cell
PID	Proportional–Integral–Derivative controller
FPID1	Type-1 Fuzzy Adaptive PID Controller
FPID2	Type-2 Fuzzy Adaptive PID Controller
Anti-windup	Mechanism to prevent integral windup in PID controllers
FOU	Footprint of Uncertainty (in Type-2 fuzzy logic)
KM Algorithm	Karnik-Mendel algorithm for type reduction
EKM Algorithm	Enhanced Karnik-Mendel algorithm
Defuzzification	Process of converting fuzzy output to crisp value
MOM	Mean of Maximum defuzzification method
Fuzzification	Conversion of crisp inputs into fuzzy sets

Symbols

A	Pre-exponential factor (kA/cm^2)
α, β	Ohmic resistance constants (Ω, K)
e	Error signal
Δe or ec	Change in error
E	Actual open-circuit voltage
E_0	Ideal standard potential / Standard open-circuit voltage,1.1 V
F	Faraday constant (96485 C/mol)
G	Gibbs free energy (kJ/mol)

Heff	Thickness of cell component,m
i	Current density (A/cm^2)
Iref	Reference current (A)
iL	Limiting current density (A/cm^2)
Kp	Proportional gain
Ki	Integral gain
Kd	Derivative gain
N	Number of cells in stack
Ni, Nj	Surface flux of gas ($mol/(m^2 \cdot s)$)
P	Global pressure of gas mixture (Pa)
Pa, Pc	Anode and cathode pressures,atm
PH_2, PO_2, PH_2O	"Partial pressures of hydrogen, oxygen and water vapor" (atm)
qH_2, qO_2, qH_2O	"Flow rates of hydrogen, oxygen and water"
R	Universal gas constant (8.314 $J/(mol \cdot K)$)
Ra, Rc	Resistance in anode / cathode (Ω)
T	Operating temperature (K)
T_0	Reference temperature (973 K)
U	Utilization factor
∇	Gradient operator
λs	Thermal conductivity ($W/(m \cdot K)$)
η	Efficiency
ρ	Density (kg/m^3)
cp	Heat capacity ($J/(kg \cdot K)$)
τa, τc	Time constants for anode and cathode pressure response (s)

GENERAL INTRODUCTION

The energy sector worldwide is, nowadays, slowly shifting to green and renewable sources to overcome greenhouse effects and use of fossil fuels in energy consumption. International Energy Agency (IEA) predicted that such hydrogen systems would contribute to reaching net-zero target by 2050 [1]. Of various hydrogen technologies, Solid Oxide Fuel Cells (SOFCs) generate considerable interest, as these cells are highly efficient, flexible in choice of fuels and produce very low pollution. design of an SOFC permits conversion of energy into electrical energy from chemical energy of hydrogen or other fuels by an electrochemical process with efficiencies of about 60% when used in stand-alone mode, but even greater in combined heat and power (CHP) applications[2].

The operational challenges faced by SOFC systems with respect to their control in most cases arise due to fuel and oxygen flow control. These systems are very sensitive to changes in operating conditions, be it temperature, pressure or flow rate. "Precise control of fuel and oxidant supply is critical for maintaining stable operation and maximizing efficiency of SOFC systems"[3]. With SOFCs, traditional methods have generally used PID controllers because of their simplicity and availability. However, they seldom account for model's nonlinear dynamic behavior and uncertainty[4]. This limitation provoked researchers to focus more on advanced control strategies such as Fuzzy Logic and Adaptive Control to increase robustness and adaptiveness.

Fuzzy logic provides useful mathematical framework to assess imprecise or uncertain data, employing such constructs as linguistic variables and rules[5]. In other words, fuzzy logic can dynamically vary control parameters of PID controllers to enhance performance of system in operation depending on varying environmental conditions. On contrary, adaptive control affects real-time adjustment based on feedback from system; hence its suitability for complex and dynamic systems like SOFC[6]. few recent examples highlight efficacy of fuzzy logic combined with adaptive PID control for system stability and efficiency[7].

The aim of this thesis is to control fuel and oxygen flow in SOFCs through design and implementation of Fuzzy Adaptive PID controllers. first type of fuzzy adaptive PID controller modifies PID parameters independently, while second type implements collective adaptation strategy. two types of controllers will be quantified by comparing their performance with classical PID type regarding response time, stability and efficiency of overall system. research offers practical guidance toward operational reliability improvements of SOFCs, thus contributing to acceptance of hydrogen-based systems.

The study serves as bridge that links theoretical developments to support practical applications in renewable energy technologies. As noted by Smith and Zhang, "the integration of advanced control techniques with renewable energy systems is essential for achieving sustainable energy solutions"[8]. Following this study, with increased efficiency in control of SOFCs, present study would complement control mechanism in achieving cleaner and more sustainable energy future.

CHAPTER 1:

FUEL CELLS

1.1 Introduction

Fuel cells represent corner stone of modern energy research, offering clean, efficient and versatile alternative to conventional power generation technologies. As devices that directly convert chemical energy into electrical energy through electrochemical reactions, fuel cells bypass inefficiencies and emissions associated with combustion-based systems[9]. This unique capability has positioned them as key technology in global transition toward sustainable energy solutions.

This chapter give as foundational exploration of fuel cell technology, providing an overview of its historical development, fundamental principle and diverse application. discussion starts with historical perspective, tracing origins of fuel cells from their initials conception in 19th century to advanced systems in use today[10]. Following this chapter delves into core component of fuel cells like as electrodes, electrolytes and bipolar plates and explains how this element work together to enable efficient energy conversion.

An important portion of this chapter is dedicating to classified fuel cells based on their electrolyte type and operating conditions. Each type of fuel cell ranging from alkaline fuel cells (AFCs) to proton exchange membrane fuel cells (PEMFCs), molten carbonate fuel cells (MCFCs), solid oxide fuel cells (SOFCs), phosphoric acid fuel cells (PAFCs) and direct methanol fuel cells (DMFCs) is examining in terms of its structure, operating principles and specific applications[11] . This classification highlights versatility of fuel cells and their ability to meet need of various sectors, including stationary power generation, transportation and portable electronics.

The chapter conclude by critically evaluating advantages and disadvantages of fuel cells, drawing on recent researches to provide as balanced perspective[12] . By addressing both potential and challenge of fuel cell technology, this introduction aim to equip reader with comprehensive understanding of subject matter and establish solid foundation for more detailed analyses presented in subsequent chapters.

1.2 History

The history of fuels cells is fascinating journey that span nearly two centuries, marked by groundbreaking discoveries, technological advancement and practical application. concepts for fuel cells are first introduced in 1839 by Sir William Grove, Welsh physicist and lawyer, who Demonte "gas voltaic battery" rudimentary precursor to modern's fuel cells[13]. Grove's experiment involved reversing process of electrolysis, showing that hydrogen and oxygen could be combined electrochemically to produce electricity and water. Although this early device lacked practical utility, it laid theoretical foundation for future developments in fuel cell technology.

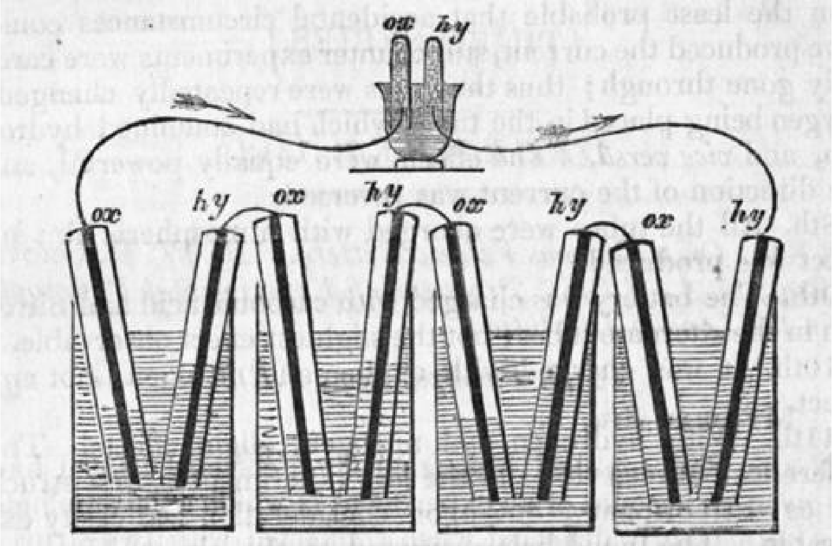


Figure 1: Grove's gaseous voltaic battery

Despite Grove's pioneering works, progress in fuels cells researches remained slow throughout 19th century due to limited understanding of electrochemistry and absence of suitable material It was not until mid of 20th century that advancement begin to emerges. in 1950s, Francis Thomas Bacon, British engineer, develop first practical alkaline fuel cell (AFC) using potassium hydroxide as electrolyte[14]. Bacon's design achieves high efficiency and stability in comparison with earlier prototypes, making suitable for real world application. his work formed basis for fuel cells used in NASA's Apollo and Space Shuttle programs, where they provided reliable power and drinking water for astronauts[15].

The success of AFCs spurred big research into another fuel cell technologies during half of 20th century. in 1960s, General Electric pioneered developments of proton exchange membrane fuel cells (PEMFCs), it utilized solid polymer electrolyte instead of liquid solutions. This innovation addressed some of limitation of AFCs such as sensitivity to carbon dioxide contamination and paved way for their use in transportation and portable applications[16].

In same times, researchers explored high temperature fuels cell technologies, like molten carbonate fuel cells (MCFCs) and solid oxide fuel cells (SOFCs).

MCFCs that operate at temperatures around 650°C, is developed in 1960s and are suited for largescale stationary power generation due to their ability to utilize variety of fuels, including natural gas and biogas[17]. SOFCs operate at even higher temperatures (700–1000°C) emerged in 1970s and gained attention for their high efficiency and potential for cogeneration of heat and electricity[18].

in late 20th and early 21st centuries an advancement in materials science and engineering accelerated commercialization of fuel cells like improvements in catalyst durability, membrane performance and electrode design have enhanced reliability and cost of PEMFCs. enabling their widespread adoption in automotive applications. Innovations in ceramic materials have improved thermal stability and longevity of SOFCs, making them increasingly viable for distributed energy systems[19].

Now fuels cells continue to evolve driven by global efforts to combat climate change and reduce dependences on fossil fuels. Governments and industries and academic institutions are investing in research and development to overcome remaining challenges such as high costs and durability issues or infrastructure requirements[12]. These efforts underscore enduring legacy of pioneers like Grove and Bacon, whose visionary ideas laid groundwork for technology that holds immense promise for future of clean energy.

1.3 General Overview of Fuel Cells

1.3.1 Definition

Fuel cells is electrochemical device that convert chemical energy of fuel, like hydrogen. and an oxidizing agent usually oxygen from air, directly into electrical energy through pair of redox (reduction-oxidation) reaction. Unlike conventional combustions based power generations system, which involved multiples energy conversions steps and suffer from efficiency losses, fuel cells achieves high efficiencies in directly producing electricity without intermediate mechanicals processes[9].

The fundamental principles underlying fuel cell operations are controlled combination of hydrogen and oxygen to form water and releasing energy in this process. This energy is harnessed as electric current which can be used to power various applications. simplicity of this concepts belies complexity of materials and designs and engineering required to optimize fuel cell performance under real world condition[11].

Fuel cells is distinguished from batteries in that they don't store energy internally but instead on continuous supply of fuel and oxidant to sustain their operations. These characteristics make them suitable for application required long term, un interrupted power generations such as stationary power plants and vehicles or portable electronics[20]. Fuel cells are modular and scalable and adaptable, capable of operating with variety of fuels including pure hydrogen, hydrocarbons and even biofuels depending on specific type of fuel cell and its intended application.

From thermodynamic perspective, maximum theoretical efficiency of fuel cell is determined by Gibbs free energy change of electrochemical reaction, which is higher than Carnot efficiency of traditional heat engines[18]. This inherent advantage positions fuel cells as cornerstone of sustainable energy systems, offering pathway to reduce greenhouse gas emissions and dependence on fossil fuels while meeting growing global energy demands.

1.3.2 General Structure

Fuel cells despites its diversity in design and application, by sharing common fundamental structure that enables it electrochemical operation. fuel cell consists of several key components each playing critical role in facilitating efficient conversion of chemical energy into electrical energy. This component includes electrode (anode and cathode), electrolyte bipolar plates and diffusion layers. we describe general structure and function of these components in detail.

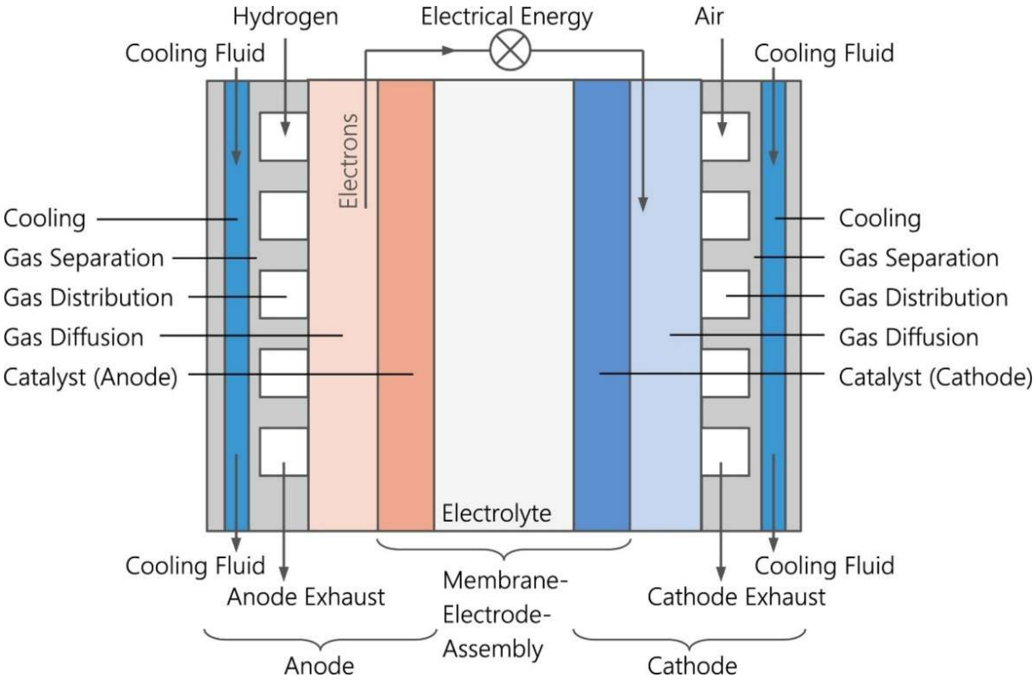


Figure 2: General Structure of Fuel cell

1.3.2.1 Electrodes

The electrodes are site where electrochemical reaction occur. Each fuel cell has two electrodes:

- ***Anode:*** anode is negative electrode in it oxidation reaction happened. In hydrogen fuel cells, hydrogen molecules (H_2) are split into protons (H^+) and electrons (e^-). protons migrate through electrolyte but electrons travel to an external circuit to generating electrical current.
- ***Cathode:*** cathode is positive electrode where reduction reaction occurred Oxygen molecules (O_2) from air combine with protons and electrons to form water (H_2O) completing electrochemical process.

Electrodes are made of porous materials to maximize surface area and enhance reaction kinetics. Catalyst often made from platinum or platinum are incorporated into electrodes to accelerate redox reactions and improve efficiency[21].

1.3.2.2 Electrolyte

The electrolyte is critical components that separate anode and cathode while allowing selective ion transport between them. It acts as medium for ion exchange, ensuring that only specific ions (protons in PEMFCs or oxygen ions in SOFCs) can pass through, there by maintaining electrochemical potential differences.

Different types of fuel cells utilize different electrolyte, which determines their operating conditions and performances characteristics:

- ***Polymer Electrolyte Membrane (PEM):*** Used in PEMFCs this solid polymer membrane conducts protons at relatively low temperatures (between 50–100°C).
- ***Liquid Alkaline Electrolyte:*** Employed in AFCs, it facilitates transport of hydroxide ions (OH^-) but is sensitive to carbon dioxide contamination.
- ***Molten Carbonate:*** Found in MCFCs, this electrolyte operates at high temperatures (600–700°C) and transports carbonate ions (CO_3^{2-}).
- ***Solid Oxide:*** in SOFCs, this ceramic material conducts oxygen ions (O^{2-}) at very high temperatures (700–1000°C).

The choice of electrolyte directly influences fuel cell's efficiency, durability and suitability for specific applications[18].

1.3.2.3 Bipolar Plates

Bipolar plates have multiple functions in fuel cell stack. They separate individual cells while distributing reactants (fuel and oxidant) across electrodes. and they collect and conducts electrons which generated in electrochemical reactions, connecting one cell to other in series to increase voltage output.

Bipolar plate is made by materials that balance electrical conductivity and mechanical strength and corrosion resistance. Common materials like graphite or coated metals and composites. Their design to includes flow channels and ensure efficient delivery of reactants and removal of reaction products such as water vapor[22].

1.3.2.4 Diffusion Layers

Also known as gas diffusion layers (GDLs) are placed adjacent to electrodes to facilitate uniform distribution of reactants and removal of byproducts. These layers are composed of porous materials such as carbon fiber paper or woven fabrics, which permit to gases to diffuse but providing structural support to electrodes.

The primary function of diffusion layer includes:

- Ensuring consistent contact between reactant and catalyst layers.
- Managing water during electrochemical reaction to prevent flooding or drying out in membrane.
- Enhancing thermals and electricals conductivity of fuel cell[16].

Integration of Components

The integrations of these components create functional fuel cell unit. multiple individual cell are stacked together in series to compose fuel cell stack, increasing of overall voltage and power outputs. precise engineering of each component of material properties geometry and interaction with other elements is crucial for achieving optimal performance, durability and costs.

1.3.3 Operating Principle

The operating principles of fuel cells is rooted in electrochemical reactions that directly converts chemical energy into electrical energies. This process occurs through pair of redox (reduction-oxidation) reaction, which are separate at anode and cathode, respectively. keys to these operations fixed in controlled flow of ions and electrons between electrodes by electrolyte and external circuit. we describe fundamental mechanisms and steps involved in operation of typical fuel cell.

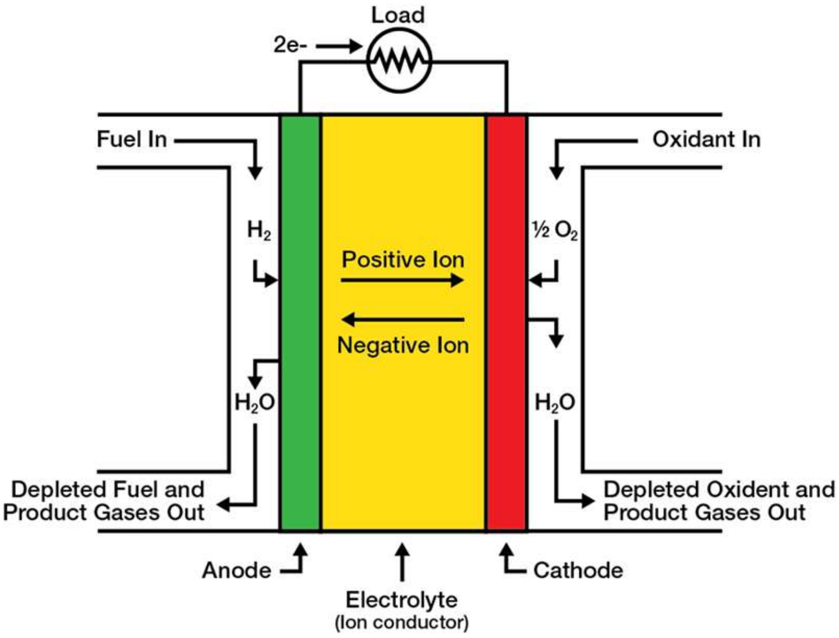


Figure 3: Working principle of Fuel Cell

1.3.3.1 Basic Electrochemical Process

At its core, fuel cell operates by combining hydrogen (H_2) and oxygen (O_2) to produce water (H_2O), releasing energy in form of electricity and heat. overall reaction can be summarized as follows:

Overall Reaction:

$$2H_2 + O_2 \rightarrow 2H_2O + Enregy(Electricity + Heat) \qquad (1.01)$$

This reaction is split into two half-reactions, occurring simultaneously at anode and cathode:

Anode Reaction (Oxidation):

At anode, hydrogen molecules are oxidized, splitting into protons (H^+) and electrons (e^-). For example in proton exchange membrane fuel cell (PEMFC), reaction is:

$$H_2 \rightarrow 2H^+ + 2e^- \qquad (1.02)$$

The protons migrate through electrolyte, while electrons travel through an external circuit, creating an electric current.

Cathode Reaction (Reduction):

At cathode, oxygen molecules react with protons and electrons to form water:

$$\frac{1}{2}O_2 + 2H^+ + 2e^- \rightarrow 2H_2O \qquad \textbf{(1.03)}$$

The combination of these two half-reactions results in continuous production of electricity, water and heat, provided there is steady supply of fuel (hydrogen) and oxidant (oxygen).

1.3.3.2 Role of Electrolyte

The electrolyte plays have role in maintaining separations of oxidation and reduction reactions by facilitating ions to transport between electrodes. Depending on type of fuel cell, electrolyte allows specific ions like as protons (H^+), hydroxide ions ($OH^–$) or oxygen ions ($O^{2–}$) to pass through it. This selective ion conduction give to electrochemical potential difference is preserved and enabling generation of electrical power.

For example:

In PEMFCs polymer electrolyte membrane conducts protons (H^+) while blocking electrons.

But in SOFCs solid oxide electrolyte conducts oxygen ions ($O^{2–}$) at high temperatures.

The choice between electrolytes determines operating temperature and efficiency or suitability of fuel cell for specific applications[18].

1.3.3.3 Electron Flow and Electrical Circuit

As electrons are released during oxidation reaction at anode, they cannot pass through electrolyte and instead travel through in external circuit to reach cathode. This flow of electrons constitutes electric current that can be used to power external devices. voltages generated by single fuel cell is in general around 0.7–1.0 volts under load but it depending on cell design and operating conditions[9].To achieve higher voltages we must use multiple fuel cells are connected in series to form fuel cell stack. bipolar plates with in stack ensure efficient electron transfer between adjacent cells while distributing reactants and removing byproducts.

1.3.3.4 Water and Heat Management

The most byproduct of electrochemical reactions in most fuel cells is water. Proper water management is need to maintaining performance and longevity of fuel cell.

In PEMFCs excess water must be removed to prevent flooding of porous electrodes, while sufficient hydration of membrane is necessary to maintain proton conductivity.

But In SOFCs water vapor is expelled along with other exhaust gases, but thermal management is need due to high operating temperatures (600–1000°C).

Heat generated during electrochemical process can be harnessed for cogeneration applications, like as heating buildings or driving additional power cycles, thereby improving overall system efficiency[17].

1.3.3.5 Efficiency Considerations

The efficiency of fuel cell determines by extent to which chemical energy of fuel is convert into good electrical energy. theoretical maximum efficiency is calculated by Gibbs free energy change (ΔG) of electrochemical reaction, which represents portion of energy available for work. In practice losses due to activation over potentials, ohmic resistance and concentration gradients reduce actual efficiency[11].

Despite these losses fuel cells generally achieve higher efficiencies than conventional combustion based systems when operated in cogeneration mode. For example:

PEMFCs exhibit efficiencies of 40–60% for electricity generation alone, rising to 85% in combined heat and power (CHP) systems.

SOFCs can achieve efficiencies exceeding 60% for electricity generation, with even higher values in cogeneration applications.

1.4 Different Types of Fuel Cells

Fuel cells are classified based on type of electrolyte they employ, which directly influences their operating conditions, efficiency and applications. Each type of fuel cell had unique characteristics that make it suitable for specific use cases. we discuss six main types of fuel cells and highlighting their structures, operating principles and typical applications.

1.4.1 Alkaline Fuel Cell (AFC)

Alkaline fuel cells (AFCs) were among first practical fuel cell technologies developed and gained prominence during NASA's Apollo space missions. They operate using an alkaline electrolyte, typically potassium hydroxide (KOH), which facilitates transport of hydroxide ions (OH^-).

Anode: $\boldsymbol{H_2 + 2(OH^-) \rightarrow 2H_2O + 2e^-}$ ***(1.04)***

Cathode: $\boldsymbol{H_2O + \frac{1}{2}O_2 + 2e^- \rightarrow 2(OH^-)}$ ***(1.05)***

Global: $\boldsymbol{H_2 + \frac{1}{2}O_2 \underset{2}{\rightarrow} H_2O}$ ***(1.06)***

Operating Temperature: 60–90°C

Efficiency: 60–70% (electrical), up to 85% in cogeneration systems

Advantages: High efficiency, rapid startup and relatively low cost compared to other high-efficiency fuel cells.

Challenges: Sensitivity to carbon dioxide (CO_2) contamination, which forms carbonate precipitates that degrade performance.

Applications: AFCs are primarily used in niche applications such as space exploration and military systems, where their high efficiency and reliability are critical[17].

1.4.2 Molten Carbonate Fuel Cell (MCFC)

Molten carbonate fuel cells (MCFCs) operate at high temperatures and use molten carbonate salt mixture as electrolyte. electrolyte conducts carbonate ions (CO_3^{2-}), enabling electrochemical reactions.

Anode: $\boldsymbol{CO_3^{-2} + H_2 \rightarrow H_2O + CO_2 + 2e^-}$ ***(1.07)***

Cathode: $\boldsymbol{CO_2 + \frac{1}{2}O_2 + 2e^- \rightarrow CO_3^{-2}}$ ***(1.08)***

Operating Temperature: 600–700°C

Efficiency: 50–60% (electrical), up to 85% in cogeneration systems

Advantages: High efficiency, ability to utilize variety of fuels (e.g., natural gas, biogas) and potential for internal reforming of hydrocarbons.

Challenges: High operating temperature leads to material degradation, corrosion and longer startup times.

Applications: MCFCs are commonly used for large-scale stationary power generation, including grid support and industrial cogeneration systems[18].

1.4.3 Solid Oxide Fuel Cell (SOFC)

Solid oxide fuel cells (SOFCs) employ solid ceramic electrolyte, typically yttria-stabilized zirconia (YSZ), which conducts oxygen ions (O^{2-}). Their high operating temperatures enable internal reforming of fuels and high electrical efficiency.

Anode: $\boldsymbol{O^{-2} + H_2 \rightarrow H_2O + 2e^-}$ ***(1.09)***

Cathode: $\boldsymbol{\frac{1}{2}O_2 + 2e^- \rightarrow O^{-2}}$ ***(1.10)***

Operating Temperature: 700–1000°C

Efficiency: 50–65% (electrical), up to 90% in cogeneration systems

Advantages: High efficiency, fuel flexibility (hydrogen, natural gas, syngas) and minimal need for expensive catalysts like platinum.

Challenges: Long startup times, thermal cycling issues and high material costs due to extreme operating temperatures.

Applications: SOFCs are ideal for stationary power generation, distributed energy systems and combined heat and power (CHP) applications[19].

1.4.4 Phosphoric Acid Fuel Cell (PAFC)

Phosphoric acid fuel cells (PAFCs) use liquid phosphoric acid as electrolyte, which conducts protons (H^+). PAFCs were among first fuel cell technologies to achieve commercial viability.

Anode: $\boldsymbol{H_2 \rightarrow 2H^+ + 2e^-}$ ***(1.11)***

Cathode: $\frac{1}{2}O_2 + 2H^+ + 2e^- \rightarrow H_2O$ *(1.12)*

Operating Temperature: 150–200°C

Efficiency: 40–50% (electrical), up to 85% in cogeneration systems

Advantages: Moderate operating temperature, relatively mature technology and tolerance to impurities in fuel stream.

Challenges: Lower efficiency compared to other high-temperature fuel cells, limited durability and higher cost.

Applications: PAFCs are widely used for stationary power generation in buildings, hospitals and utility grids[9].

1.4.5 Proton Exchange Membrane Fuel Cell (PEMFC)

Proton exchange membrane fuel cells (PEMFCs) utilize solid polymer electrolyte, typically Nafion, which conducts protons (H^+). PEMFCs are known for their compact design, fast startup times and high-power density.

Operating Temperature: 50–100°C

Efficiency: 40–60% (electrical), up to 85% in cogeneration systems

Advantages: Lightweight, compact, rapid startup and compatibility with hydrogen fuel.

Challenges: Sensitivity to impurities, reliance on expensive platinum catalysts and water management issues.

Applications: PEMFCs are extensively used in transportation (cars, buses, trucks), portable electronics and backup power systems[21].

1.4.6 Direct Methanol Fuel Cell (DMFC)

Direct methanol fuel cells (DMFCs) are subset of PEMFCs that use liquid methanol as fuel source, eliminating need for external reformers. Methanol is directly oxidized at anode.

Operating Temperature: 50–120°C

Efficiency: 20–40% (electrical), lower than other fuel cell types due to methanol crossover losses.

Advantages: Compact design, ease of fuel storage and handling and suitability for portable applications.

Challenges: Low efficiency, methanol crossover through membrane and reliance on expensive catalysts.

Applications: DMFCs are used in portable electronics, military equipment and small-scale power systems[16].

	AFC	**PAFC**	**PEMFC**	**DMFC**	**MCFC**	**SOFC**
	Low Temperature				**High Temperature**	
Temperature (°C)	*80 to 250*	*150 to 220*	*70 to 100*	*60 to 90*	*600 to 800*	*600 to 1100*
Electrolyte	*Potassium hydroxide (KOH) (liquid)*	*Phosphoric acid* (PO_4H_3) *(liquid)*	*Polymer membrane (solid)*	*Polymer membrane (solid)*	*Mixture of* Li_2CO_3 *and* K_2CO_3 *(liquid)*	*Ceramic (solid)*
Possible Fuel	H_2	H_2 *(pure or reformed)*	H_2	*Methanol* (CH_3OH)	*Hydrogen, natural gas, methanol...*	*Hydrogen, natural gas, methanol...*
Catalyst	*Platinum*	*Platinum*	*Platinum*	*Platinum*	*Nickel and nickel oxide*	*Nickel-zirconia (Cermet)*
Oxidant	O_2 *(pure)*	*Air/*O_2	*Air/*O_2	*Air/*O_2	*Air/*O_2	*Air/*O_2
Mobile Ion	OH^-	H^+	H^+	H^+	CO_3^{2-}	O^{2-}
Efficiency	*55-60%*	*35-50% (+75% with cogeneration)*	*35-45%*	*20-30%*	*50-60%*	*50-55% (+70% with cogeneration)*
Power Range (kW)	*1-100*	*50-1000*	10^{-3}*-1000*	10^{-3}*-1000*	*100-*10^5	*5-*10^5
Applications	*Military, Portable equipment*	*Transport, Portable equipment*	*Cogeneration, Mobile phone, Submarine, Automobile, Space*	*Mobile applications*	*Cogeneration, Centralized electricity production*	*Cogeneration, Centralized electricity production, Automobile, Maritime*

Table 1: Comparison of Fuel Cell Types

1.5 Applications of Fuel Cells

Fuel cells have gained big attention because to their versatility, efficiency and environmental benefits. They are deployed across wide range of applications spanning stationary power generation, transportation and portable electronics. Each application leverages unique advantages of fuel cells like as high efficiency and low emissions and modularity we explore three primary categories of fuel cell applications in detail.

1.5.1 Stationary Applications

Stationary fuel cell systems is designed for continuous or backup power generations at fixed locations like buildings or industrial facilities and utility grids. These systems is suited to combined heat and power (CHP) applications where waste heat from electricity generation is utilized for heating or cooling purposes.

Features:

- Efficiency: Stationary fuel cells may achieve electrical efficiencies of 40–60%, rising to 85% or more when integrated in CHP systems.
- Reliability: Fuel cells provided stable and uninterrupted power by marked them ideal for critical infrastructure like as hospital or data centers or emergency services
- Scalability: Systems can range from small residential units (kW scale) to large industrial installations (MW scale)

Examples of Stationary Applications:

- Backup Power Systems: Fuel cells are using in backup power sources for telecommunications, hospitals and data centers, ensuring continuous operation during grid outages. For example phosphoric acid fuel cells (PAFC) and proton exchange membrane fuel cells (PEMFC) are commonly employed in these scenarios due to their reliability and rapid startup capabilities[9].
- Combined Heat and Power (CHP): Solid oxide fuel cells (SOFC) and molten carbonate fuel cells (MCFC) are widely used in CHP systems, where waste heat generated during electricity production is capturing and using for space heating, water heating or industrial processes. This improves overall system efficiency and reduced energy costs.

- Grid Support and Distributed Energy: Fuel cells contributes at grid stability and provide decentralized power generation, reduce transmission losses and integrated renewable energy sources. SOFC and MCFC are being deployed with microgrid applications to enhance energy resilience and sustainability[18].

Figure 4: Flying Fuel Cell manufactured by MTU Aero Engines

1.5.2 Transportation Applications

Fuel cells are emerging as key technology for decarbonizing transportation sector, offer clean compared by internal combustion engines. Their ability to product zero emissions plus with their high energy density makes them attractive for vehicles ranging from cars and buses to trains and ships.

Features:

- Zero Emission: Hydrogen power fuel cell vehicles (FCVs) emitted only water vapor making them environment friendly.
- Long Range and Fast Refueling: Comparing to battery electric vehicles fuel cell vehicles offer big driving ranges and fast refueling times.
- Modularity: Fuel cells may be scaled to meeting power demand of various vehicle types begin with passenger cars to heavy duty trucks and marine vessels.

Examples of Transportation Applications:

- Passenger Vehicles: PEMFC dominating automotive sector due to their compact size fast startup times and compatibility with hydrogen fuel. Companies like Toyota Hyundai and Honda have commercializing FCV for personal use[21].
- Buses and Trucks: Fuel cell bus are deploying in urban transit systems worldwide, offering zero emission in public transportation. And fuel cell trucks are gaining traction for long freight operations where battery weight and charging times give us challenges.
- Trains and Ships: Hydrogen fuel cell trains is already operational in Europe providing emission free rail transport. In maritime applications fuel cells are being explored for ferries cargo ships and cruise liners to reduce greenhouse gas emissions from shipping[17].
- Aviation and Aerospace: Fuel cells is also be investigated for use in aerial vehicles (UAV) and auxiliary power units (APU) in aircraft. Alkaline fuel cells (AFC) were historically used in NASA Apollo missions, demonstrating their potential for aerospace applications.

Figure 5: Fuel Cell Bus

1.5.3 Portable Applications

Portable fuel cells is compact light weight and capable of delivering sustained power for mobile devices military equipment's and off grid applications. Their ability to operate in independent of electrical grid makes them ideal for situations where conventional batteries fall short.

Features:

- Extended Runtime: Portable fuel cells provides big operating time compared to traditional batteries, especially when fueled by hydrogen or methanol.
- Compact Design: Direct methanol fuel cells (DMFCs) and micro PEMFCs is designed for integration into consumer electronics and handheld devices.
- Reliability: Portable fuel cells is used in hard environments such as military operations or disaster relief scenarios, where durability is critical.

Examples of Portable Applications:

- Consumer Electronics: Portable fuel cells is developed for smartphones, laptops and cameras, offering extended battery life and quick refueling options. DMFC is suiting for these applications because their ease of fuel handling and compact design[16].
- Military and Defense: military use portable fuel cells to power communication devices sensors and vehicles in remote locations. This system reduces logistical burden of transporting heavy batteries and ensure reliable power supply in field.
- Emergency Power: Portable fuel cells are deployed in disaster zones, remote areas and outdoor activities (like camping) to provide off grid power. Their light weight design and ability to operate on liquid fuels like methanol make them highly versatile.
- Medical Devices: Fuel cells are integrated into medical equipment such as ventilators and diagnostic tools, ensuring uninterrupted operation during emergencies or in regions with unreliable grid access.

a- fuel cell powered by methanol.

b- Toshiba Fuel Cell Phone Charger

Figure 6: Examples of fuel cell applications in portable devices

Application	Fuel Cell Types	Key Advantages	Examples
Stationary	PAFC, SOFC, MCFC	High efficiency, CHP capability, reliability	Backup power, CHP systems, grid support
Transportation	PEMFC	Zero emissions, long range, fast refueling	Cars, buses, trucks, trains, ships
Portable	PEMFC, DMFC	Extended runtime, compact design, ruggedness	Consumer electronics, military, off-grid

Table 2: Comparison of Portable Applications

1.6 Advantages / Disadvantages of Fuel Cells

Fuel cells offered numerous advantages that makes them promising technology for sustainable energy solutions. However, they also face challenges that must be addressed to achieve widespread adoption. Below, we critically evaluate strengths and limitations of fuel cells, providing balanced perspective on their current state and future prospects.

1.6.1 Advantages of Fuel Cells

High Efficiency

- Fuel cells converts chemical energy directly into electrical energy with minimal losses, achieving efficiencies of 40–65% in standalone operation and up to 85–90% in combined heat and power (CHP) systems. This is higher than conventional combustion based systems, which operate at efficiencies of 30–40%[22].
- High temperature fuel cells, like as solid oxide fuel cells and molten carbonate fuel cells (MCFCs) are efficient due to their ability to utilize waste heat for cogeneration applications[18].

Environmental Benefits

- Fuel cells produce zero or near zero greenhouse gas emissions when powered by hydrogen derived from renewable sources. Even when using fossil fuels they emit less CO_2 compared to traditional power generation technologies.
- Unlike internal combustion engines, fuel cells do not produce harmful pollutants like as nitrogen oxides (NO_x), sulfur oxides (SO_x) or particulate matter, making them an environment friendly best for transportation and stationary power[9].

Modularity and Scalability

- Fuel cells are very modular and they can be scaled to meet wide range of power demand from small portable devices to large stationary power plants. This flexibility makes them better for diverse applications, including distributed energy systems and microgrids.

Quiet Operation

- Fuel cells operate silently because they do not have mechanical components like turbines or pistons. This characteristic is very advantageous for residential, urban and military applications where noise reduction is critical.

Reliability and Durability

- Fuel cells providing stable and uninterrupted power marked them ideal for critical infrastructure like as hospitals or data centers and telecommunications. Their reliability is further enhanced by absence of moving parts reduces wear and maintenances.

Fuel Flexibility

- Certain types of fuel cells SOFCs and MCFCs can using variety of fuels like hydrogen or natural gas, biogas and syngas. This flexibility reduces dependency on pure hydrogen and broadens their application in different energy systems[19].

1.6.2 Disadvantages of Fuel Cells

High Initial Costs

- The cost of manufacturing fuel cells remains big barrier to wide spread adoption. Key components like as platinum catalysts in proton exchange membrane fuel cells (PEMFCs) and high temperature materials in SOFCs contribute to high capital costs of these systems.
- The costs have decrease over years further reduction are needed to make fuel cells competitive with established technologies like batteries and internal combustion engines[21].

Durability and Lifetime Challenges

- Fuel cell face durability issues under real world operating conditions, because of high temperature systems like SOFCs and MCFCs. Factors like as thermal cycling, material degradation and catalyst poisoning can reduce their operational lifetime and increase maintenance costs.
- PEMFCs more durable but are sensitive to impurities in fuel stream and require careful water management to prevent membrane drying or flooding[16].

Hydrogen Infrastructure Limitations

- The wide spread adoption of hydrogen power fuel cells is hindered by lack of robust hydrogen production, storage and distribution infrastructure. Producing hydrogen through electrolysis or steam methane reforming remains energy intensive and very cost, while storage and transportation give us additional challenges due to hydrogen's low volumetric energy density.
- Alternative fuels like as methanol or natural gas address these issues but comes with their own limitations such as lower efficiency and emissions during reforming processes[17].

Sensitivity to Operating Conditions

- Fuel cells is sensitive to specific operating condition such temperatures humidity and fuel purity. In PEMFCs requires precise water management to maintain membrane conductivity but AFCs are susceptible to carbon dioxide contamination. These sensitivities complicate system design and operations.

Limited Commercialization and Market Penetration

- Despite decade of research and developments fuel cells have not to achieve wide spread commercial success outside niche markets. Limited consumer awareness, competition from alternative technologies (lithium-ion batteries) and policy barriers have slowed their market penetration.

Material and Resource Constraints

- The reliance on rare and expensive materials such as platinum in PEMFCs and yttria-stabilized zirconia (YSZ) in SOFCs, raises concerns about resource availability and long term sustainability. Efforts to develop alternative catalysts and materials are ongoing but remain key challenge[11].

ASPECT	ADVANTAGES	DISADVANTAGES
Efficiency	High efficiency (40–90%) in standalone and CHP systems	High initial costs and sensitivity to operating conditions
Environment	Zero or near-zero emissions, no harmful pollutants	Hydrogen production and infrastructure challenges
Scalability	Modular and scalable for diverse applications	Durability and lifetime issues in certain types (e.g., SOFCs, MCFCs)
Flexibility	Can utilize various fuels (hydrogen, natural gas, biogas)	Limited commercialization and market penetration
Operation	Quiet operation, no moving parts	Material and resource constraints (e.g., platinum, YSZ)
Reliability	Stable and uninterrupted power supply	Sensitivity to impurities and thermal cycling

Table 3: Advantages and Disadvantages of fuel cells

Conclusion

This chapter has discussed fuel cells as an emerging technology that could change face of energy in world. Fuel cells were started from their history and covered essentials, components, functional principles and applications. With various types of fuel cells alkaline fuel cells (AFCs), proton-exchange membrane fuel cells (PEMFCs), molten carbonate fuel cells (MCFCs), solid oxide fuel cells (SOFCs), phosphoric acid fuel cells (PAFCs) and direct methanol fuel cells (DMFCs)-this chapter helps in outlining peculiar properties of these types according to different use cases.

The operating principle of fuel cells is established on electrochemical reactions, where their high efficiency and low environmental impact are possible. Modular, scalable and adaptable, they can be applied in many areas: stationary production of power, transport applications and portable electronics. Fuel cells can now provide clean energy in many areas such as buildings and industries, as well as enable zero-emission vehicles and remote devices.

Fuel cells miss many opportunities due to limitations, as discussed in this chapter. Major roadblocks on path to commercialization of fuel cells are high initial costs, durability of fuel cells, gaps in hydrogen infrastructure and availability of materials. To overcome obstacles, this technological advancement will need materials science, engineering and system integration along with some policies and investments that encourage market growth.

Fuel cells have advantages over competing technologies: high efficiency, low emissions, reliability and fuel flexibility which makes them critical enabler of sustainable energy future. Keeping that in mind, therefore, as ongoing research and development initiatives lead to resolution of present challenges for this evolving area, it would be expected that fuel cells will play key roles in decarbonization of energy systems and realization of global climate goals.

CHAPTER 2:

DEVELOPMENT OF SOLID OXIDE FUEL CELLS (SOFC)

2.1 Introduction

Solid oxide fuel cells (SOFCs) have emerged as one of most promising fuel cell technologies because of their high efficiency, fuel flexibility and potential for cogeneration applications. Operating at high temperatures (in general between 700°C and 1000°C)[20], SOFCs offer unique advantages like as internal reforming of hydrocarbons and ability to using wide range of fuel like hydrogen or natural gas and biogas. These characteristics make they suitable to stationary power generation distributed energy systems and large scale industrial applications.

This chapter focuses to development of SOFC technology beginning with an exploration of reasons behind selection of SOFC over other fuel cell types. we examined various configurations of SOFCs including tubular planar and monolithic designs highlighting their structural differences and operational implications. chapter then delving at development of dynamic model for an SOFC stack considering both ideal and real performance metrics. Finally, we present control system for SOFC operation and validate developed model through simulation results.

2.2 Reasons for Selection

The selection of solid oxide fuel cells (SOFCs) as focal point of research and development in driven by their unique characteristics and advantages compared other types of fuel cells. These attributes made SOFCs well suited to wide range of applications, from stationary power generation to distributed energy systems. we outline key reasons of choosing SOFCs as technology off interest.

1. High Efficiency

One of most advantages of SOFCs is their efficiency. Operating on high temperatures (between 700°C and 1000°C)[20] SOFCs achieve electrical efficiencies of 50–65% in standalone operations. When integrated into combined heat and power (CHP) systems their overall efficiency can exceed 90% making them one of most efficient energy conversion technologies available[18]. This high efficiency stems by direct conversion of chemical energy to electrical energy with low energy losses comparing to conventional combustion based systems.

2. Fuel Flexibility

Unlike low temperature fuel cells like proton exchange membrane fuel cells (PEMFCs), which require purified hydrogen SOFCs can using big variety of fuels include:

- ***Hydrogen***: ideal fuel for all fuel cell types produced only water as byproduct.
- ***Natural Gas:*** Widely available and easy reformed in hydrogen with in SOFC due it high operating temperature.
- ***Biogas***: renewable fuel derived from organic waste which can be used directly in SOFCs without extensive purification.
- ***Syngas***: mixture of hydrogen and carbon monoxide can produce from coal or biomass gasification.

This fuel flexibility reduces dependency on pure hydrogen infrastructure and broadens applicability of SOFCs in regions where alternative fuels are more available[19].

3. High Temperature Operation

The elevated operating temperatures SOFCs (700–1000°C) offer some advantages:

- ***Internal Reforming***: can reform hydrocarbon fuels (e.g., natural gas, biogas) to hydrogen by eliminating need for external reformers and reduce system complexity.
- ***Non Precious Metal Catalysts:*** high temperatures enable using of non precious metal catalysts unlike PEMFC which rely on platinum based expensive catalysts.
- ***Cogeneration Potential:*** high temperature exhaust gases can be harnessed to heating cooling or driving additional power cycles improving overall system efficiency.

this characteristic makes SOFC preferred in stationary and industrial applications where waste heat recovery is important.

4. Durability and Stability

SOFCs have excellent durability and stability on appropriate conditions. Their solid state design eliminates issues related to electrolyte evaporation or degradation which are common in liquid electrolyte fuel cells like alkaline fuel cells (AFCs) or molten carbonate fuel cells (MCFCs). In add SOFC is less sensitive at carbon monoxide (CO) and carbon dioxide (CO_2) poisoning compared by PEMFCs allowing them operating at impure fuels without performance degradation[17].

5. Scalability and Modularity

SOFC systems are modular and scalable ranges from small residential units (kW scale) to large industrial installation (MW scale). This scalability making them suitable for variety of applications:

- Distributed energy systems
- Microgrids
- Backup power for critical infrastructure
- Large scale power plants

They able to integrate into existing energy infrastructures enhances their appeal.

6. Environmental Benefits

SOFC produce zero or near zero greenhouse gas emissions when powered by renewable fuels such hydrogen or biogas. their high efficiency results in lower CO_2 emissions comparing by traditional power generation technologies. SOFC do not emit harmful pollutants such nitrogen oxides (NO_x) sulfur oxides (SO_x) or particulate matter contributing to cleaner air and healthier environment[9].

7. Technological Maturity

While SOFC technology is evolving it has reached level of maturity makes it viable for certain applications. Companies such Bloom Energy and Siemens have successfully deployed SOFC systems for stationary generation their reliability potential for adoption. advancements in materials science manufactures techniques and system integration are expected to further enhance their performance and reduce costs[11].

Challenges

Despite these advantages there are challenges associated with SOFCs that must addressed to realize potential:

- ***Operating Temperatures:*** elevated temperatures required for operation can lead to thermal cycling issues, material degradation and longer startup times.
- ***Costs:*** While SOFCs avoid need for expensive platinum catalysts, use of advanced ceramics and interconnect materials can still result in high manufacturing costs.
- ***Durability:*** Long term durability under real world operating conditions remains concern, for applications required frequent startups and shutdowns.
- ***Infrastructures***: lack of hydrogen infrastructure limiting adoption of hydrogen powered SOFCs, necessitating using of alternative fuels.

Addressing these challenges through research and development will be critical to get full potential of SOFC technology.

2.3 Different Configurations of SOFCs

The design of solid oxide fuel cells plays big role in determining performance, efficiency and suitability for specific applications. three primary configurations have emerged: tubular, planar and monolithic. every configuration had advantages and challenges, makes them suitable for different operational requirements and use. Below, we discuss these configurations in detail.

2.3.1 Tubular Configuration

in tubular configuration, components of SOFC electrolyte, anode and cathode are arranged in concentric layers around central tube. fuel typically flows through inner tube, while air flows around outer surface. this design simplifies sealing and reduces risk of gas leakage as there are no complex interfaces between cells.

Advantages:

- ***Thermal Shock Resistance***: cylindrical geometry provides excellent resistance to thermal stresses making tubular SOFC durable under fluctuating operating conditions.
- ***Simplified Sealing***: absence of flat surfaces and interconnects reduces complexity of sealing.
- ***Scalability***: Tubular designs can be easily scaled up by stacking multiple tubes in parallel.

Challenges:

- ***Lower Power Density**:* long ion transport paths on tubular designs result by higher ohmic losses.
- ***Higher Manufacturing Costs***: tubular components require advanced manufacturing techniques increased production costs.
- ***Limited Compactness***: tubular design it less compact than planar or monolithic configuration limiting its application in space constrained environments.

Applications:

Tubular SOFC are used at stationary power generation systems such those developed at Siemens. Their durability and ease of sealing make them ideal for industrial applications

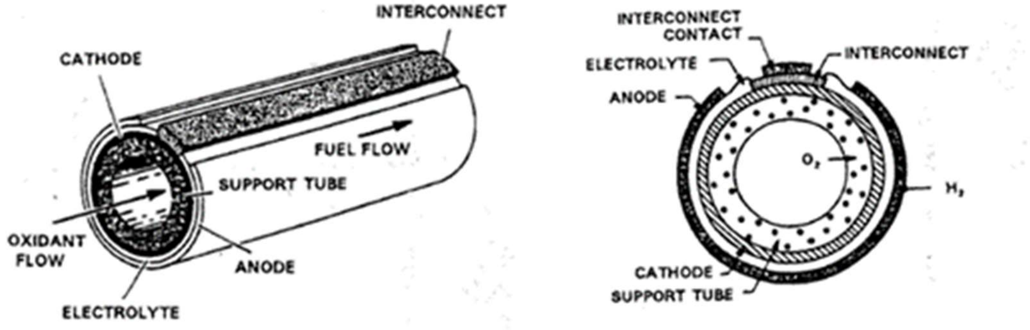


Figure 7:Principal Diagram of Tubular Geometry

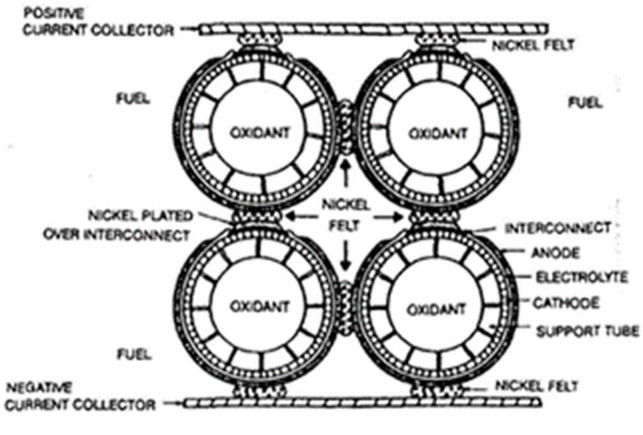


Figure 8:Principal Diagram of Assembly of Multiple Tubular Cells

2.3.2 Planar Configuration

the electrolyte in planar configuration anode and cathode are arranged in flat layered stacks. Fuel and oxidant flow through channels integrated into bipolar plates which separate adjacent cells. This design allows for shorter ion transport paths and higher power density.

Advantages:

- ***High Power Density***: Short ion transport paths minimize ohmic losses for greater power density than tubular designs.
- ***Compact Design:*** planar type is more tightly in size, and is appropriate for limited space applications.
- ***Ease of Integration***: Planar SOFCs can be integrated into stacks to produce a large amount of power for distributed energy systems.

Challenges:

- ***Complex Sealing***: gas tight seals between flat surfaces is challenge, at high operating temperatures, where thermal expansion mismatches can occur.
- ***Thermal Stress Sensitivity***: Planar designs are more susceptible to thermal stresses and mechanical failure due to their rigid structure.
- ***Durability Issues***: thermal cycling can lead to material degradation and reduced life.

Applications:

Planar SOFCs are applied for distributed energy systems, microgrids and home power generation. small and scalable size of two converters is well suited for high power density and modularity.

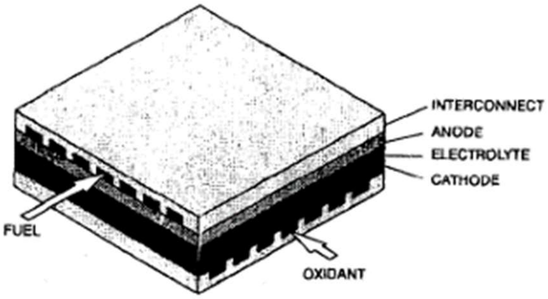


Figure 9:Planar Geometry

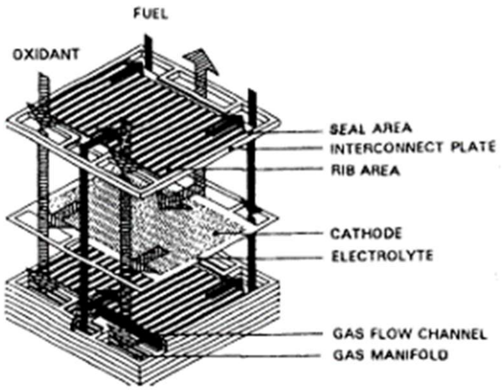


Figure 10:Stack in Planar Geometry with Counter-Current Flow

2.3.3 Monolithic Configuration

The monolithic configuration brings together multiple cells into a single, compact unit by utilizing advanced manufacturing techniques like co-sintering. This innovative design cuts down on need for interconnect materials and helps reduce ohmic losses by forming a highly integrated structure.

Advantages:

- ***Ultra High Power Density***: monolithic design achieves highest power density among SOFC configurations because to it minimal interconnect materials and optimized ion transport paths.
- ***Reduced Material Usage***: By eliminating bulky interconnects and sealing components monolithic SOFCs reduce material costs and simplify fabrication processes.
- ***Efficiency***: compact and integrated design minimizes energy losses, resulted superior overall efficiency.

Challenges:

- ***High Fabrication Complexity***: intricate manufacturing process required for monolithic designs increases production costs and limits scalability.

- ***Thermal Cycling Vulnerability***: tightly integrated structure makes monolithic SOFC susceptible to thermal stresses and mechanical failure during startup and shutdown cycles.
- ***Limited Commercialization***: Due their complexity, monolithic SOFC are used in niche applications and have not yet achieved commercial adoption.

Applications: Monolithic SOFCs are suited for specialized applications requiring ultra-high efficiency and compact designs such portable power systems and aerospace applications. Their potential for miniaturization also makes them attractive for emerging technologies like wearable electronics.

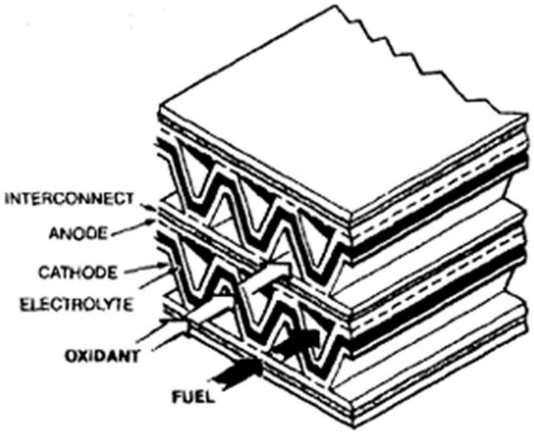


Figure 11:Monolithic Geometry with Co-Current Flow

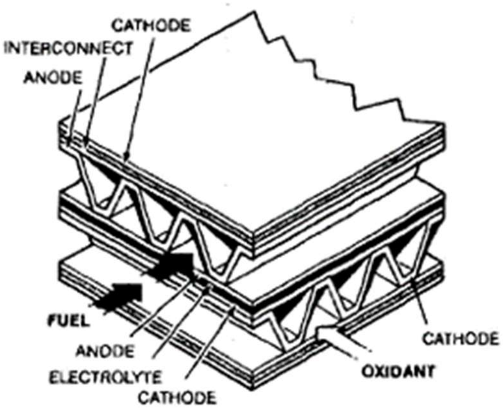


Figure 12:Monolithic Geometry with Counter-Current Flow

2.4 Development of Dynamic Model of an SOFC Stack

The development of dynamic model for solid oxide fuel cell (SOFC) stack is essential to predict its performance under various operating conditions. This section begins by examining ideal performance of an SOFC cell, focusing on key thermodynamic and electrochemical principles. Below, we explore theoretical foundations of SOFC operation, including Gibbs free energy, Nernst equation, molar flow rates and partial pressures.

2.4.1 Ideal Performance of an SOFC Cell

To understand ideal performance of an SOFC, we must consider fundamental thermodynamics and electrochemistry governing its operation. These principles provide baseline for evaluating real-world performance and identifying potential losses.

2.4.1.1 Gibbs Free Energy

The maximum theoretical voltage of an SOFC is determined by Gibbs free energy change (**ΔG**) of electrochemical reaction. Gibbs free energy represents portion of chemical energy available for conversion into electrical work. For hydrogen-oxygen reaction in an SOFC, overall reaction is:

$$H_2 + \frac{1}{2} O_2 \rightarrow H_2O \qquad \textbf{(2.01)}$$

The Gibbs free energy depends on temperature and this dependence is expressed by relation:

$$\Delta G = \Delta H - T\Delta S \qquad \textbf{(2.02)}$$

The variation of Gibbs free energy is difference between sum of free energies of products and sum of free energies of reactants.

$$\Delta G = \sum G_{products} - \sum G_{reactants} \qquad \textbf{(2.03)}$$

If we consider hydrogen/oxygen reaction of SOFC cell, we have:

$$\Delta G = G_{H_2O} - G_{H_2} - \frac{1}{2} G_{O_2} \qquad \textbf{(2.04)}$$

Where:

- G_{H_2O}**:** Gibbs free energy of steam.

- G_{H_2}: Gibbs free energy of hydrogen.
- G_{O_2}**:** Gibbs free energy of oxygen.

The theoretical open-circuit voltage (***E₀***) can be calculated using following equation:

$$E_0 = -\frac{\Delta G}{nF} \qquad \textbf{(2.05)}$$

where:

- ***ΔG***: Gibbs free energy change of reaction (J/mol),
- ***n***: Number of electrons transferred per molecule of fuel (for hydrogen, n=2)
- ***F***: Faraday constant (96485 C/mol).

2.4.1.2 Nernst Equation

The Nernst equation is cornerstone of electrochemical theory and plays critical role in determining voltage output of solid oxide fuel cell (SOFC). It accounts for effects of temperature, reactant concentrations and product formation on cell's open-circuit voltage (E). This equation provides more realistic estimation of cell voltage compared to theoretical maximum voltage derived from Gibbs free energy.

Derivation of Nernst Equation

The Nernst equation is derived from thermodynamic principles and describes how cell voltage changes with variations in reaction conditions. For hydrogen-oxygen reaction in an SOFC **(2.1).** Nernst equation can be expressed as:

$$E = E_0 + \frac{RT}{nF} ln\left(\frac{P_{H_2}P_{O_2}^{0.5}}{P_{H_2O}}\right) \qquad \textbf{(2.06)}$$

where:

- ***E***: Actual open-circuit voltage (V)
- ***E₀***: Standard open-circuit voltage (V), calculated from Gibbs free energy
- ***R***: Universal gas constant (8.314 J/mol.K)
- ***T***: Operating temperature (K)
- ***n***: Number of electrons transferred per molecule of fuel (n=2 for hydrogen)
- ***F***: Faraday constant (96485 C/mol)

- $\boldsymbol{P_{H_2}}$, $\boldsymbol{P_{O_2}}$, $\boldsymbol{P_{H_2O}}$: Partial pressures of hydrogen, oxygen and water vapor (atm).

The Nernst equation highlights several factors that influence actual cell voltage:

Temperature (T)

- The term $\boldsymbol{\frac{RT}{nF}}$ shows that higher operating temperatures increase logarithmic contribution to cell voltage. This explains why SOFCs, which operate at high temperatures (700–1000°C), achieve higher voltages compared to low-temperature fuel cells like PEMFCs.
- However, excessively high temperatures can lead to material degradation and thermal stresses, necessitating careful thermal management.

Reactant Partial Pressures (P_{H2} and P_{O2})

- Higher partial pressures of hydrogen ($\boldsymbol{P_{H2}}$) and oxygen ($\boldsymbol{P_{O2}}$) enhance driving force for ion conduction, increasing cell voltage.
- For example doubling partial pressure of hydrogen or oxygen increases logarithmic term in Nernst equation, leading to higher open circuit voltage.

Product Partial Pressure (P_{H2O})

- The presence of water vapor ($\boldsymbol{P_{H_2O}}$) reduces cell voltage due to its appearance in denominator of logarithmic term. Excessive water vapor can lower driving force for electrochemical reaction, emphasizing need for effective water management.

2.4.1.3 Molar Flow Rates

The set of molar flow rates that can be defined are:

- The hydrogen flow rate entering anodic compartment, defined by $\boldsymbol{q_{H_2}^{in}}$
- The oxygen flow rate entering cathodic compartment, defined by $\boldsymbol{q_{O_2}^{in}}$

The hydrogen flow rate consumed in reaction with oxygen to form water and electricity, defined by $\boldsymbol{q_{H_2}^{r}}$.

The hydrogen flow rate exiting stack, defined by $\boldsymbol{q_{H_2}^{out}}$.

$$q_{H_2} = q_{H_2}^{in} - q_{H_2}^{r} - q_{H_2}^{out} \qquad (2.07)$$

According to principles of electrochemistry, hydrogen flow rate participating in reaction can be calculated based on current flowing through stack I_{fc}^{r}[23].

$$q_{H_2}^{r} = \frac{N_{cell} I_{fc}^{r}}{2F} = 2K_r I_{fc}^{r} \qquad (2.08)$$

N_{cell} : number of cells constituting stack.

$K_r = \frac{N_{cell}}{4F}$: constant defined to simplify model [kmol/(s.A)].

Let us consider case of hydrogen, which is stored in pressurized bottle and will be used to power fuel cell. valve placed between storage reservoir and anode side of cell is necessary to control hydrogen flow rate entering stack. pressure in anodic compartment must also be controlled. For this purpose, another valve is placed at exit of anode (Figure 13).

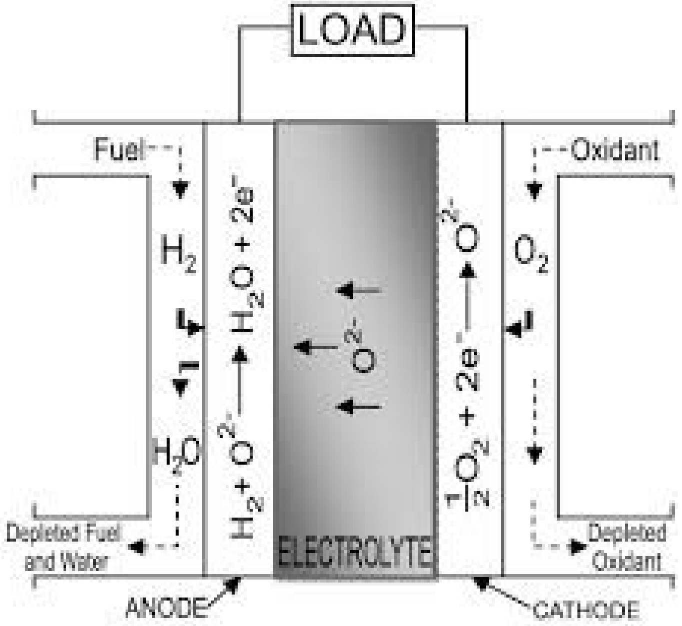


Figure 13:Operating Diagram of SOFC

2.4.1.4. Partial Pressures

In [24], calculation of partial pressures for hydrogen, oxygen and water is based on molar flow rates (inlet, outlet and consumed), operational parameters of fuel cell (temperature and pressure), physical properties and electrochemical characteristics of both anodic and cathodic compartments. These expressions for partial pressures will be used to calculate output voltage of fuel cell.

As shown in Figure (13), when charging current is being generated, hydrogen and oxygen diffuse through porous electrodes (anode and cathode, respectively) to reach reaction sites. As result, partial pressures of hydrogen, oxygen and water will gradually form along channels of anode and cathode while fuel cell is operating. Assuming that variation in partial pressures is uniform, arithmetic mean values are used to represent effective partial pressures of gases with in channels:

$$P_{H_2} = \frac{P_{H_2}^{in} + P_{H_2}^{out}}{2} \qquad \mathbf{(2.09)}$$

$$P_{H_2O} = \frac{P_{H_2O}^{in} + P_{H_2O}^{out}}{2} \qquad \mathbf{(2.10)}$$

$$P_{O_2} = \frac{P_{O_2}^{in} + P_{O_2}^{out}}{2} \qquad \mathbf{(2.11)}$$

The effective partial pressures of hydrogen and oxygen at actual reaction sites will be lower than those in gas flow channels due to mass diffusion. Conversely, partial pressure of water vapor at reaction sites will be higher than in anodic channel. To calculate output voltage of fuel cell, partial pressures at reaction sites must be determined. In gaseous mixture consisting of (***N***) species, diffusion of component (***i***) through porous electrodes can be described by Stefan-Maxwell equation[25]:

$$\nabla x_i = \frac{RT}{P} \sum_{j=1}^{N} \frac{x_j N_j - x_i N_i}{D_{i,j}} \qquad \mathbf{(2.12)}$$

where:

- ∇: gradient operator.
- x_i , x_j: Molar fractions of species.
- $D_{i,j}$: Effective binary diffusivity of pair *i-j* [m²/s].
- N_i , N_j: Surface flux of gas [mol/(m²·s)].
- ***P***: Global pressure of gas mixture [Pa].

in anodic channel, gas flow is mixture of hydrogen and water. For one-dimensional transport assumed along (x)-axis as illustrated in Figure (13), diffusion of hydrogen can be written as follows [48]:

$$\frac{dx_{H_2}}{dx} = \frac{RT}{P_a}\left(\frac{x_{H_2}N_{H_2O} - x_{H_2O}N_{H_2}}{D_{H_2,H_2O}}\right) \qquad \textbf{(2.13)}$$

With P_a being global pressure of gas at anode, we can determine molar fluxes of hydrogen and water using Faraday's law[26]:

$$\boldsymbol{N_{H_2} = -N_{H_2O} = \frac{i_{den}}{2F}} \qquad \textbf{(2.14)}$$

where $\boldsymbol{i_{den}}$ is current density [A/m²].

The gas in anodic channel is assumed to be mixture of $\boldsymbol{H_2}$ and $\boldsymbol{H_2O}$ and given that $\boldsymbol{x_{H_2} + x_{H_2O} = 1}$, $\boldsymbol{dP_{H_2} = dx_{H_2} + P_a}$ and $\boldsymbol{dP_{H_2O} = dx_{H_2O} + P_a}$, equations (2.13) and (2.14) can be combined to give partial pressures of $\boldsymbol{H_2}$ and $\boldsymbol{H_2O}$:

$$\frac{dP_{H_2}}{dx} = -\frac{RT}{D_{H_2,H_2O}}\frac{i_{den}}{2F} \qquad \textbf{(2.15)}$$

$$\frac{dP_{H_2O}}{dx} = \frac{RT}{D_{H_2,H_2O}}\frac{i_{den}}{2F} \qquad \textbf{(2.16)}$$

Integrating last two equations (2.15) and (2.16) gives:

$$\boldsymbol{P^*_{H_2} = \int_0^{l_a}\frac{dP_{H_2}}{dx} + P_{H_2} = P_{H_2} - \frac{RTl_a}{2FD_{H_2,H_2O}}i_{den}} \qquad \textbf{(2.17)}$$

$$\boldsymbol{P^*_{H_2O} = \int_0^{l_a}\frac{dP_{H_2O}}{dx} + P_{H_2O} = P_{H_2O} + \frac{RTl_a}{2FD_{H_2,H_2O}}i_{den}} \qquad \textbf{(2.18)}$$

where $\boldsymbol{l_a}$ is distance between anode surface and reaction site [m] and “*” represents effective value.

in cathodic channel, oxidant is air, which primarily consists of ($\boldsymbol{O_2}$) and ($\boldsymbol{N_2}$), i.e., $\boldsymbol{x_{O_2} + x_{N_2} = 1}$ Applying similar procedure for cathode (Stefan-Maxwell equation), diffusion of oxygen can be written as:

$$\frac{dx_{O_2}}{dx} = \frac{RT}{P_c}\left(\frac{x_{O_2}N_{N_2} - x_{N_2}N_{O_2}}{D_{O_2,N_2}}\right) \qquad \textbf{(2.19)}$$

Since nitrogen does not participate in chemical reaction, it is assumed that normal molar flux of nitrogen at cathode surface is zero: $N_{N_2}=0$

The molar flux of oxygen is determined by Faraday's law:

$$N_{O_2} = \frac{i_{den}}{4F} \qquad (2.20)$$

From equations (2.19) and (2.20), we obtain:

$$\frac{dx_{O_2}}{dx} = \frac{RTi_{den}}{4FP_cD_{O_2,N_2}}\left(x_{O_2} - 1\right) \qquad (2.21)$$

Similar to method used at anode, effective partial pressure of oxygen at reaction site can be expressed as:

$$P^*_{O_2} = P_c - \left(P_c - P_{O_2}\right) exp\left(\frac{RTl_c}{4FP_cD_{O_2,N_2}} i_{den}\right) \qquad (2.22)$$

The dynamic equations for partial pressures of hydrogen, water and oxygen in anodic and cathodic channels can be determined using equations for ideal gases as follows[27]:

$$\frac{V_a}{RT}\frac{dP_{H_2}}{dt} = q^{in}_{H_2} - q^{out}_{H_2} - \frac{i}{2F} \qquad (2.23)$$

$$\frac{V_a}{RT}\frac{dP_{H_2O}}{dt} = q^{in}_{H_2O} - q^{out}_{H_2O} + \frac{i}{2F} \qquad (2.24)$$

$$\frac{V_c}{RT}\frac{dP_{O_2}}{dt} = q^{in}_{O_2} - q^{out}_{O_2} - \frac{i}{4F} \qquad (2.25)$$

Where:

- V_a , V_c is volume of anodic (cathodic) channel [m³].
- $q^{in}_{H_2}$, $q^{in}_{O_2}$ and $q^{in}_{H_2O}$ are molar flow rates of H_2, O_2 and H_2O [mol/s].
- i is current of fuel cell [A].

The molar flow rates of H_2, O_2 and H_2O at inlet and outlet of channels (anode and cathode) can be expressed as follows:

$$\begin{cases} q_{H_2}^{in} = M_a x_{H_2}^{in} = M_a \frac{P_{H_2}^{in}}{P_a} \\ q_{H_2}^{out} = M_a x_{H_2}^{out} = M_a \frac{P_{H_2}^{out}}{P_a} \end{cases} \quad (2.26)$$

$$\begin{cases} q_{H_2O}^{in} = M_a x_{H_2O}^{in} = M_a \frac{P_{H_2O}^{in}}{P_a} \\ q_{H_2O}^{out} = M_a x_{H_2O}^{out} = M_a \frac{P_{H_2O}^{out}}{P_a} \end{cases} \quad (2.27)$$

$$\begin{cases} q_{O_2}^{in} = M_c x_{O_2}^{in} = M_c \frac{P_{O_2}^{in}}{P_c} \\ q_{O_2}^{out} = M_c x_{O_2}^{out} = M_c \frac{P_{O_2}^{out}}{P_c} \end{cases} \quad (2.28)$$

Substituting (2.26)– (2.28) into equations (2.23)– (2.25), we find:

$$\frac{dP_{H_2}}{dt} = \frac{2M_aRT}{V_aP_a}(P_{H_2}^{in} - P_{H_2} - \frac{P_a}{4FM_a}i) \quad (2.29)$$

$$\frac{dP_{H_2O}}{dt} = \frac{2M_aRT}{V_aP_a}\left(P_{H_2O}^{in} - P_{H_2O} + \frac{P_a}{4FM_a}i\right) \quad (2.30)$$

$$\frac{dP_{O_2}}{dt} = \frac{2M_cRT}{V_cP_c}(P_{O_2}^{in} - P_{O_2} - \frac{P_c}{4FM_c}i) \quad (2.31)$$

Using Laplace transform, above differential equations (2.29)– (2.31) can be written in following form:

$$P_{H_2}(s) = \frac{1}{(1+\tau_a s)}\left[P_{H_2}^{in}(s) + \tau_a P_{H_2}(0) - \frac{P_a}{4FM_a}I(s)\right] \quad (2.32)$$

$$P_{O_2}(s) = \frac{1}{(1+\tau_c s)}\left[P_{O_2}^{in}(s) + \tau_c P_{O_2}(0) - \frac{P_c}{4FM_c}I(s)\right] \quad (2.33)$$

$$P_{H_2O}(s) = \frac{1}{(1+\tau_a s)}\left[P_{H_2O}^{in}(s) + \tau_a P_{H_2O}(0) + \frac{P_a}{4FM_a}I(s)\right] \quad (2.34)$$

The time constants $\tau_a = \frac{V_aP_a}{2M_aRT}, \tau_c = \frac{V_cP_c}{2M_cRT}$ [s] are associated with pressure at anode and cathode, respectively, representing rate at which hydrogen pressure (in anodic channel) and oxygen vapor pressure (in cathodic channel) vary with changes in charge. physical significance of time constant τ_a is time required to fill reservoir of volume $\frac{V_a}{2}$ to pressure P_a if mass flow rate is M_a . similar physical interpretation applies to τ_c.

These expressions for partial pressures will be used to calculate output voltage of fuel cell.

2.4.2 Real Performance of an SOFC Cell

While ideal performance of solid oxide fuel cell (SOFC) is governed by thermodynamic principles such as Nernst equation, real-world operation deviates due to various losses. These losses reduce actual voltage output and efficiency of cell compared to its theoretical maximum. three primary types of losses in SOFCs are **ohmic losses**, **activation losses** and **concentration losses**. Below, we examine each type in detail.

2.4.2.1 Ohmic Losses

Ohmic losses arise from resistance encountered by ions and electrons as they move through various components of SOFC. These losses are directly proportional to current density and are described by Ohm's law:

$$\boldsymbol{V_{ohm} = r \cdot i} \qquad \textbf{(2.35)}$$

where:

- $\boldsymbol{i}$: Current density delivered by cell [A/cm²]
- $\boldsymbol{r}$: Total resistance of cell [Ω], which is sum of various resistances:

$$\boldsymbol{r = R_e + R_a + R_c + R_{cont}} \qquad \textbf{(2.36)}$$

where:

- $\boldsymbol{R_e}$: Resistance to flux of oxygen ions O2− in electrolyte [Ω].
- $\boldsymbol{R_a}$: Resistance to flux of electrons e− in anode [Ω].
- $\boldsymbol{R_c}$: Resistance to flux of electrons e− in cathode [Ω].
- $\boldsymbol{R_{cont}}$: Contact resistance between components of cell [Ω].

Furthermore, ohmic resistance is expressed by following equation:

$$\boldsymbol{r = r_0\, exp\left[a\left(\frac{1}{T_0} - \frac{1}{T}\right)\right]} \qquad \textbf{(2.37)}$$

where ($\boldsymbol{r_0}$) is internal resistance at temperature ($\boldsymbol{T_0}$) and (**T**) and ($\boldsymbol{a}$) are constants.

Ohmic losses increase linearly with current density and are at high operating currents. This makes them critical factor in determining power density and efficiency of SOFC systems.

2.4.2.2 Activation Losses

Activation losses occur due to reaction kinetics of electrochemical process at electrodes. These losses are associated with energy required to overcome activation energy barrier for reactions to proceed. Activation losses are modeled using **Butler-Volmer equation**, which describes relationship between current density and overpotential.

$$V_{act} = A_a \ln\left(\frac{i}{i_{0_a}}\right) + A_c \ln\left(\frac{i}{i_{0_c}}\right) \qquad (2.38)$$

Equation (2.38) can be rewritten as:

$$V_{act} = A \ln\left(\frac{i}{i_0}\right) \qquad (2.39)$$

where:

$$A = A_a + A_c = \frac{RT}{\alpha n F} \qquad (2.40)$$

and

$$i_0 = (i_{0_a})^{\frac{A_a}{A}} (i_{0_c})^{\frac{A_c}{A}} \qquad (2.41)$$

where:

- α: Charge transfer coefficient, which depends on specific reaction and electrode material.
- i: Current density supplied by cell [A/cm²].
- i_0: Exchange current density, representing minimum current driven by cell [A/cm²].

Generally, in hydrogen-fueled SOFC, activation losses at anode are negligible compared to those at cathode. These losses follow **Tafel equation**.

Activation losses are most pronounced at low current densities and decrease as current increases, making them dominant factor during startup and low-load operations.

2.4.2.3 Concentration Losses

Concentration losses, also known as mass transport losses, occur when concentration of reactants near electrode surfaces decreases due to high reaction rates. This depletion reduces driving force for electrochemical reactions, leading to drop in cell voltage. Concentration losses become at high current densities when rate of reactant consumption exceeds rate of replenishment.

Several factors contribute to concentration losses, including:

- Low gas diffusion through porous electrodes,
- Dissolution of reactants or products in electrolyte,
- Diffusion of reactants/products to or from reaction site.

At high current densities, slow reactant transport to reaction site becomes major contributor to concentration polarization. These losses are described by following equation:

$$\boldsymbol{V_{conc} = -\frac{RT}{nF} ln\left(1 - \frac{i}{i_L}\right)} \qquad \textbf{(2.42)}$$

where:

- $\boldsymbol{i_L}$ is **limiting current density** [A/cm²], representing maximum current that cell can deliver before transport limitations cause concentration saturation at electrode.

Concentration losses increase exponentially with current density and are primary factor limiting maximum power output of an SOFC.

Type of Loss	Cause	Dependence on Current Density	Mitigation Strategies
Ohmic Losses	Resistance to ion and electron flow	Linear	Thin electrolytes, advanced materials, optimized interconnects
Activation Losses	Slow reaction kinetics at electrodes	Decreases with increasing current	High-activity catalysts, elevated temperatures, porous electrodes
Concentration Losses	Depletion of reactants near electrodes	Exponential	Increased flow rates, optimized electrode design, higher pressures

Table 4: Comparison of Losses

2.4.3 Expression of Output Voltage of Fuel Cell

The output voltage ($\boldsymbol{V_{output}}$) of solid oxide fuel cell (SOFC) is determined by subtracting various losses from ideal voltage ($\boldsymbol{E}$) predicted by Nernst equation. These losses include ohmic losses, activation losses and concentration losses, which collectively reduce actual voltage generated by cell. expression for $\boldsymbol{V_{output}}$ can be written as:

$$\boldsymbol{V_{output} = E - V_{ohmic} - V_{activation} - V_{concentration}} \qquad \textbf{(2.43)}$$

where:

- $\boldsymbol{E}$: Ideal voltage calculated using Nernst equation,
- $\boldsymbol{V_{ohmic}}$: Voltage loss due to ohmic resistance,
- $\boldsymbol{V_{activation}}$: Voltage loss due to activation overpotential,
- $\boldsymbol{V_{concentration}}$: Voltage loss due to concentration overpotential.

Below, we break down each term in this expression and discuss its contribution to overall output voltage.

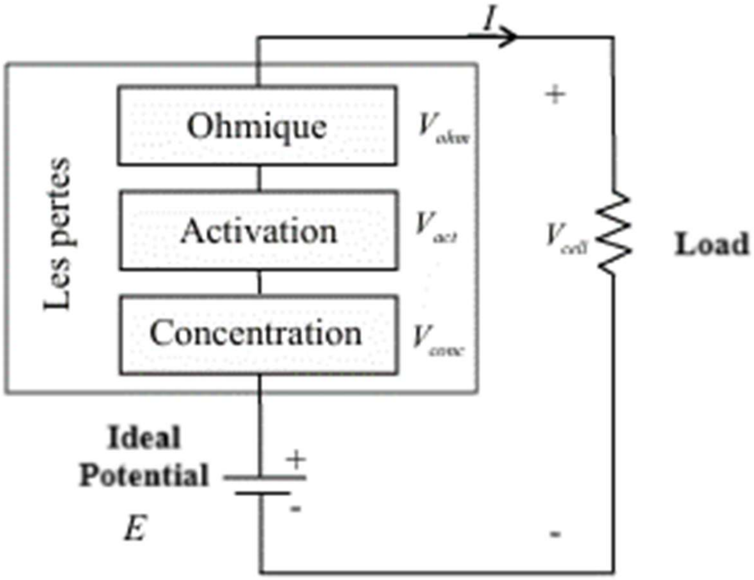


Figure 14:Equivalent Circuit of SOFC

2.5 Control System for an SOFC-Type Fuel Cell

The performance and efficiency of solid oxide fuel cell (SOFC) are highly dependent on control strategies implemented to manage its operating conditions. robust control system ensures optimal reactant supply, minimizes losses and maintains stable operation under varying load demands. In this section, we explore three key aspects of SOFC control systems: ratio between reactant flow rates, relationship between hydrogen flow rate and cell current and management of cell current.

2.5.1 Optimal Ratio Between Reactant Flow Rates

According to chemical reaction (2.01) occurring inside SOFC, stoichiometric ratio between hydrogen and oxygen is 2 to 1. Oxygen is always supplied in excess to allow more complete reaction with hydrogen. This ratio is defined by following equation:

$$r_{H/O} = \frac{\dot{q}_{H_2}^{in}}{\dot{q}_{O_2}^{in}} \qquad (2.44)$$

For system safety reasons, value of this ratio is chosen so that pressure difference between hydrogen and oxygen ($\Delta P = P_{H_2} - P_{O_2}$) does not exceed 4 kPa under normal operating conditions and 8 kPa under transient conditions.

To maintain in pressure difference below 4 kPa under normal conditions, hydrogen-to-oxygen flow rate ratio ($\boldsymbol{r_{H/O}}$) should be around 1.145[28]. This value was determined through prior feasibility simulations.

2.5.2 Relationship Between Hydrogen Flow Rate and Cell Current

In practice, not all supplied gas is consumed by cell. fuel utilization factor is defined as ratio of consumed hydrogen flow rate ($\dot{q}_{H_2}^{r}$) to supplied hydrogen flow rate ($\dot{q}_{H_2}^{in}$). It is expressed as follows:

$$U_f = \frac{\dot{q}_{H_2}^{u}}{\dot{q}_{H_2}^{in}} \qquad \textbf{(2.45)}$$

The actual current at fuel cell system output can be measured and fuel flow rate at stack inlet can be controlled by adjusting fuel utilization rate.

For an optimal utilization value ($\boldsymbol{U_{opt}}$**=85%**), using Equation (2.08), reference hydrogen flow rate supplied to cell as function of current can be defined as follows:

$$\dot{q}_{H_2}^{in} = \frac{2KI}{U_{opt}} = \frac{2KI}{0.85} \qquad \textbf{(2.46)}$$

From Equation (2.44), reference oxygen flow rate can also be determined as follows:

$$\dot{q}_{O_2}^{ref} = \frac{\dot{q}_{H_2}^{ref}}{r_{H_2/O_2}} \qquad \textbf{(2.47)}$$

It is possible to measure current at fuel cell output to determine reference flow rates of both fuel and oxidant. Through fuel processor, actual hydrogen flow injected into cell is controlled. same applies to oxidant, where its flow rate is adjusted by compressor.

The chemical reaction in fuel processor is usually slow and is associated with parameter exchange time in chemical reaction following change in reactant flow rates. This dynamic response function is modeled as first order transfer function with time constant $\boldsymbol{\tau_r}$[23]:

$$\frac{\dot{q}_{H_2}^{in}}{\dot{q}_{H_2}^{ref}} = \frac{1}{(1+\tau_r s)} \qquad \textbf{(2.48)}$$

$$\frac{\dot{q}_{O_2}^{in}}{\dot{q}_{O_2}^{ref}} = \frac{1}{(1+\tau_r s)} \qquad \textbf{(2.49)}$$

2.5.3 Cell Current

To limit transient current fluctuations, new boundaries are defined for utilization factor, which include maximum utilization rate ($U_{max}=0.9$) and minimum utilization rate ($U_{min}=0.8$). In some cases, Equation (2.46) may not hold. However, it is crucial that this ratio remains within strict limits:

$$U_{min}\dot{q}_{H_2}^{in} \leq 2KI_{ref} \leq U_{max}\dot{q}_{H_2}^{in} \tag{2.50}$$

Thus:

$$\frac{U_{min}\dot{q}_{H_2}^{in}}{2K} \leq I_{ref} \leq \frac{U_{max}\dot{q}_{H_2}^{in}}{2K} \tag{2.51}$$

The reference current of cell is calculated based on reference power (P_{ref}) that we want cell to provide:

$$I_{ref} = \frac{P_{ref}}{V_c} \tag{2.52}$$

2.6 Model Validation

In this section, MATLAB/Simulink environment is used to simulate dynamic model of tubular geometry SOFC, based on electrochemical and thermodynamic characteristics presented in previous sections.

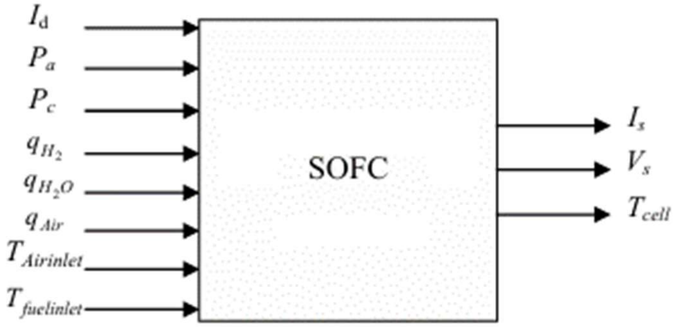


Figure 15: Inputs/Outputs of SOFC

Figure 9 represents inputs and outputs of fuel cell developed in[12]. There are eight input parameters: load current (I_d), pressures at anode and cathode (P_a and P_c), fuel flow rate (hydrogen: q_{H_2}), water vapor flow rate (q_{H_2O}), air flow rate (q_{Air}) and initial temperatures of fuel cell and air (T_{fuel_inlet} and T_{Air_inlet}).

There are three output parameters: output current and voltage (I_s and V_s) and cell temperature (T_{cell}).

2.6.1 Model Parameters

The simulation parameters are given for tubular geometry SOFC model with nominal power of 5KW. These parameters are listed in following table [29]:

Parameter	**Representation**	**Value**
K_r	Reaction constant	0.993 × 10^{-3} mol/(s A)
K_{H_2}	Valve molar constant for hydrogen	0.843 mol/(s atm)
K_{H_2O}	Valve molar constant for steam	0.281 mol/(s atm)
K_{O_2}	Valve molar constant for oxygen	2.52 mol/(s atm)
τ_{H_2}	Response time for hydrogen flow	26.1 s
τ_{H_2O}	Response time for steam flow	78.3 s
τ_{O_2}	Response time for oxygen flow	2.91 s
τ_f	Fuel processor response time	5 s
F	Faraday's constant	96486C/mol
R	Gas constant	8.31 J/(mol K)
E_0	Ideal standard potential	1.1 V
N	Number of cells in stack	384
E_{act}	Activation energy	120 kJ/mol
A	Pre-exponential factor	101.2 kA/cm²
α	Ohmic resistance constant	0.2 Ω
β	Ohmic resistance constant	-2870 K
T_0	Constant temperature	973 K
η	Efficiency	0.8
H_{eff}	Thickness	10^{-3} m
λ_s	Thermal conductivity	2 W/(m K)
t	Relaxation time	200 s
ρ	Density	6600 kg/m³
c_p	Heat capacity	400 J/(kg K)
T_{int}	Initial temperature	1273 K

Table 5:Simulation Parameters

2.7 Conclusion

In this chapter we have explored comprehensive development of Solid Oxide Fuel Cells highlighting their advantages over other fuel cell technologies. discussion began with an in depth explanation of key reasons behind selection of SOFC emphasizing their efficiency, fuel flexibility and environmental benefits. We examined three major SOFC configurations (tubular, planar and monolithic) each offering unique structural and operational suited to specific applications.

A detailed dynamic model of an SOFC stack was developed. Starting from ideal thermodynamic principles such as Gibbs free energy and Nernst equation, model incorporated electrochemical and physical variables like molar flow rates and partial pressures. Real world losses (ohmic, activation and concentration) were analyzed and integrated into model to enhance its predictive accuracy under operational conditions.

CHAPTER 3:

FUZZY ADAPTIVE PID MODELING OF SOFC FUEL CELL

3.1. Introduction

Traditional Proportional-Integral-Derivative (PID) controllers have been widely employed in industrial applications due to their simplicity and effectiveness in linear systems. dynamic behavior of SOFCs characterized by nonlinearities and time varying parameters often renders conventional PID controllers inadequate[30]. To address these limitations, adaptive control techniques have explored but they require precise system models and extensive resources which may not be feasible in real time applications[31].

In recent fuzzy logic control systems have gained attention because to them ability to handle uncertainties and nonlinearities without requiring explicit mathematical model of system[5]. Fuzzy logic provides framework of approximating human decision processes, making it suitable to systems with complex and poorly understood dynamics. By integrating fuzzy logic with PID control, hybrid approach known fuzzy adaptive PID control has been proposed to dynamically adjust controller gains based on real time system behavior[32]. This approach combines robustness of fuzzy logic with simplicity and effectiveness of PID control offering promised solution to SOFC systems.

The objective of chapter to explore application of fuzzy adaptive PID modeling on context of SOFC systems:

1. Develop comprehensive fuzzy adaptive PID controller of nonlinear dynamics of SOFCs.
2. Evaluate performance of controller under varying operating conditions and disturbances.
3. Validate effectiveness of controller by simulations and experimental results.

This chapter begins with overview of fuzzy logic and its role in adaptive control systems followed at detailed discussion of fuzzy models and their application to dynamic systems. Subsequent sections focus at design and implementation of fuzzy adaptive PID controller including gain adaptation mechanisms and numerical validation. Finally, chapter concludes with discussion of implications of this work for SOFC control systems and potential avenues for future research.

3.2. Introduction to Fuzzy Logic and Adaptive Control

Fuzzy logic and adaptive control represent two principal points in field of control systems each one address specific challenges associated with complex, nonlinear and uncertain systems. They integrate into framework like fuzzy adaptive PID control has proven be effective in handling dynamic behavior of systems like Solid Oxide Fuel Cells which is nonlinearities and time varying characteristics.

3.2.1. Fundamentals of Fuzzy Logic

Fuzzy logic introduced by Lotfi Zadeh in 1965[5], provides mathematical framework to dealing with imprecise or uncertain information on allowing partial truth values between "completely true" and "completely false" Unlike classical logic which operates on crisp sets (0 or 1), fuzzy logic using membership functions to map inputs to degrees of membership in range of [0, 1]. This enables representation of linguistic variables such "high temperature" or "low pressure" in way that close human reasoning.

The general structure of fuzzy logic system consists by four components:

1. ***Fuzzification:*** process of converting crisp inputs into fuzzy sets by membership functions.
2. ***Rule Base:*** set of "if-then" rules define relationship between inputs and outputs based on expert knowledge.
3. ***Inference Engine:*** Applies fuzzy logic operations (and or) to evaluate rules and determined appropriate output.
4. ***Defuzzification:*** Converts fuzzy output back to crisp value that can used for control purposes[33].

Fuzzy logic is advantage in systems where precise mathematical models are unavailable or complex to derive. It's able handle uncertainties and nonlinearities makes it natural for applications in energy systems robotics and industrial automation[34].

3.2.2. Adaptive Control Systems

Adaptive control systems designed to adjust them parameters at real time to compensate for changes at dynamics of controlled system. Unlike traditional controllers which rely on fixed gains, adaptive controllers continuously update control laws based on feedback from system. This adaptation is important for systems with time varying parameters external disturbances or unknown dynamics.

There are two types of adaptive control:

1. ***Model Reference Adaptive Control (MRAC):*** controller adjusts its parameters to minimize error between system output and reference model.
2. ***Self Tuning Regulators (STR):*** controller estimates system parameters online and updates control law according it[35].

While adaptive control techniques powerful, they often require accurate system models and extensive resources which limit their applicability at real time scenarios[31]. their performance may degrade in presence of uncertainties or unmodeled dynamics.

3.2.3. Integration of Fuzzy Logic and Adaptive Control

The integration of fuzzy logic with adaptive control resolve many of limitations of both approaches. Fuzzy logic provides mechanism of handling uncertainties and nonlinearities without requiring explicit system models, while adaptive control ensures that controller dynamically adjust its parameters to maintain optimal performance. This synergy valuable in systems like SOFC, where operating conditions variable and difficult to predict.

One notable example of this integration is fuzzy adaptive PID controller which combines robustness of fuzzy logic with simplicity and effectiveness of PID control. this hybrid approach fuzzy logic used dynamically adjust PID gains (proportional, integral and derivative) based on real time system behavior. This allows controller to adapt to changing conditions while maintaining stability and performance[32].

3.3.4. Advantages of Fuzzy Adaptive PID Controller

The fuzzy adaptive PID controller offers advantages over traditional PID and adaptive controllers:

- ***Robustness:*** Handles uncertainties and nonlinearities
- ***Adaptability:*** Dynamically adjusts gains to accommodate changing operating conditions.

- ***Simplicity:*** Retains intuitive structure of PID control while incorporating flexibility of fuzzy logic.
- ***Wide Applicability:*** Suitable for systems with complex dynamics such SOFC where precise mathematical models unavailable [33].

3.3.5. Challenges

Design and implementation of fuzzy adaptive PID controllers present several challenges:

- ***Rule Base Complexity***: Designing an effective rule base requires domain expertise or extensive data collection.
- ***Computational Overhead***: Real time fuzzification, inference and defuzzification may introduce delays in high speed applications.
- ***Parameter Tuning***: Selecting appropriate membership functions and scaling factors is important for optimal performance[34].

These challenges require consideration of system's requirements and constraints.

3.3.6. Fundamental Differences Between Type-1 and Type-2 Fuzzy Sets

Type-1 fuzzy sets represent classical form of fuzzy sets in which each element assigned precise membership degree with in interval [0,1] [5]. These sets used in various applications where uncertainty can model with crisp membership functions. but they don't inherently account for uncertainties in membership functions themselves. type-2 fuzzy sets offer advanced framework that enables modeling of higher order uncertainties by allowing membership degrees to be fuzzy rather than crisp[34]. This means that for each element instead of single membership value range or fuzzy set of values is considered, making type-2 fuzzy systems suitable for handling high levels of uncertainty such those found in dynamic environments, noisy data or imprecise linguistic information. type-2 fuzzy sets most used because to them computational tractability while still provide advantages over type-1 systems in terms of uncertainty modeling [34].the choice between type-1 and type-2 fuzzy logic systems depends on nature of application and extent to which uncertainty plays role in system's performance.

3.2.7. Applications in Energy Systems

Fuzzy logic and adaptive control found application in energy systems including renewable energy generation power electronics and fuel cell technologies. fuzzy logic has been applied to optimize performance of photovoltaic systems[36], wind turbines[37] and battery management systems[38]. adaptive control techniques have employed to enhance efficiency and reliability of energy storage systems and microgrids[39].

in context of SOFC systems fuzzy adaptive PID control offers promising solution to challenges posed by nonlinear dynamics, thermal management and load variations. By leveraging strengths of both fuzzy logic and adaptive control this approach enables precise regulation of critical parameters such voltage, current and temperature improving overall efficiency and lifespan of fuel cell[28].

3.3. FUZZY ADAPTIVE PID CONTROLLER

The fuzzy adaptive PID controller operates by monitoring system's error ($\mathbf{e}$) and rate of change of error ($\Delta \boldsymbol{e}$) to adjust proportional ($\boldsymbol{K_p}$), integral ($\boldsymbol{K_i}$) and derivative ($\boldsymbol{K_d}$) gains of PID controller. Unlike conventional PID controllers, where gains are fixed, fuzzy adaptive PID controller uses fuzzy logic to adapt these gains in real time based on system's current state. This dynamic adjustment ensures that controller remains effective even in presence of disturbances or changes in system dynamics.

The overall architecture of fuzzy adaptive PID controller can be summarized as follows:

- ***Input Variables***: Error ($\boldsymbol{e}$) and rate of change of error ($\Delta \boldsymbol{e}$).
- ***Fuzzy Logic System***: rule base and membership functions that map inputs to adjustments of PID gains.
- ***Output Variables***: Adjusted values of $\boldsymbol{K_p}$, $\boldsymbol{K_i}$ and $\boldsymbol{K_d}$.
- ***PID Control Law***: adjusted gains are used to compute control signal ($u(t)$) using standard PID equation:

$$\boldsymbol{u(t) = K_p e(t) + K_i \int e(t)dt + K_d \frac{e(t)}{dt}} \qquad \textbf{(3.01)}$$

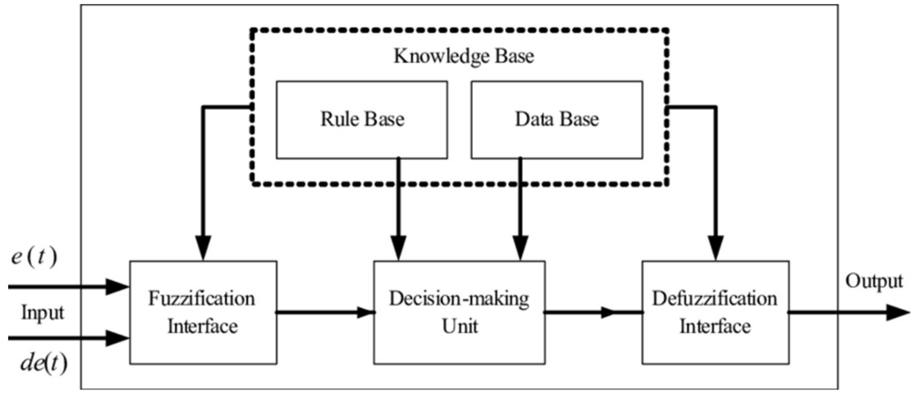


Figure 16:General structure of fuzzy controller

3.3.1. Overview of Controller Architecture

Fuzzy adaptive PID controllers represent hybrid control architecture that combines robustness of fuzzy logic with simplicity and effectiveness of traditional PID control. This structure is suited for systems with nonlinear dynamics, time varying parameters and uncertainties such as Solid Oxide Fuel Cells. general structure of fuzzy adaptive PID controller can be divided into three primary components: fuzzification, rule based adaptation and defuzzification. Each component plays critical role in dynamically adjusting PID gains to ensure optimal system performance under varying operating conditions.

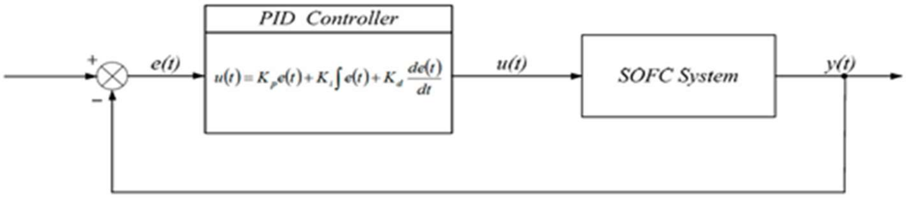


Figure 17:Block Diagram of PID Controller

The adjusted gains are applied to standard PID equation:

$$u(t) = K_p e(t) + K_i \int [e(t) + K u_2(t)] + K_d \frac{dy(t)}{t} \qquad \textbf{(3.02)}$$

$$u_2(t) = u_1(t) - u_0(t) \qquad \textbf{(3.03)}$$

Where:

$u(t)$: Represents control action or control signal.

K_p, K_i and K_d : Are proportional, integral and derivative gains, respectively.

$e(t)$: Is tracking error signal, defined as difference between desired setpoint and actual output of system.

This formulation allows PID controller to adjust its response based on current error ($K_p e(t)$), accumulation of past errors ($K_i \int e(t)$) and rate of change of error ($K_d \frac{dy(t)}{t}$).

3.3.2. Components of Fuzzy Adaptive PID Controller

3.3.2.1. Fuzzy PI Controller

The inputs to controller are error ***e*** and change in error ***ec***, which are defined as follows:

$$e(k) = r(k) - y(k) \qquad (3.04)$$

$$ec(k) = e(k) - e(k-1) \qquad (3.05)$$

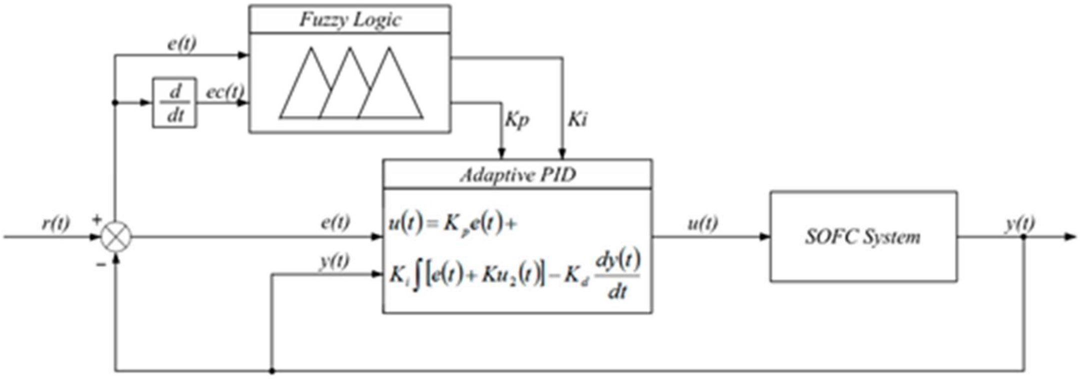


Figure 18:Bloc Diagram of Fuzzy Adaptive PID Controller

where **r(k)** denotes rated voltage and **y(k)** denotes output voltage.

The outputs of controller are modifying values of proportional gain K_{p1} and integral gain K_{i1}.

A fuzzy controller, based on Type-1 or Type-2 fuzzy logic, generally consists of three fundamental stages in its inference process: Fuzzification, Fuzzy Logic Inference (Judgment), Defuzzification.

While both types follow same general structure, they differ in how they represent and handle uncertainty in fuzzification and inference stages. Type-2 fuzzy systems offer enhanced capabilities in modeling and managing uncertainties in membership functions and linguistic variables, making them more robust in complex and noisy environments.

3.3.2.2. Fuzzification (Type-1 Fuzzy Logic)

Fuzzification is process of converting crisp input variables ($\boldsymbol{e}$ and $\Delta\boldsymbol{e}$) into linguistic variables using membership functions. These membership functions define degree to which each input belongs to specific fuzzy set such as: **NB, NM, NS, ZO, PS, PM** and **PB**, which stand for ***Negative Big, Negative Medium, Negative Small, Zero, Positive Small, Positive Medium and Positive Big***, respectively.

- The fuzzy membership functions for **NB** (**Negative Big**) and **PB** (**Positive Big**) are of Pi-type.
- The fuzzy membership functions for remaining items (**NM, NS, ZO, PS, PM**) are of triangular type.

The universe of discourse for inputs (**e, ec**) and outputs ($\boldsymbol{K_{p1}}$, $\boldsymbol{K_{i1}}$) is defined as [–3, 3]. scaling gains for these variables are as follows:

- ***e (error)***: 0.5
- ***ec (change in error)***: 0.5
- $\boldsymbol{K_{p1}}$ ***(Modifying value of proportional gain)***: 0.1
- $\boldsymbol{K_{i1}}$ ***(Modifying value of integral gain)***: 0.04

The membership function curves for inputs are displayed in Figures 19 and 20.

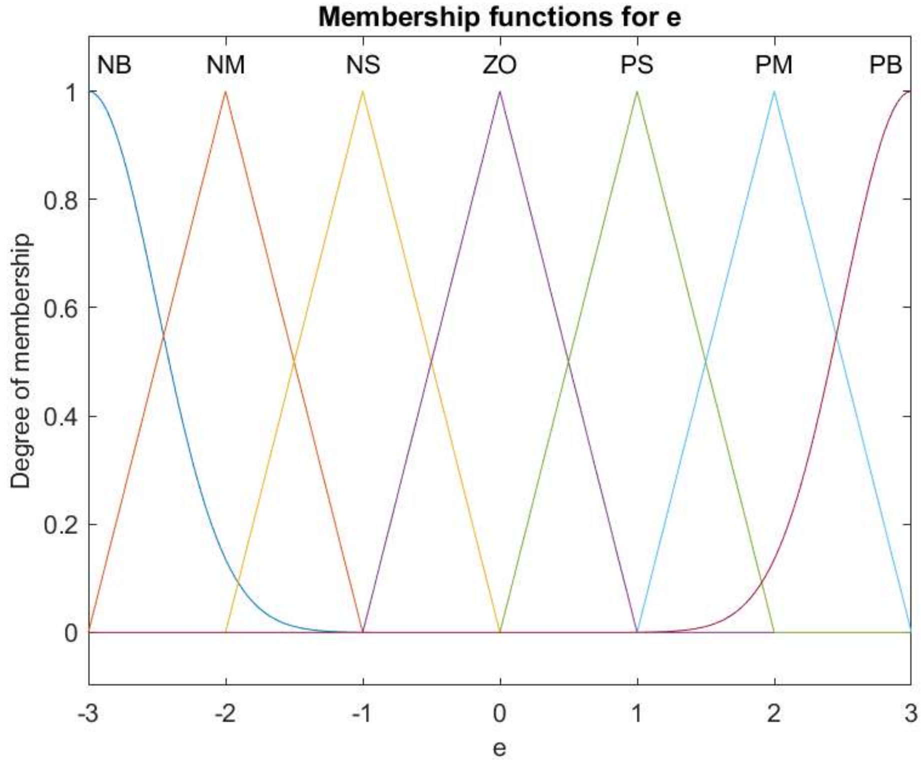


Figure 19:The membership function for e

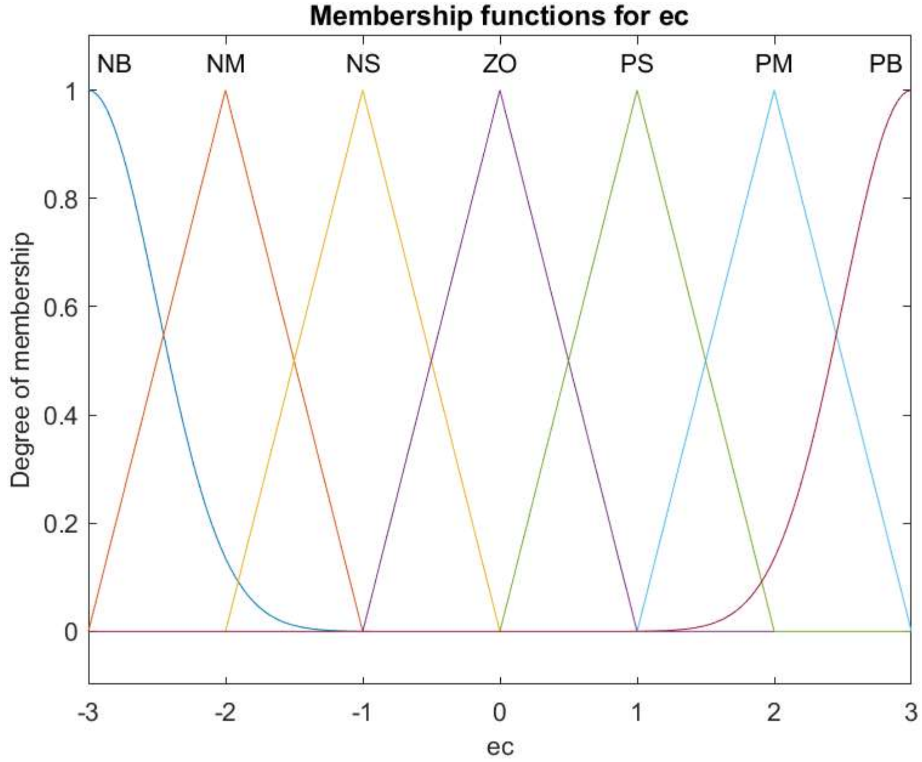


Figure 20:The membership function for ec

The membership function curves for outputs are displayed in Figures 21 and 22.

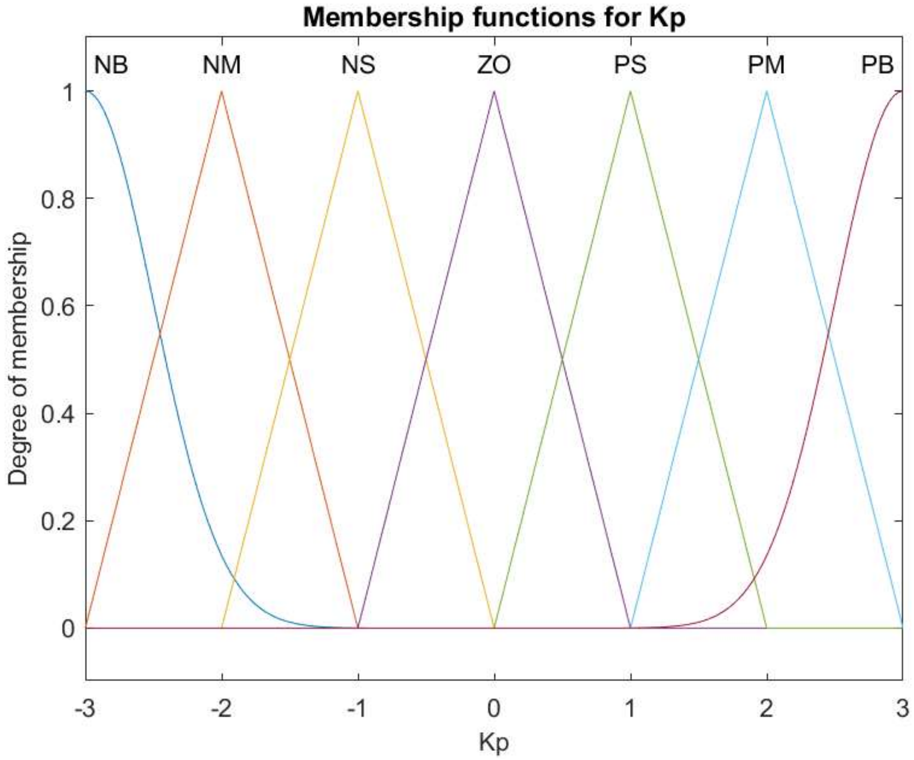


Figure 21:The membership function for Kp

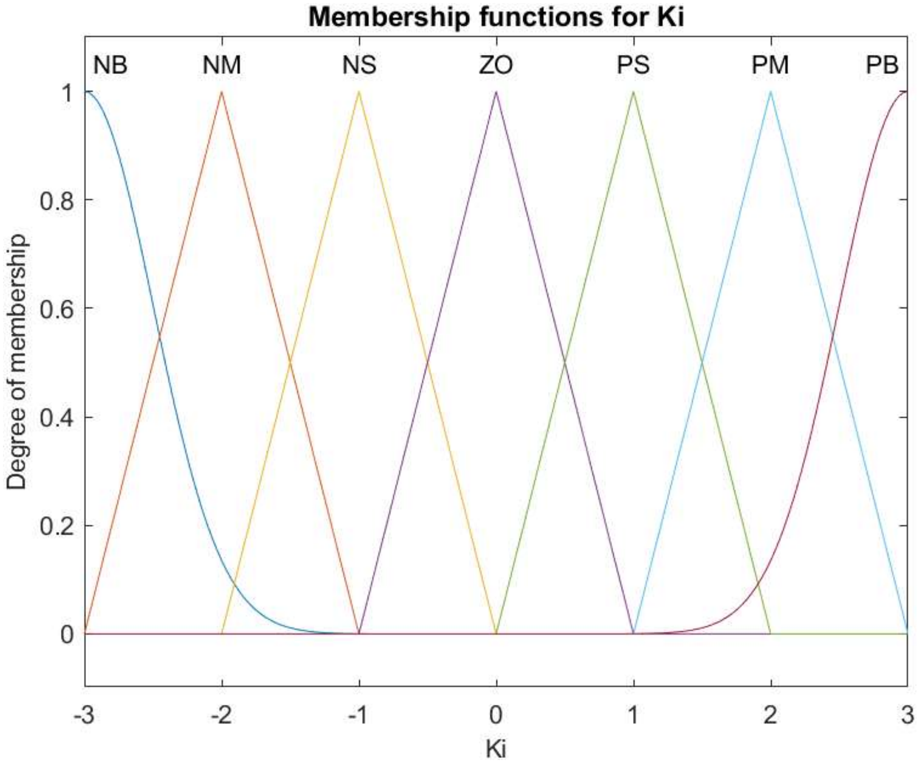


Figure 22:The membership function for Ki

3.3.2.3. Fuzzification (Type-2 Fuzzy Logic)

Fuzzification in Type-2 fuzzy logic system refers to process of transforming crisp input variables (error $\boldsymbol{e}$ and change in error $\boldsymbol{ec}$) into linguistic fuzzy variables, similar to Type-1 systems. However, unlike Type-1, where each input is mapped to single crisp membership value, Type-2 fuzzification maps each input to Footprint of Uncertainty (FOU), which represents range of possible membership values. This allows system to capture and model uncertainties in input data or in interpretation of linguistic terms such as: **NB**, **NM**, **NS**, **ZO**, **PS**, **PM** and **PB**, which stand for ***Negative Big***, ***Negative Medium***, ***Negative Small***, **Zero**, ***Positive Small***, ***Positive Medium*** and ***Positive Big***, respectively.

- The Interval Type-2 fuzzy membership functions for **NB** and **PB** are modeled using gaussian type membership functions with an associated uncertainty band.
- For remaining linguistic terms (**NM**, **NS**, **ZO**, **PS**, **PM**), triangular Interval Type-2 membership functions are used, where each term is represented by an upper and lower membership function that defines uncertainty range.

The universe of discourse for inputs (error $\boldsymbol{e}$, change in error $\boldsymbol{ec}$) and outputs (controller parameters $\boldsymbol{K_{p1}}$, $\boldsymbol{K_{i1}}$) is defined over interval [–3, 3].

This fuzzification approach enhances robustness of system by explicitly modeling uncertainties in both input measurements and linguistic definitions.

The membership function curves for inputs are displayed in Figures 23 and 24.

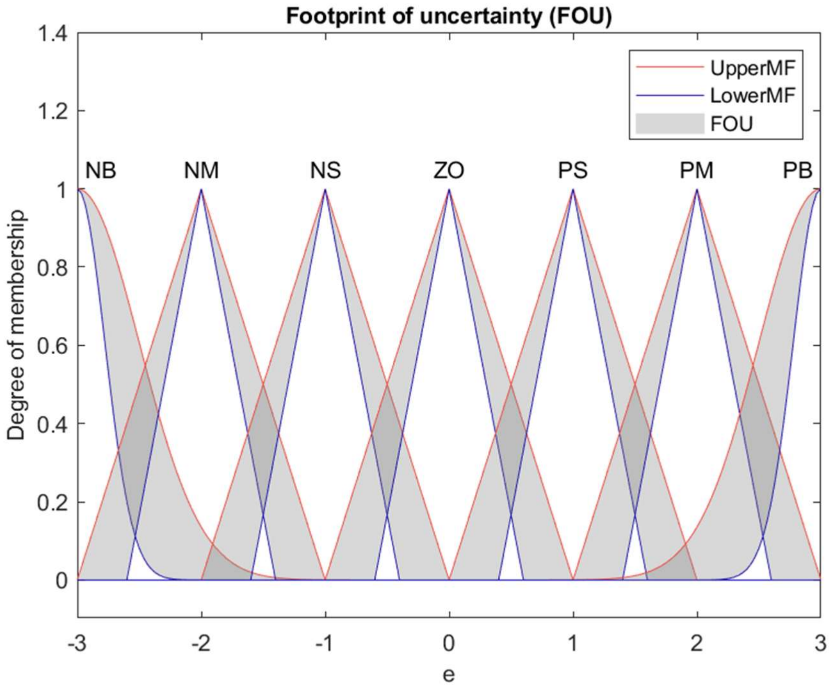


Figure 23:The membership function for e

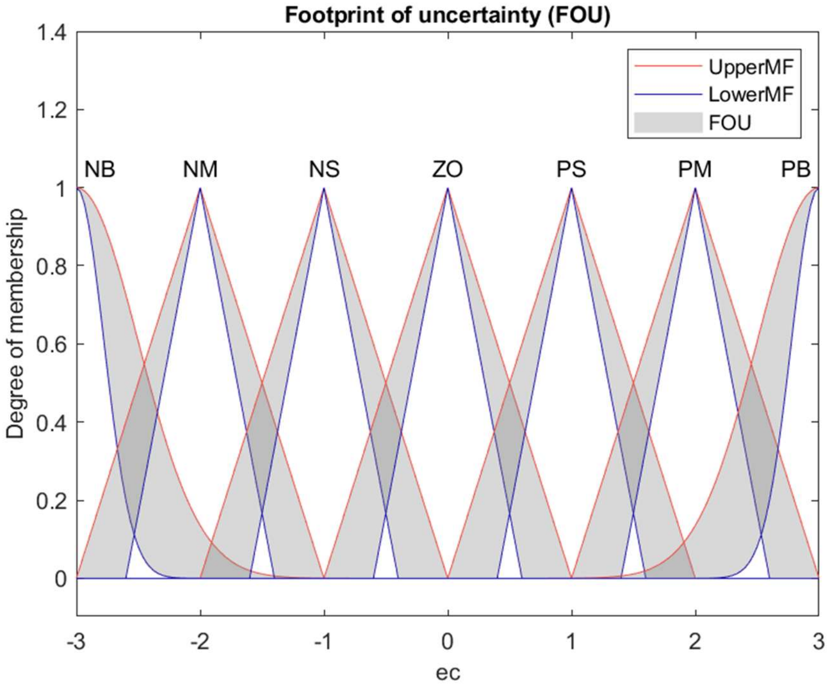


Figure 24::The membership function for ec

The membership function curves for outputs are displayed in Figures 25 and 26.

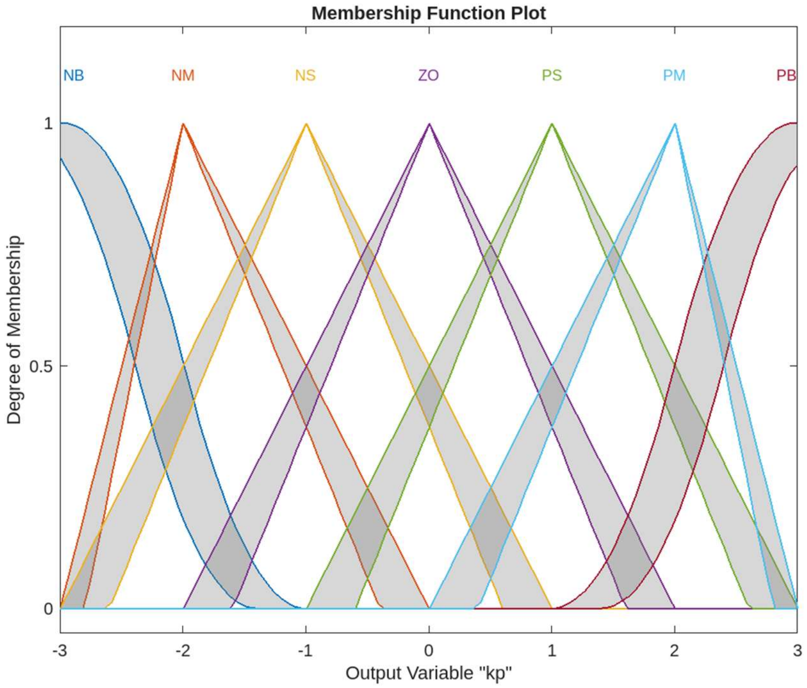


Figure 25:The membership function for Kp

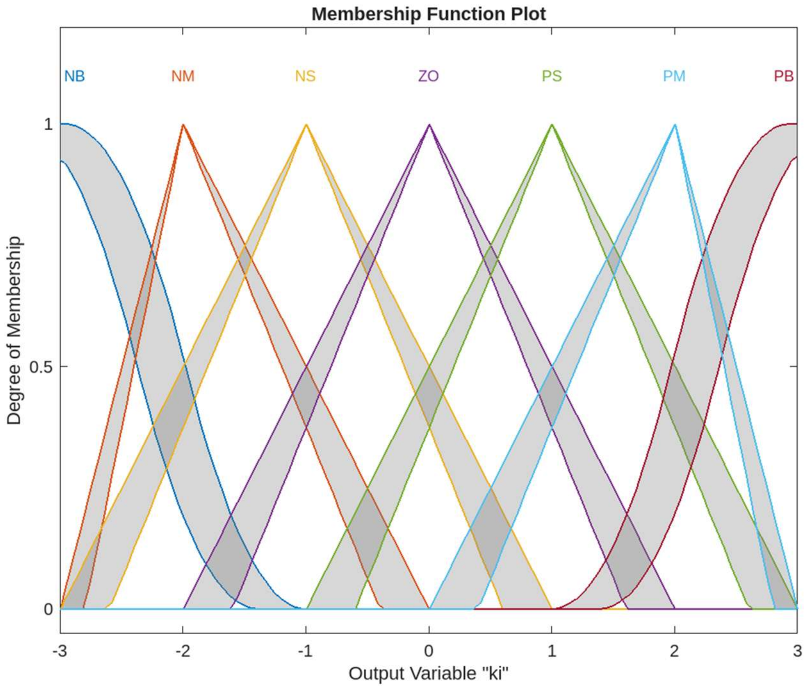


Figure 26:The membership function for Ki

3.3.2.3. Rule Base

The fuzzy logic judgment forms foundation of fuzzy control, relying on fuzzy logic rules. controller is designed using Mamdani Inference Method [27]. output membership value is computed as follows:

$$\mu_{K_{p1}}(c_p) = \mu_e(e) \wedge \mu_{ec}(e) \qquad \textbf{(3.06)}$$

$$\mu_{K_{i1}}(c_i) = \mu_e(e) \wedge \mu_{ec}(e) \qquad \textbf{(3.07)}$$

Here, c_p and c_i are elements in fuzzy sets K_{p1} and K_{i1}, respectively, while $\mu_{K_{p1}}(c_p)$ and $\mu_{K_{i1}}(c_i)$ represent membership degrees of c_p and c_i.

The fuzzy rules, totaling 49, are presented in Table 6 and Table 7. These rules are expressed as: "If e is A_i and ec is B_i, then K_{p1} is C_i and K_{i1} is D_i " where A_i , B_i, C_i and D_i are fuzzy items in universe of discourse of e, ec, K_{p1} and K_{i1}, respectively.

ec ***e***	**NB**	**NM**	**NS**	**ZO**	**PS**	**PM**	**PB**
NB	PB	PB	PM	PM	PS	ZO	ZO
NM	PB	PB	PM	PS	PS	ZO	NS
NS	PM	PM	PM	PS	ZO	NS	NS
ZO	PM	PM	PS	ZO	NS	NM	NM
PS	PS	PS	ZO	NS	NS	NM	NM
PM	PS	ZO	NS	NM	NM	NM	NB
PB	ZO	ZO	NM	NM	NM	NB	NB

Table 6 : Fuzzy rules of Kp1

and

ec / e	NB	NM	NS	ZO	PS	PM	PB
NB	NB	NB	NM	NM	NS	ZO	ZO
NM	NB	NB	NM	NS	NS	ZO	ZO
NS	NB	NM	NS	NS	ZO	PS	PS
ZO	NM	NM	NS	ZO	PS	PM	PM
PS	NM	NS	ZO	PS	PS	PM	PB
PM	ZO	ZO	PS	PS	PM	PB	PB
PB	ZO	ZO	PS	PM	PM	PB	PB

Table 7: Fuzzy rules of Ki1

3.3.2.4. Defuzzification (Type-1 Fuzzy Logic)

Defuzzification converts fuzzy output (adjusted gains) back into crisp values that can be used in PID control law. Common defuzzification methods include:

- ***Centroid Method***: Computes center of gravity of fuzzy output.
- ***Weighted Average Method***: Calculates weighted average of fuzzy outputs.
- ***Mean of Maximum Method***: Selects mean value of maximum membership function.

The choice of defuzzification method depends on desired balance between computational efficiency and accuracy.

In our System, during defuzzification, weighted Average method was employed to transform vector into single value [28], allowing determination of fuzzy controller's output.

The calculations for ($\boldsymbol{K_{p1}}$) and ($\boldsymbol{K_{i1}}$) are given by:

$$\boldsymbol{K_{p1}} = \frac{\sum_{i=1}^{m} \mu_{K_{p1_i}}(c_p) \cdot c_{p_i}}{\sum_{i=1}^{m} \mu_{K_{p1_i}}(c_p)} \qquad \textbf{(3.08)}$$

$$\boldsymbol{K_{i1}} = \frac{\sum_{j=1}^{n} \mu_{K_{i1_j}}(c_i) \cdot c_{i_j}}{\sum_{j=1}^{n} \mu_{K_{i1_j}}(c_i)} \qquad \textbf{(3.09)}$$

Using fuzzy controller's output, proportional gain ($\boldsymbol{K_p}$) and integral gain ($\boldsymbol{K_i}$) are computed as follows:

$$\boldsymbol{K_p = K_{p0} + K_1 \cdot K_{p1}} \qquad \textbf{(3.10)}$$

$$\boldsymbol{K_i = K_{i0} + K_2 \cdot K_{i1}} \qquad \textbf{(3.11)}$$

Here, (K_{p0}) represents initial value of proportional gain, (K_{i0}) is initial value of integral gain and (K_1) and (K_2) are constants that determine gain of (K_{p1}) and (K_{i1}) respectively.

3.3.2.4. Defuzzification (Type-2 Fuzzy Logic)

In Type-2 fuzzy logic systems, defuzzification process is more complex compared to Type-1 systems due to nature of Type-2 fuzzy sets, which are characterized by uncertain or interval-based membership functions. Unlike in Type-1 systems, where defuzzification directly converts fuzzy output into single crisp value, Type-2 systems require an additional intermediate step known as type reduction, followed by conventional defuzzification.

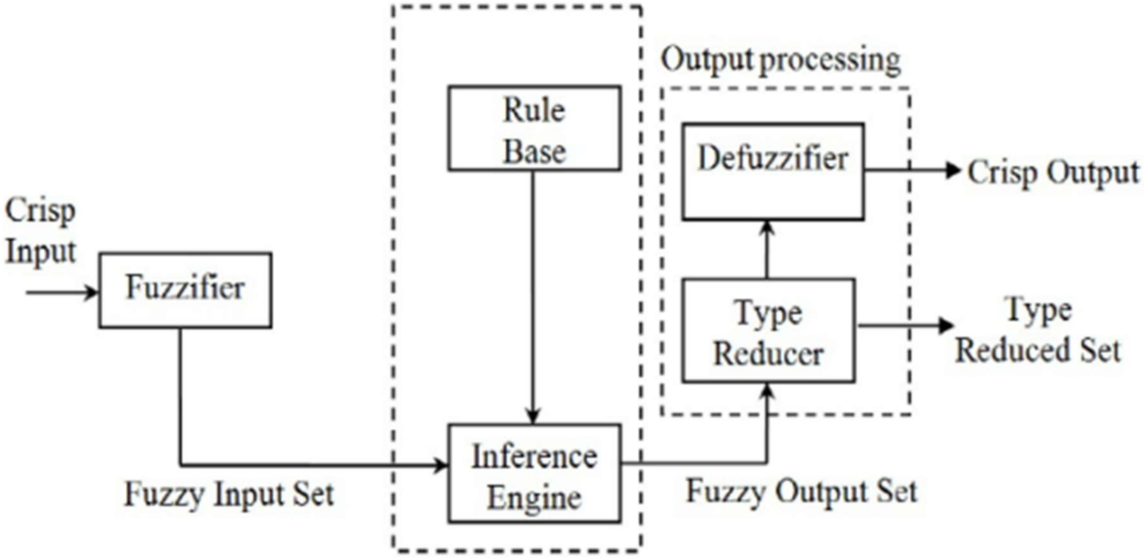


Figure 27:Type 2 fuzzy logic system

Two-Stage Process:

1. **Type Reduction:**

The Type-2 fuzzy output set is reduced to an interval-valued Type-1 fuzzy set, representing range of possible crisp values. This is typically achieved using algorithms such as:

- Karnik-Mendel (KM) Algorithm
- Enhanced Karnik-Mendel (EKM) Algorithm

The result is footprint of uncertainty (FOU) expressed as lower and upper bound, reflecting system's confidence or uncertainty in output.

2. **Defuzzification**:

Once type reduction is completed, standard defuzzification methods are applied to resulting interval set to obtain final crisp output. Common techniques include:

- Centroid Method
- Weighted Average Method
- Mean of Maximum (MOM) Method

In many applications, **average of lower and upper bounds** is used as final crisp output.

In our system, **Weighted Average Method** was employed after type reduction to transform fuzzy output vector into single representative value [28], allowing for precise determination of fuzzy controller's output while accounting for input uncertainties.

3.5. Implementation of Anti-Windup

In this study, fuel flow rate is constrained between 0 and 2 mol/s. Due to this constraint, actuator saturation can occur, potentially causing windup issues. To address this, back-calculation method is employed [40]. feedback mechanism for integrator is designed. integrator's input is calculated as follows:

$$\boldsymbol{e_1(t) = e_0(t) + K \cdot u_2(t)} \qquad \textbf{(3.12)}$$

Here, ($\boldsymbol{e_1(t)}$) represents integrator's input, ($\boldsymbol{e_0(t)}$) is tracking error signal and ($\boldsymbol{u_0(t)}$) and ($\boldsymbol{u_1(t)}$) denote controlled signal before and after saturation block, respectively. term ($\boldsymbol{u_2(t)}$) is defined as difference between ($\boldsymbol{u_1(t)}$) and ($\boldsymbol{u_0(t)}$), as shown in Equations (**3.03**) and (**3.12**).

Based on Equations (**3.03**) and (**3.12**), when upper limit is exceeded, controlled signal ($\boldsymbol{u_1(t)}$) will be less than ($\boldsymbol{u_0(t)}$), resulting in negative ($\boldsymbol{u_2(t)}$). This negative signal reduces integral action. Conversely, when controlled signal falls below lower limit, ($\boldsymbol{u_1(t)}$) will exceed ($\boldsymbol{u_0(t)}$), making ($\boldsymbol{u_2(t)}$) positive, which increases integral action. This approach enables controller to mitigate windup effectively, reducing overshoot and achieving faster stabilization.

3.6. Differential Forward Algorithm

Due to differentiator's high sensitivity to input variations [41], proposed controller incorporates differential forward algorithm to mitigate this issue. In conventional PID controller, differentiator processes tracking error signal (***e(t)***), as defined in Equation (3.13). However, when rated voltage (***r(t)***) changes, output voltage (***y(t)***) remains unchanged, leading to sudden variation in (***e(t)***). This results in "snap back" effect in differential action.

To address this, differential forward algorithm directly differentiates output voltage (***y(t)***). Specifically, differentiator employs first order actual differential algorithm, expressed as:

$$\boldsymbol{G_D(s) = \frac{s}{0.1s+1}} \qquad \textbf{(3.13)}$$

This approach ensures that variations in rated voltage do not impact differentiator. Since output voltage changes at relatively slow rate, it avoids producing large control values. Consequently, this method eliminates "snap back" issue in differential action.

3.7. Simulation

The Simulation environment was developed using MATLAB/Simulink. simulation was conducted in two main stages. First, dynamic model of Solid Oxide Fuel Cell (SOFC) was implemented based on established electrochemical and thermodynamic equations.

This model captures essential characteristics of SOFC, including voltage-current behavior, fuel utilization and thermal response, under different operating conditions.

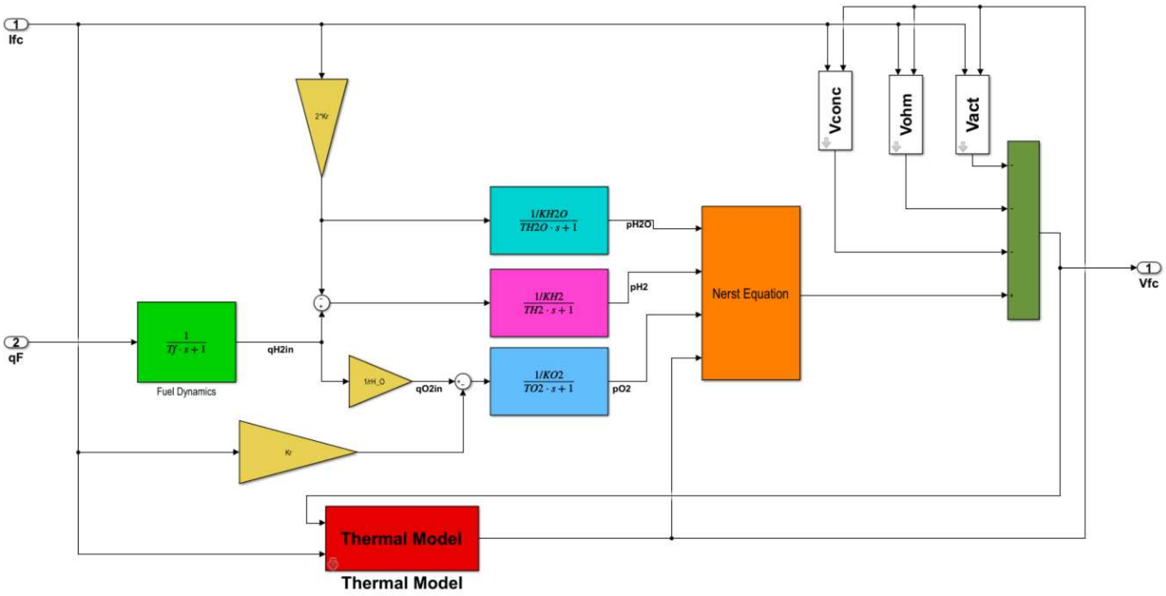


Figure 28:SOFC Dynamic Model

in second stage, Mamdani-type fuzzy logic-based PID controller was integrated with SOFC model. This controller continuously adjusts proportional and integral gains according to real-time error and its rate of change. Various scenarios, including setpoint tracking and disturbance rejection, were simulated to assess controller's effectiveness in maintaining output voltage within desired limits.

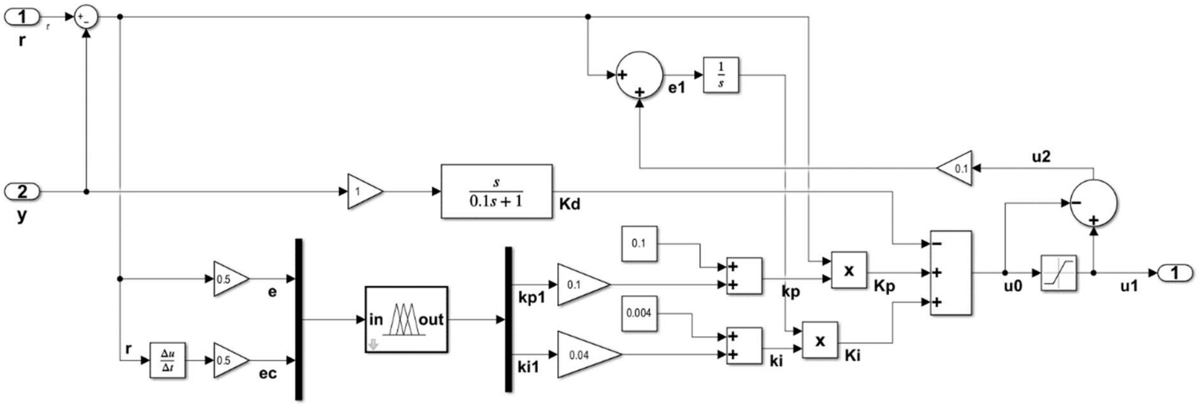
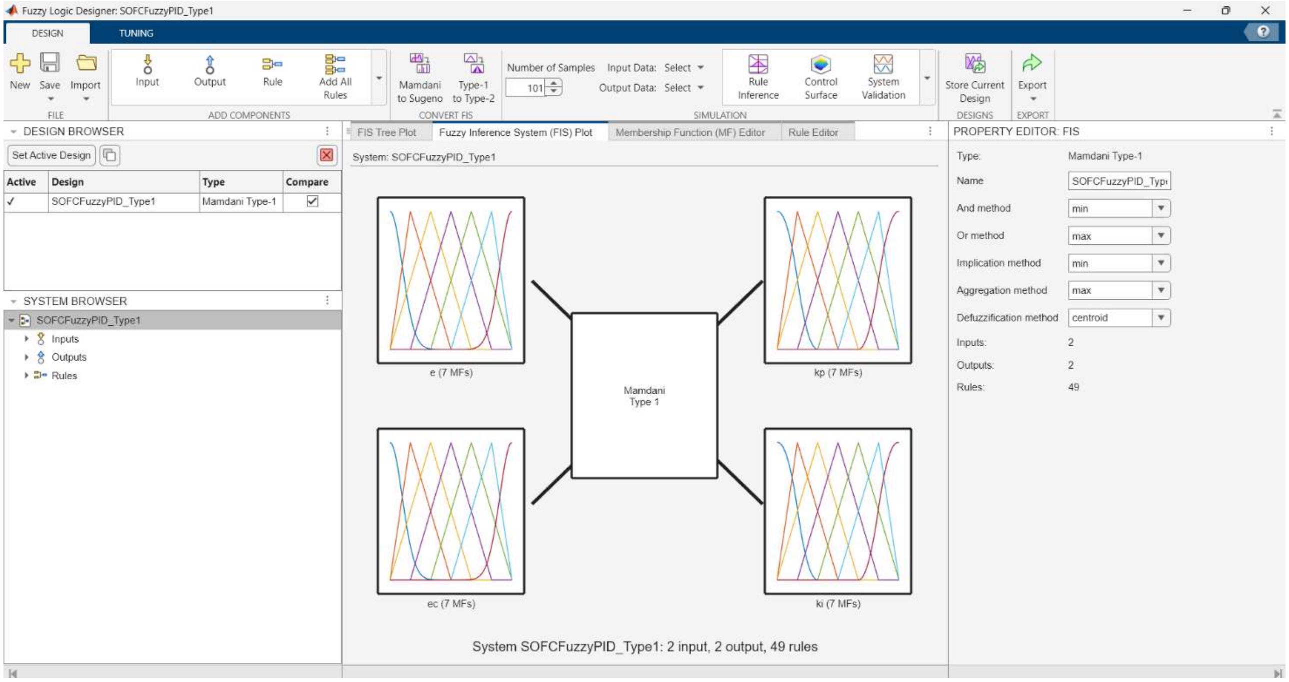


Figure 29:Fuzzy Adaptive Controller Model



Active	Design	Type	Compare
✓	SOFCFuzzyPID_Type1	Mamdani Type-1	☑

Type: Mamdani Type-1
Name: SOFCFuzzyPID_Typ
And method: min
Or method: max
Implication method: min
Aggregation method: max
Defuzzification method: centroid
Inputs: 2
Outputs: 2
Rules: 49

System SOFCFuzzyPID_Type1: 2 input, 2 output, 49 rules

Figure 30:Fuzzy Logic Designer Tool Box (TYPE 1)

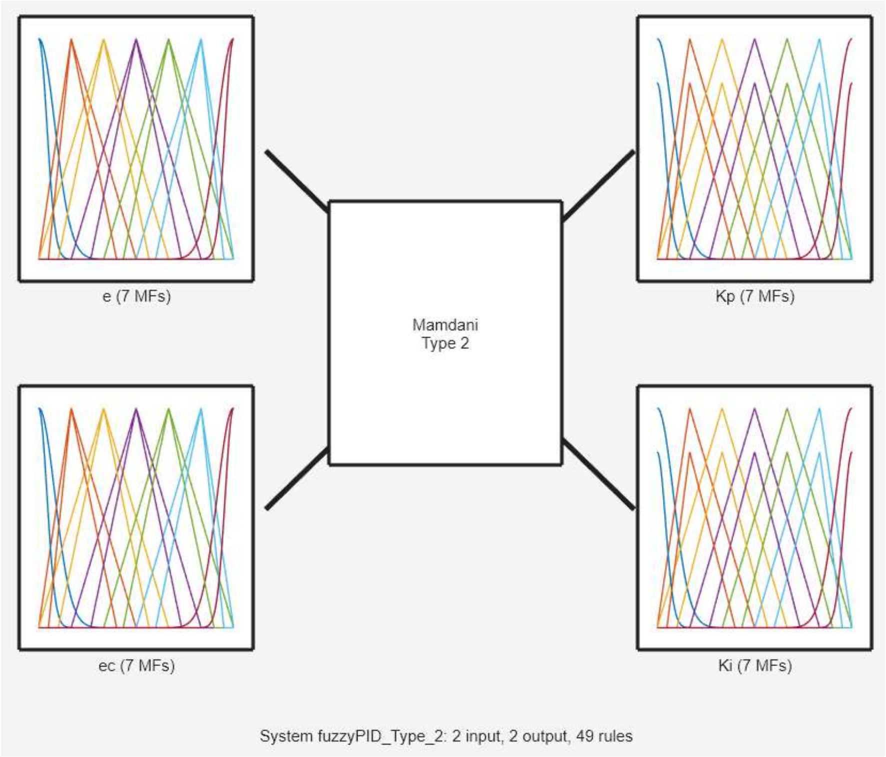


Figure 31:Fuzzy Logic Designer Tool Box (TYPE 2)

3.7.1 Simulation of Disturbance Response

For disturbance response simulation, rated voltage is fixed at 305 V and current begins at 300 A. current is then increased by 30 at 50 s and by 60 at 150 s and reach 450 at 250 s and decreased by 75 at 350 s.

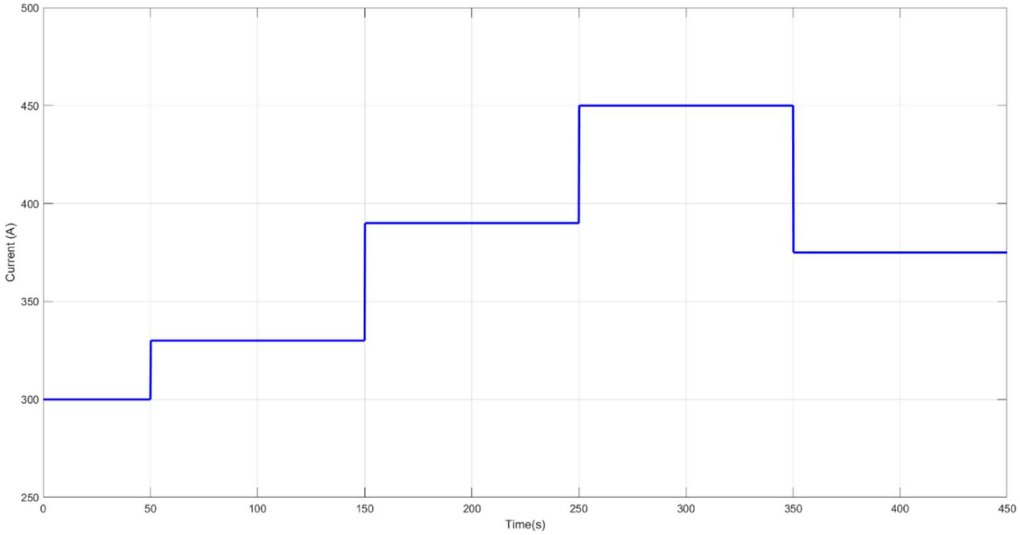


Figure 32:Current curve

Detailed data from simulation are provided in Table 8.

Number	PID				FPID1				FPID2			
	Overshoot %	Settling Time	Rise Time (s)	Saturation (s)	Overshoot %	Settling Time (s)	Rise Time (s)	Saturation (s)	Overshoot %	Settling Time	Rise Time (s)	Saturation (s)
1	0.33	2.25	3.17	1.75	0	2.33	2.3	1.9	0	2.95	3.55	1.1
2	0.43	2.87	5.7	3	0	2.93	2.55	4.3	0	2.87	3.64	3.7
3	2.07	33	17.5	28.5	0	11.61	10.1	19.2	0	11.54	10.4	17
4	5.5	5.23	4.33	0	5.48	5.27	4.01	3.4	5.44	5.26	3.3	1.4

Table 8:Detailed data of simulation result of disturbance response

Comparison of SOFC output voltage response using conventional PID (blue) and fuzzy adaptive controllers (FPID1, FPID2).

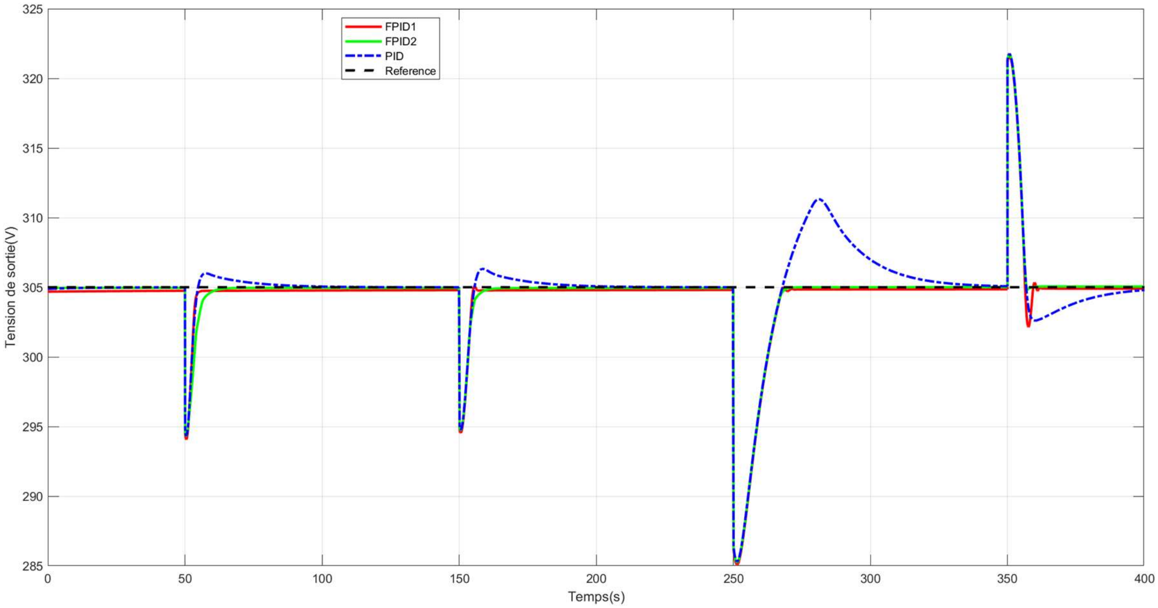


Figure 33:. Output voltage of SOFC

Close-up view of voltage regulation by FPID1 (Type-1) and FPID2 (Type-2) controllers. FPID2 demonstrates superior tracking precision and near-zero overshoot

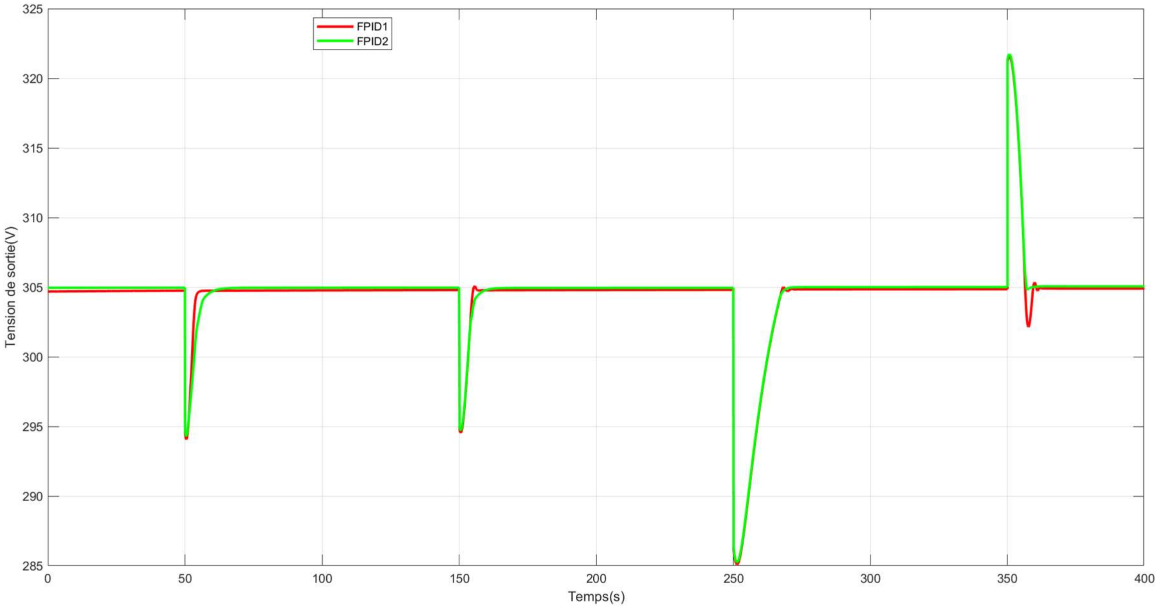


Figure 34:. Output voltages using FPID1 and FPID2 controllers

Dynamic adaptation of the proportional gain (Kp) in FPID1 and FPID2 controllers. The Type-2 fuzzy system (FPID2)

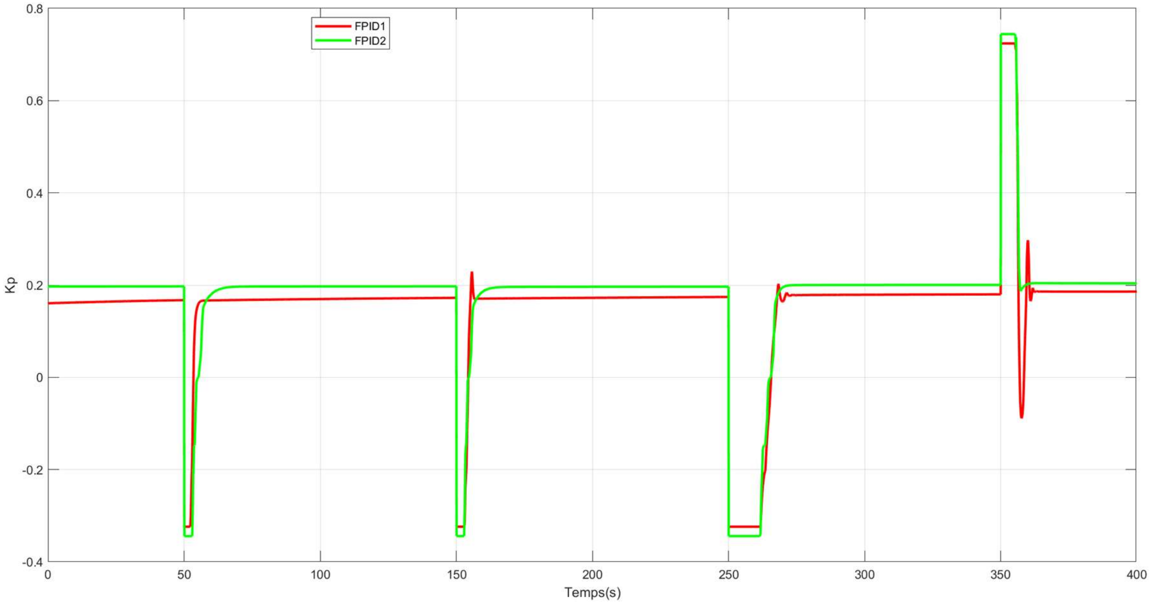


Figure 35:curve of Kp

Evolution of the integral gain (Ki) during operation. FPID2's adaptive tuning prevents integral windup and reduces steady-state error more effectively than FPID1, especially during transient phases.

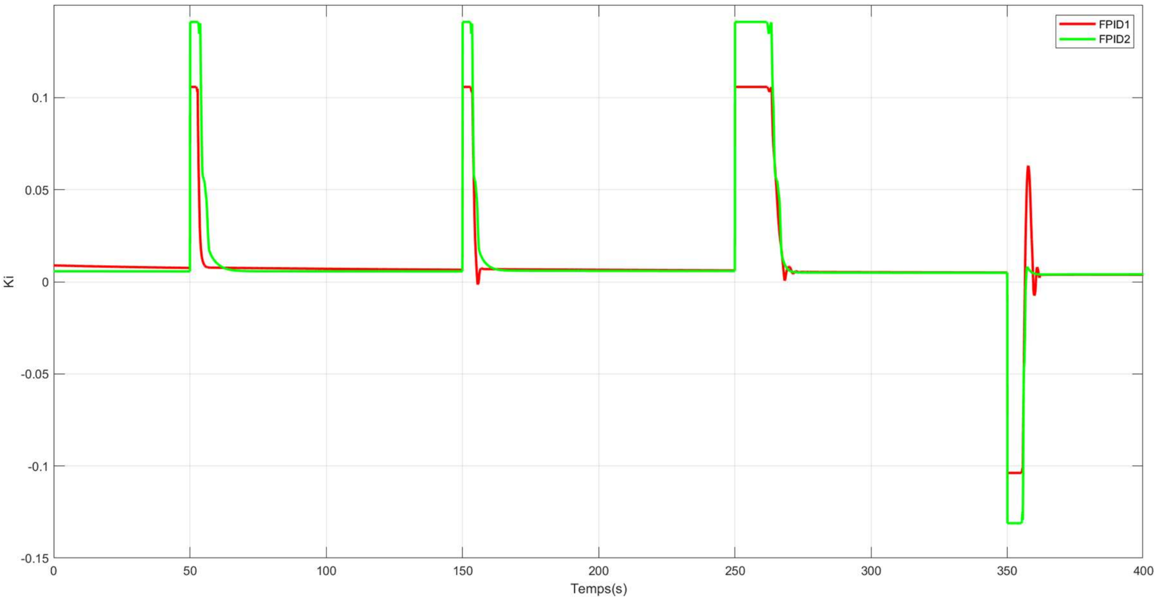


Figure 36:Ki Curve

Fuel utilization rate (U) under varying loads. The fuzzy controllers maintain U within optimal bounds (0.6–0.9), ensuring efficient fuel consumption and avoiding starvation or saturation risks

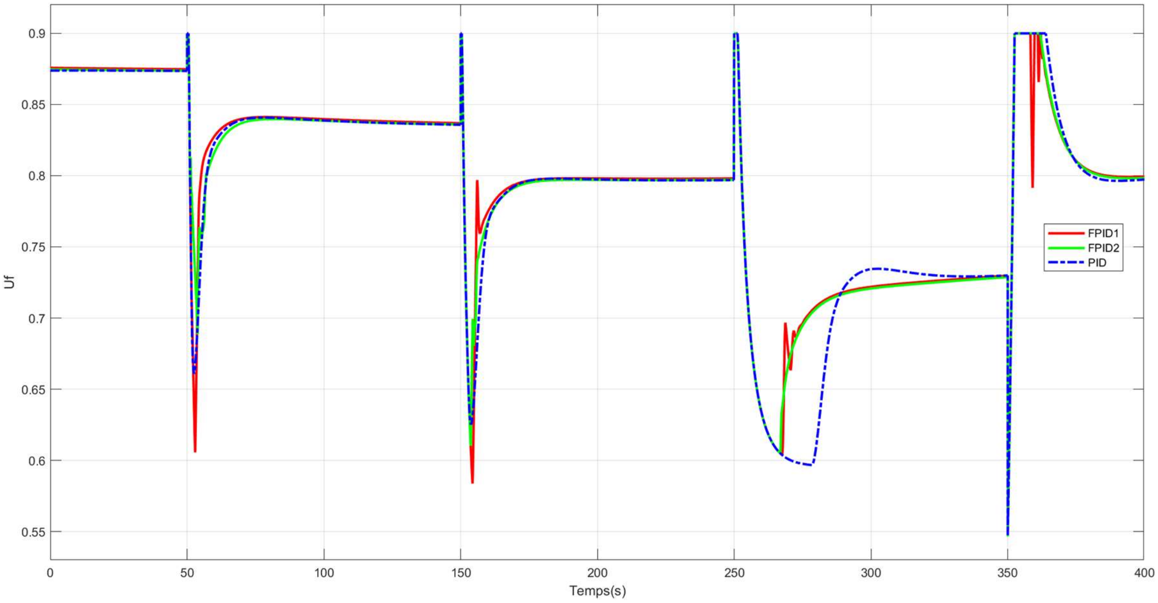


Figure 37:Trajectory of Utilization Rate by SOFC

3.8. Discussion

Apparently, simulation results presented in Table 8 reflect the fact that the fuzzy adaptive PID controllers (FPID1 and FPID2) function better when accommodating disturbances in the system as compared to the conventional PID controller; indeed, in most operating conditions, the controllers exhibited zero overshoot with the exception of the last case, contrary to the traditional PID, where an overshoot ranging between 0.33 and 5.5% was encountered, rendering the controller unstable as loads varied. Along with that, the settling time under FPID1 and FPID2 controllers was much smaller than that of the PID controller: for example, in Scenario 3, the PID needed 28.5 seconds to settle while FPID1 and FPID2 had settling times of 19.2 and 17 seconds, respectively, showing the higher speed of dynamic response.

Also, the rise time of the fuzzy controllers was less that of the conventional, confirming their enhancement in transient response. Both fuzzy controllers exhibited admirable disturbance rejection from that point forward, but FPID2 (Type-2 fuzzy logic) was the most robust in handling uncertainty because of the ability to model input noise and variations via the Footprint of Uncertainty (FOU). However, this advantage comes with increased computational complexity.

In the scenario of actuator saturation, fuzzy controllers were lesser saturation period to render smooth fuel flow regulation without entering windup. These results stood up to prove that fuzzy adaptive control, especially with Type-2 fuzzy logic augments SOFC stability, responsiveness, and efficiency during operative conditions in actuality. while FPID1 makes a reasonable choice for simpler systems, FPID2 should be preferably used for applications where high precision and adaptability to a dynamic disturbance are required. This paper adds to the development of intelligent control techniques for SOFCs, thereby enhancing their reliability within the area of sustainable energy generation.

3.9. Conclusion

In this chapter, a fuzzy-adaptive PID controller is developed for voltage regulation in SOFCs, which is difficult due to its nonlinear slow dynamics with operational restrictions. This controller dynamically varied between proportional and integral gains to meet various operating conditions, thus overcoming PID limitations. Simulation results in MATLAB/Simulink revealed that the proposed controller performs better, with zero overshoot, less settling time, and lesser saturation period under various disturbances. Also, incorporating anti-windup and differential forwarding makes it more robust by suppressing the effects of actuator saturation and instability in derivative action, respectively. The implementation allows that when comparing Type-1 and Type-2 fuzzy logic approaches, the Type-2 fuzzy-based approach proved more adaptable to uncertainty and hence suited the highly dynamic and uncertain nature of SOFCs despite its computation demand. On the other hand, Type-1 gave the simpler solution for stable working conditions. This research provides a glimpse into the possibilities of a fuzzy adaptive PID controller as a potentially efficient and robust way in voltage regulation of SOFC.

General Conclusion

In light of global shift toward clean and sustainable energy sources, Solid Oxide Fuel Cells have emerged as key enabling technology because of to their high efficiency, fuel flexibility and low environmental impact. But nonlinear dynamics and sensitivity of SOFC systems to changes in fuel and oxygen flows present challenges in ensuring stable and efficient operation. Traditional PID controllers while simple and widely implemented, often fall short in such complex and uncertain environments.

This research has focused on addressing these limitations by proposing two fuzzy adaptive PID control strategies: one based on Type-1 fuzzy logic and other on more advanced with Type-2 fuzzy logic. comprehensive dynamic model of SOFC developed allowing for us precise simulation and control of key parameters. fuzzy controllers designed to adapt PID gains for real time based on system's error and its rate of change. it enhancing control accuracy and robustness.

Simulation results are demonstrated superior performance of fuzzy adaptive PID controllers compared to classical PID methods. Type-2 fuzzy controller exhibited most outcomes, in environments with uncertainty, disturbances or noisy inputs. Its ability to incorporate and model uncertainty through Footprint of Uncertainty (FOU) allowed for smoother control actions, faster response times and improved stability.

Increased computational complexity associated with Type-2 fuzzy logic systems, their performance benefits justify their adoption in high precision applications. Type-1 fuzzy controller remains best alternative for systems with moderate complexity offering balance between adaptability and computational efficiency.

Integration of anti-windup mechanisms and differential forward algorithms further enhanced reliability and practicality of proposed controllers makes them suitable for real world deployment.

In conclusion, this study validated effectiveness of combining fuzzy logic and adaptive control to improve performance of SOFC systems. findings not only advance state of research in intelligent energy control but also contribute to goal of enabling smart and efficient hydrogen based energy solutions. As energy systems continue to evolve, such advanced control techniques will be important in achieving best sustainability, reliability and autonomy in next generation renewable technologies.

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